Chapter 24. Solution of Right Triangles [Simple 2-D Problems Involving One Right-angled Triangle]

Exercise 24

Solution 1:

(i)

From the figure we have

\[
\sin 60^\circ = \frac{20}{x}
\]

\[
\frac{\sqrt{3}}{2} = \frac{20}{x}
\]

\[
x = \frac{40}{\sqrt{3}}
\]

(ii)

From the figure we have

\[
\tan 30^\circ = \frac{20}{x}
\]

\[
\frac{1}{\sqrt{3}} = \frac{20}{x}
\]

\[
x = 20\sqrt{3}
\]

(iii)

From the figure we have

\[
\sin 45^\circ = \frac{20}{x}
\]

\[
\frac{1}{\sqrt{2}} = \frac{20}{x}
\]

\[
x = 20\sqrt{2}
\]
Solution 2:

(i)

From the figure we have

\[ \cos A = \frac{10}{20} \]
\[ \cos A = \frac{1}{2} \]
\[ \cos A = \cos 60^\circ \]
\[ A = 60^\circ \]

(ii)

From the figure we have

\[ \sin A = \frac{\sqrt{2}}{10} \]
\[ \sin A = \frac{1}{\sqrt{2}} \]
\[ \sin A = \sin 45^\circ \]
\[ A = 45^\circ \]

(iii)

From the figure we have

\[ \tan A = \frac{10\sqrt{3}}{10} \]
\[ \tan A = \sqrt{3} \]
\[ \tan A = \sin 60^\circ \]
\[ A = 60^\circ \]
Solution 3:
The figure is drawn as follows:

\[ \tan 60^\circ = \frac{30}{AD} \]
\[ \sqrt{3} = \frac{30}{AD} \]
\[ AD = \frac{30}{\sqrt{3}} \]

Again

\[ \sin x = \frac{AD}{20} \]
\[ AD = 20 \sin x \]

Now

\[ 20 \sin x = \frac{30}{\sqrt{3}} \]
\[ \sin x = \frac{30}{20\sqrt{3}} \]
\[ \sin x = \frac{\sqrt{3}}{2} \]
\[ \sin x = \sin 60^\circ \]
\[ x = 60^\circ \]
Solution 4:

(i)

From the right triangle ABE

\[ \tan 45^\circ = \frac{AE}{BE} \]

\[ 1 = \frac{AE}{BE} \]

\[ AE = BE \]

Therefore \( AE = BE = 50 \text{ m} \).

Now from the rectangle BCDE we have

\( DE = BC = 10 \text{ m} \).

Therefore the length of AD will be:

\( AD = AE + DE = 50 + 10 = 60 \text{ m} \).

(ii)

From the triangle ABD we have

\[ \sin B = \frac{AD}{AB} \]

\[ \sin 30^\circ = \frac{AD}{100} \quad \text{[Since } \angle ACD \text{ is the exterior angle of the triangle } ABC \] \]

\[ 1 = \frac{AD}{100} \]

\[ AD = 50 \text{ m} \]

Solution 5:

From right triangle ABC,

\[ \tan 60^\circ = \frac{AC}{BC} \]

\[ \sqrt{3} = \frac{AC}{40} \]

\[ AC = 40\sqrt{3} \text{ cm} \]

From right triangle BDC,

\[ \tan 45^\circ = \frac{DC}{BC} \]

\[ 1 = \frac{DC}{40} \]

\[ DC = 40 \text{ cm} \]

From the figure, it is clear that \( AD = AC - DC \)

\[ AD = 40\sqrt{3} - 40 \]

\[ AD = 40(\sqrt{3} - 1) \]

\[ AD = 29.28 \text{ cm} \]
Solution 6:
We know, diagonals of a rhombus bisect each other at right angles and also bisect the angle of vertex.

The figure is shown below:

![Rhombus Diagram]

Now

\[ OA = OC = \frac{1}{2} AC, \quad OB = OD = \frac{1}{2} BD; \quad \angle AOB = 90^\circ \]

And

\[ \angle OAB = \frac{60^\circ}{2} = 30^\circ \]

Also given \( AB = 60 \text{cm} \)

In right triangle AOB

\[ \sin 30^\circ = \frac{OB}{AB} \]

\[ \frac{1}{2} = \frac{OB}{60} \]

\[ OB = 30 \text{cm} \]

Also

\[ \cos 30^\circ = \frac{OA}{AB} \]

\[ \frac{\sqrt{3}}{2} = \frac{OA}{60} \]

\[ OA = 51.96 \text{cm} \]

Therefore,

Length of diagonal \( AC = 2 \times OA = 2 \times 51.96 = 103.92 \text{cm} \).

Length of diagonal \( BD = 2 \times OB = 2 \times 30 = 60 \text{cm} \).
Solution 7:
Consider the figure

From right triangle ACF

\[ \tan 45^\circ = \frac{20}{AC} \]

\[ 1 = \frac{20}{AC} \]

\[ AC = 20 \text{ cm} \]

From triangle DEB

\[ \tan 60^\circ = \frac{30}{BD} \]

\[ \sqrt{3} = \frac{30}{BD} \]

\[ BD = \frac{30}{\sqrt{3}} = 17.32 \text{ cm} \]

Given \( FC = 20, ED = 30 \). So \( EP = 10 \text{ cm} \)

Therefore

\[ \tan 60^\circ = \frac{FP}{EP} \]

\[ \sqrt{3} = \frac{FP}{10} \]

\[ FP = 10\sqrt{3} = 17.32 \text{ cm} \]

Thus \( AB = AC + CD + BD = 54.64 \text{ cm} \).

Solution 8:
First draw two perpendiculars to AB from the point D and C respectively. Since AB|| CD therefore PMCD will be a rectangle.
Consider the figure,
(i)

From right triangle ADP we have

\[ \cos 60^\circ = \frac{AP}{AD} \]

\[ \frac{1}{2} = \frac{AP}{20} \]

\[ AP = 10 \]

Similarly from the right triangle BMC we have BM = 10 cm.

Now from the rectangle PMCD we have CD = PM = 20 cm.

Therefore

\[ AB = AP + PM + MB = 10 + 20 + 10 = 40 \text{ cm}. \]

(ii)

Again from the right triangle APD we have

\[ \sin 60^\circ = \frac{PD}{20} \]

\[ \frac{\sqrt{3}}{2} = \frac{PD}{20} \]

\[ PD = 10 \sqrt{3} \]

Therefore the distance between AB and CD is \( 10 \sqrt{3} \).
Solution 9:
From right triangle AQP
\[
\tan 30^\circ = \frac{AQ}{AP} = \frac{1}{\sqrt{3}} = \frac{10}{AP} \Rightarrow AP = 10\sqrt{3}
\]
Also from triangle PBR
\[
\tan 45^\circ = \frac{PB}{BR} = \frac{1}{8} \Rightarrow PB = 8
\]
Therefore,
\[
AB = AP + PB = 10\sqrt{3} + 8
\]

Solution 10:
From right triangle ADE
\[
\tan 45^\circ = \frac{AE}{DE} = \frac{1}{30} \Rightarrow AE = 30 \text{ cm}
\]
Also, from triangle DBE
\[
\tan 60^\circ = \frac{BE}{DE} = \frac{\sqrt{3}}{30} \Rightarrow BE = 30\sqrt{3} \text{ cm}
\]
Therefore \(AB = AE + BE = 30 + 30\sqrt{3} = 30(1 + \sqrt{3}) \text{ cm}\)
Solution 11:

(i)

From the triangle $ADC$ we have

$$\tan 45^0 = \frac{AD}{DC}$$

$$1 = \frac{2}{DC}$$

$$DC = 2$$

Since $AD \parallel DC$ and $AD \perp BC$, $ABCD$ is a parallelogram and hence opposite sides are equal.

Therefore $AB = DC = 2\, \text{cm}$

(ii)

Again

$$\sin 45^0 = \frac{AD}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{2}{AC}$$

$$AC = 2\sqrt{2}$$

(iii)

From the right triangle $ADE$ we have

$$\sin 60^0 = \frac{AD}{AE}$$

$$\frac{\sqrt{3}}{2} = \frac{2}{AE}$$

$$AE = \frac{4}{\sqrt{3}}$$
Solution 12:

Let $BE = x$, and $EC = 25 - x$

In $\triangle ABC$,

$$\sin 60^\circ = \frac{AE}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AE}{8}$$

$$\Rightarrow AE = 8 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow AE = 4\sqrt{3} \text{ cm}$$

(i) $BE^2 = AB^2 - AE^2$

$$\Rightarrow BE^2 = 8^2 - (4\sqrt{3})^2$$

$$\Rightarrow BE^2 = 64 - 48$$

$$\Rightarrow BE^2 = 16$$

$$\Rightarrow BE = 4 \text{ cm}$$

(ii) $EC = BC - BE$

$$= 25 - 4$$

$$= 21$$

In right $\triangle AEC$,

$$AC^2 = AE^2 + EC^2$$

$$\Rightarrow AC^2 = (4\sqrt{3})^2 + 21^2$$

$$\Rightarrow AC^2 = 48 + 441$$

$$\Rightarrow AC^2 = 489$$

$$\Rightarrow AC = 22.11 \text{ cm}$$
Solution 13:

(i) From right triangle ABC

\[ \tan 30^0 = \frac{AB}{BC} \]
\[ \frac{1}{\sqrt{3}} = \frac{12}{BC} \]
\[ BC = 12\sqrt{3} \text{ cm} \]

(ii) From the right triangle ABD

\[ \cos A = \frac{AD}{AB} \]
\[ \cos 60^0 = \frac{AD}{AB} \]
\[ \frac{1}{2} = \frac{AD}{12} \]
\[ AD = 6 \text{ cm} \]

(iii) From right triangle ABC

\[ \sin B = \frac{AB}{AC} \]
\[ \sin 30^0 = \frac{AB}{AC} \]
\[ \frac{1}{2} = \frac{12}{AC} \]
\[ AC = 24 \text{ cm} \]
Solution 14:
Consider the figure

(i) Here \( AB \) is \( \sqrt{3} \) times of \( BC \) means

\[
\frac{AB}{BC} = \sqrt{3}
\]
\[
\cot \theta = \cot 30^\circ
\]
\[
\theta = 30^\circ
\]

(ii)
Again from the figure

\[
\frac{BC}{AB} = \sqrt{3}
\]
\[
\tan \theta = \sqrt{3}
\]
\[
\theta = \tan 60^\circ
\]
\[
\theta = 60^\circ
\]

Therefore, magnitude of angle \( A \) is \( 30^\circ \)

Solution 15:
Given that the ladder makes an angle of 30o with the ground and reaches upto a height
of 15 m of the tower which is shown in the figure below:

![Diagram of a ladder reaching 15 m]

Suppose the length of the ladder is \( x \) m

From the figure

\[
\frac{15}{x} = \sin 30^\circ \quad \text{[} \therefore \frac{\text{Perp.}}{\text{Hypot.}} = \sin \text{]}
\]

\[
\frac{15}{x} = \frac{1}{2} \quad \Rightarrow \quad x = 30 \text{ m}
\]

Therefore the length of the ladder is 30 m.

**Solution 16:**

![Diagram of a kite with string and angles]

Suppose that the greatest height is \( x \) m.

From the figure

\[
\frac{x}{100} = \sin 60^\circ \quad \text{[} \therefore \frac{\text{Perp.}}{\text{Hypot.}} = \sin \text{]}
\]

\[
\frac{x}{100} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad x = 86.6 \text{ m}
\]

Therefore the greatest height reached by the kite is 86.6 m.
Solution 17:

(i) Let $BC = xm$

$BD = BC + CD = (x+20) cm$

In $\triangle ABD$,

$\tan 30^\circ = \frac{AB}{BD}$

$$1 = \frac{\sqrt{3} \cdot AB}{x + 20}$$

$x + 20 = \sqrt{3} \cdot AB$ ... (1)

In $\triangle ABC$

$\tan 45^\circ = \frac{AB}{BC}$

$1 = \frac{AB}{x}$

$AB = x$ ... (2)

From (1)

$AB + 20 = \sqrt{3} \cdot AB$

$AB(\sqrt{3} - 1) = 20$

$AB = \frac{20}{(\sqrt{3} - 1)}$

$= \frac{20}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$

$= \frac{20(\sqrt{3} + 1)}{3 - 1} = 27.32 \text{ cm}$
From (2)

\[ AB = x = 27.32 \text{cm} \]

Therefore \( BC = x = AB = 27.32 \text{cm} \)

Therefore, \( AB = 27.32 \text{cm}, BC = 27.32 \text{cm} \)

(ii)

Let \( BC = x \text{m} \)

\( BD = BC + CD = (x + 20) \text{ cm} \)

In \( \triangle ABD \),

\[
\tan 30° = \frac{AB}{BD}
\]

\[
\frac{1}{\sqrt{3}} = \frac{AB}{x + 20}
\]

\[ x + 20 = \sqrt{3} \cdot AB \quad ...(1) \]

In \( \triangle ABC \)

\[
\tan 60° = \frac{AB}{BC}
\]

\[
\sqrt{3} = \frac{AB}{x}
\]

\[ x = \frac{AB}{\sqrt{3}} \quad ...(2) \]

From (1)

\[
\frac{AB}{\sqrt{3}} + 20 = \sqrt{3} \cdot AB
\]

\[ AB + 20\sqrt{3} = 3AB \]

\[ 2AB = 20\sqrt{3} \]

\[ AB = \frac{20\sqrt{3}}{2} \]

\[ = 10\sqrt{3} = 17.32 \text{cm} \]
From (2)
\[
x = \frac{AB}{\sqrt{3}} = \frac{17.32}{\sqrt{3}} = 10\text{cm}
\]

Therefore $BC = x = 10\text{cm}$

Therefore,

$AB = 17.32\text{cm, } BC = 10\text{cm}$

(iii)

Let $BC = xm$

$BD = BC + CD = (x + 20)\text{cm}$

In $\triangle ABD$,

\[
tag 45^\circ = \frac{AB}{BD}
\]

\[
1 = \frac{AB}{x + 20}
\]

\[
x + 20 = AB \quad (1)
\]

In $\triangle ABC$

\[
tag 60^\circ = \frac{AB}{BC}
\]

\[
\sqrt{3} = \frac{AB}{x}
\]

\[
x = \frac{AB}{\sqrt{3}} \quad (2)
\]
From (1)

\[
\frac{AB}{\sqrt{3}} + 20 = AB
\]

\[
AB + 20\sqrt{3} = \sqrt{3}AB
\]

\[
AB(\sqrt{3} - 1) = 20\sqrt{3}
\]

\[
AB = \frac{20\sqrt{3}}{\left(\sqrt{3} - 1\right)}
\]

\[
= \frac{20\sqrt{3}}{\left(\sqrt{3} - 1\right)} \times \left(\frac{\sqrt{3} + 1}{\sqrt{3} + 1}\right)
\]

\[
= \frac{20\sqrt{3}(\sqrt{3} + 1)}{3 - 1} = 47.32\text{cm}
\]

From (2)

\[
x = \frac{AB}{\sqrt{3}} = \frac{47.32}{\sqrt{3}} = 27.32\text{cm}
\]

\[\therefore BC = x = 27.32\text{cm}\]

Therefore,

\[AB = 47.32\text{cm}, BC = 27.32\text{cm}\]
Solution 18:

(i) From $\triangle APB$

\[
\tan 30^\circ = \frac{AB}{PB}
\]

\[
1 = \frac{150}{\sqrt{3}} = \frac{PB}{PB}
\]

\[PB = 150\sqrt{3} = 259.80 \text{ m}\]

Also, from $\triangle ABQ$

\[
\tan 45^\circ = \frac{AB}{BQ}
\]

\[
1 = \frac{150}{BQ}
\]

\[BQ = 150 \text{ m}\]

Therefore,

\[PQ = PB + BQ = 259.80 + 150 = 409.80 \text{ m}\]

(ii) From $\triangle APB$

\[
\tan 30^\circ = \frac{AB}{PB}
\]

\[
1 = \frac{150}{\sqrt{3}} = \frac{PB}{PB}
\]

\[PB = 150\sqrt{3} = 259.80 \text{ m}\]

Also, from $\triangle ABQ$

\[
\tan 45^\circ = \frac{AB}{BQ}
\]

\[
1 = \frac{150}{BQ}
\]

\[BQ = 150 \text{ m}\]

Therefore,

\[PQ = PB - BQ = 259.80 - 150 = 109.80 \text{ m}\]
Solution 19:

Given \( \tan x^\circ = \frac{5}{12} \) and \( \tan y^\circ = \frac{3}{4} \) and \( AB = 48 \text{ m} \);

Let length of \( BC = x \text{ m} \)

From \( \triangle ADC \)

\[
\tan x^\circ = \frac{DC}{AC} \Rightarrow \frac{5}{12} = \frac{DC}{48 + x}
\]

\( 240 + 5x = 12CD \) \( \ldots (1) \)

Also, from \( \triangle BDC \)

\[
\tan y^\circ = \frac{CD}{BC} \Rightarrow \frac{3}{4} = \frac{CD}{x}
\]

\( x = \frac{4CD}{3} \) \( \ldots (2) \)

From (1)

\[
240 + 5\left(\frac{4CD}{3}\right) = 12CD
\]

\[
240 + \frac{20CD}{3} = 12CD
\]

\[
720 + 20CD = 36CD
\]

\[
16CD = 720
\]

\[
CD = 45
\]

Therefore, length of CD is 45 m.
Solution 20:
Since in a rhombus all sides are equal.

The diagram is shown below:

Therefore \[ PQ = \frac{96}{4} = 24 \text{ cm} \text{. Let } \angle PQR = 120^\circ. \]

We also know that in rhombus diagonals bisect each other perpendicularly and diagonal bisect the angle at vertex.
Hence POR is a right angle triangle and
\[ POR = \frac{1}{2} \angle PQR = 60^\circ \]

\[ \sin 60^\circ = \frac{\text{Perp.}}{\text{Hypot.}} = \frac{PO}{PQ} = \frac{PO}{24} \]

But
\[ \sin 60^\circ = \frac{\sqrt{3}}{2} \]
\[ \frac{PO}{24} = \frac{\sqrt{3}}{2} \]
\[ PO = 12 \sqrt{3} \approx 20.784 \]

Therefore,
\[ PR = 2PO = 2 \times 20.784 = 41.568\text{ cm} \]

Also,
\[ \cos 60^\circ = \frac{\text{Base}}{\text{Hypot.}} = \frac{OQ}{24} \]

But
\[ \cos 60^\circ = \frac{1}{2} \]
\[ \frac{OQ}{24} = \frac{1}{2} \]
\[ OQ = 12 \]

Therefore, \[ SQ = 2 \times OQ = 2 \times 12 = 24\text{ cm} \]

So, the length of the diagonal \( PR = 41.568\text{ cm} \) and \( SQ = 24\text{ cm} \).