Exercise 15A

Question 1.
In the figure, given below, straight lines AB and CD intersect at P; and AC // BD. Prove that:

(i) ΔAPC and ΔBPD are similar.
(ii) If BD = 2.4 cm AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm; find the lengths of PA and PC.

Solution:

(i) 
In ΔAPC and ΔBPD,
∠APC = ∠BPD .....(vertically opposite angles)
∠ACP = ∠BDP .....(alternate angles since AC||BD)
∴ ΔAPC ~ ΔBPD .....(AA criterion for similarity)

(ii) 
In ΔAPC and ΔBPD,
∠APC = ∠BPD .....(vertically opposite angles)
∠ACP = ∠BDP .....(alternate angles since AC||BD)
∴ ΔAPC ~ ΔBPD .....(AA criterion for similarity)
So, \( \frac{PA}{PB} = \frac{PC}{PD} = \frac{AC}{BD} \)
∴ \( \frac{PA}{3.2} = \frac{PC}{4} = \frac{3.6}{2.4} \)
In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that:

(i) $\triangle APB$ is similar to $\triangle CPD$
(ii) $PA \times PD = PB \times PC$

Solution:

(i)

In $\triangle APB$ and $\triangle CPD$,
$\angle APB = \angle CPD$ .... (vertically opposite angles)
$\angle ABP = \angle CDP$ ..... (alternate angles since $AB \parallel DC$)
$\therefore \triangle APB \sim \triangle CPD$ .... (AA criterion for similarity)

(ii)
In $\triangle APB$ and $\triangle CPD,$

$\angle APB = \angle CPD$ .....(vertically opposite angles)

$\angle ABP = \angle CDP$ .....(alternate angles since $AB \parallel DC$)

$\therefore \triangle APB \sim \triangle CPD$ ...(AA criterion for similarity)

$\Rightarrow \frac{PA}{PC} = \frac{PB}{PD}$ .....(Since corresponding sides of similar triangles are equal.)

$\Rightarrow PA \times PD = PB \times FC$

**Question 3.**
P is a point on side $BC$ of a parallelogram $ABCD.$ If $DP$ produced meets $AB$ produced at point $L,$ prove that:

(i) $DP: PL = DC: BL.$

(ii) $DL: DP = AL: DC.$

**Solution:**

(i) Since $AD \parallel BC,$ that is, $AD \parallel BP,$

by the Basic Proportionality Theorem, we get

$\frac{DL}{DP} = \frac{AL}{AB}$

Since $ABCD$ is a parallelogram, $AB = DC.$

So, $\frac{DL}{DP} = \frac{AL}{DC}.$
Question 4.
In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O. If AO = 2CO and BO = 2DO; show that:

(i) \( \Delta AOB \) is similar to \( \Delta COD \).
(ii) \( OA \times OD = OB \times OC \).

Solution:

(i)

\[
\text{Since } AD \parallel BC, \text{ that is, } AD \parallel BP, \text{ by the Basic Proportionality theorem, we get:}
\]

\[
\frac{DP}{PL} = \frac{AB}{BL}
\]

\[
\text{Since } ABCD \text{ is a parallelogram, } AB = DC.
\]

\[
\text{So, } \frac{DP}{PL} = \frac{DC}{BL}.
\]
Question 5.
In \( \triangle ABC \), angle \( \angle ABC \) is equal to twice the angle \( \angle ACB \), and bisector of angle \( \angle ABC \) meets the opposite side at point \( P \). Show that:
(i) \( CB: BA = CP: PA \)
(ii) \( AB \times BC = BP \times CA \)

Solution:

(i) 

\[
\begin{align*}
\text{Since } AO &= 2CO \text{ and } BO = 2DO, \\
\frac{AO}{CO} &= 2 \cdot \frac{BO}{DO} \\
\text{Also, } \angle AOB &= \angle DOC \quad \text{(vertically opposite angles)} \\
\text{So, } \triangle AOB \sim \triangle COD \quad \text{(SAS criterion for similarity)}
\end{align*}
\]
In \( \triangle ABC \),
\[ \angle ABC = 2 \angle ACB \]
Let \( \angle ACB = x \)
\[ \Rightarrow \angle ABC = 2 \angle ACB = 2x \]
Given BP is bisector of \( \angle ABC \).
Hence \( \angle ABP = \angle PBC = x \).
Using the angle bisector theorem,
that is, the bisector of an angle divides the side opposite to it in the ratio of other two sides.
Hence, \( CB : BA = CP : PA \).

(ii)
In $\triangle ABC$,
\[\angle ABC = 2\angle ACB\]
Let $\angle ACB = x$
\[\Rightarrow \angle ABC = 2\angle ACB = 2x\]
Given BP is bisector of $\angle ABC$.
Hence $\angle ABP = \angle PBC = x$.
Using the angle bisector theorem,
that is, the bisector of an angle divides the side opposite to it
in the ratio of other two sides.
Hence, $CB : BA = CP : FA$.
Consider $\triangle ABC$ and $\triangle APB$,
\[\angle ABC = \angle APB \quad \text{[Exterior angle property]}\]
\[\angle BCP = \angle ABP \quad \text{[Given]}\]
\[\therefore \triangle ABC \sim \triangle APB \quad \text{[AA criterion for Similarity]}\]
\[
\frac{CA}{CB} = \frac{BC}{BP} \quad \text{...(Corresponding sides of similar triangles are proportional.)}
\]
\[\Rightarrow AB \times BC = BP \times CA\]

**Question 6.**

In $\triangle ABC$; BM $\perp$ AC and CN $\perp$ AB; show that:

\[
\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}
\]

**Solution:**

![Diagram of triangle ABC with perpendiculars BM and CN and bisector BP.]
In the given figure, DE//BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.

(i) Write all possible pairs of similar triangles.
(ii) Find lengths of ME and DM.

**Solution:**

(i) 

In \(\triangle AME\) and \(\triangle ANC\),

\[
\angle AME = \angle ANC \quad \text{(Since } DE \parallel BC \text{ that is, } ME \parallel NC.\text{)}
\]

\[
\angle MAE = \angle NAC \quad \text{(common angle)}
\]

\(\Rightarrow \triangle AME \sim \triangle ANC \quad \text{(AA criterion for Similarity)}\)

(ii) 

In \(\triangle ADM\) and \(\triangle ABN\),

\[
\angle ADM = \angle ABN \quad \text{(Since } DE \parallel BC \text{ that is, } DM \parallel BN.\text{)}
\]

\[
\angle DAM = \angle BAN \quad \text{(common angle)}
\]

\(\Rightarrow \triangle ADM \sim \triangle ABN \quad \text{(AA criterion for Similarity)}\)

In \(\triangle ADE\) and \(\triangle ABC\),

\[
\angle ADE = \angle ABC \quad \text{(Since } DE \parallel BC \text{ that is, } ME \parallel NC.\text{)}
\]

\[
\angle AED = \angle ACB \quad \text{(Since } DE \parallel BC.\text{)}
\]

\(\Rightarrow \triangle ADE \sim \triangle ABC \quad \text{(AA criterion for Similarity)}\)
In $\triangle AME$ and $\triangle ANC$,

$\angle AME = \angle ANC \quad \text{(Since } DE \parallel BC \text{ that is, } ME \parallel NC.\text{)}$

$\angle MAE = \angle NAC \quad \text{(common angle)}$

$\Rightarrow \triangle AME \sim \triangle ANC \quad \text{(AA criterion for Similarity)}$

$\Rightarrow \ \frac{ME}{NC} = \frac{AE}{AC}$

$\Rightarrow \ \frac{ME}{6} = \frac{15}{24}$

$\Rightarrow \ ME = 3.75 \text{ cm}$

In $\triangle ADE$ and $\triangle ABN$,

$\angle ADE = \angle ABC \quad \text{(Since } DE \parallel BC \text{ that is, } ME \parallel NC.\text{)}$

$\angle AED = \angle ACB \quad \text{(Since } DE \parallel BC.\text{)}$

$\Rightarrow \triangle ADE \sim \triangle ABC \quad \text{(AA criterion for Similarity)}$

$\Rightarrow \ \frac{AD}{AB} = \frac{AE}{AC} = \frac{15}{24} \quad \text{......(i)}$

In $\triangle ADM$ and $\triangle ABN$,

$\angle ADM = \angle ABN \quad \text{(Since } DE \parallel BC \text{ that is, } DM \parallel BN.\text{)}$

$\angle DAM = \angle BAN \quad \text{(common angle)}$

$\Rightarrow \triangle ADM \sim \triangle ABN \quad \text{(AA criterion for Similarity)}$

$\Rightarrow \ \frac{DM}{BN} = \frac{AD}{AB} = \frac{15}{24} \quad \text{......from (i)}$

$\Rightarrow \ DM = 15 \text{ cm}$

**Question 8.**

In the given figure, $AD = AE$ and $AD^2 = BD \times EC$

Prove that: triangles $ABD$ and $CAE$ are similar.
Question 9.

In the given figure, AB // DC, BO = 6 cm and DQ = 8 cm; find: BP x DO.

Solution:

In \( \triangle ABD \) and \( \triangle CAE \),
\[ \angle ADE = \angle AED \quad \text{...(Angles opposite equal sides are equal.)} \]
So, \( \angle ADB = \angle AEC \)
\[ \text{.....(Since } \angle ADB + \angle ADE = 180^\circ \text{ and } \angle AEC + \angle AED = 180^\circ \text{)} \]
Also, \( AB^2 = BD \times EC \)
\[ \Rightarrow \frac{AD}{BD} = \frac{EC}{AD} \]
\[ \Rightarrow \frac{AD}{BD} = \frac{EC}{AE} \]
\[ \Rightarrow \triangle ABD \sim \triangle CAE \quad \text{.....(SAS criterion for Similarity)} \]

Question 10.

Angle BAC of triangle ABC is obtuse and AB = AC. P is a point in BC such that PC = 12 cm. PQ and PR are perpendiculrars to sides AB and AC respectively. If PQ = 15 cm and
PR=9 cm; find the length of PB

Solution:

In \( \triangle ABC \),
\[ AC = AB \quad \text{(Given)} \]
\[ \Rightarrow \angle ABC = \angle ACB \quad \text{(Angles opposite equal sides are equal.)} \]
In \( \triangle PRC \) and \( \triangle PQB \),
\[ \angle ABC = \angle ACB \]
\[ \angle PRC = \angle PQB \quad \text{(Both are right angles.)} \]
\[ \Rightarrow \triangle PRC \sim \triangle PQB \quad \text{(AA criterion for similarity)} \]

\[ \Rightarrow \frac{PR}{PQ} = \frac{PC}{QB} = \frac{PB}{PB} \]

\[ \Rightarrow \frac{9}{15} = \frac{12}{PB} \]
\[ \Rightarrow PB = 20 \text{ cm} \]

**Question 11.**
State, true or false:

(i) Two similar polygons are necessarily congruent.
(ii) Two congruent polygons are necessarily similar.
(iii) All equiangular triangles are similar.
(iv) All isosceles triangles are similar.
(v) Two isosceles-right triangles are similar.
(vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.
(vii) The diagonals of a trapezium, divide each other into proportional segments.
Solution:

(i) False
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) True

Question 12.

Given $\angle GHE = \angle DFE = 90^\circ$, $DH = 8$, $DF = 12$, $DG = 3x + 1$ and $DE = 4x + 2$.

Find the lengths of segments DG and DE.

Solution:

In $\triangle DHG$ and $\triangle DFE$,

$\angle GHD = \angle DFE = 90^\circ$

$\angle D = \angle D$ (Common)

$\therefore \triangle DHG \sim \triangle DFE$

$\Rightarrow \frac{DH}{DF} = \frac{DG}{DE}$

$\Rightarrow \frac{8}{12} = \frac{3x - 1}{4x + 2}$

$\Rightarrow 32x + 16 = 36x - 12$

$\Rightarrow 28 = 4x$

$\Rightarrow x = 7$

$\therefore DG = 3 \times 7 - 1 = 20$

$DE = 4 \times 7 + 2 = 30$
Question 13.
D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that $CA^2 = CB \times CD$.

Solution:

In $\triangle ADB$ and $\triangle BAC$,
\[
\angle ADC = \angle BAC \quad \text{(Given)} \\
\angle ACD = \angle ACB \quad \text{(Common)}
\]
\[\therefore \triangle ADB \sim \triangle BAC \]
\[
\therefore \frac{CA}{CB} = \frac{CD}{CA}
\]
Hence, $CA^2 = CB \times CD$

Question 14.
In the given figure, $\triangle ABC$ and $\triangle AMP$ are right angled at B and M respectively. Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(i) Prove that $\triangle ABC \sim \triangle AMP$
(ii) Find AB and BC.

Solution:

(i) In $\triangle ABC$ and $\triangle AMP$,
\[\angle BAC = \angle PAM \quad \text{[Common]} \]
\[\angle ABC = \angle PMA \quad \text{[Each = 90°]} \]
\[\triangle ABC \sim \triangle AMP \quad \text{[AA Similarity]} \]

(ii)
\[AM = \sqrt{AP^2 - PM^2} = \sqrt{15^2 - 12^2} = 11 \]
Since $\triangle ABC \sim \triangle AMP$, 

Given : RS and PT are altitudes of \( \triangle PQR \) prove that:

(i) \( \triangle PQT \sim \triangle QRS \),

(ii) \( PQ \times QS = RQ \times QT \).

Solution:

\[
\begin{align*}
\frac{AB}{AM} &= \frac{BC}{PM} = \frac{AC}{AP} \\
\Rightarrow \frac{AB}{11} &= \frac{BC}{12} = \frac{AC}{15} \\
\Rightarrow \frac{AB}{11} &= \frac{BC}{12} = \frac{10}{15} \\
\Rightarrow AB &= \frac{10 \times 11}{15} = 7.33 \\
BC &= \frac{10}{15} = 8 \text{ cm}
\end{align*}
\]
Question 16.

Given: ABCD is a rhombus, DPR and CBR are straight lines

Prove that: $DP \times CR = DC \times PR$.

Solution:

In $\triangle DPA$ and $\triangle RPC$,

- $\angle DPA = \angle RPC$ (Vertically opposite angles)
- $\angle PAD = \angle PCR$ (Alternate angles)

$\triangle DPA \sim \triangle RPC$

\[ \frac{DP}{PR} = \frac{AD}{CR} \]

\[ \frac{DP}{PR} = \frac{DC}{CR} \quad (AD = DC, \text{ as } ABCD \text{ is rhombus}) \]

Hence, $DP \times CR = DC \times PR$. 
Question 17.
Given: FB = FD, AE ⊥ FD and FC ⊥ AD. Prove : \( \frac{FB}{AD} = \frac{BC}{ED} \)

Solution:

Given, FB = FD
\[ \therefore \angle FDB = \angle FBD \quad \ldots (1) \]
In \( \triangle AED \) and \( \triangle FCB \),
\[ \angle AED = \angle FCB = 90^\circ \]
\[ \angle ADE = \angle FBC \quad \text{[Using (1)]} \]
\( \triangle AED \sim \triangle FCB \quad \text{[By AA similarity]} \]
\[ \therefore \frac{AD}{FB} = \frac{ED}{BC} \]
\[ \frac{FB}{AD} = \frac{BC}{ED} \]

Question 18.
In \( \triangle PQR \), \( \angle Q = 90^\circ \) and QM is perpendicular to PR, Prove that:

(i) \( PQ^2 = PM \times PR \)
(ii) \( QR^2 = PR \times MR \)
(iii) \( PQ^2 + QR^2 = PR^2 \)

Solution:
Question 19.
In Δ ABC, ∠B = 90° and BD × AC.
(i) If CD = 10 cm and BD = 8 cm; find AD.
(ii) If AC = 18 cm and AD = 6 cm; find BD.
(iii) If AC = 9 cm, AB = 7 cm; find AD.

Solution:
(i) In \( \triangle CDB \),
\[
\angle 1 + \angle 2 + \angle 3 = 180^\circ
\]
\[
\angle 1 + \angle 3 = 90^\circ \quad \text{(1) (Since, } \angle 2 = 90^\circ)\]
\[
\angle 3 + \angle 4 = 90^\circ \quad \text{(2) (Since, } \angle ABC = 90^\circ)\]
From (1) and (2),
\[
\angle 1 + \angle 3 = \angle 3 + \angle 4
\]
\[
\angle 1 = \angle 4
\]
Also, \( \angle 2 = \angle 5 = 90^\circ \)
\[
\therefore \triangle CDB \sim \triangle BDA \quad \text{(By AA similarity)}
\]
\[
\frac{CD}{BD} = \frac{BD}{AD}
\]
\[
\Rightarrow BD^2 = AD \times CD
\]
\[
\Rightarrow (8)^2 = AD \times 10
\]
\[
\Rightarrow AD = 6.4
\]
Hence, \( AD = 6.4 \text{ cm} \)

(ii) Also, by similarity, we have:
\[
\frac{BD}{DA} = \frac{CD}{BD}
\]
\[
BD^2 = 6 \times (18 - 6)
\]
\[
BD^2 = 72
\]
Hence, \( BD = 8.5 \text{ cm} \)

(iii)
Clearly, \( \triangle ADB \sim \triangle ABC \)
\[
\therefore \frac{AD}{AB} = \frac{BD}{AC}
\]
\[
AD = \frac{7 \times 7}{9} = \frac{49}{9} = 5 \frac{4}{9}
\]
Hence, \( AD = 5 \frac{4}{9} \text{ cm} \)

Question 20.

In the figure, \( PQRS \) is a parallelogram with \( PQ = 16 \text{ cm} \) and \( QR = 10 \text{ cm} \). \( L \) is a point on \( PR \) such that \( RL : LP = 2 : 3 \). \( QL \) produced meets \( RS \) at \( M \) and \( PS \) produced at \( N \).
Find the lengths of PN and RM.

**Solution:**

In $\triangle RLQ$ and $\triangle PLN$,
\[ \angle RLQ = \angle PLN \quad \text{(Vertically opposite angles)} \]
\[ \angle LRQ = \angle LPN \quad \text{(Alternate angles)} \]
\[ \triangle RLQ \sim \triangle PLN \quad \text{(AA similarity)} \]
\[ \therefore \frac{RL}{LP} = \frac{RQ}{PN} \]
\[ \frac{2}{3} = \frac{10}{PN} \]
\[ PN = 15 \text{ cm} \]

In $\triangle RLM$ and $\triangle PLQ$,
\[ \angle RLM = \angle PLQ \quad \text{(Vertically opposite angles)} \]
\[ \angle LRM = \angle LPQ \quad \text{(Alternate angles)} \]
\[ \triangle RLM \sim \triangle PLQ \quad \text{(AA similarity)} \]
\[ \therefore \frac{RM}{PQ} = \frac{RL}{LP} \]
\[ \frac{RM}{PQ} = \frac{2}{3} \]
\[ RM = \frac{32}{3} = 10\frac{2}{3} \text{ cm} \]

**Question 21.**

In quadrilateral ABCD, diagonals AC and BD intersect at point E. Such that

AE : EC = BE : ED.

Show that ABCD is a parallelogram
Solution:

Given, AE: EC = BE: ED
Draw EF \parallel AB

In \triangle ABD, EF \parallel AB
Using Basic Proportionality theorem,
\[
\frac{DF}{FA} = \frac{DE}{EB}
\]

But, given
\[
\frac{DE}{EB} = \frac{CE}{EA}
\]

\[
\therefore \frac{DF}{FA} = \frac{CE}{EA}
\]

Thus, in \triangle DCA, E and F are points on CA and DA respectively such that
\[
\frac{DF}{FA} = \frac{CE}{EA}
\]

Thus, by converse of Basic proportionality theorem, FE \parallel DC.
But, FE \parallel AB.
Hence, AB \parallel DC.
Thus, ABCD is a trapezium.

Question 22.

In \triangle ABC, AD is perpendicular to side BC and \(AD^2 = BD \times DC\).

Show that angle \(BAC = 90^\circ\)
Solution:

Given, \( AD^2 = BD \times DC \)
\[
\frac{AD}{BD} = \frac{BD}{DC}
\]
\( \angle DAB = \angle ADC = 90^\circ \)
\( \therefore \triangle DBA \sim \triangle DAC \) (SAS similarity)
So, these two triangles will be equiangular.
\( \therefore \angle 1 = \angle C \) and \( \angle 2 = \angle B \)
\( \angle 1 + \angle 2 = \angle B + \angle C \)
\( \angle A = \angle B + \angle C \)
By angle sum property,
\( \angle A + \angle B + \angle C = 180^\circ \)
\( \angle A + \angle A = 180^\circ \)
\( 2\angle A = 180^\circ \)
\( \angle A = \angle BAC = 90^\circ \)

Question 23.

In the given figure \( AB // EF // DC; AB \sim 67.5 \text{ cm} \). \( DC = 40.5 \text{ cm} \) and \( AE = 52.5 \text{ cm} \).

(i) Name the three pairs of similar triangles.
(ii) Find the lengths of \( EC \) and \( EF \).
(i) The three pair of similar triangles are:
\[ \triangle BEF \text{ and } \triangle BDC \]
\[ \triangle CEF \text{ and } \triangle CAB \]
\[ \triangle ABE \text{ and } \triangle CDE \]
(ii) Since, \( \triangle ABE \text{ and } \triangle CDE \) are similar,
\[
\frac{AB}{CD} = \frac{AE}{CE} \\
\frac{67.5}{40.5} = \frac{52.5}{CE} \\
CE = 31.5 \text{ cm}
\]
Since, \( \triangle CEF \text{ and } \triangle CAB \) are similar,
\[
\frac{CE}{CA} = \frac{EF}{AB} \\
\frac{31.5}{52.5 + 31.5} = \frac{EF}{67.5} \\
\frac{31.5}{84} = \frac{EF}{67.5} \\
EF = \frac{2126.25}{84} \\
EF = \frac{405}{16} = 25\frac{5}{16} \text{ cm}
\]

**Question 24.**

In the given figure, QR is parallel to AB and DR is parallel to QB.

Prove that \( PQ^2 = PD \times PA \).
Solution:

Given, QR is parallel to AB. Using Basic proportionality theorem,
\[ \frac{PQ}{PA} = \frac{PR}{PB} \quad \ldots (1) \]

Also, DR is parallel to QB. Using Basic proportionality theorem,
\[ \frac{PD}{PQ} = \frac{PR}{PB} \quad \ldots (2) \]

From (1) and (2), we get,
\[ \frac{PQ}{PA} = \frac{PD}{PQ} \]
\[ PQ^2 = PD \times PA \]

**Question 25.**
Through the mid-point M of the side CD of \( \text{a parallelogram} \ ABCD \), the line BM is drawn \( \text{intersecting diagonal} \ AC \) in L and AD produced in E.
Prove that: \( EL = 2BL \).

**Solution:**

\[ \angle 1 = \angle 6 \text{ (Alternate interior angles)} \]
\[ \angle 2 = \angle 3 \text{ (Vertically opposite angles)} \]
\[ DM = MC \text{ (M is the mid-point of CD)} \]
\[ \therefore \triangle DEM \cong \triangle CBM \text{ (AAS congruence criterion)} \]

So, \( DE = BC \) (Corresponding parts of congruent triangles)

Also, \( AD = BC \) (Opposite sides of a parallelogram)

\[ \Rightarrow AE = AD + DE = 2BC \]

Now, \( \angle 1 = \angle 6 \text{ and } \angle 4 = \angle 5 \)
\[ \therefore \triangle EAL \sim \triangle BLC \text{ (AA similarity)} \]

\[ \Rightarrow \frac{EL}{BL} = \frac{EA}{BC} \]

\[ \Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = 2 \]

\[ \Rightarrow EL = 2BL \]
Question 26.

In the figure given below P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC.

(i) Calculate the ratio PQ : AC, giving reason for your answer.
(ii) In triangle ARC, $\angle ARC = 90^\circ$ and in triangle PQS, $\angle PSQ = 90^\circ$. Given QS = 6 cm, calculate the length of AR. [1999]

Solution:

(i) Given, AP: PB = 4:3.
Since, PQ || AC. Using Basic Proportionality theorem,
\[
\frac{AP}{PB} = \frac{CQ}{QB}
\Rightarrow \frac{CQ}{QB} = \frac{4}{3}
\Rightarrow \frac{BQ}{BC} = \frac{3}{7} \quad \ldots (1)
\]
Now, $\angle PQB = \angle ACB$ (Corresponding angles)
$\angle QPB = \angle CAB$ (Corresponding angles)
\[\therefore \triangle PBQ \sim \triangle ABC \quad \text{(AA similarity)}\]
\[
\Rightarrow \frac{PQ}{BQ} = \frac{AC}{BC}
\Rightarrow \frac{PQ}{AC} = \frac{3}{7} \quad \text{[Using (1)]}
\]

(ii) $\angle ARC = \angle QSP = 90^\circ$
$\angle ACR = \angle SPQ$ (Alternate angles)
\[\therefore \triangle ARC \sim \triangle QSP \quad \text{(AA similarity)}\]
\[
\Rightarrow \frac{AR}{QS} = \frac{AC}{PQ}
\Rightarrow \frac{AR}{QS} = \frac{7}{3}
\Rightarrow AR = \frac{7 \times 6}{3} = 14 \text{ cm}
**Question 27.**
In the right angled triangle QPR, PM is an altitude.

![Diagram of a right angled triangle QPR with altitude PM]

Given that QR = 8 cm and MQ = 3.5 cm. Calculate, the value of PR., [2000]

Given— In right angled $\Delta$ QPR, $\angle P = 90^\circ$ PM $\perp$ QR, QR = 8 cm, MQ = 3.5 cm
Calculate— PR

**Solution:**

We have:

$\angle QPR = \angle PMR = 90^\circ$

$\angle PRQ = \angle PRM$ (Common)

$\triangle PQR \sim \triangle MPR$ (AA similarity)

\[
\frac{QR}{PR} = \frac{PR}{MR}
\]

\[
PR^2 = 8 \times 4.5 = 36
\]

$PR = 6$ cm

**Question 28.**
In the figure given below, the medians BD and CE of a triangle ABC meet at G. Prove that—

(i) $\Delta EGD \sim \Delta CGB$

(ii) $BG = 2GD$ from (i) above.
Solution:

(i) Since, BD and CE are medians.
AD = DC
AE = BE
Hence, by converse of Basic Proportionality theorem,
ED || BC
In Δ EGD and Δ CGB,
∠DEG = ∠GCB (Alternate angles)
∠EGD = ∠BGC (Vertically opposite angles)
Δ EGD ~ Δ CGB (AA similarity)

(ii) Since, Δ EGD ~ Δ CGB
\[
\frac{GD}{GB} = \frac{ED}{BC} \quad \ldots (1)
\]
In Δ AED and Δ ABC,
∠ AED = ∠ ABC (Corresponding angles)
∠ AED = ∠ BAC (Common)
Δ AED ~ Δ BAC (AA similarity)

\[
\therefore \frac{ED}{AB} = \frac{AE}{BC} = \frac{1}{2} \quad \text{(Since, E is the mid-point of AB)}
\]
\[
\Rightarrow \frac{ED}{BC} = \frac{1}{2}
\]
From (1),
GD = 1
GB = 2
GB = 2GD

Exercise 15B

Question 1.
In the following figure, point D divides AB in the ratio 3:5. Find:

\[
(i) \frac{AE}{EC} \quad (ii) \frac{AD}{AB} \quad (iii) \frac{AE}{AC}
\]

Also, if:
(iv) DE = 2.4 cm, find the length of BC.
(v) BC = 4.8 cm, find the length of DE.

Solution:

(i).

Given that \( \frac{AD}{DB} = \frac{3}{5} \).

So, \( \frac{AD}{AB} = \frac{3}{8} \).

In \( \triangle ADE \) and \( \triangle ABC \),
\[ \angle ADE = \angle ABC \quad \text{(Since DE \parallel BC, so the angles are corresponding angles.)} \]
\[ \angle A = \angle A \quad \text{(Common angle)} \]
\[ \therefore \triangle ADE \sim \triangle ABC \quad \text{(AA criterion for Similarity)} \]

\[ \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \]

\[ \Rightarrow \frac{AE}{AC} = \frac{3}{8} \]

(ii)

Given that \( \frac{AD}{DB} = \frac{3}{5} \).

So, \( \frac{AD}{AB} = \frac{3}{8} \).
(iii)

Given that \( \frac{AD}{DB} = \frac{3}{5} \).

So, \( \frac{AD}{AB} = \frac{3}{8} \).

In \( \triangle ADE \) and \( \triangle ABC \),
\( \angle ADE = \angle ABC \) .... (Since \( DE \parallel BC \), so the angles are corresponding angles.)
\( \angle A = \angle A \) .... (Common angle)

:. \( \triangle ADE \sim \triangle ABC \) ...(AA criterion for Similarity)

\( \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \)
\( \Rightarrow \frac{AE}{AC} = \frac{3}{8} \)

(iv)

Given that \( \frac{AD}{DB} = \frac{3}{5} \).

So, \( \frac{AD}{AB} = \frac{3}{8} \).

In \( \triangle ADE \) and \( \triangle ABC \),
\( \angle ADE = \angle ABC \) .... (Since \( DE \parallel BC \), so the angles are corresponding angles.)
\( \angle A = \angle A \) .... (Common angle)

:. \( \triangle ADE \sim \triangle ABC \) ...(AA criterion for Similarity)

\( \Rightarrow \frac{AD}{AB} = \frac{DE}{BC} \)
\( \Rightarrow \frac{3}{8} = \frac{2.4}{BC} \)
\( \Rightarrow BC = 6.4 \text{ cm} \)
(v)

Given that \( \frac{AD}{DB} = \frac{3}{5} \).

So, \( \frac{AD}{AB} = \frac{3}{8} \).

In \( \triangle ADE \) and \( \triangle ABC \),
\[
\angle ADE = \angle ABC \quad \text{(Since DE || BC, so the angles are corresponding angles.)}
\]
\[
\angle A = \angle A \quad \text{(Common angle)}
\]
\[
\therefore \triangle ADE \sim \triangle ABC \quad \text{(AA criterion for Similarity)}
\]
\[
\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}
\]
\[
\Rightarrow \frac{3}{8} = \frac{DE}{4.8}
\]
\[
\Rightarrow DE = 1.8 \text{ cm}
\]

Question 2.
In the given figure, PQ // AB;
CQ = 4.8 cm QB = 3.6 cm and AB = 6.3 cm. Find:

(i) \( \frac{CP}{PA} \)

(ii) PQ

(iii) If AP = x, then the value of AC in terms of x.
(i)

In \( \triangle CPQ \) and \( \triangle CAB \),
\[ \angle PCQ = \angle ACB \quad \text{....(Since \( PQ \parallel AB \), so the angles are corresponding angles.)} \]
\[ \angle C = \angle C \quad \text{....(Common angle)} \]
\[ \therefore \triangle CPQ \sim \triangle CAB \quad \text{(AA criterion for Similarity)} \]

\[ \Rightarrow \frac{CP}{CA} = \frac{CQ}{CB} \]
\[ \Rightarrow \frac{CP}{CA} = \frac{4.8}{8.4} = \frac{4}{7} \]

So, \( \frac{CP}{PA} = \frac{4}{3} \).

(ii)

In \( \triangle CPQ \) and \( \triangle CAB \),
\[ \angle PCQ = \angle ACB \quad \text{....(Since \( PQ \parallel AB \), so the angles are corresponding angles.)} \]
\[ \angle C = \angle C \quad \text{....(Common angle)} \]
\[ \therefore \triangle CPQ \sim \triangle CAB \quad \text{(AA criterion for Similarity)} \]

\[ \Rightarrow \frac{PQ}{AB} = \frac{CQ}{CB} \]
\[ \Rightarrow \frac{PQ}{6.3} = \frac{4.8}{8.4} \]
\[ \Rightarrow PQ = 3.6 \text{ cm} \]

(iii)

In \( \triangle CPQ \) and \( \triangle CAB \),
\[ \angle PCQ = \angle ACB \quad \text{....(Since \( PQ \parallel AB \), so the angles are corresponding angles.)} \]
\[ \angle C = \angle C \quad \text{....(Common angle)} \]
\[ \therefore \triangle CPQ \sim \triangle CAB \quad \text{(AA criterion for Similarity)} \]

\[ \Rightarrow \frac{CP}{AC} = \frac{CQ}{CB} \]
\[ \Rightarrow \frac{CP}{AC} = \frac{4.8}{8.4} = \frac{4}{7} \]

So, if \( AC \) is 7 parts, and \( CP \) is 4 parts, then \( PA \) is 3 parts.

Thus, \( AC = \frac{7}{3} \) \( PA = \frac{7}{3} \times \).
Question 3.
A line PQ is drawn parallel to the side BC of \( \triangle ABC \) which cuts side AB at P and side AC at Q. If \( AB = 9.0 \text{ cm} \), \( CA = 6.0 \text{ cm} \) and \( AQ = 4.2 \text{ cm} \), find the length of AP.

Solution:

In \( \triangle APQ \) and \( \triangle ABC \),

\[ \angle ACQ = \angle ABC \quad \text{(Since PQ || BC, so the angles are corresponding angles.)} \]
\[ \angle PAQ = \angle BAC \quad \text{(Common angle)} \]

\[ \therefore \triangle APQ \sim \triangle ABC \quad \text{(AA criterion for Similarity)} \]

\[ \Rightarrow \frac{AP}{AB} = \frac{AQ}{AC} \]
\[ \Rightarrow \frac{AP}{9} = \frac{4.2}{6} \]
\[ \Rightarrow AP = 6.3 \text{ cm} \]

Question 4.
In \( \triangle ABC \), D and E are the points on sides AB and AC respectively. Find whether DE // BC, if:
(i) \( AB = 9 \text{ cm} \), \( AD = 4 \text{ cm} \), \( AE = 6 \text{ cm} \) and \( EC = 7.5 \text{ cm} \).
(ii) \( AB = 63 \text{ cm} \), \( EC = 11.0 \text{ cm} \), \( AD = 0.8 \text{ cm} \) and \( AE = 1.6 \text{ cm} \).

Solution:
(i).

In $\triangle ADE$ and $\triangle ABC$,

\[
\frac{AE}{EC} = \frac{6}{7.5} = \frac{4}{5}
\]

\[
\frac{AD}{BD} = \frac{4}{5} \quad \text{(Since } AB = 9 \text{ cm and } AD = 4 \text{ cm)}
\]

So,

\[
\frac{AE}{EC} = \frac{AD}{BD}.
\]

$\therefore$ $DE \parallel BC \quad \text{(By the Converse of Mid-point theorem)}$
Question 5.
In the given figure, $\triangle ABC \sim \triangle ADE$. If $AE: EC = 4:7$ and $DE = 6.6$ cm, find $BC$. If ‘$x$’ be the length of the perpendicular from $A$ to $DE$, find the length of perpendicular from $A$ to $DE$. 

\[ \frac{AE}{EC} = \frac{1.6}{0.8} = \frac{4}{2} = 2 \quad \frac{AD}{BD} = \frac{0.8}{\frac{6.3 - 8}{5.5}} = \frac{0.8}{5.5} \]

So, \[ \frac{AE}{EC} = \frac{AD}{BD} \]

\[ \therefore DE \parallel BC \ldots \text{(By the Converse of Mid-point theorem)} \]
A to DE find the length of perpendicular from A to BC in terms of ‘x’.

Solution:

Given that \( \triangle ABC \sim \triangle ADE \).
\( \angle ABC = \angle ADE \) and \( \angle ACB = \angle AED \)
So, \( DE \parallel BC \)
Also, \( \frac{AB}{AD} = \frac{AC}{AE} = \frac{11}{4}. \) \( \) .... \( \) (Since \( \frac{AE}{EC} = \frac{4}{7} \))
In \( \triangle ADP \) and \( \triangle ABQ \),
\( \angle ADP = \angle ABQ \) ...(Since \( DP \parallel BQ \).)
\( \angle APD = \angle AQB \) ...(Since \( DP \parallel BQ \).)
So, \( \triangle ADP \sim \triangle ABQ \) ...(AA Criterion for Similarity)
\( \Rightarrow \frac{AD}{AB} = \frac{AP}{AQ} \)
\( \Rightarrow \frac{4}{11} = \frac{x}{AQ} \)
\( \Rightarrow AQ = \frac{11}{4}x \)

Question 6.
A line segment DE is drawn parallel to base BC of \( \triangle ABC \) which cuts AB at point D and AC at point E. If \( AB = 5 \) BD and \( EC = 3.2 \) cm, find the length of AE.
Solution:

Since DE \parallel BC, \triangle ADE \sim \triangle ABC

\[ \frac{AD}{BD} = \frac{AE}{EC} \]
\[ \frac{AB - BD}{BD} = \frac{AE}{EC} \]
\[ \frac{5BD - BD}{BD} = \frac{AE}{EC} \]
\[ \frac{4BD}{BD} = \frac{AE}{3.2} \]
\[ AE = 4 \times 3.2 = 12.8 \text{ cm} \]

Question 7.
In the figure, given below, AB, CD and EF are parallel lines. Given AB = 7.5 cm, DC = y cm, EF = 4.5 cm, BC = x cm and CE = 3 cm, calculate the values of x and y.
Solution:

In $\triangle BEF$, $DC || EF$.

$\Rightarrow \frac{BD}{DF} = \frac{BC}{CE}$

$\Rightarrow \frac{BD}{DF} = \frac{x}{3}$

So, $BD = x$ and $DF = 3$.

In $\triangle AFB$, $DC || AB$.

$\Rightarrow \frac{FD}{CD} = \frac{FB}{AB}$

$\Rightarrow \frac{FD}{CD} = \frac{FD + DB}{AB}$

$\Rightarrow \frac{3}{y} = \frac{x + 3}{7.5}$ ... (i)

In $\triangle BFE$, $DC || EF$.

$\Rightarrow \frac{BC}{CD} = \frac{BE}{EF}$

$\Rightarrow \frac{BC}{CD} = \frac{BC + CE}{EF}$

$\Rightarrow \frac{x}{y} = \frac{x + 3}{4.5}$

$\Rightarrow y = \frac{4.5x}{x + 3}$ ... (ii)

Substituting (ii) in (i), we get

$\frac{3}{4.5x} - \frac{x + 3}{7.5}$

$\Rightarrow \frac{3x + 9}{4.5x} = \frac{x + 3}{7.5}$

$\Rightarrow 22.5x + 67.5 = 4.5x^2 + 13.5x$

$\Rightarrow 4.5x^2 + 13.5x - 22.5x - 67.5 = 0$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow (x - 5)(x + 3) = 0$

So, $x = 5$ and $x = -3$. 
Question 8.
In the figure, given below, PQR is a right- angle triangle right angled at Q. XY is parallel to QR, PQ = 6 cm, PY=4 cm and PX : XQ = 1:2. Calculate the lengths of PR and QR.

Solution:

Given that \( \frac{PX}{XQ} = \frac{1}{2} \) and XY || QR.

So, \( \frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2} \).

Since PY = 4 cm, YR = 8 cm.
Hence, PR = 12 cm.
Since \( \triangle PQR \) is a right-angled triangle.
By Pythagoras theorem,
\( QR^2 = PR^2 - PQ^2 \)
\( QR^2 = 12^2 - 6^2 \)
\( QR^2 = 144 - 36 \)
\( QR^2 = 108 \)
\( QR = 10.39 \text{ cm} \)
Question 9.
In the following figure, M is mid-point of BC of a parallelogram ABCD. DM intersects the
diagonal AC at P and AB produced at E. Prove that PE = 2PD.

Solution:

In $\triangle BME$ and $\triangle DMC$,
$\angle BME = \angle CMD \quad \text{(vertically opposite angles)}$
$\angle MCD = \angle MBE \quad \text{(alternate angles)}$
BM = BC \quad \text{...(M is the mid-point of BC)}$
So, $\triangle BME \cong \triangle DMC \quad \text{(AAS congruence criterion)}$
\[\Rightarrow BE = DC = AB\]
In $\triangle DCP$ and $\triangle EPA$,
$\angle DPC = \angle EPA \quad \text{(vertically opposite angles)}$
$\angle CDP = \angle AEP \quad \text{(alternate angles)}$
$\triangle DCP \sim \triangle EPA \quad \text{(AA criterion for Similarity)}$
\[\Rightarrow \frac{DC}{EA} = \frac{CP}{AP} = \frac{PD}{EP}\]
\[\Rightarrow \frac{DC}{EA} = \frac{PD}{EP}\]
\[\Rightarrow \frac{EA}{PE} = \frac{PD}{DC}\]
\[\Rightarrow \frac{PE}{PD} = \frac{AB + EA}{DC}\]
\[\Rightarrow \frac{PE}{PD} = \frac{2DC}{DC}\]
\[\Rightarrow PE = 2PD\]
**Question 10.**
The given figure shows a parallelogram ABCD. E is a point in AD and CE produced meets BA produced at point F. If AE=4 cm, AF = 8 cm and AB = 12 cm, find the perimeter of the parallelogram ABCD.

![Parallelogram](image)

**Solution:**

\[ AF = 8 \text{ cm and } AB = 12 \text{ cm} \]
So, FB = 20 cm.

In \( \triangle DEC \) and \( \triangle EAF \),
\[ \angle DEC = \angle EAF \text{ ...(vertically opposite angles)} \]
\[ \angle EDC = \angle EAF \text{ ...(alternate angles)} \]
So, \( \triangle DEC \sim \triangle EAF \text{ ...(AA criterion for Similarity)} \)

\[
\Rightarrow \frac{DE}{AE} = \frac{EC}{EF} = \frac{DC}{AF} \\
\Rightarrow \frac{DE}{AE} = \frac{DC}{AF} \\
\Rightarrow \frac{DE}{AE} = \frac{AB}{AF} \\
\Rightarrow \frac{DE}{4} = \frac{12}{8} \\
\Rightarrow DE = 6 \text{ cm} \\
\]
So, \( AD = AE + ED = 4 + 6 = 10 \text{ cm} \)

Perimeter of the parallelogram ABCD
\[ = AB + BC + CD + AD \]
\[ = 12 + 10 + 12 + 10 \]
\[ = 44 \text{ cm} \]

**Exercise 15C**

**Question 1.**
(i) The ratio between the corresponding sides of two similar triangles is 2 to 5. Find the ratio between the areas of these triangles.

(ii) Areas of two similar triangles are 98 sq. cm and 128 sq. cm. Find the ratio between
the lengths of their corresponding sides.

**Solution:**

We know that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

(i) Required ratio = \( \frac{2^2}{5^2} = \frac{4}{25} \)

(ii) Required ratio = \( \sqrt{\frac{98}{128}} = \sqrt{\frac{49}{64}} = \frac{7}{8} \)

**Question 2.**
A line PQ is drawn parallel to the base BC, of \( \Delta ABC \) which meets sides AB and AC at points P and Q respectively. If AP = \( \frac{1}{3} \) PB; find the value of:

\[
\begin{align*}
(i) \quad & \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta APQ} , \\
(ii) \quad & \frac{\text{Area of } \Delta APQ}{\text{Area of trapezium } PBCQ}
\end{align*}
\]

**Solution:**
Question 3.
The perimeters of two similar triangles are 30 cm and 24 cm. If one side of first triangle is 12 cm, determine the corresponding side of the second triangle.

Solution:

(i) \[ \frac{AP}{PB} = \frac{1}{3} \implies \frac{AP}{PB} = \frac{1}{3} \]

In \( \triangle APQ \) and \( \triangle ABC \),
As \( PQ \parallel BC \), corresponding angles are equal
\( \angle APQ = \angle ABC \)
\( \angle AQP = \angle ACB \)
\( \triangle APQ \sim \triangle ABC \)
\[
\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle APQ} = \frac{AB^2}{\frac{AP^2}{4}} = \frac{16}{1} = 16 : 1
\]
\[
\left( \frac{AP}{PB} = \frac{1}{3} \implies \frac{AB}{AP} = \frac{4}{1} \right)
\]

\[
\frac{\text{Area of } \triangle APQ}{\text{Area of trapezium PBCQ}} = \frac{\frac{1}{16} - 1}{\text{Area of } \triangle APQ} = \frac{1}{16 - 1} = 1 : 15
\]

(ii) Let \( \triangle ABC \sim \triangle DEF \)

Then, \[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB + BC + AC}{DE + EF + DF}
\]

\[
= \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF}
\]

\[
\Rightarrow \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE}
\]

\[
\Rightarrow \frac{30}{24} = \frac{12}{DE}
\]

\[
\Rightarrow DE = 9.6 \text{ cm}
\]
Question 4.
In the given figure AX : XB = 3 : 5

Find:
(i) the length of BC, if length of XY is 18 cm.
(ii) ratio between the areas of trapezium XBCY and triangle ABC.

Solution:

Given, \[
\frac{AX}{XB} = \frac{3}{5} \Rightarrow \frac{AX}{AB} = \frac{3}{8} \quad ... (1)
\]

(i)
In \(\triangle AXY\) and \(\triangle ABC\),
As \(XY \parallel BC\), corresponding angles are equal
\(\angle AXY = \angle ABC\)
\(\angle AYX = \angle ACB\)
\(\triangle AXY \sim \triangle ABC\)
\(\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}\)
\(\Rightarrow \frac{3}{8} = \frac{18}{BC}\)
\(\Rightarrow BC = 48 \text{ cm}\)

(ii)
Area of \(\triangle AXY\) = \(\frac{AX^2}{2} = \frac{9}{2}\)
Area of \(\triangle ABC\) = \(\frac{AB^2}{2} = \frac{64}{2}\)
Area of \(\triangle ABC - \) Area of \(\triangle AXY\) = \(\frac{64 - 9}{64}\) = \(\frac{55}{64}\)
Area of trapezium XBCY = \(\frac{55}{64}\)
Question 5.
ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that PQ || BC and divides triangle ABC into two parts equal in area. Find the value of ratio BP : AB.

Given— In ∆ ABC, PQ || BC in such away that area APQ = area PQCB
To Find— The ratio ol' BP : AB

Solution:

From the given information, we have:
\[
\frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \frac{1}{2}
\]
\[
\Rightarrow \frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \frac{1}{2}
\]
\[
\Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2}
\]
\[
\Rightarrow \frac{AP}{AB} = \frac{1}{\sqrt{2}}
\]
\[
\Rightarrow \frac{AB - BP}{AB} \cdot \frac{1}{\sqrt{2}}
\]
Question 6.
In the given triangle PQR, LM is parallel to QR and PM : MR = 3 : 4

Solution:

(i) \[ \frac{PL}{PQ} \] and then \[ \frac{LM}{QR} \]

(ii) \[ \frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle MNR} \]

(iii) \[ \frac{\text{Area of } \triangle LQM}{\text{Area of } \triangle LQN} \]

(i)
In \( \triangle PLM \) and \( \triangle PQR \),
As \( LM \parallel QR \), corresponding angles are equal
\[ \angle PLM = \angle PQR \]
\[ \angle PML = \angle PRQ \]
\( \triangle PLM \sim \triangle PQR \)
\[ \Rightarrow \frac{PM}{PR} = \frac{LM}{QR} \]
\[ \Rightarrow 3 \quad \frac{LM}{QR} \quad \left( \therefore \frac{PM}{MR} = \frac{3}{4} \Rightarrow \frac{PR}{MR} = \frac{7}{4} \right) \]

Also, by using Basic Proportionality theorem, we have:
\[ \frac{PL}{LQ} = \frac{PM}{MR} = \frac{3}{4} \]
\[ \frac{LQ}{PL} = \frac{4}{3} \]
\[ \Rightarrow 1 + \frac{LQ}{PL} = 1 + \frac{4}{3} \]
\[ \Rightarrow \frac{PL + LQ}{PL} = \frac{3 + 4}{3} \]
\[ \Rightarrow \frac{PQ}{PL} = \frac{7}{3} \]
\[ \Rightarrow \frac{PL}{PQ} = \frac{3}{7} \]

(i) Since \( \triangle LMN \) and \( \triangle MNR \) have common vertex at M and their bases LN and NR are along the same straight line

\[ \frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle MNR} = \frac{LN}{NR} \]

Now, in \( \triangle LNQ \) and \( \triangle RNR \),
\[ \angle NLQ = \angle NRQ \quad \text{(Alternate angles)} \]
\[ \angle LMN = \angle NQR \quad \text{(Alternate angles)} \]
\[ \triangle LMN \sim \triangle NQR \quad \text{(AA similarity)} \]
\[ \therefore \frac{MN}{LN} = \frac{LQ}{QR} = \frac{3}{7} \]
\[ \therefore \frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle NQR} = \frac{LN}{QR} = \frac{3}{7} \]

(ii) Since \( \triangle LQM \) and \( \triangle LQN \) have common vertex at L and their bases QM and QN are along the same straight line

\[ \frac{\text{Area of } \triangle LQM}{\text{Area of } \triangle LQN} = \frac{QM}{QN} = \frac{10}{7} \]

\[ \therefore \frac{MN}{QN} = \frac{3}{7} \Rightarrow \frac{QM}{QN} = \frac{10}{7} \]

Question 7.
The given diagram shows two isosceles triangles which are similar also. In the given diagram, PQ and BC are not parallel:

PC = 4, AQ = 3, QB = 12, BC = 15 and AP = PQ.

\[ \begin{align*}
\text{Calculate—} \\
\text{(i) the length of AP} \\
\text{(ii) the ratio of the areas of triangle APQ and triangle ABC.}
\end{align*} \]
Solution:

(i) 
Given, $\Delta AQP \sim \Delta ACB$

\[ \frac{AQ}{AC} = \frac{AP}{AB} \]

\[ \Rightarrow \frac{3}{4+AP} = \frac{AP}{3+12} \]

\[ \Rightarrow AP^2 + 4AP - 45 = 0 \]

\[ \Rightarrow (AP + 9)(AP - 5) = 0 \]

\[ \Rightarrow AP = 5 \text{ units} \quad \text{(as length cannot be negative)} \]

(ii) 
Since, $\Delta AQP \sim \Delta ACB$

\[ \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ACB)} = \frac{PQ^2}{BC^2} \]

\[ \Rightarrow \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \frac{AP^2}{BC^2} \quad (PQ = AP) \]

\[ \Rightarrow \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \left(\frac{5}{15}\right)^2 = \frac{1}{9} \]

Question 8.
In the figure, given below, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1:2. DP produced meets AB produced at Q. Given the area of triangle CPQ = 20 cm².

Calculate—
(i) area of triangle CDP
(ii) area of parallelogram ABCD [1996]
(i) In $\triangle BPQ$ and $\triangle CPD$

\[ \angle BPQ = \angle CPD \quad \text{(Vertically opposite angles)} \]
\[ \angle BQP = \angle PDC \quad \text{(Alternate angles)} \]
\[ \triangle BPQ \sim \triangle CPD \quad \text{(AA similarity)} \]

\[ \therefore \frac{BP}{PC} = \frac{PQ}{PD} = \frac{BQ}{CD} = \frac{1}{2} \quad \therefore \frac{BP}{PC} = \frac{1}{2} \]

Also,

\[ \frac{\text{ar}(\triangle BPQ)}{\text{ar}(\triangle CPD)} = \left( \frac{BP}{PC} \right)^2 \]

\[ \Rightarrow \frac{10}{\text{ar}(\triangle CPD)} = \frac{1}{4} \quad \text{ar}(\triangle BPQ) = \frac{1}{2} \times \text{ar}(\triangle CPQ), \text{ar}(\triangle CPQ) = 20 \]

\[ \Rightarrow \text{ar}(\triangle CPD) = 40 \text{ cm}^2 \]

(ii) In $\triangle BPQ$ and $\triangle AQD$

As $BP \parallel AD$, corresponding angles are equal

\[ \angle QBP = \angle QAD \]
\[ \angle BQP = \angle QAQ \quad \text{(Common)} \]
\[ \triangle BPQ \sim \triangle AQD \quad \text{(AA similarity)} \]

\[ \therefore \frac{AQ}{EQ} = \frac{QD}{QP} = \frac{AD}{BP} = 3 \quad \therefore \frac{BP}{PC} = \frac{PQ}{QD} = \frac{1}{2} \quad \Rightarrow \frac{PQ}{QD} = \frac{1}{3} \]

Also,

\[ \frac{\text{ar}(\triangle AQD)}{\text{ar}(\triangle BPQ)} = \left( \frac{AQ}{BQ} \right)^2 \]

\[ \Rightarrow \frac{\text{ar}(\triangle AQD)}{10} = 9 \]

\[ \Rightarrow \text{ar}(\triangle AQD) = 90 \text{ cm}^2 \]

\[ \therefore \text{ar}(\triangle ADPB) = \text{ar}(\triangle AQD) - \text{ar}(\triangle BPQ) = 90 \text{ cm}^2 - 10 \text{ cm}^2 = 80 \text{ cm}^2 \]

\[ \text{ar}(\triangle ABC) = \text{ar}(\triangle CDP) + \text{ar}(\triangle ADPB) = 40 \text{ cm}^2 + 80 \text{ cm}^2 = 120 \text{ cm}^2 \]

**Question 9.**

In the given figure, $BC$ is parallel to $DE$. Area of triangle $ABC = 25 \text{ cm}^2$.
Area of trapezium $BCED = 24 \text{ cm}^2$ and $DE = 14 \text{ cm}$. Calculate the length of $BC$.
Also, find the area of triangle $BCD$. 
Solution:

In $\triangle ABC$ and $\triangle ADE$,
As $BC \parallel DE$, corresponding angles are equal
$\angle ABC = \angle ADE$
$\angle ACB = \angle AED$
$\triangle ABC \sim \triangle ADE$

\[ \therefore \frac{ar(\triangle ABC)}{ar(\triangle ADE)} = \frac{BC^2}{DE^2} \]

\[ \frac{25}{49} = \frac{BC^2}{14^2} \quad (ar(\triangle ADE) = ar(\triangle ABC) + ar(\text{trapezium BCED})) \]

$BC^2 = 100$

$BC = 10 \text{ cm}$

In trapezium $BCED$,

Area = $\frac{1}{2}(\text{Sum of parallel sides}) \times h$

Given: Area of trapezium $BCED = 24 \text{ cm}^2$, $BC = 10\text{cm}$, $DE = 14\text{cm}$

\[ \therefore 24 = \frac{1}{2}(10 + 14) \times h \]

$\Rightarrow h = \frac{48}{(10 + 14)}$

$\Rightarrow h = \frac{48}{24}$

$\Rightarrow h = 2$

Area of $\triangle BCD = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times BC \times h$

$= \frac{1}{2} \times 10 \times 2$

$\therefore \text{Area of } \triangle BCD = 10 \text{ cm}^2$
**Question 10.**
The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If AP : CP = 3 : 5.

Find:
(i) \( \triangle APB : \triangle CPB \)
(ii) \( \triangle DPC : \triangle APB \)
(iii) \( \triangle ADP : \triangle APB \)
(iv) \( \triangle APB : \triangle ADB \)

**Solution:**

(i) Since \( \triangle APB \) and \( \triangle CPB \) have common vertex at B and their bases AP and PC are along the same straight line
\[
\frac{\text{ar}(\triangle APB)}{\text{ar}(\triangle CPB)} = \frac{AP}{PC} = \frac{3}{5}
\]

(ii) Since \( \triangle DPC \) and \( \triangle BPA \) are similar
\[
\frac{\text{ar}(\triangle DPC)}{\text{ar}(\triangle BPA)} = \left( \frac{PC}{AP} \right)^2 = \left( \frac{5}{3} \right)^2 = \frac{25}{9}
\]

(iii) Since \( \triangle ADP \) and \( \triangle APB \) have common vertex at A and their bases DP and PB are along the same straight line
\[
\frac{\text{ar}(\triangle ADP)}{\text{ar}(\triangle APB)} = \frac{DP}{PB} = \frac{5}{3}
\]
\[
\left( \triangle APB \sim \triangle CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \right)
\]

(iv) Since \( \triangle APB \) and \( \triangle ADB \) have common vertex at A and their bases BP and BD are along the same straight line
\[
\frac{\text{ar}(\triangle APB)}{\text{ar}(\triangle ADB)} = \frac{PB}{BD} = \frac{3}{8}
\]
\[
\left( \triangle APB \sim \triangle CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \Rightarrow \frac{BP}{BD} = \frac{3}{8} \right)
\]

**Question 11.**
In the given figure, ARC is a triangle. DE is parallel to BC and \( \frac{AD}{DB} = \frac{3}{2} \).

(i) Determine the ratios \( \frac{AD}{AB} : \frac{DE}{BC} \).
(ii) Prove that ∆DEF is similar to ∆CBF. Hence, find \( \frac{EF}{FB} \).

(iii) What is the ratio of the areas of ∆DEF and ∆BFC?

**Solution:**

(i) Given, \( DE \parallel BC \) and \( \frac{AD}{DB} = \frac{3}{2} \).

In ∆ADE and ∆ABC,

\[ \angle A = \angle A \text{(Corresponding Angles)} \]

\[ \angle ADE = \angle ABC \text{(Corresponding Angles)} \]

∴ ∆ADE ~ ∆ABC (By AA: similarity)

\[ \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \] \( \text{(1)} \)

Now, \[ \frac{AD}{AB} = \frac{AD}{AD + DB} = \frac{3}{3 + 2} = \frac{3}{5} \]

Using (1), we get \[ \frac{AD}{AB} = \frac{3}{5} = \frac{DE}{BC} \] \( \text{(2)} \)
Question 12.
In the given figure, \( \angle B = \angle E, \angle ACD = \angle BCE, AB=10.4 \text{ cm and } DE=7.8 \text{ cm}. \) Find the ratio between areas of the \( \triangle ABC \) and \( \triangle DEC \).

Solution:

\[ \text{Given, } \angle ACD = \angle BCE \]
\[ \angle ACD + \angle BCD = \angle BCE + \angle BCD \]
\[ \angle ACB = \angle DCE \]

Also, given \( \angle B = \angle E \)

\[ \therefore \triangle ABC \sim \triangle DEC \]

\[ \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEC)} = \left( \frac{AB}{DE} \right)^2 = \left( \frac{10.4}{7.8} \right)^2 = \left( \frac{4}{3} \right)^2 = \frac{16}{9} \]

Question 13.
Triangle ABC is an isosceles triangle in which \( AB = AC = 13 \text{ cm and } BC = 10 \text{ cm}. \) AD is perpendicular to BC. If CE = 8 cm and EF \( \perp AB \), find:
Solution:

(i) \( AB = AC \text{(Given)} \)
\[ \therefore \angle FBE = \angle ACD \]
\[ \angle BFE = \angle ADC \]
\[ \triangle EFB \sim \triangle ADC \quad \text{(AA similarity)} \]
\[ \therefore \frac{\text{ar}(\triangle ADC)}{\text{ar}(\triangle EFB)} = \left( \frac{AC}{BE} \right)^2 \]
\[ = \left( \frac{AC}{BC + CE} \right)^2 \]
\[ = \left[ \frac{13}{18} \right]^2 = \frac{169}{324} \quad \ldots \text{(1)} \]

(ii) Similarly, it can be proved that \( \triangle ADB \sim \triangle EFB \)
\[ \therefore \frac{\text{ar}(\triangle ADB)}{\text{ar}(\triangle EFB)} = \left( \frac{AB}{BE} \right)^2 \]
\[ = \left( \frac{13}{18} \right)^2 \]
\[ = \frac{169}{324} \quad \ldots \text{(2)} \]

From (1) and (2),
\[ \frac{\text{ar}(\triangle ABC)}{324} = \frac{169 + 169}{324} = \frac{338}{324} = \frac{169}{162} \]
\[ \therefore \frac{\text{ar}(\triangle EFB)}{\text{ar}(\triangle ABC)} = \frac{162}{169} \]

Exercise 15D

Question 1.
A triangle \( ABC \) has been enlarged by scale factor \( m = 2.5 \) to the triangle \( A'B'C' \).
Calculate:
(i) the length of $AB$, if $A'B' = 6$ cm.
(ii) the length of $C'A'$ if $CA = 4$ cm.

Solution:

(i)
Given that $ABC$ is a triangle that has been enlarged by scale factor $m = 2.5$ to the triangle $A'B'C'$.
$A'B' = 6$ cm
So, $AB(2.5) = A'B'$
$\Rightarrow AB(2.5) = 6$
$\Rightarrow AB = 2.4$ cm

(ii)
Given that $ABC$ is a triangle that has been enlarged by scale factor $m = 2.5$ to the triangle $A'B'C'$.
$A'B' = 6$ cm
So, $AB(2.5) = A'B'$
$\Rightarrow AB(2.5) = 6$
$\Rightarrow AB = 2.4$ cm
If $CA = 4$ cm.
So, $CA(2.5) = C'A'$
$\Rightarrow (4)(2.5) = C'A'$
$\Rightarrow C'A' = 10$ cm

Question 2.
A triangle $LMN$ has been reduced by scale factor $0.8$ to the triangle $L'M'N'$. Calculate:
(i) the length of $M'N'$, if $MN = 8$ cm.
(ii) the length of $LM$, if $L'M' = 5.4$ cm.

Solution:

(i)
Given that $LMN$ is a triangle that has been reduced by scale factor $m = 0.8$ to the triangle $L'M'N'$.
$MN = 6$ cm
So, $MN(0.8) = M'N'$
$\Rightarrow (6)(0.8) = M'N'$
$\Rightarrow M'N' = 6.4$ cm
Question 3.
A triangle ABC is enlarged, about the point O as centre of enlargement, and the scale factor is 3. Find:

(i) $A'B'$, if $AB = 4$ cm.
(ii) $BC$, if $B'C' = 15$ cm.
(iii) $OA$, if $OA' = 6$ cm.
(iv) $OC'$, if $OC = 21$ cm.

Also, state the value of:

\[
\begin{align*}
(a) & \quad \frac{OB'}{OB} \\
(b) & \quad \frac{C'A'}{CA}
\end{align*}
\]

Solution:

(i)
Given that $ABC$ is enlarged and the scale factor $m = 3$ to the triangle $AB'C'$.

$AB = 4$ cm

So, $AB(3) = A'B'$

$\Rightarrow (4)(3) = A'B'$

$\Rightarrow A'B' = 12$ cm

(ii)
Given that $ABC$ is enlarged and the scale factor $m = 3$ to the triangle $AB'C'$.

$B'C' = 15$ cm

So, $BC(3) = B'C'$

$\Rightarrow BC(3) = 15$

$\Rightarrow BC = 5$ cm
Given that $ABC$ is enlarged and the scale factor $m = 3$ to the triangle $A'B'C'$.

$OA' = 6 \text{ cm}$

So, $OA(3) = OA'$

$\Rightarrow OA(3) = 6$

$\Rightarrow OA = 2 \text{ cm}$

Given that triangle $ABC$ is enlarged and the scale factor is $m = 3$ to the triangle $A'B'C'$.

$OC = 21 \text{ cm}$

So, $(OC)3 = OC'$

i.e. $21 \times 3 = OC'$

i.e. $OC' = 63 \text{ cm}$

The ratio of the lengths of two corresponding sides of two similar triangles.

(a) Given that $ABC$ is enlarged and the scale factor $m = 3$ to the triangle $A'B'C'$.

$\Rightarrow \frac{OB'}{OB} = 3$

(b) Given that $ABC$ is enlarged and the scale factor $m = 3$ to the triangle $A'B'C'$.

$\Rightarrow \frac{C'A'}{CA} = 3$

**Question 4.**

A model of an aeroplane is made to a scale of 1:400. Calculate:

(i) the length, in cm, of the model; if the length of the aeroplane is 40 m.

(ii) the length, in m, of the aeroplane, if length of its model is 16 cm.

**Solution:**

The ratio of the lengths of two corresponding sides of two similar triangles.

A model of an aeroplane is made to a scale of 1:400.

So, the length of the model $= \frac{1}{400} \times 4000 = 10 \text{ cm}$

(ii)

The ratio of the lengths of two corresponding sides of two similar triangles.

A model of an aeroplane is made to a scale of 1:400.

So, the length of the aeroplane $= 400 \times \frac{16}{100} = 64 \text{ m}$
Question 5.
The dimensions of the model of a multistory building are 1.2 m × 75 cm × 2 m. If the scale factor is 1:30; find the actual dimensions of the building.

Solution:
The ratio of the lengths of two corresponding sides of two similar triangles.
The scale factor is 1:30.
The actual dimensions of the building = \frac{30}{1} \times \text{(dimensions of the model of the building)}
⇒ The actual dimensions of the building = \frac{30}{1} \times \left(\frac{1.2 \times 75}{100} \times 2\right)
⇒ The actual dimensions of the building = 36 \text{ m} \times 22.5 \text{ m} \times 60 \text{ m}

Question 6.
On a map drawn to a scale of 1: 2,50,000; a triangular plot of land has the following measurements : AB = 3 cm, BC = 4 cm and angle ABC = 90°.
Calculate:
(i) the actual lengths of AB and BC in km.
(ii) the area of the plot in sq. km.

Solution:
The ratio of the lengths of two corresponding sides of two similar triangles.
The scale factor is 1:2,50,000.
The length of AB on the map = \frac{1}{2,50,000} \times \text{(the actual length of AB)}
⇒ 3 = \frac{1}{2,50,000} \times \text{(the actual length of AB)}
⇒ the actual length of AB = 3 \times 2,50,000
⇒ the actual length of AB = 750,000 = 7.5 \text{ km}

The length of BC on the map = \frac{1}{2,50,000} \times \text{(the actual length of BC)}
⇒ 4 = \frac{1}{2,50,000} \times \text{(the actual length of BC)}
⇒ the actual length of BC = 4 \times 2,50,000
⇒ the actual length of BC = 1,00,000 = 10 \text{ km}
(ii) The area of the plot in sq. km

\[ \frac{1}{2} \times AB \times BC \]

\[ = \frac{1}{2} \times 7.5 \times 10 \]

\[ = 37.5 \text{ sq. km} \]

**Question 7.**

A model of a ship of made to a scale 1 : 300

(i) The length of the model of ship is 2 m. Calculate the lengths of the ship.

(ii) The area of the deck ship is 180,000 m\(^2\). Calculate the area of the deck of the model.

(iii) The volume of the model in 6.5 m\(^3\). Calculate the volume of the ship. (2016)

**Solution:**

i. Scale factor \( k = \frac{1}{300} \)

Length of the model of the ship = \( k \times \text{Length of the ship} \)

\[ \Rightarrow 2 = \frac{1}{300} \times \text{Length of the ship} \]

\[ \Rightarrow \text{Length of the ship} = 600 \text{ m} \]

ii. Area of the deck of the model = \( k^2 \times \text{Area of the deck of the ship} \)

\[ \Rightarrow \text{Area of the deck of the model} = \left( \frac{1}{300} \right)^2 \times 180,000 \]

\[ = \frac{1}{90000} \times 180,000 \]

\[ = 2 \text{ m}^2 \]

iii. Volume of the model = \( k^3 \times \text{Volume of the ship} \)

\[ \Rightarrow 6.5 = \left( \frac{1}{300} \right)^3 \times \text{Volume of the ship} \]

\[ \Rightarrow \text{Volume of the ship} = 6.5 \times 27000000 = 17,5000,000 \text{ m}^3 \]

**Question 7(old).**

A model of ship is made to a scale of 1: 200.

(i) The length of the model is 4 m; calculate the length of the ship.

(ii) The area of the deck of the ship is 160000 m\(^2\); find the area of the deck of the model.

(iii) The volume of the model is 200 litres; calculate the volume of the ship in m\(^3\).
Solution:

Scale factor \( k = \frac{1}{200} \)

(i) Length of model = \( k \times \) actual length of the ship
\[ \Rightarrow \text{Actual length of the ship} = 4 \times 200 = 800 \text{ m} \]

(ii) Area of the deck of the model = \( k^2 \times \) area of the deck of the ship
\[ = \left( \frac{1}{200} \right)^2 \times 160000 \text{ m}^2 = 4 \text{ m}^2 \]

(iii) Volume of the model = \( k^3 \times \) volume of the ship
Volume of the ship
\[ = \frac{1}{k^3} \times 200 \text{ litres} \]
\[ = (200)^3 \times 200 \text{ litres} \]
\[ = 1600000000 \text{ litres} \]
\[ = 160000 \text{ m}^3 \]

Question 8.
An aeroplane is 30 in long and its model is 15 cm long. If the total outer surface area of the model is 150 cm\(^2\), find the cost of painting the outer surface of the aeroplane at the rate of ₹ 120 per sq. m. Given that 50 sq. m of the surface of the aeroplane is left for windows.

Solution:

15 cm represents = 30 m
1 cm represents \( \frac{30}{15} = 2 \text{ m} \)

1 cm\(^2\) represents \( 2 \text{ m} \times 2 \text{ m} = 4 \text{ m}^2 \)
Surface area of the model = 150 cm\(^2\)

Actual surface area of aeroplane = \( 150 \times 2 \times 2 \text{ m}^2 = 600 \text{ m}^2 \)
50 m\(^2\) is left out for windows
Area to be painted = 600 - 50 = 550 m\(^2\)
Cost of painting per m\(^2\) = Rs. 120
Cost of painting 550 m\(^2\) = 120 \times 550 = Rs. 66000
Exercise 15E

Question 1.
In the following figure, XY is parallel to BC, AX = 9 cm, XB = 4.5 cm and BC = 18 cm.

Find:
(i) \( \frac{AY}{YC} \)  (ii) \( \frac{YC}{AC} \)  (iii) XY

Solution:

(i)
Given that \( XY \parallel BC \).
So, \( \triangle AXY \sim \triangle ABC \).
\[
\frac{AX}{AB} = \frac{AY}{AC}
\]
\[
\Rightarrow \frac{9}{13.5} = \frac{AY}{AC}
\]
\[
\Rightarrow \frac{AY}{AC} = \frac{2}{1}
\]

(ii)
Given that \( XY \parallel BC \).
So, \( \triangle AXY \sim \triangle ABC \).
\[
\frac{AX}{AB} = \frac{AY}{AC}
\]
\[
\Rightarrow \frac{9}{13.5} = \frac{AY}{AC}
\]
\[
\Rightarrow \frac{YC}{AC} = \frac{4.5}{13.5} = \frac{1}{3}
\]
Question 2.
In the following figure, ABCD is a trapezium with AB // DC. If AB = 9 cm, DC = 18 cm, CF = 13.5 cm, AP = 6 cm and BE = 15 cm.

Calculate:
(i) EC
(ii) AF
(iii) PE

Solution:
(i) In \(\triangle AEB\) and \(\triangle FEC\),
\[ \angle AEB = \angle FEC \quad \text{(vertically opposite angles)} \]
\[ \angle BAE = \angle CFE \quad \text{(Since AB // DC)} \]
\(\triangle AEB \sim \triangle FEC\) \((\text{AA criterion for similarity})\)
\[ \Rightarrow \frac{AE}{FE} = \frac{BE}{CE} = \frac{AB}{FC} \]
\[ \Rightarrow 15 \cdot \frac{9}{31.5} \]
\[ \Rightarrow CE = 22.5 \text{ cm} \]

(ii) In \(\triangle APB\) and \(\triangle FPD\),
\[ \angle APB = \angle FPD \quad \text{(vertically opposite angles)} \]
\[ \angle BAP = \angle FDP \quad \text{(Since AB // DF)} \]
\(\triangle APB \sim \triangle FPD\) \((\text{AA criterion for similarity})\)
\[ \Rightarrow \frac{AP}{FP} = \frac{AB}{FD} \]
\[ \Rightarrow 6 \cdot \frac{9}{31.5} \]
\[ \Rightarrow FP = 21 \text{ cm} \]
So, \(AF = AP + PF = 6 + 21 = 27 \text{ cm}\).
(iii)

In $\triangle APB$ and $\triangle FPD$,
\[ \angle APB = \angle FPD \quad \text{...(vertically opposite angles)} \]
\[ \angle BAP = \angle DFP \quad \text{...(Since } AB \parallel DF) \]
\[ \triangle APB \sim \triangle FPD \quad \text{...(AA criterion for Similarity)} \]
\[ \Rightarrow \frac{AP}{FP} = \frac{AB}{FD} \]
\[ \Rightarrow \frac{6}{FP} = \frac{9}{31.5} \]
\[ \Rightarrow FP = 21 \text{ cm} \]
So, $AF = AP + PF = 6 + 21 = 27 \text{ cm}$.

In $\triangle AEB$ and $\triangle FEC$,
\[ \angle AEB = \angle FEC \quad \text{...(vertically opposite angles)} \]
\[ \angle BAE = \angle CFE \quad \text{...(Since } AB \parallel DC) \]
\[ \triangle AEB \sim \triangle FEC \quad \text{...(AA criterion for Similarity)} \]
\[ \Rightarrow \frac{AE}{FE} = \frac{BE}{CE} = \frac{AB}{FC} \]
\[ \Rightarrow \frac{AE}{FE} = \frac{9}{13.5} \]
\[ \frac{AF - EF}{EF} = \frac{9}{13.5} \]
\[ \Rightarrow \frac{AF}{EF} - 1 = \frac{9}{13.5} \]
\[ \Rightarrow \frac{27}{EF} = \frac{9}{13.5} + 1 = \frac{22.5}{13.5} \]
\[ \Rightarrow EF = \frac{27 \times 13.5}{22.5} = 16.2 \text{ cm} \]
Now, $PE = PF - EF = 21 - 16.2 = 4.8 \text{ cm}$

**Question 3.**

In the following figure, $AB$, $CD$ and $EF$ are perpendicular to the straight line $BDF$. 

![Diagram of perpendicular lines]
If \( AB = x \) and \( CD = z \) unit and \( EF = y \) unit, prove that \( \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \).

**Solution:**

In \( \triangle FDC \) and \( \triangle FBA \),
\[ \angle FDC = \angle FBA \quad \text{(Since \( DC \parallel AB \))} \]
\[ \angle DFC = \angle BFA \quad \text{(common angle)} \]
\( \triangle FDC \sim \triangle FBA \quad \text{(AA criterion for Similarity)} \)

\[ \Rightarrow \frac{DC}{AB} = \frac{DF}{BF} \]

\[ \Rightarrow \frac{z}{x} = \frac{DF}{BF} \quad \text{....(i)} \]

In \( \triangle BDC \) and \( \triangle BFE \),
\[ \angle BDC = \angle BFE \quad \text{(Since \( DC \parallel FE \))} \]
\[ \angle DBC = \angle FBE \quad \text{(common angle)} \]
\( \triangle BDC \sim \triangle BFE \quad \text{(AA criterion for Similarity)} \)

\[ \Rightarrow \frac{BD}{BF} = \frac{DC}{EF} \]

\[ \Rightarrow \frac{BD}{BF} = \frac{z}{y} \quad \text{....(ii)} \]

Adding (i) and (ii), we get

\[ \frac{BD}{BF} + \frac{DF}{BF} = \frac{z}{y} + \frac{z}{x} \]

\[ \Rightarrow 1 = \frac{z}{y} + \frac{z}{x} \]

\[ \Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y} \]

Hence proved.

**Question 4.**

Triangle \( ABC \) is similar to triangle \( PQR \). If \( AD \) and \( PM \) are corresponding medians of the two triangles, prove that:

\[ \frac{AB}{PQ} = \frac{AD}{PM}. \]
Solution:

\[ \triangle ABC \sim \triangle PQR. \]

\[ \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} \]

\[ \angle ABC = \angle PQR, \text{ that is, } \angle ABD = \angle PMQ \]

Also, \( \angle ADB = \angle PMQ \) ....(both are right angles)

So, \( \triangle ABD \sim \triangle PQM \) ....(AA criterion for Similarity)

\[ \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} \]

**Question 5.**

Triangle ABC is similar to triangle PQR. If AD and PM are altitudes of the two triangles, prove that: \( \frac{AB}{PQ} = \frac{AD}{PM} \)

**Solution:**

Given that \( \triangle ABC \sim \triangle PQR. \)
Question 6.
Triangle $ABC$ is similar to triangle $PQR$. If bisector of angle $BAC$ meets $BC$ at point $D$ and bisector of angle $QPR$ meets $QR$ at point $M$, prove that:

Solution:

\[ \angle ABC = \angle PQR, \text{ that is, } \angle ABD = \angle PQM \]
Also, \[ \angle ADB = \angle PMQ \] ....(both are right angles)
So, \[ \triangle ABD \sim \triangle PQM \] .....(AA criterion for Similarity)
\[ \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} \]
Question 7.
In the following figure, $\angle AXY = \angle AYX$. If $\frac{BX}{AX} = \frac{CY}{AY}$, show that triangle $ABC$ is isosceles.

Solution:

Given that $\angle AXY = \angle AYX$.
So, $AX = AY$....(Sides opposite equal angles are equal.)

Also, $\frac{BX}{AX} = \frac{CY}{AY}$ .....(By the Basic Proportionality theorem)

So, $BX = CY$

Thus, $AX + BX = AY + CY$

$\Rightarrow AB = AC$

Hence, $\triangle ABC$ is an isosceles triangle.

Question 8.
In the following diagram, lines $l$, $m$ and $n$ are parallel to each other. Two transversals $p$ and $q$ intersect the parallel lines at points $A, B, C$ and $P, Q, R$ as shown.

Prove that: $\frac{AB}{BC} = \frac{PQ}{QR}$
Solution:

Join AR.

In $\triangle ACR$, $BX \parallel CR$. By Basic Proportionality theorem,

$$\frac{AB}{BC} = \frac{AX}{XR} \quad \ldots (1)$$

In $\triangle APR$, $XQ \parallel AP$. By Basic Proportionality theorem,

$$\frac{PQ}{QR} = \frac{AX}{XR} \quad \ldots (2)$$

From (1) and (2), we get,

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

Question 9.

In the following figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that: $\frac{BE}{EC} = \frac{BC}{CP}$

Solution:
Question 10.
In the figure given below, AB//EF//CD. If AB = 22.5 cm, EP = 7.5 cm, PC = 15 cm and DC = 27 cm.

Calculate:
(i) EF  
(ii) AC

Solution:

(i)

In $\triangle PCD$ and $\triangle PEF$,
$\angle CPD = \angle EPF$ ....(vertically opposite angles)
$\angle DCE = \angle FEP$ ....(Since DC||EF.)

$\triangle PCD \sim \triangle PEF$ ....(AA criterion for Similarity)

$\Rightarrow \frac{27}{EF} = \frac{15}{7.5}$

$\Rightarrow EF = 13.5$ cm
Question 11.
In $\triangle ABC$, $\angle ABC = \angle DAC$. $AB = 8 \text{ cm, } AC = 4 \text{ cm, } AD = 5 \text{ cm}$.

(i) Prove that $\triangle ACD$ is similar to $\triangle BCA$.
(ii) Find $BC$ and $CD$.
(iii) Find area of $\triangle ACD$: area of $\triangle ABC$. (2014)

Solution:

(i)
In $\triangle ACD$ and $\triangle BCA$,
$\angle DAC = \angle ABC \quad (\text{given})$
$\angle ACD = \angle BCA \quad (\text{common angles})$
$\triangle ACD \sim \triangle BCA \quad (\text{AA criterion for Similarity})$
Question 12.
In the given triangle P, Q and R are the midpoints of sides AB, BC and AC respectively. Prove that triangle PQR is similar to triangle ABC.

(ii)
In $\triangle ACD$ and $\triangle BCA$,

$\angle DAC = \angle ABC \ldots (\text{given})$

$\angle ACD = \angle BCA \ldots (\text{common angles})$

$\triangle ACD \sim \triangle BCA \ldots (\text{AA criterion for Similarity})$

\[
\Rightarrow \frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{AB}
\]

\[
\Rightarrow \frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}
\]

\[
\Rightarrow \frac{4}{BC} = \frac{5}{8}
\]

\[
\Rightarrow BC = \frac{32}{5} \approx 6.4 \text{ cm}
\]

\[
\Rightarrow \frac{CD}{4} = \frac{5}{8}
\]

\[
\Rightarrow CD = \frac{20}{8} = 2.5 \text{ cm}
\]

(iii)
In $\triangle ACD$ and $\triangle BCA$,

$\angle DAC = \angle ABC \ldots (\text{given})$

$\angle ACD = \angle BCA \ldots (\text{common angles})$

$\triangle ACD \sim \triangle BCA \ldots (\text{AA criterion for Similarity})$

\[
\Rightarrow \frac{\text{ar}(\triangle ACD)}{\text{ar}(\triangle ABC)} = \frac{AD^2}{AB^2}
\]

\[
\Rightarrow \frac{\text{ar}(\triangle ACD)}{\text{ar}(\triangle ABC)} = \frac{5^2}{8^2} = \frac{25}{64}
\]
Solution:

In \( \triangle ABC \), PR \parallel BC. By Basic proportionality theorem,
\[
\frac{AP}{PB} = \frac{AR}{RC}
\]

Also, in \( \triangle PAR \) and \( \triangle ABC \),
\[
\angle PAR = \angle BAC \quad \text{(Common)}
\]
\[
\angle APR = \angle ABC \quad \text{(Corresponding angles)}
\]
\( \triangle PAR \sim \triangle BAC \quad \text{(AA similarity)}\)

\[
\frac{PR}{BC} = \frac{AP}{AB}
\]

\[
\frac{PR}{BC} = \frac{1}{2} \quad \text{(As P is the mid-point of AB)}
\]

\[
PR = \frac{1}{2} BC
\]

Similarly, \( PQ = \frac{1}{2} AC \)
\[
RQ = \frac{1}{2} AB
\]

Thus,
\[
\frac{PR}{BC} = \frac{PQ}{AC} = \frac{RQ}{AB}
\]
\[\Rightarrow \triangle QRP \sim \triangle ABC \quad \text{(SSS similarity)}\]

**Question 13.**

In the following figure, AD and CE are medians of \( \triangle ABC \). DF is drawn parallel to CE. Prove that:

(i) \( EF = FB \);
(ii) \( AG : GD = 2 : 1 \)
Solution:

(i)

In \( \triangle BFD \) and \( \triangle BEC \),
\[ \angle BFD = \angle BEC \quad \text{(Corresponding angles)} \]
\[ \angle FBD = \angle EBC \quad \text{(Common)} \]
\[ \triangle BFD \sim \triangle BEC \quad \text{(AA similarity)} \]
\[ \therefore \frac{BF}{BD} = \frac{BE}{BC} \]
\[ \frac{BF}{BE} = \frac{1}{2} \quad \text{(As D is the mid-point of BC)} \]
\[ BE = 2BF \]
\[ BF = FE = 2BF \]

Hence, \( EF = FB \)

(ii) In \( \triangle AFD \), \( EG \parallel FD \). Using Basic Proportionality theorem,
\[ \frac{AE}{EF} = \frac{AG}{GD} \quad \text{... (1)} \]

Now, \( AE = EB \) (as \( E \) is the mid-point of \( AB \))
\[ AE = 2EF \] (Since, \( EF = FB \), by (i))

From (1),
\[ \frac{AG}{GD} = \frac{2}{1} \]

Hence, \( AG : GD = 2 : 1 \).

Question 14.
The two similar triangles are equal in area. Prove that the triangles are congruent.

Solution:

Let us assume two similar triangles as \( \triangle ABC \sim \triangle PQR \)

Now
\[ \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \]

Since \( \text{area}(\triangle ABC) = \text{area}(\triangle PQR) \)

Therefore \( AB = PQ \)
\[ BC = QR \]
\[ AC = PR \]

So, respective sides of two similar triangles are also of same length

So, \( \triangle ABC \cong \triangle PQR \quad \text{(by SSS rule)} \)
Question 15.
The ratio between the altitudes of two similar triangles is 3 : 5; write the ratio between their:

(i) medians
(ii) perimeters
(iii) areas

**Solution:**

The ratio between the altitudes of two similar triangles is same as the ratio between their sides.
(i) The ratio between the medians of two similar triangles is same as the ratio between their sides.
   \[\text{Required ratio} = 3 : 5\]
(ii) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.
   \[\text{Required ratio} = 3 : 5\]
(iii) The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.
   \[\text{Required ratio} = (3)^2 : (5)^2 = 9 : 25\]

Question 16.
The ratio between the areas of two similar triangles is 16 : 25. Find the ratio between their:

(i) perimeters
(ii) altitudes
(iii) medians.

**Solution:**

The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides. So, the ratio between the sides of the two triangles = 4 : 5
(i) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.
   \[\text{Required ratio} = 4 : 5\]
(ii) The ratio between the altitudes of two similar triangles is same as the ratio between their sides.
   \[\text{Required ratio} = 4 : 5\]
(iii) The ratio between the medians of two similar triangles is same as the ratio between their sides.
   \[\text{Required ratio} = 4 : 5\]

Question 17.
The following figure shows a triangle PQR in which XY is parallel to QR. If PX: XQ = 1:3 and QR = 9 cm, find the length of XY.

Further, if the area of \(\Delta PXY = x \text{ cm}^2\); find in terms of x, the area of:

(i) triangle PQR.
(ii) trapezium XQRY.
Solution:

In $\triangle PXY$ and $\triangle PQR$, $XY$ is parallel to $QR$, so corresponding angles are equal.

$\angle PXY = \angle PQR$

$\angle PYX = \angle PRQ$

Hence, $\triangle PXY \sim \triangle PQR$ (By AA similarity criterion)

$\frac{PX}{XY} = \frac{QR}{PQ}$

$\Rightarrow \frac{1}{4} = \frac{XY}{QR}$

$(PX : XQ = 1 : 3 \Rightarrow PX : PQ = 1 : 4)$

$\Rightarrow \frac{1}{4} = \frac{XY}{9}$

$\Rightarrow XY = 2.25 \text{ cm}$

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$\frac{Ar(\triangle PXY)}{Ar(\triangle PQR)} = \left(\frac{PX}{PQ}\right)^2$

$\Rightarrow \frac{Ar(\triangle PXY)}{Ar(\triangle PQR)} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$Ar(\triangle PQR) = 16x \text{ cm}^2$

(ii) $Ar(\text{trapezium XQRY}) = Ar(\triangle PQR) - Ar(\triangle PXY)$

$= (16x - x) \text{ cm}^2$

$= 15x \text{ cm}^2$

Question 18.

On a map, drawn to a scale of 1 : 20000, a rectangular plot of land $ABCD$ has $AB = 24 \text{ cm}$, and $BC = 32 \text{ cm}$. Calculate:

(i) The diagonal distance of the plot in kilometre

(ii) The area of the plot in sq. km.
**Solution:**

Scale :: 1 : 20000

1 cm represents 20000 cm = \( \frac{20000}{1000 \times 100} = 0.2 \text{ km} \)

(i)
\[ AC^2 = AB^2 + BC^2 \]
\[ = 24^2 + 32^2 \]
\[ = 576 + 1024 = 1600 \]
\[ AC = 40 \text{ cm} \]

Actual length of diagonal = 40 \( \times \) 0.2 km = 8 km

(ii)
1 cm represents 0.2 km
1 cm² represents 0.2 \( \times \) 0.2 \( \text{km}^2 \)

The area of the rectangle ABCD = AB \( \times \) BC
\[ = 24 \times 32 = 768 \text{ cm}^2 \]

Actual area of the plot = 0.2 \( \times \) 0.2 \( \times \) 768 \( \text{km}^2 \) = 30.72 \( \text{km}^2 \)

**Question 19.**
The dimensions of the model of a multi-storeyed building are 1m by 60 cm by 1.20 m. If the scale factor is 1 : 50, find the actual dimensions of the building. Also, find:

(i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq cm.

(ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is 90 \( \text{m}^3 \).
Solution:

The dimensions of the building are calculated as below.
Length = 1 × 50 m = 50 m
Breadth = 0.60 × 50 m = 30 m
Height = 1.20 × 50 m = 60 m
Thus, the actual dimensions of the building are 50 m × 30 m × 60 m.

(i)
Floor area of the room of the building = 50 × \( \left( \frac{50}{1} \right)^2 \) = 125000 cm\(^2\) = \( \frac{125000}{100 \times 100} \) = 12.5 m\(^2\)

(ii)
Volume of the model of the building
\[ = 90 \times \left( \frac{1}{50} \right)^3 = 90 \times \left( \frac{1}{50} \right) \times \left( \frac{1}{50} \right) = 90 \times \left( \frac{100 \times 100 \times 100}{50 \times 50 \times 50} \right) \text{ cm}^3 \]
\[ = 720 \text{ cm}^3 \]

Question 20.

In ∆ABC, ∠ACB = 90° and CD ⊥ AB. Prove that \( \frac{BC^2}{AC^2} = \frac{BD}{AD} \).

Solution:

(i)
In ∆PQL and ∆RMP
\[ \angle LPQ = \angle QRP \quad \text{(Given)} \]
\[ \angle QRP = \angle RPM \quad \text{(Given)} \]
\[ \angle PQL \sim \angle RMP \quad \text{(AA similarity)} \]

(ii)
As ∆PQL ∼ ∆RMP (Proved above)
\[ \frac{PQ}{RP} = \frac{QL}{PM} = \frac{PL}{RM} \]
\[ \Rightarrow QL \times RM = PL \times PM \]

(iii)
\[ \angle LPQ = \angle QRP \quad \text{(Given)} \]
\[ \angle Q = \angle Q \quad \text{(Common)} \]
\[ \angle PQL \sim \angle RQP \quad \text{(AA similarity)} \]
\[ \Rightarrow \frac{PQ}{RQ} = \frac{QL}{QP} = \frac{PL}{PR} \]
\[ \Rightarrow PQ^2 = QR \times QL \]
**Question 21.**
A triangle ABC with AB = 3 cm, BC = 6 cm and AC = 4 cm is enlarged to ∆DEF such that the longest side of ∆DEF = 9 cm. Find the scale factor and hence, the lengths of the other sides of ∆DEF.

**Solution:**

Triangle ABC is enlarged to DEF. So, the two triangles will be similar.

\[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}
\]

Longest side in \( \triangle ABC = BC = 6 \) cm

Corresponding longest side in \( \triangle DEF = EF = 9 \) cm

Scale factor = \( \frac{EF}{BC} = \frac{9}{6} = \frac{3}{2} \)

\[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}
\]

\[
DE = \frac{3}{2} \times AB = \frac{9}{2} = 4.5 \text{ cm}
\]

\[
DF = \frac{3}{2} \times AC = \frac{12}{2} = 6 \text{ cm}
\]

**Question 22.**
Two isosceles triangles have equal vertical angles. Show that the triangles are similar. If the ratio between the areas of these two triangles is 16 : 25, find the ratio between their corresponding altitudes.

**Solution:**

Let \( \triangle ABC \) and \( \triangle PQR \) be two isosceles triangles.

Then, \( \frac{AB}{AC} = \frac{1}{1} \) and \( \frac{PQ}{PR} = \frac{1}{1} \)

Also, \( \angle A = \angle P \) (Given)

\( \therefore \triangle ABC \sim \triangle PQR \) \( \text{(SAS similarity)} \)
In \( \triangle ABC \), \( AP : PB = 2 : 3 \). PO is parallel to BC and is extended to Q so that CQ is parallel to BA.

Find:
(i) area \( \triangle APO \): area \( \triangle ABC \).
(ii) area \( \triangle APO \): area \( \triangle CQO \).

Solution:

In triangle ABC, PO \( \parallel \) BC. Using Basic proportionality theorem,
\[
\frac{AP}{PB} = \frac{AO}{OC}
\]
\[
\Rightarrow \frac{AO}{OC} = \frac{2}{3} \quad \ldots (1)
\]

(i)
\( \angle PAO = \angle BAC \) (Common)
\( \angle APO = \angle ABC \) (Corresponding angles)
\( \triangle APO \sim \triangle ABC \) (AA similarity)

\[
\therefore \frac{Ar(\triangle APO)}{Ar(\triangle ABC)} = \left(\frac{AO}{AC}\right)^2 = \left(\frac{\frac{2}{2+3}}{\frac{2}{5}}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}
\]
Question 24.
The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.

Show that:
(i) $\triangle ADC \sim \triangle BEC$  
(ii) $CA \times CE = CB \times CD$  
(iii) $\triangle ABC \sim \triangle DEC$  
(iv) $CD \times AB = CA \times DE$

Solution:

(i) $\angle ADC = \angle BEC = 90^\circ$
$\angle ACD = \angle BCE$  (Common)
$\triangle ADC \sim \triangle BEC$  (AA similarity)

(ii) From part (i),
$\frac{AC}{BC} = \frac{CD}{EC}$  \ldots (1)
$\Rightarrow CA \times CE = CB \times CD$

(iii) In $\triangle ABC$ and $\triangle DEC$,
From (1),
$\frac{AC}{BC} = \frac{AC}{CD} \Rightarrow \frac{AC}{EC} = \frac{BC}{CD} \Rightarrow \frac{AC}{CD} = \frac{BC}{EC}$
Question 25.
In the given figure, ABC is a triangle with \( \angle EDB = \angle ACB \).
Prove that \( \triangle ABC \sim \triangle EBD \).
If \( BE = 6 \text{ cm} \), \( EC = 4 \text{ cm} \),
\( BD = 5 \text{ cm} \) and area of \( \triangle BDE = 9 \text{ cm}^2 \). Calculate the
(i) length of \( AB \)
(ii) area of \( \triangle ABC \)

\[ \angle DCE = \angle BCA \quad \text{(Common)} \]
\[ \triangle ABC \sim \triangle DEC \quad \text{(SAS similarity)} \]
(iv) From part (iii),
\[ \frac{AC}{AB} = \frac{DC}{DE} \]
\[ \Rightarrow CD \times AB = CA \times DE \]

\[ \text{Solution:} \]

In \( \triangle ABC \) and \( \triangle EBD \),
\[ \angle ACB = \angle EDB \text{ (given)} \]
\[ \angle ABC = \angle EBD \text{ (common)} \]
\[ \triangle ABC \sim \triangle EBD \text{ (by AA\- similarity)} \]

(i) We have,
\[ \frac{AB}{BE} = \frac{BC}{BD} \quad \Rightarrow \quad AB = \frac{6 \times 10}{5} = 12 \text{ cm} \]

(ii) \[ \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BDE} = \left( \frac{AB}{BE} \right)^2 \]
\[ \Rightarrow \text{Area of } \triangle ABC = \left( \frac{12}{6} \right)^2 \times 9 \text{ cm}^2 \]
\[ = 4 \times 9 \text{ cm}^2 = 36 \text{ cm}^2 \]
Question 26.
In the given figure, ABC is a right-angled triangle with ZBAC = 90°.
(i) Prove \(\triangle ADB \sim \triangle CDA\).
(ii) If BD = 18 cm, CD = 8 cm, find AD.
(iii) Find the ratio of the area of \(\triangle ADB\) is to area of \(\triangle CDA\).

Solution:

(i) Let \(\angle CAD = x\)
\[\Rightarrow m\angle DAB = 90^\circ - x \]
\[\Rightarrow m\angle DBA = 180^\circ - (90^\circ + 90^\circ - x) = x \]
\[\Rightarrow \angle CAD = \angle DBA \quad \text{....(1)} \]

In \(\triangle ADB\) and \(\triangle CDA\),
\[\angle ADB = \angle CDA \quad \text{....[Each 90°]}\]
\[\angle ABD = \angle CAD \quad \text{....[From (1)]}\]
\[\therefore \triangle ADB \sim \triangle CDA \quad \text{....[By A.A.]} \]

(ii) Since the corresponding sides of similar triangles are proportional, we have
\[
\frac{BD}{AD} = \frac{AD}{CD}
\]
\[\Rightarrow \frac{18}{AD} = \frac{AD}{8} \]
\[\Rightarrow AD^2 = 18 \times 8 = 144 \]
\[\Rightarrow AD = 12 \text{ cm} \]

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
\[
\Rightarrow \frac{\text{Ar}(\triangle ADB)}{\text{Ar}(\triangle CDA)} = \frac{AD^2}{CD^2} = \frac{12^2}{8^2} = \frac{144}{64} = \frac{9}{4} = 9 : 4
\]
Question 27.
In the given figure, AB and DE are perpendicular to BC.

(i) Prove that \( \triangle ABC \sim \triangle DEC \)
(ii) If \( AB = 6 \) cm: \( DE = 4 \) cm and \( AC = 15 \) cm. Calculate \( CD \).
(iii) Find the ratio of the area of \( \triangle ABC \) : area of \( \triangle DEC \).

Solution:

(i)
In \( \triangle ABC \) and \( \triangle DEC \),
\( \angle ABC = \angle DEC \) ....(both are right angles)
\( \angle ACB = \angle DCE \) ....(common angles)
\( \triangle ABC \sim \triangle DEC \) ....(AA criterion for Similarity)

(ii)
In \( \triangle ABC \) and \( \triangle DEC \),
\( \angle ABC = \angle DEC \) ....(both are right angles)
\( \angle ACB = \angle DCE \) ....(common angles)
\( \triangle ABC \sim \triangle DEC \) ....(AA criterion for Similarity)
\[
\frac{AB}{DE} = \frac{AC}{CD}
\]
\[
\Rightarrow \frac{6}{4} = \frac{15}{CD}
\]
\[
\Rightarrow CD = 10 \text{ cm}
\]
Question 28.

ABC is a right angled triangle with \( \angle ABC = 90^\circ \). D is any point on AB and DE is perpendicular to AC. Prove that:

(i) \( \triangle ADE \sim \triangle ACB \).
(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.
(iii) Find, area of \( \triangle ADE \) : area of quadrilateral BCED. (2015)

Solution:

(i)

In \( \triangle ADE \) and \( \triangle ACB \),
\( \angle AED = \angle ABC \) \( \ldots \) (both are right angles)
\( \angle DAE = \angle CAB \) \( \ldots \) (common angles)
\( \triangle ADE \sim \triangle ACB \) \( \ldots \) (AA criterion for similarity)
(ii)
In \( \triangle ADE \) and \( \triangle ACB \),
\[ \angle AED = \angle ABC \quad \text{....(both are right angles)} \]
\[ \angle DAE = \angle CAB \quad \text{....(common angles)} \]
\( \triangle ADE \sim \triangle ACB \quad \text{....(AA criterion for Similarity)} \)

\[ \frac{AE}{AB} = \frac{DE}{BC} = \frac{AD}{AC} \]
\[ \Rightarrow \frac{4}{AB} = \frac{DE}{5} = \frac{AD}{13} \quad \text{....(i)} \]

In right \( \triangle ABC \),
\[ \Rightarrow AB^2 + BC^2 = AC^2 \]
\[ \Rightarrow AB^2 + 5^2 = 13^2 \]
\[ \Rightarrow AB^2 = 144 \]
\[ \Rightarrow AB = 12 \text{ cm} \]

From (i), we get
\[ \frac{4}{12} = \frac{DE}{5} = \frac{AD}{13} \]

So,
\[ DE = \frac{20}{12} = \frac{5}{3} = 1 \frac{2}{3} \text{ cm} \]
\[ \frac{AD}{13} = \frac{4}{12} \Rightarrow AD = \frac{52}{12} = 4 \frac{1}{3} \text{ cm} \]
Question 29.

Given: AB // DE and BC // EF. Prove that:

(i) \( \frac{AD}{DG} = \frac{CF}{FG} \)
(ii) \( \triangle DFG \sim \triangle ACG \).

Solution:

We need to find the area of \( \triangle ADE \) and quadrilateral BCED.

Area of \( \triangle ADE = \frac{1}{2} \times AE \times DE = \frac{1}{2} \times 4 \times \frac{5}{3} = \frac{10}{3} \) cm²

Area of quad. BCED = Area of \( \triangle ABC \) - Area of \( \triangle ADE \)

\[
\frac{1}{2} \times BC \times AB - \frac{10}{3}
\]

\[
= \frac{1}{2} \times 5 \times 12 - \frac{10}{3}
\]

\[
= 30 - \frac{10}{3}
\]

\[
= \frac{80}{3} \text{ cm}^2
\]

Thus ratio of areas of \( \triangle ADE \) to quadrilateral BCED = \( \frac{10}{\frac{80}{3}} = \frac{1}{8} \)

\[
\frac{AD}{DG} = \frac{10}{3} = \frac{1}{8}
\]
(i) In $\triangle AGB$, $DE \parallel AB$, by Basic proportionality theorem,
\[
\frac{GD}{DA} = \frac{GE}{EB} \quad \ldots (1)
\]

In $\triangle GBC$, $EF \parallel BC$, by Basic proportionality theorem,
\[
\frac{GE}{EB} = \frac{GF}{FC} \quad \ldots (2)
\]

From (1) and (2), we get,
\[
\frac{GD}{DA} = \frac{GF}{FC}
\]
\[
\frac{AD}{CF} = \frac{DG}{FG}
\]

(ii)
From (i), we have:
\[
\frac{AD}{DG} = \frac{CF}{FG}
\]
\[
\angle DGF = \angle AGC \quad \text{(Common)}
\]
\[
\therefore \triangle DFG \sim \triangle ACG \quad \text{(SAS similarity)}
\]

**Question 30.**

i.
In $\triangle PQR$ and $\triangle SPR$,
\[
\angle PSR = \angle QPR \quad \text{... given}
\]
\[
\angle PRQ = \angle PRS \quad \text{... common angle}
\]
\[
\Rightarrow \triangle PQR \sim \triangle SPR \quad \text{(AA Test)}
\]

ii. Find the lengths of $QR$ and $PS$.
Since $\triangle PQR \sim \triangle SPR$ ... from (i)
\[
\Rightarrow \frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR} \quad \ldots (a)
\]
\[
\frac{QR}{PR} = \frac{PR}{SR} \quad \ldots \text{from (a)}
\]
\[
\Rightarrow \frac{QR}{6} = \frac{6}{3}
\]
\[ QR = \frac{6 \times 6}{3} = 12 \text{ cm} \]

\[
\frac{PQ}{SP} = \frac{PR}{SR} \quad \text{... from (a)}
\]

\[ \Rightarrow \frac{8}{SP} = \frac{6}{3} \]

\[ \Rightarrow SP = \frac{8 \times 3}{6} = 4 \text{ cm} \]

iii.

\[
\frac{\text{area of } \triangle PQR}{\text{area of } \triangle SPR} = \frac{PQ^2}{SP^2} = \frac{8^2}{4^2} = \frac{64}{16} = 4
\]