Linear Inequations (in one variable)

Exercise 4A

Question 1.
State, true or false:
(i) $x < -y \Rightarrow -x > y$
(ii) $-5x \geq 15 \Rightarrow x \geq -3$
(iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$
(iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

Solution:
(i) $x < -y \Rightarrow -x > y$
   The given statement is true.
(ii) $-5x \geq 15 \Rightarrow \frac{-5x}{5} \geq \frac{15}{5} \Rightarrow x \leq -3$
   The given statement is false.
(iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$
   The given statement is true.
(iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$
   The given statement is true.

Question 2.
State, whether the following statements are true or false:
(i) $a < b$, then $a - c < b - c$
(ii) If $a > b$, then $a + c > b + c$
(iii) If $a < b$, then $ac > bc$
(iv) If $a > b$, then $\frac{a}{c} < \frac{b}{c}$
(v) If $a - c > b - d$, then $a + d > b + c$
(vi) If $a < b$, and $c > 0$, then $a - c > b - c$
Where $a, b, c$ and $d$ are real numbers and $c \neq 0$. 
Solution:
(i) $a < b \Rightarrow a - c < b - c$ The given statement is true.
(ii) If $a > b \Rightarrow a + c > b + c$
The given statement is true.
(iii) If $a < b \Rightarrow ac < bc$ The given statement is false.
(iv) If $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$
The given statement is false.
(v) If $a - c > b - d \Rightarrow a + d > b + c$
The given statement is true.
(vi) If $a < b \Rightarrow a - c < b - c$ (Since, $c > 0$)
The given statement is false.

Question 3.
If $x \in \mathbb{N}$, find the solution set of inequalities.
(i) $5x + 3 \leq 2x + 18$
(ii) $3x - 2 < 19 - 4x$

Solution:
(i) $5x + 3 \leq 2x + 18$
$5x - 2x \leq 18 - 3$
$3x \leq 15$
$x \leq 5$

Since, $x \in \mathbb{N}$, therefore solution set is $\{1, 2, 3, 4, 5\}$.
(ii) $3x - 2 < 19 - 4x$
$3x + 4x < 19 + 2$
$7x < 21$
$x < 3$
Since, $x \in \mathbb{N}$, therefore solution set is $\{1, 2\}$.

Question 4.
If the replacement set is the set of whole numbers, solve:
(i) $x + 7 \leq 11$
(ii) $3x - 1 > 8$
(iii) $8 - x > 5$
(iv) $7 - 3x \geq -\frac{1}{2}$
(v) $x - \frac{3}{2} < \frac{3}{2} - x$
(vi) $18 \leq 3x - 2$
Solution:

(i) \( x + 7 \leq 11 \)
\( x \leq 11 - 7 \)
\( x \leq 4 \)

Since, the replacement set = \( W \) (set of whole numbers)
\( \Rightarrow \) Solution set = \( \{0, 1, 2, 3, 4\} \)

(ii) \( 3x - 1 > 8 \)
\( 3x > 8 + 1 \)
\( x > 3 \)
Since, the replacement set = \( W \) (set of whole numbers)
\( \Rightarrow \) Solution set = \( \{4, 5, 6, \ldots\} \)

(iii) \( 8 - x > 5 \)
\( -x > 5 - 8 \)
\( -x > -3 \)
\( x < 3 \)
Since, the replacement set = \( W \) (set of whole numbers)
\( \Rightarrow \) Solution set = \( \{0, 1, 2\} \)

(iv) \( 7 - 3x \geq -\frac{1}{2} \)
\( -3x \geq -\frac{1}{2} - 7 \)
\( -3x \geq -\frac{15}{2} \)
\( x \leq \frac{5}{2} \)
Since, the replacement set = \( W \) (set of whole numbers)
\( \therefore \) Solution set = \( \{0, 1, 2\} \)

(v) \( x - \frac{3}{2} < \frac{3}{2} - x \)
\( x + x < \frac{3}{2} + \frac{3}{2} \)
\( 2x < 3 \)
\( x < \frac{3}{2} \)
Since, the replacement set = W (set of whole numbers)  
∴ Solution set = {0, 1}

(vi) \[ 18 \leq 3x - 2 \]
\[ 18 + 2 \leq 3x \]
\[ 20 \leq 3x \]
\[ \frac{20}{3} \leq x \]

Since, the replacement set = W (set of whole numbers)  
∴ Solution set = \{7, 8, 9, ...\}

**Question 5.**
Solve the inequation:
\[ 3 - 2x \geq x - 12 \] given that \( x \in \mathbb{N} \).

**Solution:**
\[ 3 - 2x \geq x - 12 \]
\[ -2x - x \geq -12 - 3 \]
\[ -3x \geq -15 \]
\[ x \leq 5 \]
Since, \( x \in \mathbb{N} \), therefore,
Solution set = \{1, 2, 3, 4, 5\}

**Question 6.**
If \( 25 - 4x \leq 16 \), find:
(i) the smallest value of \( x \), when \( x \) is a real number,
(ii) the smallest value of \( x \), when \( x \) is an integer.

**Solution:**
\[ 25 - 4x \leq 16 \]
\[ -4x \leq 16 - 25 \]
\[ -4x \leq -9 \]
\[ \frac{9}{4} \geq x \]
\[ x \geq 2.25 \]

(i) The smallest value of \( x \), when \( x \) is a real number, is 2.25.
(ii) The smallest value of \( x \), when \( x \) is an integer, is 3.
Question 7.
If the replacement set is the set of real numbers, solve:
(i) \(-4x \geq -16\)
(ii) \(8 - 3x \leq 20\)
(iii) \(5 + \frac{x}{4} > \frac{x}{5} + 9\)
(iv) \(\frac{x + 3}{8} < \frac{x - 3}{5}\)

Solution:

(i) \(-4x \geq -16\)
\(x \leq 4\)
Since, the replacement set of real numbers.
\(\therefore\) Solution set = \(\{x : x \in \mathbb{R} \text{ and } x \leq 4\}\)

(ii) \(8 - 3x \leq 20\)
\(-3x \leq 20 - 8\)
\(-3x \leq 12\)
\(x \geq -4\)
Since, the replacement set of real numbers.
\(\therefore\) Solution set = \(\{x : x \in \mathbb{R} \text{ and } x \geq -4\}\)

(iii) \(5 + \frac{x}{4} > \frac{x}{5} + 9\)
\(\frac{x}{4} - \frac{x}{5} > 9 - 5\)
\(\frac{x}{20} > 4\)
\(x > 80\)
Since, the replacement set of real numbers.
\(\therefore\) Solution set = \(\{x : x \in \mathbb{R} \text{ and } x > 80\}\)

(iv) \(\frac{x + 3}{8} < \frac{x - 3}{5}\)
\(5x + 15 < 8x - 24\)
\(5x - 8x < -24 - 15\)
\(-3x < -39\)
\(x > 13\)
Since, the replacement set of real numbers.
\(\therefore\) Solution set = \(\{x : x \in \mathbb{R} \text{ and } x > 13\}\)
Question 8.
Find the smallest value of $x$ for which $5 - 2x < 5 \frac{1}{2} - 5 \frac{5}{3}x$, where $x$ is an integer.

Solution:

\[
5 - 2x < 5 \frac{1}{2} - 5 \frac{5}{3}x \\
-2x + \frac{5}{3}x < 5 \frac{1}{2} - 5 \\
\frac{-x}{3} < \frac{1}{2} \\
-x < 3 \\
x > -\frac{3}{2} \\
x > -1.5
\]

Thus, the required smallest value of $x$ is -1.

Question 9.
Find the largest value of $x$ for which $2(x - 1) \leq 9 - x$ and $x \in W$.

Solution:

\[
2(x - 1) \leq 9 - x \\
2x - 2 \leq 9 - x \\
2x + x \leq 9 + 2 \\
3x \leq 11 \\
x \leq \frac{11}{3} \\
x \leq 3.67
\]

Since, $x \in W$, thus the required largest value of $x$ is 3.

Question 10.
Solve the inequation: $12 + \frac{5}{6}x \leq 5 + 3x$ and $x \in R$. 

Question 11.
Given \( x \in \{\text{integers}\} \), find the solution set of:
\[-5 \leq 2x - 3 < x + 2\]

Solution:
\[
\begin{align*}
-5 & \leq 2x - 3 < x + 2 \\
\Rightarrow -5 & \leq 2x - 3 \\
\Rightarrow -5 + 3 & \leq 2x \\
\Rightarrow -2 & \leq 2x \\
\Rightarrow x & \geq -1
\end{align*}
\]
Since, \( x \in \{\text{integers}\} \)
\[
\therefore \text{Solution set} = \{-1, 0, 1, 2, 3, 4\}
\]

Question 12.
Given \( x \in \{\text{whole numbers}\} \), find the solution set of:
\[-1 \leq 3 + 4x < 23\]

Solution:
\[
\begin{align*}
-1 & \leq 3 + 4x < 23 \\
\Rightarrow -1 & \leq 3 + 4x \\
\Rightarrow -4 & \leq 4x \\
\Rightarrow x & \geq -1
\end{align*}
\]
Since, \( x \in \{\text{whole numbers}\} \)
\[
\therefore \text{Solution set} = \{0, 1, 2, 3, 4\}
\]
Exercise 4B

Question 1.

Represent the following inequalities on real number lines:
(i) \(2x - 1 < 5\)
(ii) \(3x + 1 \geq -5\)
(iii) \(2(2x - 3) \leq 6\)
(iv) \(-4 < x < 4\)
(v) \(-2 \leq x < 5\)
(vi) \(8 \geq x > -3\)
(vii) \(-5 < x \leq -1\)

Solution:

(i) \(2x - 1 < 5\)
\(2x < 6\)
\(x < 3\)
Solution on number line is:

(ii) \(3x + 1 \geq -5\)
\(3x \geq -6\)
\(x \geq -2\)
Solution on number line is:

(iii) \(2(2x - 3) \leq 6\)
\(2x - 3 \leq 3\)
\(2x \leq 6\)
\(x \leq 3\)
Solution on number line is:
(iv) \(-4 < x < 4\)
Solution on number line is:

(v) \(-2 \leq x < 5\)
Solution on number line is:

(vi) \(8 \geq x > -3\)
Solution on number line is:

(vii) \(-5 < x \leq -1\)
Solution on number line is:

Question 2.

For each graph given, write an inequation taking x as the variable:

(i)

(ii)
Solution:

(i) $x \leq -1, x \in \mathbb{R}$
(ii) $x \geq 2, x \in \mathbb{R}$
(iii) $-4 \leq x < 3, x \in \mathbb{R}$
(iv) $-1 < x \leq 5, x \in \mathbb{R}$

Question 3.

For the following inequations, graph the solution set on the real number line:
(i) $-4 \leq 3x - 1 < 8$
(ii) $x - 1 < 3 - x \leq 5$

Solution:

(i) $-4 \leq 3x - 1 < 8$
$-4 \leq 3x - 1$ and $3x - 1 < 8$
$-1 \leq x$ and $x < 3$

The solution set on the real number line is: 

(ii) $x - 1 < 3 - x \leq 5$
$x - 1 < 3 - x$ and $3 - x \leq 5$
$2x < 4$ and $-x \leq 2$
$x < 2$ and $x \geq -2$

The solution set on the real number line is:
Question 4.

Represent the solution of each of the following inequalities on the real number line:

(i) $4x - 1 > x + 11$

(ii) $7 - x \leq 2 - 6x$

(iii) $x + 3 \leq 2x + 9$

(iv) $2 - 3x > 7 - 5x$

(v) $1 + x \geq 5x - 11$

(vi) \( \frac{2x + 5}{3} > 3x - 3 \)

Solution:

(i) $4x - 1 > x + 11$

$3x > 12$

$x > 4$

The solution on the number line is:

(ii) $7 - x \leq 2 - 6x$

$5x \leq -5$

$x \leq -1$

The solution on the number line is:

(iii) $x + 3 \leq 2x + 9$

$-6 \leq x$

The solution on the number line is:
Question 5.

$x \in \{\text{real numbers}\}$ and $-1 < 3 - 2x \leq 7$, evaluate $x$ and represent it on a number line.

Solution:

- $-1 < 3 - 2x \leq 7$
- $-1 < 3 - 2x$ and $3 - 2x \leq 7$
- $2x < 4$ and $-2x \leq 4$
Question 6.
List the elements of the solution set of the inequation
\[-3 < x - 2 \leq 9 - 2x; x \in \mathbb{N}.\]

Solution:
\[-3 < x - 2 \leq 9 - 2x\]
\[-3 < x - 2 \text{ and } x - 2 \leq 9 - 2x\]
\[-1 < x \text{ and } 3x \leq 11\]
\[-1 < x \leq \frac{11}{3}\]
Since, \(x \in \mathbb{N}\)
\[\therefore \text{ Solution set } = \{1, 2, 3\}\]

Question 7.
Find the range of values of \(x\) which satisfies
\[-2 \frac{2}{3} \leq x + \frac{1}{3} < 3 \frac{1}{3}; x \in \mathbb{R}.\]
Graph these values of \(x\) on the number line.

Solution:
\[-2 \frac{2}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < 3 \frac{1}{3}\]
\[\Rightarrow -\frac{8}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < \frac{10}{3}\]
\[\Rightarrow -\frac{8}{3} - \frac{1}{3} \leq x \text{ and } x < \frac{10}{3} - \frac{1}{3}\]
\[\Rightarrow -\frac{9}{3} \leq x \text{ and } x < \frac{9}{3}\]
Question 8.

Find the values of x, which satisfy the inequation:
\[-2 \leq \frac{1}{2} - \frac{2x}{3} < \frac{5}{6}; \quad x \in \mathbb{N}.
\]
Graph the solution on the number line.

Solution:
\[-2 \leq \frac{1}{2} - \frac{2x}{3} < \frac{5}{6}
\]
\[-2 \leq \frac{1}{2} - \frac{2x}{3} \quad \text{and} \quad \frac{1}{2} - \frac{2x}{3} < \frac{5}{6}
\]
\[-\frac{5}{2} \leq -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} < \frac{5}{6}
\]
\[-\frac{15}{4} \geq x \quad \text{and} \quad x > -2
\]
\[3.75 \geq x \quad \text{and} \quad x > -2
\]
Since, \( x \in \mathbb{N} \)
\[\therefore \text{Solution set} = \{1, 2, 3\}
\]
The required graph of the solution set is:

Question 9.
Given \( x \in \{\text{real numbers}\} \), find the range of values of \( x \) for which \(-5 \leq 2x - 3 < x + 2\) and represent it on a number line.

Solution:
\[-5 \leq 2x - 3 < x + 2
\]
\[-5 \leq 2x - 3 \quad \text{and} \quad 2x - 3 < x + 2
\]
\[-2 \leq 2x \quad \text{and} \quad x < 5
\]
\[-1 \leq x \quad \text{and} \quad x < 5
\]
Required range is \(-1 \leq x < 5\).
The required graph is:

![Graph Image]

**Question 10.**
If $5x - 3 \leq 5 + 3x \leq 4x + 2$, express it as $a \leq x \leq b$ and then state the values of $a$ and $b$.

**Solution:**
\[
5x - 3 \leq 5 + 3x \leq 4x + 2
\]
\[
5x - 3 \leq 5 + 3x \quad \text{and} \quad 5 + 3x \leq 4x + 2
\]
\[
2x \leq 8 \quad \text{and} \quad -x \leq -3
\]
\[
x \leq 4 \quad \text{and} \quad x \geq 3
\]
Thus, $3 \leq x \leq 4$.
Hence, $a = 3$ and $b = 4$.

**Question 11.**
Solve the following inequation and graph the solution set on the number line:
$2x - 3 < x + 2 \leq 3x + 5$, $x \in \mathbb{R}$.

**Solution:**
\[
2x - 3 < x + 2 \leq 3x + 5
\]
\[
2x - 3 < x + 2 \quad \text{and} \quad x + 2 \leq 3x + 5
\]
\[
x < 5 \quad \text{and} \quad -3 \leq 2x
\]
\[
x < 5 \quad \text{and} \quad -1.5 \leq x
\]
Solution set = \{-1.5 \leq x < 5\}
The solution set can be graphed on the number line as:

![Graph Image]

**Question 12.**
Solve and graph the solution set of:
(i) $2x - 9 < 7$ and $3x + 9 \leq 25$, $x \in \mathbb{R}$
(ii) $2x - 9 \leq 7$ and $3x + 9 > 25$, $x \in \mathbb{I}$
(iii) $x + 5 \geq 4(x - 1)$ and $3 - 2x < -7$, $x \in \mathbb{R}$
Solution:

(i) \(3x - 9 < 7\) and \(3x + 9 \leq 25\)
\(2x < 16\) and \(3x \leq 16\)
\(x < 8\) and \(x \leq 5\frac{1}{3}\)

\(\therefore\) Solution set = \(\{x \leq 5\frac{1}{3}, x \in \mathbb{R}\}\)

The required graph on number line is:

(ii) \(2x - 9 \leq 7\) and \(3x + 9 > 25\)
\(2x \leq 16\) and \(3x > 16\)
\(x \leq 8\) and \(x > 5\frac{1}{3}\)

\(\therefore\) Solution set = \(\{5\frac{1}{3} \leq x \leq 8, x \in \mathbb{R}\} = \{6, 7, 8\}\)

The required graph on number line is:

(iii) \(x + 5 \geq 4(x - 1)\) and \(3 - 2x \leq -7\)
\(9 \geq 3x\) and \(-2x \leq -10\)
\(3 \geq x\) and \(x > 5\)

\(\therefore\) Solution set = Empty set

Question 13.
Solve and graph the solution set of:
(i) \(3x - 2 > 19\) or \(3 - 2x \geq -7\), \(x \in \mathbb{R}\)
(ii) \(5 > p - 1 > 2\) or \(7 \leq 2p - 1 \leq 17\), \(p \in \mathbb{R}\)

Solution:
(i) \(3x - 2 > 19\) or \(3 - 2x \geq -7\)
\(3x > 21\) or \(-2x \geq -10\)
\(x > 7\) or \(x \leq 5\)

Graph of solution set of \(x > 7\) or \(x \leq 5\) = Graph of points which belong to \(x > 7\) or \(x \leq 5\) or both.

Thus, the graph of the solution set is:
(ii) $5 > p - 1 > 2$ or $7 \leq 2p - 1 \leq 17$
$6 > p > 3$ or $8 \leq 2p \leq 18$
$6 > p > 3$ or $4 \leq p \leq 9$

Graph of solution set of $6 > p > 3$ or $4 \leq p \leq 9$
= Graph of points which belong to $6 > p > 3$ or $4 \leq p \leq 9$ or both
= Graph of points which belong to $3 < p \leq 9$

Thus, the graph of the solution set is:

**Question 14.**

The diagram represents two inequations $A$ and $B$ on real number lines:

(i) Write down $A$ and $B$ in set builder notation.
(ii) Represent $A \cup B$ and $A \cap B'$ on two different number lines.

**Solution:**

(i) $A = \{x \in \mathbb{R}: -2 \leq x < 5\}$
$B = \{x \in \mathbb{R}: -4 \leq x < 3\}$
(ii) $A \cap B = \{x \in \mathbb{R}: -2 \leq x < 5\}$

It can be represented on number line as:

$B' = \{x \in \mathbb{R}: 3 < x \leq -4\}$
$A \cap B' = \{x \in \mathbb{R}: 3 \leq x < 5\}$
It can be represented on number line as:

Question 15.
Use real number line to find the range of values of \(x\) for which:
(i) \(x > 3\) and \(0 < x < 6\)
(ii) \(x < 0\) and \(-3 \leq x < 1\)
(iii) \(-1 < x \leq 6\) and \(-2 \leq x \leq 3\)

Solution:
(i) \(x > 3\) and \(0 < x < 6\)

Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:

From both graphs, it is clear that their common range is \(3 < x < 6\)

(ii) \(x < 0\) and \(-3 \leq x < 1\)

Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:
From both graphs, it is clear that their common range is
\[-3 \leq x < 0\]

(iii) \(-1 < x \leq 6\) and \(-2 \leq x \leq 3\)
Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:

From both graphs, it is clear that their common range is
\[-1 < x \leq 3\]

**Question 16.**
Illustrate the set \(\{x: -3 \leq x < 0 \text{ or } x > 2, x \in \mathbb{R}\}\) on the real number line.

**Solution:**
Graph of solution set of \(-3 \leq x < 0 \text{ or } x > 2\)
= Graph of points which belong to \(-3 \leq x < 0 \text{ or } x > 2\) or both

Thus, the required graph is:

**Question 17.**
Given \(A = \{x: -1 < x \leq 5, x \in \mathbb{R}\}\) and \(B = \{x: -4 \leq x < 3, x \in \mathbb{R}\}\)

Represent on different number lines:
(i) \(A \cap B\)
(ii) \(A' \cap B\)
(iii) \(A - B\)
Solution:

(i) \( A \cap B = \{x: -1 < x < 3, x \in \mathbb{R}\} \)
It can be represented on a number line as:

(ii) Numbers which belong to B but do not belong to A
\( A' \cap B = \{x: -4 \leq x \leq -1, x \in \mathbb{R}\} \)
It can be represented on a number line as:

(iii) \( A - B = \{x: 3 \leq x \leq 5, x \in \mathbb{R}\} \)
It can be represented on a number line as:

**Question 18.**

P is the solution set of \( 7x - 2 > 4x + 1 \) and Q is the solution set of \( 9x - 45 \geq 5(x - 5) \); where \( x \in \mathbb{R} \). Represent:

(i) \( P \cap Q \)
(ii) \( P - Q \)
(iii) \( P \cap Q' \)
on different number lines.

**Solution:**

\[ P = \{x : 7x - 2 > 4x + 1, x \in \mathbb{R}\} \]
\[ 7x - 2 > 4x + 1 \]
\[ 7x - 4x > 1 + 2 \]
\[ 3x > 3 \]
\[ x > 1 \]
and
\[ Q = \{x : 9x - 45 \geq 5(x - 5), x \in \mathbb{R}\} \]
\[ 9x - 45 \geq 5x - 25 \]
\[ 9x - 5x \geq -25 + 45 \]
\[ 4x \geq 20 \]
\[ x \geq 5 \]
(i) \( P \cap Q = \{x : x \geq 5, x \in \mathbb{R}\} \)
Question 19.

Find the range of values of \( x \), which satisfy:
\[-\frac{1}{3} \leq \frac{x}{2} + \frac{2}{3} < \frac{5}{6} \]

Graph, in each of the following cases, the values of \( x \) on the different real number lines:
(i) \( x \in \mathbb{W} \) (ii) \( x \in \mathbb{Z} \) (iii) \( x \in \mathbb{R} \)

Solution:

\[-\frac{1}{3} \leq \frac{x}{2} + \frac{2}{3} < \frac{5}{6} \]
\[-\frac{1}{3} - \frac{5}{3} \leq \frac{x}{2} < \frac{31}{6} - \frac{5}{3} \]
\[-\frac{6}{3} \leq \frac{x}{2} < \frac{21}{6} \]
\[-4 \leq x < 7 \]

(i) If \( x \in \mathbb{W} \), range of values of \( x \) is \( \{0, 1, 2, 3, 4, 5, 6\} \).

(ii) If \( x \in \mathbb{Z} \), range of values of \( x \) is \( \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\} \).
Question 20.
Given: \(A = \{x: -8 < 5x + 2 \leq 17, x \in I\}, B = \{x: -2 \leq 7 + 3x < 17, x \in R\}\)
Where \(R = \{\text{real numbers}\}\) and \(I = \{\text{integers}\}\). Represent \(A\) and \(B\) on two different number lines. Write down the elements of \(A \cap B\).

Solution:

\(A = \{x: -8 < 5x + 2 \leq 17, x \in I\}\)  
\(= \{x: -10 < 5x \leq 15, x \in I\}\)  
\(= \{x: -2 < x \leq 3, x \in I\}\)

It can be represented on number line as:

\(B = \{x: -2 \leq 7 + 3x < 17, x \in R\}\)  
\(= \{x: -9 \leq 3x < 10, x \in R\}\)  
\(= \{x: -3 \leq x < 3.33, x \in R\}\)

It can be represented on number line as:

\(A \cap B = \{-1, 0, 1, 2, 3\}\)

Question 21.

Solve the following inequation and represent the solution set on the number line \(2x - 5 \leq 5x + 4 < 11, \text{ where } x \in I\)

Solution:
Given that \( x \in \mathbb{I} \), solve the inequation and graph the solution on the number line:

\[ 3 \geq \frac{x - 4}{2} + \frac{x}{3} \geq 2 \]

**Solution:**

\[
3 \geq \frac{x - 4}{2} + \frac{x}{3} \geq 2 \\
3 \geq \frac{3x - 12 + 2x}{6} \geq 2 \\
18 \geq 5x - 12 \geq 12 \\
30 \geq 5x \geq 24 \\
6 \geq x \geq 4.8 \\
\text{Solution set} = \{5, 6\} \\
\text{It can be graphed on number line as:}
\]

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**Question 23.**

**Given:**

\[ A = \{x: 11x - 5 > 7x + 3, x \in \mathbb{R}\} \text{ and} \]

\[ B = \{x: 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}. \]

Find the range of set \( A \cap B \) and represent it on number line.

**Solution:**

\[ A = \{x: 11x - 5 > 7x + 3, x \in \mathbb{R}\} \]

\[ = \{x: 4x > 8, x \in \mathbb{R}\} \]
\[
= \{ x: x > 2, x \in \mathbb{R} \}
\]
\[
B = \{ x: 18x - 9 \geq 15 + 12x, x \in \mathbb{R} \}
\]
\[
= \{ x: 6x \geq 24, x \in \mathbb{R} \}
\]
\[
= \{ x: x \geq 4, x \in \mathbb{R} \}
\]
\[
A \cap B = \{ x: x \geq 4, x \in \mathbb{R} \}
\]
It can be represented on number line as:

**Question 24.**

Find the set of values of \( x \), satisfying:

\[
7x + 3 \geq 3x - 5 \quad \text{and} \quad \frac{x}{4} - 5 \leq \frac{5}{4} - x,
\]
where \( x \in \mathbb{N} \).

**Solution:**

\[
\begin{align*}
7x + 3 & \geq 3x - 5 \\
4x & \geq -8 \\
x & \geq -2 \\
\frac{x}{4} - 5 & \leq \frac{5}{4} - x \\
\frac{x}{4} + x & \leq \frac{5}{4} + 5 \\
\frac{5x}{4} & \leq \frac{25}{4} \\
x & \leq 5
\end{align*}
\]

Since, \( x \in \mathbb{N} \)

\[\therefore \text{Solution set} = \{1, 2, 3, 4, 5\}\]

**Question 25.**

Solve:

(i) \( \frac{x}{2} + 5 \leq \frac{x}{3} + 6 \), where \( x \) is a positive odd integer.

(ii) \( \frac{2x + 3}{3} \geq \frac{3x - 1}{4} \), where \( x \) is a positive even integer.
Solution:

(i) \( \frac{x}{2} + 5 \leq \frac{x}{3} + 6 \)
\( \frac{x}{2} - \frac{x}{3} \leq 6 - 5 \)
\( \frac{x}{6} \leq 1 \)
\( x \leq 6 \)

Since, \( x \) is a positive odd integer
\( \therefore \) Solution set = \( \{1, 3, 5\} \)

(ii) \( \frac{2x + 3}{3} \geq \frac{3x - 1}{4} \)
\[8x + 12 \geq 9x - 3\]
\( -x \geq -15 \)
\( x \leq 15 \)

Since, \( x \) is a positive even integer
\( \therefore \) Solution set = \( \{2, 4, 6, 8, 10, 12, 14\} \)

Question 26.

Solve the inequation:
\( -2 \frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x \), \( x \in W \). Graph the solution set on the number line.

Solution:

\( -2 \frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x \)
\( -2 \frac{1}{2} \leq \frac{4x}{5} - 2x \leq \frac{4}{3} \)
\( -\frac{5}{2} \leq -\frac{6x}{5} \leq \frac{4}{3} \)
\( \frac{25}{2} \geq x \geq -\frac{10}{9} \)
Question 27.
Find three consecutive largest positive integers such that the sum of one-third of first, one-fourth of second and one-fifth of third is atmost 20.

Solution:
Let the required integers be \(x, x + 1\) and \(x + 2\).
According to the given statement,

\[
\frac{1}{3}x + \frac{1}{4}(x + 1) + \frac{1}{5}(x + 2) \leq 20
\]

\[
20x + 15x + 15 + 12x + 24 \leq 20 \times 60
\]

\[
47x + 39 \leq 1200
\]

\[
x \leq 24.702
\]

Thus, the largest value of the positive integer \(x\) is 24.
Hence, the required integers are 24, 25 and 26.

Question 28.
Solve the given inequation and graph the solution on the number line.
\[2y - 3 < y + 1 \leq 4y + 7, \ y \in \mathbb{R}\]

Solution:
\[
2y - 3 < y + 1 \leq 4y + 7, \ y \in \mathbb{R}
\]
\[
\Rightarrow 2y - 3 - y < y + 1 - y \leq 4y + 7 - y
\]
\[
\Rightarrow y - 3 < 1 \leq 3y + 7
\]
\[
\Rightarrow y - 3 < 1 \text{ and } 1 \leq 3y + 7
\]
\[
\Rightarrow y < 4 \text{ and } 3y \geq 6 \Rightarrow y \geq -2
\]
\[
\Rightarrow -2 \leq y < 4
\]

The graph of the given equation can be represented on a number line as:
Question 29.
Solve the inequation:
$3z - 5 \leq z + 3 < 5z - 9, z \in \mathbb{R}$.
Graph the solution set on the number line.

Solution:
$3z - 5 \leq z + 3$ and $z + 3 < 5z - 9$
$2z \leq 8$ and $12 < 4z$
$z \leq 4$ and $3 < z$
Since, $z \in \mathbb{R}$
∴ Solution set = $\{3 < z \leq 4, x \in \mathbb{R}\}$
It can be represented on a number line as:

![Number Line Representation](image1)

Question 30.
Solve the following inequation and represent the solution set on the number line.
$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in \mathbb{R}$

Solution:
$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$
Multiply by 6, we get
$-18 < -3 - 4x \leq 5$  
$\Rightarrow -15 < -4x \leq 8$
Dividing by $-4$, we get
$\Rightarrow \frac{-15}{-4} > x \geq \frac{8}{-4}$
$\Rightarrow -2 \leq x < \frac{15}{4}$
$\Rightarrow x \in \left[-2, \frac{15}{4}\right)$
The solution set can be represented on a number line as:

![Number Line Representation](image2)
Question 31.
Solve the following inequation and represent the solution set on the number line:

\[ 4x - 19 < 3x \cdot \frac{5}{5} - 2 \leq \frac{-2}{5} + x, x \in \mathbb{R} \]

Solution:

Consider the given inequation:

\[ 4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in \mathbb{R} \]

\[ \Rightarrow 4x - 19 + 2 < \frac{3x}{5} - 2 + 2 \leq \frac{-2}{5} + x + 2, x \in \mathbb{R} \]

\[ \Rightarrow 4x - 17 < \frac{3x}{5} \leq x + \frac{8}{5}, x \in \mathbb{R} \]

\[ \Rightarrow 4x - \frac{3x}{5} < 17 \quad \text{and} \quad \frac{-8}{5} \leq x - \frac{3x}{5}, x \in \mathbb{R} \]

\[ \Rightarrow \frac{20x - 3x}{5} < 17 \quad \text{and} \quad \frac{-8}{5} \leq \frac{5x - 3x}{5}, x \in \mathbb{R} \]

\[ \Rightarrow \frac{17x}{5} < 17 \quad \text{and} \quad \frac{-8}{5} \leq \frac{2x}{5}, x \in \mathbb{R} \]

\[ \Rightarrow \frac{x}{5} < 1 \quad \text{and} \quad -4 \leq x, x \in \mathbb{R} \]

\[ \Rightarrow x < 5 \quad \text{and} \quad -4 \leq x, x \in \mathbb{R} \]

\[ \Rightarrow -4 \leq x < 5; \text{ where } x \in \mathbb{R} \]

The solution set can be represented on a number line as follows:

\[ -4 \leq x < 5 \]

Question 32.

Solve the following in equation, write the solution set and represent it on the number line:

\[ -\frac{x}{3} \leq \frac{x}{2} - 1 \frac{1}{3} < \frac{1}{6}, x \in \mathbb{R} \]
Solution:

The given inequation is
\[- \frac{x}{3} \leq \frac{x}{2} - 1 \frac{1}{3} < \frac{1}{6}, x \in \mathbb{R}\]

\[ \Rightarrow - \frac{x}{3} \leq \frac{x}{2} - \frac{4}{3} < \frac{1}{6} \]

Now,
\[- \frac{x}{3} \leq \frac{x}{2} - \frac{4}{3} \quad \Rightarrow \quad \frac{x}{2} \leq \frac{1}{6} + \frac{4}{3} \]
\[\Rightarrow \quad \frac{2x + 3x}{6} \geq \frac{4}{3} \quad \Rightarrow \quad \frac{x}{2} < \frac{1 + 4 \times 2}{6} \]
\[\Rightarrow \quad \frac{5x}{6} \geq \frac{4}{3} \quad \Rightarrow \quad \frac{x}{2} < \frac{1 + 8}{6} \]
\[\Rightarrow \quad 5x \geq 8 \quad \Rightarrow \quad \frac{x}{2} < \frac{9}{6} \]
\[\Rightarrow \quad x \geq \frac{8}{5} \quad \Rightarrow \quad \frac{x}{2} < \frac{3}{2} \]
\[\Rightarrow \quad x \geq 1.6 \quad \Rightarrow \quad x < 3 \]

\[ \therefore \text{Solution set} = \{ x : 1.6 \leq x < 3 \} \]

It can be represented on a number line as follows:

```
1 2 3
```

Question 33.

Find the values of \(x\), which satisfy the inequation
\[-2 \frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in \mathbb{R}.\]

Graph the solution set on the number line.

Solution:
We need to find the values of $x$, such that $x$ satisfies the inequality \(-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2\), $x \in \mathbb{Z}$.

Consider the given inequality:

\[-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2\]

\[\Rightarrow - \frac{17}{6} < - \frac{3}{6} - \frac{4x}{6} \leq \frac{12}{6}\]

\[\Rightarrow \frac{17}{6} > \frac{4x - 3}{6} \geq -\frac{12}{6}\]

\[\Rightarrow 17 > 4x - 3 \geq 12\]

\[\Rightarrow -12 \leq 4x - 3 < 17\]

\[\Rightarrow -12 + 3 \leq 4x - 3 + 3 < 17 + 3\]

\[\Rightarrow -9 \leq 4x < 20\]

\[\Rightarrow -\frac{9}{4} \leq x < \frac{20}{4}\]

\[\Rightarrow -\frac{9}{4} \leq x < 5\]

Since $x \in \mathbb{Z}$, the values of $x$ are 0, 1, 2, 3, 4.

And the required line is

---

**Question 34.**

Solve the following in equation and write the solution set:

$13x - 5 < 15x + 4 < 7x + 12$, $x \in \mathbb{R}$

**Solution:**

$13x - 5 < 15x + 4 < 7x + 12$, $x \in \mathbb{R}$

We have

\[13x - 5 < 15x + 4 \quad \text{and} \quad 15x + 4 < 7x + 12\]

\[\Rightarrow 13x < 15x + 9 \quad \Rightarrow 15x < 7x + 8\]

\[\Rightarrow 0 < 2x + 9 \quad \Rightarrow 8x < 8\]

\[\Rightarrow -9 < 2x \quad \Rightarrow x < 1\]

\[\Rightarrow \frac{-9}{2} < x \quad \Rightarrow x < 1\]
Question 35.
Solve the following inequation, write the solution set and represent it on the number line.
\[-3(x - 7) \geq 15 - 7x > x + \frac{1}{3}, x \in \mathbb{R}.\]

Solution:

\[-3(x - 7) \geq 15 - 7x \quad \text{and} \quad 15 - 7x > \frac{x + 1}{3}, x \in \mathbb{R}\]

\[\Rightarrow -3(x - 7) \geq 15 - 7x \quad \text{and} \quad 15 - 7x > \frac{x + 1}{3}\]

\[\Rightarrow -3x + 21 \geq 15 - 7x \quad \text{and} \quad 45 - 21x > x + 1\]

\[\Rightarrow -3x - 7x \geq 15 - 21 \quad \text{and} \quad 45 - 1 > x + 21x\]

\[\Rightarrow 4x \geq -6 \quad \text{and} \quad 44 > 22x\]

\[\Rightarrow x \geq -\frac{3}{2} \quad \text{and} \quad 2 > x\]

\[\Rightarrow x \geq -1.5 \quad \text{and} \quad 2 > x\]

\[\therefore \text{The solution set is } \{x : x \in \mathbb{R}, -1.5 \leq x < 2\}.\]

The solution set is represented on number line as follows:

Question 36.
Solve the following inequation and represent the solution set on a number line.
\[-8 \frac{1}{2} < -\frac{1}{2} - 4x \leq 7 \frac{1}{2}, \ x \in \mathbb{I}\]

**Solution:**

\[-8 \frac{1}{2} < -\frac{1}{2} - 4x \leq 7 \frac{1}{2}, \ x \in \mathbb{I}\]

\[-8 \frac{1}{2} < -\frac{1}{2} - 4x\]
\[\Rightarrow -\frac{15}{2} < -\frac{1}{2} - 4x\]
\[\Rightarrow -\frac{15}{2} + \frac{1}{2} < -4x\]
\[\Rightarrow -\frac{14}{2} < -4x\]
\[\Rightarrow -7 < -4x\]
\[\Rightarrow 7 > 4x\]
\[\Rightarrow x < \frac{7}{4}\]

\[-\frac{1}{2} - 4x \leq 7 \frac{1}{2}\]
\[\Rightarrow -\frac{1}{2} - 4x \leq \frac{15}{2}\]
\[\Rightarrow -4x \leq \frac{15}{2} + \frac{1}{2}\]
\[\Rightarrow -4x \leq 8\]
\[\Rightarrow x \geq -2\]

So,
\[\frac{7}{4} > x \geq -2\]

As, \[x \in \mathbb{I}\]

\[x = \{-2, -1, 0, 1\}\]