Question 1.
Find, which of the following sequence form a G.P.:
(i) 8, 24, 72, 216, .......
(ii) \(\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}, .......
(iii) 9, 12, 16, 24, .......

Solution 1(i).
Given sequence: 8, 24, 72, 216.......
Now,
\[
\frac{24}{8} = 3, \quad \frac{72}{24} = 3, \quad \frac{216}{72} = 3
\]
Since \(\frac{24}{8} = \frac{72}{24} = \frac{216}{72} = ....... = 3\), the given sequence is a G.P.
with common ratio 3.

Solution 1(ii).
Given sequence: \(\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}, .......
Now,
\[
\frac{\frac{1}{24}}{\frac{1}{8}} = \frac{1}{3}, \quad \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{1}{3}, \quad \frac{\frac{1}{216}}{\frac{1}{72}} = \frac{1}{3}
\]
Since \(\frac{\frac{1}{24}}{\frac{1}{8}} = \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{\frac{1}{216}}{\frac{1}{72}} = ....... = \frac{1}{3}\), the given sequence is a G.P.
with common ratio \(\frac{1}{3}\).

Solution 1(iii).
Given sequence: 9, 12, 16, 24.......
Now,
\[
\frac{12}{9} = \frac{4}{3}, \quad \frac{16}{12} = \frac{4}{3}, \quad \frac{24}{16} = 2
\]
Since \(\frac{\frac{24}{8}}{\frac{72}{24}} = \frac{216}{72}\), the given sequence is not a G.P.
Question 2.
Find the 9th term of the series:
1, 4, 16, 64 .....

Solution:

Given sequence: 1, 4, 16, 64 ........

Now,
\[ \frac{4}{1} = 4, \quad \frac{16}{4} = 4, \quad \frac{64}{16} = 4 \]

Since \[ \frac{4}{1} = \frac{16}{4} = \frac{64}{16} = \ldots = 4 \], the given sequence is a G.P.

with first term, \( a = 1 \) and common ratio, \( r = 4 \).

Now, \( t_n = ar^{n-1} \)

\[ \Rightarrow t_9 = 1 \times 4^8 = 65536 \]

Question 3.
Find the seventh term of the G.P.:
1, \( \sqrt{3} \), 3, \( 3\sqrt{3} \) .....

Solution:

Given G.P.: 1, \( \sqrt{3} \), 3, \( 3\sqrt{3} \), ....

Here,
First term, \( a = 1 \)

Common ration, \( r = \frac{\sqrt{3}}{1} = \sqrt{3} \)

Now, \( t_n = ar^{n-1} \)

\[ \Rightarrow t_7 = 1 \times (\sqrt{3})^6 = 27 \]

Question 4.

Find the 8th term of the sequence:
\[ \frac{3}{4}, 1 \frac{1}{2}, 3, \ldots \]
Solution:

Given sequence: $\frac{3}{4}, 1\frac{1}{2}, 3, \ldots$

i.e. $\frac{3}{4}, \frac{3}{2}, 3, \ldots$

Now,

\[\frac{3}{2} \div \frac{3}{4} = 2, \quad \frac{3}{3/2} = 2,\]

Since $\frac{3}{2} \div \frac{3}{4} \div \frac{3}{3/2} \ldots = 2$, the given sequence is a G.P.

with first term, $a = \frac{3}{4}$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

⇒ $t_8 = \frac{3}{4} \times 2^7 = \frac{3}{4} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^5 = 96$

**Question 5.**

Find the 10th term of the G.P.:

**Solution:**

Given G.P.: 12, 4, 1$\frac{1}{3}$, .......

Here,

First term, $a = 12$

Common ratio, $r = \frac{4}{12} = \frac{1}{3}$

Now, $t_n = ar^{n-1}$

⇒ $t_{10} = 12 \times \left(\frac{1}{3}\right)^9 = 12 \times \frac{1}{19683} = \frac{4}{6561}$

**Question 6.**

Find the nth term of the series:
Question 7.
Find the next three terms of the sequence:
\[ \sqrt{5}, 5, 5\sqrt{5}, \ldots \]

Solution:

Given sequence: \( \sqrt{5}, 5, 5\sqrt{5}, \ldots \)

Now,
\[ \frac{5}{\sqrt{5}} = \sqrt{5}, \quad \frac{5\sqrt{5}}{5} = \sqrt{5} \]

Since \( \frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \ldots = \sqrt{5} \), the given sequence is a G.P. with first term, \( a = \sqrt{5} \) and common ratio, \( r = \sqrt{5} \).

Now, \( t_n = ar^{n-1} \)

\[ t_n = 1 \times 2^{n-1} = 2^{n-1} \]

Question 8.
Find the sixth term of the series:
\[ 2^2, 2^3, 2^4, \ldots \]
Solution:
Given sequence: \(2^2, 2^3, 2^4, \ldots\)

Now,
\[
\frac{2^3}{2^2} = 2, \quad \frac{2^4}{2^3} = 2
\]

Since \(\frac{2^2}{2^2} = \frac{2^4}{2^3} = \ldots = 2\), the given sequence is a G.P.

with first term, \(a = 2^2 = 4\) and common ratio, \(r = 2\).

Now, \(t_n = ar^{n-1}\)

\[\therefore t_6 = 4 \times (2)^5 = 4 \times 32 = 128\]

Question 9.

Find the seventh term of the G.P.:
\[\sqrt{3}+1,1, \frac{\sqrt{3}-1}{2}\], ...............

Solution:

Given G.P.: \(\sqrt{3} + 1, 1, \frac{\sqrt{3}-1}{2}\), .......

Here,
First term, \(a = \sqrt{3} + 1\)

Common ratio, \(r = \frac{1}{\sqrt{3} + 1}\)

Now, \(t_n = ar^{n-1}\)

\[\Rightarrow t_7 = \left(\sqrt{3} + 1\right) \times \left(\frac{1}{\sqrt{3} + 1}\right)^6\]

\[= \left(\frac{1}{\sqrt{3} + 1}\right)^5\]

[additional algebraic steps follow, similar to the previous solution]
**Question 10.**
Find the G.P. whose first term is 64 and next term is 32.

**Solution:**

First term, \(a = 64\)
Second term, \(t_2 = 32\)

\[\Rightarrow ar = 32\]
\[\Rightarrow 64 \times r = 32\]
\[\Rightarrow r = \frac{32}{64} = \frac{1}{2}\]

\[\therefore \text{Required G.P.} = a, ar, ar^2, \ldots\]
\[= 64, 32, 64 \times \left(\frac{1}{2}\right)^2, 64 \times \left(\frac{1}{2}\right)^3, \ldots\]
\[= 64, 32, 16, 8, \ldots\]

**Question 11.**
Find the next three terms of the series:

\[\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\]

**Solution:**

Given sequence: \[\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\]

Now,

\[\frac{2}{9} \times \frac{2}{27} = 3, \quad \frac{2}{3} \times \frac{2}{9} = 3\]

Since \[\frac{2}{9} \times \frac{2}{27} = \frac{2}{3} \times \frac{2}{9} = \ldots = 3\], the given sequence is a G.P.

with first term, \(a = \frac{2}{27}\) and common ratio, \(r = 3\).

Now, \(t_n = ar^{n-1}\)

\[\therefore \text{Next three terms:}\]
4\textsuperscript{th} term = \frac{2}{27} \times (3)^3 = \frac{2}{27} \times 27 = 2 \\
5\textsuperscript{th} term = \frac{2}{27} \times (3)^4 = \frac{2}{27} \times 27 \times 3 = 6 \\
6\textsuperscript{th} term = \frac{2}{27} \times (3)^5 = \frac{2}{27} \times 27 \times 9 = 18 

Question 12.  
Find the next two terms of the series  
2 − 6 + 18 − 54 ...........

Solution:

Given series: 2 − 6 + 18 − 54......  
Now,  
\[\frac{-6}{2} = -3, \quad \frac{18}{-6} = -3, \quad \frac{-54}{18} = -3\]

Since \[\frac{-6}{2} = \frac{18}{-6} = \frac{-54}{18} = ....... = -3\], the given sequence is a G.P. with first term, a = 2 and common ratio, r = -3.

Now, \[t_n = ar^{n-1}\]

.: Next two terms:  
5\textsuperscript{th} term = 2 \times (-3)^4 = 2 \times 81 = 162 \\
6\textsuperscript{th} term = 2 \times (-3)^5 = 2 \times (-243) = -486 

Exercise 11B

Question 1.  
Which term of the G.P. :  
\[-10, \frac{5}{\sqrt{3}}, \frac{-5}{6}, ....... \text{ .... is } \frac{-5}{72} ?\]
Solution:

For the given G.P.:
First term, \( a = -10 \)

Common ratio, \( r = \frac{\frac{5}{\sqrt{3}}}{-10} = -\frac{1}{2\sqrt{3}} \)

If \( -\frac{5}{72} \) is the \( n \)th term of the given G.P., then

\[
-\frac{5}{72} = ar^{n-1}
\]

\[
\Rightarrow -\frac{5}{72} = -10 \times \left(\frac{1}{2\sqrt{3}}\right)^{n-1}
\]

\[
\Rightarrow \frac{1}{144} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}
\]

\[
\Rightarrow \frac{1}{2\sqrt{3}} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}
\]

\[
\Rightarrow n - 1 = 4
\]

\[
\Rightarrow n = 5
\]

Question 2.

The fifth term of a G.P. is 81 and its second term is 24. Find the geometric progression.

Solution:

Let the first term of the G.P. be \( a \) and its common ratio be \( r \).

5th term = 81 \( \Rightarrow ar^4 = 81 \)

2nd term = 24 \( \Rightarrow ar = 24 \)

Now, \( \frac{ar^4}{ar} = \frac{81}{24} \)

\( \Rightarrow r^3 = \frac{27}{8} \)

\( \Rightarrow r = \frac{3}{2} \)

\( ar = 24 \)
Let the first term of the G.P. be $a$ and its common ratio be $r$.

$4^{th}$ term $= \frac{1}{18} \Rightarrow ar^3 = \frac{1}{18}$

$7^{th}$ term $= -\frac{1}{486} \Rightarrow ar^6 = -\frac{1}{486}$

Now, \[ \frac{ar^6}{ar^3} = \frac{-\frac{1}{486}}{\frac{1}{18}} \]

$\Rightarrow r^3 = -\frac{1}{27}$

$\Rightarrow r = -\frac{1}{3}$

$ar^3 = \frac{1}{18}$

$\Rightarrow a \times \left(-\frac{1}{3}\right)^3 = \frac{1}{18}$

$\Rightarrow a = -\frac{27}{18} = -\frac{3}{2}$

$\therefore$ G.P. $= a, ar, ar^2, ar^3, \ldots$

$= -\frac{3}{2}, -\frac{3}{2} \times \left(-\frac{1}{3}\right), -\frac{3}{2} \times \left(-\frac{1}{3}\right)^2, \frac{1}{18}, \ldots$

$= -\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, \ldots$
Question 4.
If the first and the third terms of a G.P. are 2 and 8 respectively, find its second term.

Solution:

Let the first term of the G.P. be \(a\) and its common ratio be \(r\).

\[ \therefore 1^{\text{st}} \text{ term} = a = 2 \]

And, \(3^{\text{rd}} \text{ term} = 8 \Rightarrow ar^2 = 8 \)

Now, \(\frac{ar^2}{a} = \frac{8}{2} \)
\[ \Rightarrow r^2 = 4 \]
\[ \Rightarrow r = \pm 2 \]

When \(a = 2\) and \(r = 2\)
\(2^{\text{nd}} \text{ term} = ar = 2 \times 2 = 4 \)

When \(a = 2\) and \(r = -2\)
\(2^{\text{nd}} \text{ term} = ar = 2 \times (-2) = -4 \)

Question 5.
The product of 3rd and 8th terms of a G.P. is 243. If its 4th term is 3, find its 7th term.

Solution:

Let the first term of the G.P. be \(a\) and its common ratio be \(r\).

Now,
\[ t_3 \times t_8 = 243 \]
\[ \Rightarrow ar^2 \times ar^7 = 243 \]
\[ \Rightarrow a^2r^9 = 243 \quad \text{....(i)} \]

Also,
\[ t_4 = 3 \]
\[ \Rightarrow ar^3 = 3 \]
\[ \Rightarrow a = \frac{3}{r^3} \]

Substituting the value of \(a\) in (i), we get
\[ \left(\frac{3}{r^3}\right)^2 \times r^9 = 243 \]
\[ \frac{9}{r^8} \times r^9 = 243 \]
\[ \Rightarrow r^3 = 27 \]
\[ \Rightarrow r = 3 \]
\[ \Rightarrow a = \frac{3}{3^3} = \frac{3}{27} = \frac{1}{9} \]
\[ \therefore 7^{th} \text{ term} = t_7 = ar^6 = \frac{1}{9} \times (3)^6 = 81 \]

**Question 6.**
Find the geometric progression with 4\(^{th}\) term = 54 and 7\(^{th}\) term = 1458.

**Solution:**

Let the first term of the G.P. be \(a\) and its common ratio be \(r\).

4\(^{th}\) term = 54 \(\Rightarrow\) \(ar^3 = 54\)

7\(^{th}\) term = 1458 \(\Rightarrow\) \(ar^6 = 1458\)

Now, \[ \frac{ar^6}{ar^3} = \frac{1458}{54} \]
\[ \Rightarrow r^3 = 27 \]
\[ \Rightarrow r = 3 \]
\[ ar^3 = 54 \]
\[ \Rightarrow a \times (3)^3 = 54 \]
\[ \Rightarrow a = \frac{54}{27} = 2 \]

\[ \therefore \text{GP.} = a, \ ar, \ ar^2, \ ar^3, \ldots \]
\[ = 2, \ 2 \times 3, \ 2 \times (3)^2, \ 54, \ldots \]
\[ = 2, \ 6, \ 18, \ 54, \ldots \]

**Question 7.**

Second term of a geometric progression is 6 and its fifth term is 9 times of its third term. Find the geometric progression. Consider that each term of the G.P. is positive.
Solution:

Let the first term of the G.P. be $a$ and its common ratio be $r$.
Now, $2^{nd}$ term = $t_2 = 6 \Rightarrow ar = 6$
Also, $t_5 = 9 \times t_3$
$\Rightarrow ar^4 = 9 \times ar^2$
$\Rightarrow r^2 = 9$
$\Rightarrow r = \pm 3$
Since, each term of a G.P. is positive, we have $r = 3$
$ar = 6$
$\Rightarrow a \times 3 = 6 \Rightarrow a = 2$

$:.\text{ G.P.} = a, ar, ar^2, ar^3, \ldots$
$= 2, 6, 2 \times (3)^2, 2 \times (3)^3, \ldots$
$= 2, 6, 18, 54, \ldots$

Question 8.

The fourth term, the seventh term and the last term of a geometric progression are 10, 80 and 2560 respectively. Find its first term, common ratio and number of terms.

Solution:

Let the first term of the G.P. be $a$ and its common ratio be $r$.
Now,
$4^{th}$ term = $t_4 = 10 \Rightarrow ar^3 = 10$
$7^{th}$ term = $t_7 = 80 \Rightarrow ar^6 = 80$
$\frac{ar^6}{ar^3} = \frac{80}{10}$
$\Rightarrow r^3 = 8$
$\Rightarrow r = 2$

$ar^3 = 10$
$\Rightarrow a \times (2)^3 = 10$
$\Rightarrow a = \frac{10}{8} = \frac{5}{4}$
Question 9.
If the 4th and 9th terms of a G.P. are 54 and 13122 respectively, find the GP. Also, find its general term.

Solution:

Let the first term of the G.P. be $a$ and its common ratio be $r$.

Now, 
4th term $= t_4 = 54 \Rightarrow ar^3 = 54$

9th term $= t_9 = 13122 \Rightarrow ar^8 = 13122$

\[
\frac{ar^8}{ar^3} = \frac{13122}{54}
\]

$\Rightarrow r^5 = 243$

$\Rightarrow r = 3$

$ar^3 = 54$

$\Rightarrow a \times 3^3 = 54$

$\Rightarrow a = \frac{54}{27} = 2$

$\therefore$ Required G.P. $= a, ar, ar^2, ar^3, \ldots$

$= 2, 2 \times 3, 2 \times (3)^2, 54$

$= 2, 6, 18, 54$

General term $= t_n = ar^{n-1} = 2 \times (3)^{n-1}$
Question 10.
The fifth, eight and eleventh terms of a geometric progression are p, q and r respectively. Show that: \( q^2 = pr \).

Solution:

Let the first term of the G.P. be \( a \) and its common ratio be \( r \).

\[ 5^{th} \text{ term} = t_5 = p \]
\[ \Rightarrow ar^4 = p \]

\[ 8^{th} \text{ term} = t_8 = q \]
\[ \Rightarrow ar^7 = q \]

\[ 11^{th} \text{ term} = t_{11} = r \]
\[ \Rightarrow ar^{10} = r \]

Now,
\[ pr = ar^4 \times ar^{10} = a^2 \times r^{14} = (a \times r^7)^2 = q^2 \]
\[ \Rightarrow q^2 = pr \]

Exercise 11C

Question 1.
Find the seventh term from the end of the series: \( \sqrt{2}, 2, 2\sqrt{2}, \ldots, 32 \).

Solution:

Given series: \( \sqrt{2}, 2, 2\sqrt{2}, \ldots, 32 \)

Now, \( \frac{2}{\sqrt{2}} = \sqrt{2}, \quad \frac{2\sqrt{2}}{2} = \sqrt{2} \)

So, the given series is a G.P. with common ratio, \( r = \sqrt{2} \)

Here, last term, \( l = 32 \)

\( \therefore 7^{th} \text{ term from an end} = \frac{l}{r^6} = \frac{32}{(\sqrt{2})^6} = \frac{32}{8} = 4 \)
**Question 2.**
Find the third term from the end of the GP.

\[
\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots, 162
\]

**Solution:**

Given G.P.: \(\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots, 162\)

Here,

Common ratio, \(r = \frac{\frac{2}{9}}{\frac{2}{27}} = 3\)

Last term, \(l = 162\)

\[\therefore \text{3rd term from an end} = \frac{l}{r^2} = \frac{162}{(3)^2} = \frac{162}{9} = 18\]

**Question 3.**

For the \(\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots, 81\);
find the product of fourth term from the beginning and the fourth term from the end.

**Solution:**

Given G.P.: \(\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots, 81\)

Here,

Common ratio, \(r = \frac{\frac{1}{9}}{\frac{1}{27}} = 3\)

First term, \(a = \frac{1}{27}\) and Last term, \(l = 81\)

\[\therefore \text{4th term from the beginning} = ar^3 = \frac{1}{27} \times (3)^3 = \frac{1}{27} \times 27 = 1\]

And, \(\text{4th term from an end} = \frac{l}{r^3} = \frac{81}{(3)^3} = \frac{81}{27} = 3\)

Thus, required product = \(1 \times 3 = 3\)
Question 4.

If for a G.P., $p^{th}$, $q^{th}$ and $r^{th}$ terms are $a$, $b$ and $c$ respectively; prove that:

$$(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$$

Solution:

Let the first term of the G.P. be $A$ and its common ratio be $R$.

Then,

- $p^{th}$ term = $a \Rightarrow AR^{p-1} = a$
- $q^{th}$ term = $b \Rightarrow AR^{q-1} = b$
- $r^{th}$ term = $c \Rightarrow AR^{r-1} = c$

Now,

$$a^{p-r} \times b^{r-p} \times c^{p-q} = (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q}$$

$$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$$

$$= A^{q-r+p-r+q-p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$

$$= A^0 \times R^0$$

$$= 1$$

Taking log on both the sides, we get

$$\log \{a^{p-r} \times b^{r-p} \times c^{p-q}\} = \log 1$$

$$\Rightarrow (q - r)\log a + (r - p)\log b + (p - q)\log c = 0 \quad \ldots \text{(proved)}$$

Question 5.

If $a$, $b$ and $c$ in G.P., prove that: $\log a^n$, $\log b^n$ and $\log c^n$ are in A.P.

Solution:

Here, $a$, $b$, $c$ are in G.P.

$\Rightarrow b^2 = ac$

Taking log on both sides, we get

$$\log (b^2) = \log (ac)$$

$$\Rightarrow 2\log b = \log a + \log c$$

$$\Rightarrow \log b + \log b = \log a + \log c$$

$$\Rightarrow \log b - \log a = \log c - \log b$$

$$\Rightarrow \log a, \log b \text{ and } \log c \text{ are in A.P.}$$
Question 6.
If each term of a G.P. is raised to the power x, show that the resulting sequence is also a G.P.

Solution:

Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ be a G.P. with common ratio $r$.

$\Rightarrow \frac{a_{n+1}}{a_n} = r$ for all $n \in \mathbb{N}$

If each term of a G.P. is raised to the power $x$, we get the sequence

$a_1^x, a_2^x, a_3^x, \ldots, a_n^x, \ldots$

Now, $\left(\frac{a_{n+1}}{a_n}\right)^x = \left(\frac{a_{n+1}}{a_n}\right)^x = r^x$ for all $n \in \mathbb{N}$

Hence, $a_1^x, a_2^x, a_3^x, \ldots, a_n^x, \ldots$ is also a G.P.

Question 7.
If $a$, $b$ and $c$ are in A.P. $a$, $x$, $b$ are in G.P. whereas $b$, $y$ and $c$ are also in G.P. Show that : $x^2$, $b^2$, $y^2$ are in A.P.

Solution:

$a$, $b$ and $c$ are in A.P.

$\Rightarrow 2b = a + c$

$a$, $x$ and $b$ are in G.P.

$\Rightarrow x^2 = ab$

$b$, $y$ and $c$ are in G.P.

$\Rightarrow y^2 = bc$

Now,

$x^2 + y^2 = ab + bc$

$= b(a + c)$

$= b \times 2b$

$= 2b^2$

$\Rightarrow x^2$, $b^2$ and $y^2$ are in A.P.

Question 8.
If $a$, $b$, $c$ are in G.P. and $a$, $x$, $b$, $y$, $c$ are in A.P., prove that :
\[
(i) \quad \frac{1}{x} + \frac{1}{y} = \frac{2}{b} \\
(ii) \quad \frac{a + c}{x + y} = 2
\]

Solution 8(i).

a, b and c are in G.P.
\[\Rightarrow b^2 - ac\]
a, x, b, y and c are in A.P.
\[\Rightarrow 2x = a + b \Rightarrow x = \frac{a + b}{2}\]
\[2b = x + y \Rightarrow b = \frac{x + y}{2}\]
\[2y = b + c \Rightarrow y = \frac{b + c}{2}\]

Now,
\[
\frac{1}{x} + \frac{1}{y} = \frac{2}{a + b} + \frac{2}{b + c}
\]
\[= \frac{2b + 2c + 2a + 2b}{ab + ac + b^2 + bc}\]
\[= \frac{2a + 2c + 4b}{ab + b^2 + b^2 + bc}\]
\[= \frac{2a + 2c + 4b}{ab + 2b^2 + bc}\]
\[= \frac{2(a + c + 2b)}{b(a + 2b + c)}\]
\[= \frac{2}{b}\]

Solution 8(ii).

a, b and c are in G.P.
\[\Rightarrow b^2 - ac\]
a, x, b, y and c are in A.P.
\[\Rightarrow 2x = a + b \Rightarrow x = \frac{a + b}{2}\]
\[2b = x + y \Rightarrow b = \frac{x + y}{2}\]
\[2y = b + c \Rightarrow y = \frac{b + c}{2}\]
Now,
\[
\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}
\]
\[
= \frac{2a(b+c) + 2c(a+b)}{(a+b)(b+c)}
\]
\[
= \frac{2ab + 2ac + 2ac + 2bc}{ab + ac + b^2 + bc}
\]
\[
= \frac{2ab + 4ac + 2bc}{ab + b^2 + b^2 + bc}
\]
\[
= 2\frac{2(ab + 2ac + bc)}{ab + 2b^2 + bc}
\]
\[
= 2\frac{2(ab + 2ac + bc)}{ab + 2ac + bc}
\]
\[
= 2
\]

Question 9.
If a, b and c are in A.P. and also in G.P., show that: \(a = b = c\).

Solution:
a, b and c are in AP.

\[\Rightarrow 2b = a + c\]

\[\Rightarrow b = \frac{a+c}{2}\]

a, b and c are also in GP.

\[\Rightarrow b^2 = ac\]

\[\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac\]

\[\Rightarrow \frac{a^2 + c^2 + 2ac}{4} = ac\]

\[\Rightarrow a^2 + c^2 + 2ac = 4ac\]

\[\Rightarrow a^2 + c^2 - 2ac = 0\]

\[\Rightarrow (a - c)^2 = 0\]

\[\Rightarrow a - c = 0\]

\[\Rightarrow a = c\]

Now, \(2b = a + c\)

\[\Rightarrow 2b = a + a\]

\[\Rightarrow 2b = 2a\]

\[\Rightarrow b = a\]

Thus, we have \(a = b = c\)
Question 10.

The first term of a G.P. is \(a\) and its \(n^{th}\) term is \(b\), where \(n\) is an even number. If the product of first \(n\) numbers of this G.P. is \(P\); prove that : \(p^2 = (ab)^n\).

Solution:

For a G.P.,
First term = \(a\)
Let the common ratio = \(r\)
\(n^{th}\) term = \(b\)
\[\Rightarrow ar^{n-1} = b\]
\(P = \) Product of first \(n\) numbers of the given G.P.
\[\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots \times ar^{n-1}\]
\[\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots \times \frac{b}{r^2} \times \frac{b}{r} \times b\]
\[\Rightarrow P = (ab) \times \left(\frac{ar}{r}\right) \times \left(\frac{ar^2}{r^2}\right) \times \ldots \times \frac{n}{2} \text{ terms}\]
\[\Rightarrow P = (ab) \times (ab) \times (ab) \times \ldots \times \frac{n}{2} \text{ terms}\]
\[\Rightarrow P = (ab)^n\]
\[\Rightarrow P = \sqrt[2]{ab^n}\]
\[\Rightarrow p^2 = ab^n\]

Question 11.

If \(a, b, c\) and \(d\) are consecutive terms of a G.P.; prove that : \((a^2 + b^2), (b^2 + c^2)\) and \((c^2 + d^2)\) are in GP.
Solution:

Let \( r \) be the common ratio of this G.P.

Given: \( a, b, c, d \) are in G.P.

\[ \Rightarrow 1^{st} = a, \]

\[ 2^{nd} \text{ term} = b = ar, \]

\[ 3^{rd} \text{ term} = c = ar^2, \]

\[ 4^{th} \text{ term} = d = ar^3. \]

Now,

\[ (b^2 + c^2)^2 = [(ar)^2 + (ar^2)^2]^2 \]

\[ = [a^2r^2 + a^2r^4]^2 \]

\[ = [a^2r^2(1 + r^2)]^2 \]

\[ = a^4r^4(1 + r^2)^2 \]

And,

\[ (a^2 + b^2) \times (c^2 + d^2) = \left( a^2 + (ar)^2 \right) \times \left( (ar)^2 + (ar^2)^2 \right) \]

\[ = \left( a^2 + a^2r^2 \right) \times \left( a^2r^4 + a^2r^6 \right) \]

\[ = a^2(1 + r^2) \times a^2r^4(1 + r^2) \]

\[ = a^4r^4(1 + r^2)^2 \]

\[ \Rightarrow (b^2 + c^2)^2 = (a^2 + b^2) \times (c^2 + d^2) \]

i.e.

\[ \frac{b^2 + c^2}{a^2 + b^2} = \frac{c^2 + d^2}{b^2 + c^2} \]

Hence, \( \{a^2 + b^2\}, \{b^2 + c^2\} \) and \( \{c^2 + d^2\} \) are in G.P.

Question 12.

If \( a, b, c \) and \( d \) are consecutive terms of a G.P. To prove:

\[ \frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2} \text{ and } \frac{1}{c^2 + d^2} \text{ are in } G.P. \]
Solution:

Let \( r \) be the common ratio of this G.P.
Given: \( a, b, c, d \) are in G.P.

\[ \Rightarrow 1^{\text{st}} = a, \]
\[ \text{2}^{\text{nd}} \text{ term} = b = ar, \]
\[ \text{3}^{\text{rd}} \text{ term} = c = ar^2, \]
\[ \text{4}^{\text{th}} \text{ term} = d = ar^3. \]

Now,
\[ \left( \frac{1}{b^2 + c^2} \right)^2 = \left( \frac{1}{(ar)^2 + (ar^2)^2} \right)^2 \]
\[ = \left( \frac{1}{a^2r^2 + a^2r^4} \right)^2 \]
\[ = \frac{1}{a^4r^4} \left( \frac{1}{1 + r^2} \right)^2 \]
\[ = \frac{1}{a^4r^4} \times \frac{1}{(1 + r^2)^2} \]

And,
\[ \left( \frac{1}{a^2 + b^2} \right) \times \left( \frac{1}{c^2 + d^2} \right) = \left( \frac{1}{a^2 + (ar)^2} \right) \times \left( \frac{1}{(ar)^2 + (ar^2)^2} \right) \]
\[ = \left( \frac{1}{a^2 + a^2r^2} \right) \times \left( \frac{1}{a^2r^4 + a^2r^6} \right) \]
\[ = \frac{1}{a^2} \left( \frac{1}{1 + r^2} \right) \times \frac{1}{a^2r^4} \left( \frac{1}{1 + r^2} \right) \]
\[ = \frac{1}{a^4r^4} \times \frac{1}{(1 + r^2)^2} \]
\[ \Rightarrow \left( \frac{1}{b^2 + c^2} \right)^2 = \left( \frac{1}{a^2 + b^2} \right) \times \left( \frac{1}{c^2 + d^2} \right) \]

Hence, \( \frac{1}{a^2 + b^2}, \frac{1}{b^2 + c^2} \) and \( \frac{1}{c^2 + d^2} \) are in G.P.

Exercise 11D

Question 1.

Find the sum of G.P.:
(i) \( 1 + 3 + 9 + 27 + \ldots \ldots \ldots \) to 12 terms.
(ii) \( 0.3 + 0.03 + 0.003 + 0.0003 + \ldots \) to 8 terms.
(iii) \[ 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots \text{ to 9 terms.} \]

(iv) \[ 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots \text{ to } n \text{ terms.} \]

(v) \[ \frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \ldots \text{ upto } n \text{ terms.} \]

(vi) \[ \sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \ldots \text{ to } n \text{ terms.} \]

Solution 1(i).

Given GP.: 1 + 3 + 9 + 27 + \ldots 

Here,

first term, \( a = 1 \)

common ratio, \( r = \frac{3}{1} = 3 \) (\( r > 1 \))

number of terms to be added, \( n = 12 \)

\[ S_n = \frac{a(r^n - 1)}{r - 1} \]

\[ S_{12} = \frac{1(3^{12} - 1)}{3 - 1} = \frac{3^{12} - 1}{2} = \frac{531441 - 1}{2} = \frac{531440}{2} = 265720 \]

Solution 1(ii).

Given GP.: 0.3 + 0.03 + 0.003 + 0.0003 + \ldots 

Here,

first term, \( a = 0.3 \)

common ratio, \( r = \frac{0.03}{0.3} = 0.1 \) (\( r < 1 \))

number of terms to be added, \( n = 8 \)

\[ S_n = \frac{a(1-r^n)}{1-r} \]

\[ S_8 = \frac{0.3(1-(0.1)^8)}{1-0.1} = \frac{0.3(1-(0.1)^8)}{0.9} = \frac{1-(0.1)^8}{3} = \frac{1}{3}(1-\frac{1}{10^8}) \]
Solution 1(iii).

Given GP: \(1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots\)

Here,
first term, \(a = 1\)
common ratio, \(r = \frac{-\frac{1}{2}}{1} = -\frac{1}{2} (r < 1)\)
number of terms to be added, \(n = 9\)

\[ S_n = \frac{a(1-r^n)}{1-r} \]
\[ \Rightarrow S_8 = \frac{1\left(1-\left(-\frac{1}{2}\right)^9\right)}{1-\left(-\frac{1}{2}\right)} \]
\[ = \frac{1-\left(-\frac{1}{2}\right)^9}{1+\frac{1}{2}} \]
\[ = \frac{1+\frac{1}{2^9}}{\frac{3}{2}} \]
\[ = \frac{2}{3} \left(1+\frac{1}{2^9}\right) \]
\[ = \frac{2}{3} \left(1+\frac{1}{512}\right) \]
\[ = \frac{2}{3} \cdot \frac{513}{512} \]
\[ = \frac{171}{256} \]
Solution 1(iv).

Given G.P.: \(1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots \) upto n terms

Here,
first term, \(a = 1\)

common ratio, \(r = \frac{-\frac{1}{3}}{1} = -\frac{1}{3} \quad (r < 1)\)

number of terms to be added = \(n\)

\[ S_n = \frac{a(1-r^n)}{1-r} \]

\[ = \frac{1 \left(1 - \left(-\frac{1}{3}\right)^n\right)}{1 - \left(-\frac{1}{3}\right)} \]

\[ = \frac{1 \left(1 - \left(-\frac{1}{3}\right)^n\right)}{1 + \frac{1}{3}} \]

\[ = \frac{1 - \left(-\frac{1}{3}\right)^n}{\frac{4}{3}} \]

\[ = \frac{3}{4} \left[1 - \left(-\frac{1}{3}\right)^n\right] \]
Solution 1(v).

Given G.P.: \( \frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \ldots \) upto n terms

Here,

first term, \( a = \frac{x+y}{x-y} \)

common ratio, \( r = \frac{1}{\frac{x+y}{x+y}} = \frac{x-y}{x+y} \) \((r < 1)\)

number of terms to be added = n

\[ S_n = \frac{a(1-r^n)}{1-r} \]

\[ S_n = \frac{\frac{x+y}{x-y} \left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{1-\left(\frac{x-y}{x+y}\right)} \]

\[ = \frac{x+y}{x-y} \left(1-\left(\frac{x-y}{x+y}\right)^n\right) \]

\[ = \frac{x+y}{x+y-x+y} \]

\[ = \frac{x+y}{x+y} \left(1-\left(\frac{x-y}{x+y}\right)^n\right) \]

\[ = \frac{2y}{x+y} \left(1-\left(\frac{x-y}{x+y}\right)^n\right) \]

\[ = \frac{(x+y)^2}{2y(x-y)} \]
Solution 1(vi).

Given G.P.: \( \sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \ldots \ldots \) upto \( n \) terms

Here,

first term, \( a = \sqrt{3} \)

common ratio, \( r = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3} \) \( (r < 1) \)

number of terms to be added = \( n \)

\[ S_n = \frac{a(1-r^n)}{1-r} \]

\[ \Rightarrow S_n = \frac{\sqrt{3}(1-\left(\frac{1}{3}\right)^n)}{1-\frac{1}{3}} \]

\[ = \frac{\sqrt{3}(1-\frac{1}{3^n})}{\frac{2}{3}} \]

\[ = \frac{3\sqrt{3}}{2} \left(1 - \frac{1}{3^n}\right) \]

Question 2.
How many terms of the geometric progression 1+4 + 16 + 64 + ......... must be added to get sum equal to 5461?

Solution:

Given G.P.: 1 + 4 + 16 + 64 + .......

Here,

first term, \( a = 1 \)

common ratio, \( r = \frac{4}{1} = 4 \) \( (r > 1) \)

Let the number of terms to be added = \( n \)

Then, \( S_n = 5461 \)

\[ \Rightarrow \frac{a(r^n - 1)}{r - 1} = 5461 \]
Question 3.
The first term of a G.P. is 27 and its 8th term is \( \frac{1}{81} \). Find the sum of its first 10 terms.

Solution:
Given,  
First term, \( a = 27 \)  
8th term = \( ar^7 = \frac{1}{81} \)  
\( n = 10 \)
Now,  
\[ \frac{ar^7}{a} = \frac{\frac{1}{81}}{27} \]
\[ r^7 = \frac{1}{2187} \]
\[ r^7 = \left( \frac{1}{3} \right)^7 \]
\[ r = \frac{1}{3} \] (\( r < 1 \))
\[ S_n = \frac{a(1-r^n)}{1-r} \]
\[ S_{10} = \frac{27 \left(1- \left(\frac{1}{3}\right)^{10}\right)}{1-\frac{1}{3}} \]
\[ S_{10} = \frac{27 \left(1- \frac{1}{3^{10}}\right)}{2/3} \]
\[ S_{10} = \frac{81}{2} \left(1- \frac{1}{3^{10}}\right) \]
Question 4.

A boy spends ₹ 10 on first day, ₹ 20 on second day, ₹ 40 on third day and so on. Find how much, in all, will he spend in 12 days?

Solution:

Amount spent on 1st day = Rs. 10
Amount spent on 2nd day = Rs. 20
Amount spent on 3rd day = Rs. 40 and so on

Now, \[ \frac{20}{10} = 2, \frac{40}{20} = 2, \]
Thus, 10, 20, 40, ..... is a G.P. with first term, \( a = 10 \)
and common ratio, \( r = 2 \ (r > 1) \)

\( \therefore \) Total amount spent in 12 days = \( S_{12} \)

\[ S_n = \frac{a(r^n - 1)}{r - 1} \]

\[ \Rightarrow S_{12} = \frac{10(2^{12} - 1)}{2-1} = 10(2^{12} - 1) = 10(4096 - 1) = 10 \times 4095 = 40950 \]

Hence, the total amount spent in 12 days is Rs. 40950.

Question 5.

The 4th and the 7th terms of a G.P. are \( \frac{1}{27} \) and \( \frac{1}{729} \) respectively. Find the sum of n terms of this G.P.

Solution:

For a G.P.,

4th term = \( ar^3 = \frac{1}{27} \)

7th term = \( ar^6 = \frac{1}{729} \)

Now, \( \frac{ar^6}{ar^3} = \frac{\frac{1}{729}}{\frac{1}{27}} = \frac{1}{27} \)

\[ \Rightarrow r^3 = \frac{1}{27} = \left(\frac{1}{3}\right)^3 \]
A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728; find its first term.

**Solution:**

For a G.P.,
Common ratio, $r = 3 \ (r > 1)$
Last term, $l = 486$

$S = 728$

\[ \Rightarrow \frac{l r - a}{r - 1} = 728 \]
\[ \Rightarrow \frac{486 \times 3 - a}{3 - 1} = 728 \]
\[ \Rightarrow \frac{1458 - a}{2} = 728 \]
\[ \Rightarrow 1458 - a = 1456 \]

Hence, the first term is 2.
Question 7.
Find the sum of G.P. : 3, 6, 12, ..........., 1536.

Solution:

Given G.P. : 3, 6, 12, ......., 1536
Here,
First term, \( a = 3 \)
Common ratio, \( r = \frac{6}{3} = 2 \quad (r > 1) \)
Last term, \( l = 1536 \)

\[
\begin{align*}
\text{Required sum} & = \frac{lr - a}{r - 1} \\
& = \frac{1536 \times 2 - 3}{2 - 1} \\
& = 3072 - 3 \\
& = 3069
\end{align*}
\]

Question 8.
How many terms of the series 2 + 6 + 18 + ............ must be taken to make the sum equal to 728 ?

Solution:

Given series: 2 + 6 + 18 + .......

Now, \( \frac{6}{2} = 3, \frac{18}{6} = 3 \)

Thus, given series is a G.P. with first term, \( a = 2 \)
and common ratio, \( r = 3 \quad (r > 1) \)
Let the number of terms to be added = \( n \)
Then, \( S_n = 728 \)

\[
\begin{align*}
\Rightarrow \quad \frac{a(r^n - 1)}{r - 1} & = 728 \\
\Rightarrow \quad \frac{2(3^n - 1)}{3 - 1} & = 728 \\
\Rightarrow \quad 3^n - 1 & = 728 \\
\Rightarrow \quad 3^n & = 729 \\
\Rightarrow \quad 3^n & = 3^6 \\
\Rightarrow \quad n & = 6
\end{align*}
\]

Hence, required number of terms = 6
Question 9.
In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125 : 152.

Find its common ratio.

Solution:

Let \( a \) be the first term and \( r \) be the common ratio of given G.P.

Now, sum of first three terms = \( S_3 = \frac{a(r^3 - 1)}{r - 1} \)

Now, sum of first six terms = \( S_6 = \frac{a(r^6 - 1)}{r - 1} \)

It is given that
\[
\frac{a(r^3 - 1)}{r - 1} = \frac{125}{152}
\]

\[
\Rightarrow \frac{r^3 - 1}{r^6 - 1} = \frac{125}{152}
\]

\[
\Rightarrow \frac{r^3 - 1}{(r^3 - 1)(r^3 + 1)} = \frac{125}{152}
\]

\[
\Rightarrow \frac{1}{r^3 + 1} = \frac{125}{152}
\]

\[
\Rightarrow r^3 + 1 = \frac{152}{125}
\]

\[
\Rightarrow r^3 = \frac{152}{125} - 1 = \frac{152 - 125}{125} = \frac{27}{125}
\]

\[
\Rightarrow r^3 = \left(\frac{3}{5}\right)^3
\]

\[
\Rightarrow r = \frac{3}{5}
\]

Hence, the common ratio is \( \frac{3}{5} \).
Question 10.
Find how many terms of G.P. \( \frac{2}{9} - \frac{1}{3} + \frac{1}{2} \ldots \ldots \) must be added to get the sum equal to \( \frac{55}{72} \)?

Solution:

Given G.P.: \( \frac{2}{9} - \frac{1}{3} + \frac{1}{2} \ldots \ldots \)

Here,

First term, \( a = \frac{2}{9} \)

Common ratio, \( r = \frac{-\frac{1}{3}}{\frac{2}{9}} = -\frac{3}{2} < 1 \)

Let required number of terms be \( n \).

\[ \Rightarrow S_n = \frac{55}{72} \]

\[ \Rightarrow \frac{a(1-r^n)}{1-r} = \frac{55}{72} \]

\[ \Rightarrow \frac{\frac{2}{9} \left(1 - \left(-\frac{3}{2}\right)^n\right)}{1 - \left(-\frac{3}{2}\right)} = \frac{55}{72} \]

\[ \Rightarrow \frac{\frac{2}{9} \left(1 - \left(-\frac{3}{2}\right)^n\right)}{\frac{5}{2}} = \frac{55}{72} \]

\[ \Rightarrow \frac{2}{9} \left(1 - \left(-\frac{3}{2}\right)^n\right) = \frac{55}{72} \times \frac{5}{2} \]

\[ \Rightarrow 1 - \left(-\frac{3}{2}\right)^n = \frac{55}{72} \times \frac{5}{2} \times \frac{9}{2} \]

\[ \Rightarrow 1 - \left(-\frac{3}{2}\right)^n = \frac{275}{32} \]

\[ \Rightarrow 1 - \frac{275}{32} = \left(-\frac{3}{2}\right)^n \]

\[ \Rightarrow -\frac{243}{32} = \left(-\frac{3}{2}\right)^n \]

\[ \Rightarrow \left(-\frac{3}{2}\right)^5 = \left(-\frac{3}{2}\right)^n \]

\[ \Rightarrow n = 5 \]

\[ \therefore \text{Required number of terms} = 5 \]
**Question 11.**
If the sum $1 + 2 + 2^2 + \ldots + 2^{n-1}$ is 255, find the value of $n$.

**Solution:**

Required series: $1 + 2 + 2^2 + \ldots + 2^{n-1}$

Now, $\frac{2}{1} = 2$, $\frac{2^2}{2} = 2$

Thus, given series is a G.P. with

first term, $a = 1$

common ratio, $r = 2$ ($r > 1$)

Last term, $l = 2^{n-1}$

Let there be $n$ terms in the series.

Then, $S_n = 255$

$\Rightarrow \frac{l - a}{r - 1} = 255$

$\Rightarrow \frac{2^{n-1} \times 2 - 1}{2 - 1} = 255$

$\Rightarrow 2^{n-1} \times 2 - 1 = 255$

$\Rightarrow 2^{n-1} \times 2 = 256$

$\Rightarrow 2^{n-1} = 128$

$\Rightarrow 2^{n-1} = 2^7$

$\Rightarrow n - 1 = 7$

$\Rightarrow n = 8$

**Question 12.**
Find the geometric mean between:

(i) $\frac{4}{9}$ and $\frac{9}{4}$

(ii) 14 and $\frac{7}{32}$

(iii) $2a$ and $8a^3$

**Solution 12(i).**

Geometric mean between $\frac{4}{9}$ and $\frac{9}{4} = \sqrt{\frac{4}{9} \times \frac{9}{4}} = \sqrt{1} = 1$

**Solution 12(ii).**

Geometric mean between 14 and $\frac{7}{32} = \sqrt{14 \times \frac{7}{32}} = \sqrt{\frac{49}{16}} = \frac{7}{4} = 1\frac{3}{4}$
Solution 12(iii).
Geometric mean between $2a$ and $8a^3 = \sqrt{2a \times 8a^3} = \sqrt{16a^4} = 4a^2$

Question 13.
The sum of three numbers in G.P. is $\frac{39}{10}$ and their product is 1. Find the numbers.

Solution:

Let the numbers be $\frac{a}{r}$, $a$ and $ar$.

\[
\Rightarrow \frac{a}{r} \times a \times ar = 1
\]
\[
\Rightarrow a^3 = 1
\]
\[
\Rightarrow a = 1
\]

Now, $\frac{a}{r} + a + ar = \frac{39}{10}$

\[
\Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10}
\]
\[
\Rightarrow \frac{1 + r + r^2}{r} = \frac{39}{10}
\]
\[
\Rightarrow 10 + 10r + 10r^2 = 39r
\]
\[
\Rightarrow 10r^2 - 29r + 10 = 0
\]
\[
\Rightarrow 10r^2 - 25r - 4r + 10 = 0
\]
\[
\Rightarrow 5(2r - 5) - 2(2r - 5) = 0
\]
\[
\Rightarrow (2r - 5)(5r - 2) = 0
\]
\[
\Rightarrow r = \frac{5}{2} \text{ or } r = \frac{2}{5}
\]

Thus, required terms are:

\[
\frac{a}{r}, a, ar = \frac{1}{5}, 1, 1 \times \frac{5}{2} \text{ OR } \frac{1}{5}, 1, 1 \times \frac{2}{5}
\]
\[
= \frac{2}{5}, 1, \frac{5}{2} \text{ OR } \frac{5}{2}, 1, \frac{2}{5}
\]

Question 14.
The first term of a G.P. is -3 and the square of the second term is equal to its 4th term. Find its 7th term.

Solution:
For a G.P.,
First term, \( a = -3 \)
It is given that,
\[
(\text{2}^{\text{nd}} \text{ term})^2 = 4^{\text{th}} \text{ term}
\]
\[
\Rightarrow (ar)^2 = ar^3
\]
\[
\Rightarrow a^2r^2 = ar^3
\]
\[
\Rightarrow a = r
\]
\[
\Rightarrow r = -3
\]
Now, \( 7^{\text{th}} \) term = \( ar^6 = -3 \times (-3)^6 = -3 \times 729 = -2187 \)

**Question 15.**
Find the 5th term of the G.P. \( \frac{5}{2}, 1, \ldots \)

**Solution:**

\[
\text{First term (a)} = \frac{5}{2}
\]

And, common ratio \( (r) = \frac{1}{5} = \frac{2}{5} \)

Now, \( t_n = ar^{n-1} \)

\[
\Rightarrow 5^{\text{th}} \text{ term} = t_5 = \frac{5}{2} \times \left(\frac{5}{2}\right)^{5-1} = \frac{5}{2} \times \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}
\]

**Question 16.**
The first two terms of a G.P. are 125 and 25 respectively. Find the 5th and the 6th terms of the G.P.

**Solution:**

First term (a) = 125

And, common ratio \( (r) = \frac{25}{125} = \frac{1}{5} \)

Now, \( t_n = ar^{n-1} \)

\[
\Rightarrow 5^{\text{th}} \text{ term} = t_5 = 125 \times \left(\frac{1}{5}\right)^{5-1} = 125 \times \left(\frac{1}{5}\right)^4 = 125 \times \frac{1}{625} = \frac{1}{5}
\]

\[
\Rightarrow 6^{\text{th}} \text{ term} = t_6 = 125 \times \left(\frac{1}{5}\right)^{6-1} = 125 \times \left(\frac{1}{5}\right)^5 = 125 \times \frac{1}{3125} = \frac{1}{25}
\]
Question 17.
Find the sum of the sequence $\frac{1}{3}, 1, -3, 9, \ldots$ up to 8 terms.

Solution:

Here,
$$\frac{1}{3} = -3 = \frac{9}{-3} = -3$$

Thus, the given sequence is a G.P. with first term (a) = $-\frac{1}{3}$ and common ratio (r) = $-3$ (r < 1).

Number of terms to be added, n = 8

\[ S_n = \frac{a(1 - r^n)}{1 - r} \]

\[ \Rightarrow S_8 = \frac{-\frac{1}{3}(1 - (-3)^8)}{1 - 3} = \frac{-1 + 3^8}{12} = \frac{1}{12} (3^8 - 1) \]

Question 18.
The first term of a G.P. is 27. If the 8th term be $\frac{1}{81}$, what will be the sum of 10 terms?

Solution:

Given,
First term, a = 27

8th term = $ar^7 = \frac{1}{81}$

n = 10

Now,

\[ \frac{ar^7}{a} = \frac{\frac{1}{81}}{27} \]

\[ \Rightarrow r^7 = \frac{1}{2187} \]

\[ \Rightarrow r^7 = \left(\frac{1}{3}\right)^7 \]

\[ \Rightarrow r = \frac{1}{3} \ (r < 1) \]

\[ \therefore S_n = \frac{a(1 - r^n)}{1 - r} \]
Questi

Question 19.
Find a G.P. for which the sum of first two terms is -4 and the fifth term is 4 times the third term.

Solution:

Let the five terms of the given G.P. be

\[ \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2 \]

Given, sum of first two terms = -4

\[ \frac{a}{r^2} + \frac{a}{r} = -4 \]
\[ \Rightarrow \frac{a + ar}{r^2} = -4 \]
\[ \Rightarrow a + ar = -4r^2 \]
\[ \Rightarrow a(1 + r) = -4r^2 \]
\[ \Rightarrow a = -\frac{4r^2}{1 + r} \]

And, 5th term = 4(3rd term)
\[ \Rightarrow ar^2 = 4(a) \]
\[ \Rightarrow r^2 = 4 \]
\[ \Rightarrow r = \pm 2 \]

When \( r = 2 \),
\[ a = -\frac{4(2)^2}{1 + 2} = -\frac{16}{3} \]
When \( r = -2, \)

\[
a = \frac{-4(-2)^2}{1 - 2} = 16
\]

Thus, the required terms are \( \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2. \)

\[
i.e. \frac{16}{3}, \frac{16}{2}, -\frac{16}{3}, -\frac{16}{3} \times 2, -\frac{16}{3} \times 4 \text{ OR } \frac{16}{4}, \frac{16}{-2}, 16, 16(-2), 16 \times 4
\]

\[
i.e. \frac{4}{3}, \frac{8}{3}, -\frac{16}{3}, -\frac{32}{3}, -\frac{64}{3} \text{ OR } 4, -8, 16, -32, 64
\]

### Additional Questions

**Question 1.**

Find the sum of \( n \) terms of the series:

(i) \( 4 + 44 + 444 + \ldots \ldots \)

(ii) \( 0.8 + 0.88 + 0.888 + \ldots \ldots \ldots \)

**Solution 1(i).**

Required sum = \( 4 + 44 + 444 + \ldots \ldots \text{ up to } n \text{ terms} \)

\[
= 4\left(1 + 11 + 111 + \ldots \ldots \text{ up to } n \text{ terms}\right)
\]

\[
= \frac{4}{9}\left(9 + 99 + 999 + \ldots \ldots \text{ up to } n \text{ terms}\right)
\]

\[
= \frac{4}{9}\left[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots \ldots \text{ up to } n \text{ terms}\right]
\]

\[
= 4\left[\frac{10 + 10^2 + 10^3 + \ldots \ldots \text{ up to } n \text{ terms}}{9} - (1 + 1 + 1 + \ldots \ldots \text{ up to } n \text{ terms})\right]
\]

\[
= 4\left[\frac{10(10^n - 1)}{10 - 1} - n\right]
\]

\[
= 4\left[\frac{10}{9}(10^n - 1) - n\right]
\]
Solution 1(ii).

Required sum = $0.8 + 0.88 + 0.888 + \ldots \ldots \text{upto n terms}$

$= \frac{8}{9}(0.9 + 0.99 + 0.999 + \ldots \ldots \text{upto n terms})$

$= \frac{8}{9}[1 - 0.1 + (1 - 0.01) + (1 - 0.001) + \ldots \ldots \text{upto n terms}]$

$= \frac{8}{9} \left[ (1 + 1 + 1 + \ldots \ldots \text{upto n terms}) - (0.1 + 0.01 + 0.001 + \ldots \ldots \text{upto n terms}) \right]$

$= \frac{8}{9} \left[ (1 + 1 + 1 + \ldots \ldots \text{upto n terms}) \left( \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \ldots \text{upto n terms} \right) \right]$

$= \frac{8}{9} \left[ n - \frac{10}{9} \left( \frac{1 - \left( \frac{1}{10} \right)^n}{1 - \frac{1}{10}} \right) \right]$

$= \frac{8}{9} \left[ n - \frac{10}{9} \cdot \frac{1}{10} \left( 1 - \frac{1}{10^n} \right) \right]$

$= \frac{8}{9} \left[ n - \frac{1}{9} \left( 1 - \frac{1}{10^n} \right) \right]$

$\because r = \frac{1}{10} < 1$

Question 2.

Find the sum of infinite terms of each of the following geometric progression:

(i) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots \ldots$

(ii) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots \ldots$

(iii) $\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots \ldots$

(iv) $\sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \ldots \ldots$

(v) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \frac{1}{9\sqrt{3}} + \ldots \ldots$
Solution 2(i).

Given GP: \(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\)

Here,
First term, \(a = 1\)
Common ratio, \(r = \frac{\frac{1}{3}}{1} = \frac{1}{3} \left( |r| = \left| \frac{1}{3} \right| = \frac{1}{3} < 1 \right)\)
\[ \therefore \text{Required sum} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} \]

Solution 2(ii).

Given GP: \(1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots\)

Here,
First term, \(a = 1\)
Common ratio, \(r = \frac{-\frac{1}{2}}{1} = -\frac{1}{2} \left( |r| = |\frac{-1}{2}| = \frac{1}{2} < 1 \right)\)
\[ \therefore \text{Required sum} = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3} \]

Solution 2(iii).

Given GP: \(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\)

Here,
First term, \(a = \frac{1}{3}\)
Common ratio, \(r = \frac{\frac{1}{3^2}}{\frac{1}{3}} = \frac{1}{3} \left( |r| = \left| \frac{1}{3} \right| = \frac{1}{3} < 1 \right)\)
\[ \therefore \text{Required sum} = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \]
Solution 2(iv).

Given GP: \( \sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \ldots \)

Here,
First term, \( a = \sqrt{2} \)

Common ratio, \( r = -\frac{\sqrt{2}}{2} = -\frac{1}{2} \left( \left| r \right| = \left| -\frac{1}{2} \right| = \frac{1}{2} < 1 \right) \)

\[ \therefore \text{Required sum} = \frac{a}{1-r} = \frac{\sqrt{2}}{1-\left(-\frac{1}{2}\right)} = \frac{\sqrt{2}}{1+\frac{1}{2}} = \frac{2\sqrt{2}}{3} \]

Solution 2(v).

Given GP: \( \sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} - \frac{1}{9\sqrt{3}} + \ldots \)

Here,
First term, \( a = \sqrt{3} \)

Common ratio, \( r = -\frac{\sqrt{3}}{3} = \frac{1}{3} \left( \left| r \right| = \left| \frac{1}{3} \right| = \frac{1}{3} < 1 \right) \)

\[ \therefore \text{Required sum} = \frac{a}{1-r} = \frac{\sqrt{3}}{1-\left(-\frac{1}{3}\right)} = \frac{\sqrt{3}}{\frac{2}{3}} = \frac{3\sqrt{3}}{2} \]

Question 3.
The second term of a G.P. is 9 and sum of its infinite terms is 48. Find its first three terms.

Solution:

Let \( a \) be the first term and \( r \) be the common ratio of a G.P.

\( 2^{nd} \) term, \( t_2 = ar = 9 \)

\( \Rightarrow r = \frac{9}{a} \)

Sum of its infinite terms, \( S = 48 \)

\( \Rightarrow \frac{a}{1-r} = 48 \)
Question 4.
Find three geometric means between \(\frac{1}{3}\) and 432.

Solution:

\[
\Rightarrow \frac{a}{\frac{9}{a}} = 48 \\
\Rightarrow \frac{a^2}{a - 9} = 48 \\
\Rightarrow a^2 = 48a - 432 \\
\Rightarrow a^2 - 48a + 432 = 0 \\
\Rightarrow a^2 - 36a - 12a + 432 = 0 \\
\Rightarrow a(a - 36) - 12(a - 36) = 0 \\
\Rightarrow (a - 36)(a - 12) = 0 \\
\Rightarrow a = 36 \text{ or } a = 12 \\
\]

When \(a = 36\), \(r = \frac{9}{36} = \frac{1}{4}\)

\(\Rightarrow 1^{st} \text{ term } = 36,\)

\(2^{nd} \text{ term } = ar = 36 \times \frac{1}{4} = 9\)

\(3^{rd} \text{ term } = ar^2 = 36 \times \frac{1}{16} = \frac{9}{4}\)

When \(a = 12\), \(r = \frac{9}{12} = \frac{3}{4}\)

\(\Rightarrow 1^{st} \text{ term } = 12,\)

\(2^{nd} \text{ term } = ar = 12 \times \frac{3}{4} = 9\)

\(3^{rd} \text{ term } = ar^2 = 12 \times \frac{9}{16} = \frac{27}{4}\)
Let $G_1, G_2, G_3$ be three geometric means between $a = \frac{1}{3}$ and $b = 432$.

Then, $\frac{1}{3}, G_1, G_2, G_3, 432$ is a G.P.

Thus, we have

First term $= a = \frac{1}{3}$

$5^{th}$ term of the G.P. $= ar^4 = 432$

$\Rightarrow \frac{1}{3} x r^4 = 432$

$\Rightarrow r^4 = 1296$

$\Rightarrow r^4 = 6^4$

$\Rightarrow r = 6$

$\therefore G_1 = ar = \frac{1}{3} x 6 = 2$

$G_2 = ar^2 = \frac{1}{3} x 6 x 6 = 12$

$G_3 = ar^3 = \frac{1}{3} x 6 x 6 x 6 = 72$

Question 5.

Find:

(i) two geometric means between 2 and 16
(ii) four geometric means between 3 and 96.
(iii) five geometric means between $\frac{35}{9}$ and $\frac{401}{2}$

Solution 5(i).

Let $G_1, G_2$ be two geometric means between $a = 2$ and $b = 16$.

Then, $2, G_1, G_2, 16$ is a G.P.

Thus, we have

First term $= a = 2$

$4^{th}$ term of the G.P. $= ar^3 = 16$

$\Rightarrow 2 x r^3 = 16$

$\Rightarrow r^3 = 8$

$\Rightarrow r^3 = 2^3$

$\Rightarrow r = 2$

$\therefore G_1 = ar = 2 x 2 = 4$

$G_2 = ar^2 = 2 x 2 x 2 = 8$
Solution 5(ii).

Let $G_1, G_2, G_3, G_4$ be four geometric means between $a = 3$ and $b = 96$.
Then, $3, G_1, G_2, G_3, G_4, 96$ is a G.P.
Thus, we have
First term $= a = 3$
$6^{th}$ term of the G.P. $= ar^5 = 96$
$\Rightarrow 3 \times r^5 = 96$
$\Rightarrow r^5 = 32$
$\Rightarrow r^3 = 2^3$
$\Rightarrow r = 2$
$\therefore G_1 = ar = 3 \times 2 = 6$
$G_2 = ar^2 = 3 \times 4 = 12$
$G_3 = ar^3 = 3 \times 8 = 24$
$G_4 = ar^4 = 3 \times 16 = 48$

Solution 5(iii).

Let $G_1, G_2, G_3, G_4, G_5$ be five geometric means between $a = \frac{3\frac{5}{9}}{9} = \frac{32}{9}$ and $b = 40 \frac{1}{2} = \frac{81}{2}$.
Then, $\frac{32}{9}, G_1, G_2, G_3, G_4, G_5, \frac{81}{2}$ is a G.P.
Thus, we have
First term $= a = \frac{32}{9}$
$7^{th}$ term of the G.P. $= ar^6 = \frac{81}{2}$
$\Rightarrow \frac{32}{9} \times r^6 = \frac{81}{2}$
$\Rightarrow r^6 = \frac{81}{2} \times \frac{9}{32}$
$\Rightarrow r^6 = \frac{729}{64}$
$\Rightarrow r^6 = \left(\frac{3}{2}\right)^6$
$\Rightarrow r = \frac{3}{2}$
Question 6.
The sum of three numbers in G.P. is \(\frac{39}{10}\) and their product is 1. Find the numbers.

Solution:
Sum of three numbers in G.P. = \(\frac{39}{10}\) and their product = 1

Let number be \(\frac{a}{r}\), \(a\), \(ar\), then

\[
\frac{a}{r} \times a \times ar = 1 \Rightarrow a^3 = 1 = (1)^3
\]

\[
\therefore \; \; a = 1
\]

and \(\frac{a}{r} + a + ar = \frac{39}{10}\)

\[
\Rightarrow a \left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}
\]

\[
\frac{1}{r} + 1 + r = \frac{39}{10} \times 1 = \frac{39}{10}
\]

\[
r + \frac{1}{r} = \frac{39}{10} - 1 = \frac{39 - 10}{10} = \frac{29}{10}
\]

\[
r^2 + 1 = \frac{29}{10} r
\]
Question 7.
Find the numbers in G.P. whose sum is 52 and the sum of whose product in pairs is 624.

Solution:

Let the numbers be $a$, $ar$ and $ar^2$.

$\Rightarrow a + ar + ar^2 = 52 \quad \text{(i)}$

And, $(a \times ar) + (ar \times ar^2) + (ar^2 \times a) = 624$

$\Rightarrow a^2 + a^2r^3 + a^2r^2 = 624$

$\Rightarrow ar(a + ar^2 + ar) = 624$

$\Rightarrow ar \times 52 = 624 \quad \Rightarrow [\text{From (i)}]$

$\Rightarrow ar = 12$

$\Rightarrow a = \frac{12}{r}$

Substituting in (i), we get

$\frac{12}{r} + \frac{12}{r} \times r + \frac{12}{r} \times r^2 = 52$

$\Rightarrow \frac{12}{r} + 12 + 12r = 52$

$\Rightarrow \frac{12 + 12r + 12r^2}{r} = 52$

$\Rightarrow 12 + 12r + 12r^2 = 52r$

$\Rightarrow 12r^2 - 40r + 12 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0$
Question 8.
The sum of three numbers in G.P. is 21 and the sum of their squares is 189. Find the numbers.

Solution:

Let the numbers be $a$, $ar$, and $ar^2$.

$\Rightarrow (a)^2 + (ar)^2 + (ar^2)^2 = 189$

$\Rightarrow a^2 + a^2r^2 + a^2r^4 = 189$

And, $a + ar + ar^2 = 21$

$\Rightarrow (a + ar + ar^2)^2 = 21^2$

$\Rightarrow a^2 + a^2r^2 + a^2r^4 + 2a^2r + 2ar^3 + 2a^2r^2 - 441$

$\Rightarrow 189 + 2ar(a + ar^2 + ar) = 441$

$\Rightarrow 2ar \times 21 = 441 - 189$

$\Rightarrow 42ar = 252$

$\Rightarrow ar = 6$

$\Rightarrow r = \frac{6}{a}$

Now, $a + ar + ar^2 = 21$

$\Rightarrow a + a \times \frac{6}{a} + a \times \frac{36}{a^2} = 21$

$\Rightarrow a + 6 \times \frac{36}{a} = 21$

$\Rightarrow a^2 + 6a + 36 = 21a$

$\Rightarrow a^2 - 15a + 36 = 0$

$\Rightarrow a^2 - 12a - 3a + 36 = 0$

$\Rightarrow a^2 - 9a + 4a + 36 = 0$

$\Rightarrow a(a - 9) + 4(a - 9) = 0$

$\Rightarrow (a + 4)(a - 9) = 0$

$\Rightarrow a = -4$ or $a = 9$

For $a = -4$, $ar = 6$, $r = \frac{6}{-4} = -\frac{3}{2}$

For $a = 9$, $ar = 6$, $r = \frac{6}{9} = \frac{2}{3}$

Thus, required terms are:

$a, ar, ar^2 = 36, 36 \times \frac{1}{3}, 36 \times \frac{1}{9}$ OR $4, 4 \times 3, 4 \times 9$

$= 36, 12, 4$ OR $4, 12, 36$
\[ \Rightarrow a(a-12) - 3(a-12) = 0 \]
\[ \Rightarrow (a-12)(a-3) = 0 \]
\[ \Rightarrow a = 12 \text{ or } a = 3 \]
\[ \Rightarrow r = \frac{6}{12} = \frac{1}{2} \text{ or } r = \frac{6}{3} = 2 \]

Thus, required terms are:

\[ a, ar, ar^2 = 12, 12 \times \frac{1}{2}, 12 \times \frac{1}{4} \text{ OR } 3, 3 \times 2, 3 \times 4 \]

\[ = 12, 6, 3 \text{ OR } 3, 6, 12 \]