Chapter 16. Area Theorems [Proof and Use]

Exercise 16(A)

Solution 1:

(i) \(\triangle ADE\) and parallelogram \(ABED\) are on the same base \(AB\) and between the same parallels \(DE\parallel AB\), so area of the triangle \(\triangle ADE\) is half the area of parallelogram \(ABED\).

Area of \(ABED = 2 \times \text{(Area of } ADE) = 120 \text{ cm}^2\)

(ii) Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e., between the same parallels.

Area of \(ABCF = \text{Area of } ABED = 120 \text{ cm}^2\)

(iii) We know that area of triangles on the same base and between same parallel lines are equal.

Area of \(ABE = \text{Area of } ADE = 60 \text{ cm}^2\)

Solution 2:

After drawing the opposite sides of \(AB\), we get

Since from the figure, we get \(CD\parallel FE\) therefore \(FC\) must parallel to \(DE\). Therefore it is proved that the quadrilateral \(CDEF\) is a parallelogram.
Area of parallelogram on same base and between same parallel lines is always equal and area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e., between same parallel lines.
So Area of \(CDEF = \text{Area of } ABDC + \text{Area of } ABEF\)
Hence Proved
Solution 3:

(i)

Since POS and parallelogram PMLS are on the same base PS and between the same parallels i.e. SP/LM.

As O is the center of LM and Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.

The area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels.

So 2(Area of PSO)=Area of PMLS

Hence Proved.

(ii)

Consider the expression \( \text{Area}(\triangle POS) + \text{Area}(\triangle QOR) \).

LM is parallel to PS and PS is parallel to RQ, therefore, LMI is.

Since triangle POS lie on the base PS and in between the parallels PS and LM, we have, \( \text{Area}(\triangle POS) = \frac{1}{2} \text{Area}(\square PSLM) \).

Since triangle QOR lie on the base QR and in between the parallels LM and RQ, we have,

\[
\text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\square LMQR)
\]

\[
\text{Area}(\triangle POS) + \text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\square PSLM) + \frac{1}{2} \text{Area}(\square LMQR)
\]

\[
= \frac{1}{2} \left[ \text{Area}(\square PSLM) + \text{Area}(\square LMQR) \right]
\]

\[
= \frac{1}{2} \left[ \text{Area}(\square PQRS) \right]
\]

(iii)

In a parallelogram, the diagonals bisect each other.

Therefore, OS = OQ.

Consider the triangle PQS, since OS = OQ, OP is the median of the triangle PQS.

We know that median of a triangle divides it into two triangles of equal area.

Therefore,

\[
\text{Area}(\triangle POS) = \text{Area}(\triangle POQ) \ldots (1)
\]

Similarly, since OR is the median of the triangle QRS, we have,

\[
\text{Area}(\triangle QOR) = \text{Area}(\triangle SOR) \ldots (2)
\]

Adding equations (1) and (2), we have,

\[
\text{Area}(\triangle POS) + \text{Area}(\triangle QOR) = \text{Area}(\triangle POQ) + \text{Area}(\triangle SOR)
\]

Hence Proved.

Solution 4:

Given ABCD is a parallelogram. P and Q are any points on the sides AB and BC respectively, join diagonals AC and BD.

proof:

since triangles with same base and between same set of parallel lines have equal areas

area (CPD)=area(BCD)..... (1)

again, diagonals of the parallelogram bisects area in two equal parts

area (BCD)=(1/2) area of parallelogram ABCD..... (2)
from (1) and (2)
area(CPD)=1/2 area(ABCD)...... (3)
similarly area (AQD)=area(ABD)=1/2 area(ABCD)...... (4)
from (3) and (4)
area(CPD)=area(AQD),
hence proved.

(ii)
We know that area of triangles on the same base and between same parallel lines are equal
So Area of AQD = Area of ACD = Area of PDC = Area of BDC = Area of ABC = Area of APD + Area of BPC
Hence proved.

Solution 5:
(i) Since triangle BEC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.
So Area( △BEC) = \frac{1}{2} \times Area( □ABCD) = \frac{1}{2} \times 48 = 24 \text{ cm}^2

(ii)
Area( □ANMD) = Area( □BNMC)
= \frac{1}{2} Area( □ABCD)
= \frac{1}{2} \times 2 \times Area( △BEC)
= Area( △BEC)

Therefore, parallelograms ANMD and NBCM have areas equal to triangle BEC.

Solution 6:
Since △DCB and △DEB are on the same base DB and between the same parallels i.e. DB//CE, therefore we get
Ar( △DCB) = Ar( △DEB)
Ar( △DCB + △ADB) = Ar( △DEB + △ADB)
Ar( □ABCD) = Ar( △ADB)
Hence proved.

Solution 7:
△APB and parallelogram ABCD are on the same base AB and between the same parallel lines AB and CD.

:: Ar (△APB) = \frac{1}{2} Ar (parallelogram AB CD) ......(i)

△ADQ and parallelogram ABCD are on the same base AD and between the same parallel lines AD and BQ.

:: Ar (△ADQ) = \frac{1}{2} Ar (parallelogram AB CD) ...... (ii)

Adding equation (i) and (ii), we get

:: Ar (△APB) + Ar (△ADQ) = Ar (parallelogram AB CD)
Ar (quad. ADQP) - Ar (△BCQ) = Ar (parallelogram AB CD)
Ar (quad. ADQP) - Ar (△BCQ) = Ar (quad. ADQP) - Ar (△DCQ)
\therefore Ar (△BCQ) = Ar (△DCQ)

Subtracting Ar. △PCQ from both sides, we get

\therefore Ar (△BCP) = Ar (△PCQ)

Hence proved.
Solution 8:

Since triangle EDG and EGA are on the same base EG and between the same parallel lines EG and DA, therefore

\[ \text{Ar.} \{\triangle EDO\} = \text{Ar.} \{\triangle EGA\} \]

Subtracting \( \triangle EGO \) from both sides, we have

\[ \text{Ar.} \{\triangle BOD\} = \text{Ar.} \{\triangle GOA\} \quad \text{(i)} \]

Similarly

\[ \text{Ar.} \{\triangle DPC\} = \text{Ar.} \{\triangle BPF\} \quad \text{(ii)} \]

Now

\[ \text{Ar.} \{\triangle CDE\} = \text{Ar.} \{\triangle GOA\} + \text{Ar.} \{\triangle GEF\} + \text{Ar.} \{\triangle APFDC\} \\
= \text{Ar.} \{\triangle BOD\} + \text{Ar.} \{\triangle DPC\} + \text{Ar.} \{\triangle APBDC\} \\
= \text{Ar.} \{\text{pen} \ AB \ CDE\} \]

Hence proved.

Solution 9:

Joining PC we get

\( \triangle ABC \) and \( \triangle BPC \) are on the same base BC and between the same parallel lines AP and BC.  \[ \therefore \text{Ar.} \{\triangle ABC\} = \text{Ar.} \{\triangle BPC\} \quad \text{(i)} \]

\( \triangle BPC \) and \( \triangle BPQ \) are on the same base BP and between the same parallel lines BP and CQ.  \[ \therefore \text{Ar.} \{\triangle BPC\} = \text{Ar.} \{\triangle BPQ\} \quad \text{(ii)} \]

From (i) and (ii), we get

\[ \therefore \text{Ar.} \{\triangle ABC\} = \text{Ar.} \{\triangle BPQ\} \]

Hence proved.
Solution 10:

(i) 
\[
\angle BAC = \angle BAE + \angle BAC \\
\angle EAC = 90^\circ + \angle BAC \\
\angle BAB = \angle FAC + \angle BAC \\
\angle BAB = 90^\circ + \angle BAC \\
\]

From (i) and (ii), we get 
\[
\angle BAC = \angle BAF
\]

In \( \triangle EAC \) and \( \triangle BAF \), we have, \( EA = AB \)

\[
\angle EAC = \angle BAF \quad \text{and} \quad AC = AF
\]

\( \therefore \) \( \triangle EAC \cong \triangle BAF \) (SAS axiom of congruency)

(ii) 
Since \( \triangle ABC \) is a right triangle, we have,

\[
AC^2 = AB^2 + BC^2 \quad \text{[Using Pythagoras Theorem in } \triangle ABC]\]

\[
\Rightarrow AB^2 = AC^2 - BC^2
\]

\[
\Rightarrow AB^2 = (AR + RC)^2 - (BR^2 + RC^2) \quad \text{[Since } AC = AR + RC \text{ and Using Pythagoras Theorem in } \triangle BRC]\]

\[
\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2) \quad \text{[Using the identity]}\]

\[
\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - AB^2 - AR^2 + RC^2 \quad \text{[Using Pythagoras Theorem in } \triangle ABR]\]

\[
\Rightarrow 2AB^2 = 2AR^2 + 2RC^2
\]

\[
\Rightarrow AB^2 = AR(AR + RC)
\]

\[
\Rightarrow AB^2 = AR \times AC
\]

\[
\Rightarrow AB^2 = AR \times AF
\]

\[
\Rightarrow \text{Area}(\square ABDE) = \text{Area}(\text{rectangle } ARHF)
\]

Solution 11:

(i) 
In \( \triangle ABC \), \( D \) is midpoint of \( AB \) and \( E \) is the midpoint of \( AC \).

\[
\frac{AD}{AB} = \frac{AE}{AC}
\]

\( DE \) is parallel to \( BC \).

\( \therefore \) \( \text{Area}(\triangle ADC) = \text{Area}(\triangle DBE) = \frac{1}{2} \text{Area}(\triangle ABC) \)

Again

\( \therefore \) \( \text{Area}(\triangle AED) = \text{Area}(\triangle DBC) = \frac{1}{2} \text{Area}(\triangle ABC) \)

From the above two equations, we have

\[
\text{Area}(\triangle ADC) = \text{Area}(\triangle AED) + \text{Area}(\triangle DBC) = \frac{1}{2} \text{Area}(\triangle ABC)
\]

Hence Proved

(ii) 
We know that the area of triangles on the same base and between same parallel lines are equal

\[
\text{Area}(\triangle DBC) = \text{Area}(\triangle BCE)
\]

\[
\text{Area}(\triangle DOB) + \text{Area}(\triangle BOC) + \text{Area}(\triangle BOC) + \text{Area}(\triangle COE)
\]

So \( \text{Area}(\triangle DOB) + \text{Area}(\triangle COE) \)
Solution 12:

(i) Since $\triangle EBC$ and parallelogram $ABCD$ are on the same base $BC$ and between the same parallels i.e. $BC//AD$.

\[ \text{Area} \ (\triangle EBC) = \frac{1}{2} \times \text{Area} \ (\text{parallelogram } ABCD) \]

\[ (\text{parallelogram } ABCD) = 2 \times \text{Area} \ (\triangle EBC) \]

\[ = 2 \times 480 \text{ cm}^2 \]

\[ = 960 \text{ cm}^2 \]

(ii) Parallelograms on same base and between same parallels are equal in area

Area of $BCFE = \text{Area of } ABCD = 960 \text{ cm}^2$

(iii)

Area of triangle $ACD = 480 + \frac{1}{2} \times 30 \times \text{Altitude}$

Altitude = 32 cm

(iv)

The area of a triangle is half that of a parallelogram on the same base and between the same parallels.

Therefore,

\[ \text{Area} \ (\triangle ECF) = \frac{1}{2} \times \text{Area} \ (\square CBEF) \]

Similarly, \[ \text{Area} \ (\triangle BCE) = \frac{1}{2} \times \text{Area} \ (\square CBEF) \]

\[ \Rightarrow \text{Area} \ (\triangle ECF) = \text{Area} \ (\triangle BCE) = 480 \text{ cm}^2 \]

Solution 13:

Here $AD=DB$ and $EC=DB$, therefore $EC=AD$

Again, $\angle BFC = \angle APD$ (opposite angles)

Since $ED$ and $CB$ are parallel lines and $AC$ cut this line, therefore

$\angle BCF = \angle FAD$

From the above conditions, we have

$\triangle EFC = \triangle APD$

Adding quadrilateral $CBDF$ in both sides, we have

Area of $\square ABC$ = Area of $\triangle ABC$

Solution 14:

In Parallelogram $PQRS$, $AC // PS // QR$ and $PQ // DB // SR$.

Similarly, $AQRC$ and $APSC$ are also parallelograms.

Since $\triangle ABC$ and parallelogram $AQRC$ are on the same base $AC$ and between the same parallels, then

\[ \text{Area} \ (\triangle ABC) = \frac{1}{2} \times \text{Area} \ (\triangle AQRC) \ldots \ldots (i) \]

Similarly,

\[ \text{Area} \ (\triangle ADC) = \frac{1}{2} \times \text{Area} \ (\triangle APSC) \ldots \ldots (ii) \]

Adding (i) and (ii), we get

Area of quadrilateral $PQRS = 2 \times \text{Area of quad. } ABCD$
Solution 15:
Given: ABCD is a trapezium

Join C and M
We know that area of triangles on the same base and between same parallel lines are equal.

So Area of ΔAMD = Area of ΔAMC

Similarly, consider AMNC quadrilateral where MN || AC.

ΔACM and ΔACN are on the same base and between the same parallel lines. So areas are equal.

So, Area of ΔACM = Area of ΔCAN

From the above two equations, we can say

Area of ΔADM = Area of ΔCAN

Hence Proved.

Solution 16:
We know that area of triangles on the same base and between same parallel lines are equal.
Consider ABEFD quadrilateral; AD||BE
With common base, BE and between AD and BE parallel lines, we have
Area of ΔABE = Area of ΔBDE

Similarly, in BEFC quadrilateral, BE||CF
With common base BC and between BE and CF parallel lines, we have
Area of ΔBEC = Area of ΔBEF

Adding both equations, we have

Area of ΔABE + Area of ΔBEC = Area of ΔBEF + Area of ΔBDE

=> Area of AEC = Area of DBF

Hence Proved.

Solution 17:
Given: ABCD is a parallelogram.
We know that
Area of ΔABC = Area of ΔACD

Consider ΔABX,
Area of ΔABX = Area of ΔABC + Area of ΔACX

We also know that area of triangles on the same base and between same parallel lines are equal.
Area of ΔACX = Area of ΔCXD

From above equations, we can conclude that
Area of ΔABX = Area of ΔABC + Area of ΔACX = Area of ΔACD + Area of ΔCXD = Area of ACXD Quadrilateral

Hence Proved.

Solution 18:
Join B and R and P and R.
We know that the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram
are on the same base and between the parallels
Consider ABCD parallelogram:
Since the parallelogram ABCD and the triangle ABR lie on AB and between the parallels AB and DC, we have
\[ \text{Area}(\square ABCD) = 2 \times \text{Area}(\triangle ABR) \ldots (1) \]

We know that the area of triangles with same base and between the same parallel lines are equal.
Since the triangles ABR and APR lie on the same base AR and between the parallels AR and PQ, we have,
\[ \text{Area}(\triangle ABR) = \text{Area}(\triangle APR) \ldots (2) \]

From equations (1) and (2), we have,
\[ \text{Area}(\square ABCD) = 2 \times \text{Area}(\triangle APR) \ldots (3) \]
Also, the triangle APR and the parallelogram ARQP lie on the same base AR and between the parallels, AR and PQ,
\[ \text{Area}(\triangle APR) = \frac{1}{2} \times \text{Area}(\square ARQP) \ldots (4) \]

Using (4) in equation (3), we have,
\[ \text{Area}(\square ABCD) = 2 \times \frac{1}{2} \times \text{Area}(\square ARQP) \]
\[ \text{Area}(\square ABCD) = \text{Area}(\square ARQP) \]
Hence proved.

Exercise 16(B)

Solution 1:

(i) Suppose ABCD is a parallelogram (given)

Consider the triangles ABC and ADC:
\[ AB = CD \quad [\text{ABCD is a parallelogram}] \]
\[ AD = BC \quad [\text{ABCD is a parallelogram}] \]
\[ AD = AD \quad [\text{common}] \]
By Side – Side – Side criterion of congruence, we have,
\[ \triangle ABC \cong \triangle ADC \]
Area of congruent triangles are equal.
Therefore, \[ \text{Area of } ABC = \text{Area of } ADC \]

(ii) Consider the following figure:

Here \[ AP \perp BC \]
Solution 2:

Since \[ \text{Area}(\triangle ABD) = \frac{1}{2} BD \times AP \]

And, \[ \text{Area}(\triangle ADC) = \frac{1}{2} DC \times AP \]

\[ \therefore \frac{\text{Area}(\triangle ABD)}{\text{Area}(\triangle ADC)} = \frac{\frac{1}{2} BD \times AP}{\frac{1}{2} DC \times AP} = \frac{BD}{DC}, \]

hence proved

(III) Consider the following figure:

![Figure](image)

Here

\[ \text{Area}(\triangle ABC) = \frac{1}{2} BM \times AC \]

And, \[ \text{Area}(\triangle ADC) = \frac{1}{2} DN \times AC \]

\[ \therefore \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle ADC)} = \frac{\frac{1}{2} BM \times AC}{\frac{1}{2} DN \times AC} = \frac{BM}{DN}, \]

hence proved

Solution 2:

AD is the median of \( \triangle ABC \). Therefore it will divide \( \triangle ABC \) into two triangles of equal areas.

\[ \therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle ACD) \quad (\text{I}) \]

ED is the median of \( \triangle EBC \)

\[ \therefore \text{Area}(\triangle EBD) = \text{Area}(\triangle ECD) \quad (\text{II}) \]

Subtracting equation (II) from (I), we obtain

\[ \text{Area}(\triangle ABD) - \text{Area}(\triangle EBD) = \text{Area}(\triangle ACD) - \text{Area}(\triangle ECD) \]

\[ \text{Area}(\triangle ABE) = \text{Area}(\triangle ACE). \text{ Hence proved} \]
Solution 3:
AD is the median of \( \triangle ABC \). Therefore it will divide \( \triangle ABC \) into two triangles of equal areas.

\[
\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle ACD)
\]

\[
\text{Area}(\triangle ABD) = \frac{1}{2} \times \text{Area}(\triangle ABC) \quad \text{(i)}
\]

In \( \triangle ABD \), E is the mid-point of AD. Therefore BE is the median.

\[
\therefore \text{Area}(\triangle BED) = \text{Area}(\triangle ABE)
\]

\[
\text{Area}(\triangle BED) = \frac{1}{2} \times \text{Area}(\triangle ABD)
\]

\[
\text{Area}(\triangle BED) = \frac{1}{4} \times \text{Area}(\triangle ABC) \quad \text{[from equation (i)]}
\]

\[
\text{Area}(\triangle BED) = \frac{1}{4} \times \text{Area}(\triangle ABC)
\]

Solution 4:
We have to join PD and BD.

BD is the diagonal of the parallelogram ABCD. Therefore it divides the parallelogram into two equal parts.

\[
\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle BCD)
\]

\[
= \frac{1}{2} \times \text{Area (parallelogram ABCD)} \quad \text{(i)}
\]

DP is the median of \( \triangle ABD \). Therefore it will divide \( \triangle ABD \) into two triangles of equal areas.

\[
\therefore \text{Area}(\triangle APD) = \text{Area}(\triangle DPB)
\]

\[
= \frac{1}{2} \times \text{Area}(\triangle ABD)
\]

\[
= \frac{1}{8} \times \text{Area (parallelogram ABCD)} \quad \text{[from equation (i)]}
\]

\[
= \frac{1}{4} \times \text{Area (parallelogram ABCD)} \quad \text{(ii)}
\]

In \( \triangle APD \), Q is the mid-point of AD. Therefore PQ is the median.

\[
\therefore \text{Area}(\triangle APQ) = \text{Area}(\triangle DPQ)
\]

\[
= \frac{1}{2} \times \text{Area}(\triangle APD)
\]

\[
= \frac{1}{2} \times \frac{1}{4} \times \text{Area (parallelogram ABCD)} \quad \text{[from equation (ii)]}
\]

\[
\text{Area}(\triangle APQ) = \frac{1}{8} \times \text{Area (parallelogram ABCD)}, \text{hence proved}
\]
**Solution 5:**

In \( \triangle ABC \), \( \frac{BD}{DC} = \frac{1}{2} \) \( \implies \frac{BD}{DC} = \frac{1}{2} \)

\[ \therefore \text{Area}(\triangle ABD) : \text{Area}(\triangle ADC) = 1:2 \]

But \( \text{Area}(\triangle ABD) + \text{Area}(\triangle ADC) = \text{Area}(\triangle ABC) \)

\[ \text{Area}(\triangle ABD) + 2\text{Area}(\triangle ABD) = \text{Area}(\triangle ABC) \]

\[ 3\text{Area}(\triangle ABD) = \text{Area}(\triangle ABC) \]

\[ \text{Area}(\triangle ABD) = \frac{1}{3} \text{Area}(\triangle ABC) \]

**Solution 6:**

Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases. So, we have

\[ \frac{\text{Area of } \triangle DPB}{\text{Area of } \triangle PCB} = \frac{3}{2} \]

Given: Area of \( \triangle DPB = 30 \text{ sq. cm} \)

Let 'x' be the area of the triangle \( \triangle PCB \)

Therefore, we have,

\[ \frac{30}{x} = \frac{3}{2} \]

\[ \Rightarrow x = \frac{30}{3} \times 2 = 20 \text{ sq. cm.} \]

So area of \( \triangle PCB = 20 \text{ sq. cm} \)

Consider the following figure.

From the diagram, it is clear that,

\[ \text{Area}(\triangle CDB) = \text{Area}(\triangle DPB) + \text{Area}(\triangle CPB) \]

\[ = 30 + 20 \]

\[ = 50 \text{ sq. cm} \]

Diagonal of the parallelogram divides it into two triangles \( \triangle ADB \) and \( \triangle CDB \) of equal area.

Therefore,

\[ \text{Area}(\parallel \text{gm } ABCD) = 2 \times \text{Area}(\triangle CDB) \]

\[ = 2 \times 50 = 100 \text{ sq. cm} \]
Solution 7:

BC = CE (given)
Also, in parallelogram ABCD, BC = AD
⇒ AD = CE
Now, in ΔADF and ΔECF, we have
AD = CE
∠ADF = ∠ECF (Alternate angles)
∠DAF = ∠CEF (Alternate angles)
⇒ ΔADF ≅ ΔECF (ASA Criterion)
⇒ Area(ΔADF) = Area(ΔECF) .... (1)
Also, in ΔFBE, FC is the median (Since BC = CE)
⇒ Area(ΔBCF) = Area(ΔECF) .... (2)
From (1) and (2),
Area(ΔADF) = Area(ΔBCF) .... (3)
Again, ΔADF and ΔABD are on the base DF and between parallels DF and AB.
⇒ Area(ΔABD) = Area(ΔADF) .... (4)
From (3) and (4),
Area(ΔABD) = Area(ΔBCF) = 30 cm²
⇒ Area(ΔBCD) = Area(ΔABD) + Area(ΔBCF) = 30 + 30 = 60 cm²
Hence, Area of parallelogram ABCD = 2 × Area(ΔBCD) = 2 × 30 = 120 cm²
Solution 8:

In ΔABC,

R and Q are the mid-points of AC and BC respectively.

⇒ RQ ∥ AB

that is RQ ∥ PB

So, area(ΔPBQ) = area(ΔAPR) ...(i) (Since AP = PB and triangles on the same base and between the same parallels are equal in area)

Since P and R are the mid-points of AB and AC respectively,

⇒ FR ∥ BC

that is FR ∥ BQ

So, quadrilateral PMQR is a parallelogram.

Also, area(ΔPBQ) = area(ΔPQR) ...(ii) (diagonal of a parallelogram divide the parallelogram into two triangles with equal area)

From (i) and (ii),

area(ΔPQR) = area(ΔPBQ) = area(ΔAPR) ...(iii)

Similarly, P and Q are the mid-points of AB and BC respectively.

⇒ PQ ∥ AC

that is PQ ∥ RC

So, quadrilateral PQCR is a parallelogram.

Also, area(ΔRQC) = area(ΔPQR) ...(iv) (diagonal of a parallelogram divide the parallelogram into two triangles with equal area)

From (iii) and (iv),

area(ΔPQR) = area(ΔPBQ) = area(ΔRQC) = area(ΔAPR)

So, area(ΔPBQ) = \(\frac{1}{4}\) area(ΔABC) ...(v)

Also, since S is the mid-point of PQ,

BS is the median of ΔPQB

So, area(ΔQSB) = \(\frac{1}{2}\) area(ΔPBQ)

From (v),

area(ΔQSB) = \(\frac{1}{2}\) \(\frac{1}{4}\) area(ΔABC)

⇒ area(ΔABC) = 8 area(ΔQSB)

Exercise 16(C)
Solution 1:

(i) 
Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases. So, we have:

\[
\frac{\text{Area of } \triangle DOC}{\text{Area of } \triangle BOC} = \frac{DO}{BO} = 1 \quad \quad (1)
\]

Similarly,

\[
\frac{\text{Area of } \triangle DOA}{\text{Area of } \triangle BOA} = \frac{DO}{BO} = 1 \quad \quad (2)
\]

We know that the area of triangles on the same base and between the same parallel lines are equal.

\[
\text{Area of } \triangle ACD = \text{Area of } \triangle BCD
\]

\[
\text{Area of } \triangle AOD + \text{Area of } \triangle DOC = \text{Area of } \triangle DOC + \text{Area of } \triangle BOC
\]

\[
\Rightarrow \text{Area of } \triangle AOD = \text{Area of } \triangle BOC \quad \quad (3)
\]

From 1, 2 and 3 we have

\[
\text{Area (} \triangle DOC \text{)} = \text{Area (} \triangle AOB \text{)}
\]

Hence Proved.

(ii) 
Similarly, from 1, 2 and 3, we also have

\[
\text{Area of } \triangle DBC = \text{Area of } \triangle DOC + \text{Area of } \triangle BOC = \text{Area of } \triangle AOB + \text{Area of } \triangle BOC = \text{Area of } \triangle ABC
\]

So, Area of \( \triangle DBC \) = Area of \( \triangle ABC \)

Hence Proved.

(iii) 
We know that the area of triangles on the same base and between the same parallel lines are equal.

Given: Triangles are equal in area on the common base. So it indicates \( AD \parallel BC \).

So, \( ABCD \) is a parallelogram.

Hence Proved.
Solution 2:
Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.

So, we have

\[
\frac{\text{Area of } \triangle APD}{\text{Area of } \triangle BPD} = \frac{AP}{BP} = \frac{1}{2}
\]

Area of parallelogram ABCD = 324 sq.cm
Area of the triangles with the same base and between the same parallels are equal.

We know that area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.

Therefore, we have,

\[
\text{Area}(\triangle ABD) = \frac{1}{2} \times \text{Area}(\parallelgm \ ABOD)
\]

\[
= \frac{324}{2} = 162 \text{ sq. cm}
\]

From the diagram it is clear that,

\[
\text{Area}(\triangle ABD) = \text{Area}(\triangle APD) + \text{Area}(\triangle BPD)
\]

\[
\Rightarrow 162 = \text{Area}(\triangle APD) + 2 \times \text{Area}(\triangle APD)
\]

\[
\Rightarrow 162 = 3 \times \text{Area}(\triangle APD)
\]

\[
\Rightarrow \text{Area}(\triangle APD) = \frac{162}{3}
\]

\[
\Rightarrow \text{Area}(\triangle APD) = 54 \text{ sq. cm}
\]

Consider the triangles \( \triangle AOP \) and \( \triangle COD \)

\( \angle AOP = \angle COD \) [vertically opposite angles]

\( \angle CDO = \angle APD \) [\( AB \) and \( DC \) are parallel and \( DP \) is the transversal, alternate interior angles are equal]

Thus, by Angle – Angle similarity, \( \triangle AOP \sim \triangle COD \).

Hence the corresponding sides are proportional.

\[
\frac{AP}{CD} = \frac{OP}{OD} = \frac{AB}{\overline{AP} + \overline{PB}} = \frac{\overline{AP}}{3\overline{AP}} = \frac{1}{3}
\]
Solution 3:
E and F are the midpoints of the sides AB and AC.

Consider the following figure.

Therefore, by midpoint theorem, we have, EF \parallel BC

Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC

Therefore, \( \text{Ar.} (\triangle BEF) = \text{Ar.} (\triangle CEF) \)
\( \Rightarrow \text{Ar.} (\triangle BOE) + \text{Ar.} (\triangle EOF) = \text{Ar.} (\triangle EOF) + \text{Ar.} (\triangle COF) \)
\( \Rightarrow \text{Ar.} (\triangle BOE) = \text{Ar.} (\triangle COF) \)

Now BF and CE are the medians of the triangle ABC

Medians of the triangle divides it into two equal areas of triangles.

Thus, we have, \( \text{Ar.} \triangle ABF = \text{Ar.} \triangle CBF \)

Subtracting \( \text{Ar.} \triangle BOE \) on both the sides, we have
\( \text{Ar.} \triangle ABF - \text{Ar.} \triangle BOE = \text{Ar.} \triangle CBF - \text{Ar.} \triangle BOE \)

Since, \( \text{Ar.} (\triangle BOE) = \text{Ar.} (\triangle COF) \)

\( \text{Ar.} \triangle ABF - \text{Ar.} \triangle BOE = \text{Ar.} \triangle CBF - \text{Ar.} \triangle COF \)

\( \text{Ar.} (\text{quad. AEOF}) = \text{Ar.} (\triangle OBC) \), hence proved

Solution 4:
(i) Joining AC we have the following figure

Consider the triangles \( \triangle POB \) and \( \triangle COD \)
\( \angle POB = \angle DOC \) [vertically opposite angles]
\( \angle OPB = \angle ODC \) [\( AB \) and \( DC \) are parallel, \( CP \) and \( BD \) are the transversals, alternate interior angles are equal]

Therefore, by Angle – Angle similarity criterion of congruence,
\( \triangle POB \sim \triangle COD \)

Since \( P \) is the midpoint \( AP = BP \), and \( AB = CD \), we have \( CD = 2BP \)

Therefore, we have,
\[
\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2}
\]
\( \Rightarrow OP:OC = 1:2 \)
Since from part (i), we have
\[ \frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2}, \]
Ratio between the areas of two similar triangles is equal to the ratio between the squares of the corresponding sides.
Here, \( \triangle DOC \) and \( \triangle POB \) are similar triangles.
Thus, we have,
\[ \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = \frac{DC^2}{PB^2} \]
\[ \Rightarrow \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = \frac{(2PB)^2}{PB^2} \]
\[ \Rightarrow \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = \frac{4PB^2}{PB^2} \]
\[ \Rightarrow \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = 4 \]
\[ \Rightarrow \text{Ar.}(\triangle DOC) = 4 \times \text{Ar.}(\triangle POB) \]
\[ \text{= 160 cm}^2 \]
Now consider \( \text{Ar.}(\triangle DBC) = \text{Ar.}(\triangle DOC) + \text{Ar.}(\triangle BOC) \]
\[ \text{= 160 + 80} \]
\[ \text{= 240 cm}^2 \]
Two triangles are equal in area if they are on the equal bases and between the same parallels.
Therefore, \( \text{Ar.}(\triangle DBC) = \text{Ar.}(\triangle ABC) = 240 \text{ cm}^2 \)
Median divides the triangle into areas of two equal triangles.
Thus, \( CP \) is the median of the triangle \( ABC \).
Hence, \( \text{Ar.}(\triangle ABC) = 2 \times \text{Ar.}(\triangle PBC) \)
\[ \Rightarrow \text{Ar.}(\triangle PBC) = \frac{\text{Ar.}(\triangle ABC)}{2} \]
\[ \Rightarrow \text{Ar.}(\triangle PBC) = 120 \text{ cm}^2 \]

(iii)

From part (ii) we have,
\( \text{Ar.}(\triangle ABC) = 2 \times \text{Ar.}(\triangle PBC) = 240 \text{ cm}^2 \)
Area of a triangle is half the area of the Parallelogram
if both are on equal bases and between the same parallels.
Thus, \( \text{Ar.}(\triangle ABC) = \frac{1}{2} \times \text{Ar.}(\text{Parallelogram } ABCD) \)
\[ \Rightarrow \text{Ar.}(\text{Parallelogram } ABCD) = 2 \times \text{Ar.}(\triangle ABC) \]
\[ \Rightarrow \text{Ar.}(\text{Parallelogram } ABCD) = 2 \times 240 \]
\[ \Rightarrow \text{Ar.}(\text{Parallelogram } ABCD) = 480 \text{ cm}^2 \]
Solution 5:

(i) The figure is shown below

Medians intersect at centroid.
Given that $G$ is the point of intersection of medians and hence $G$ is the centroid of the triangle $ABC$.
Centroid divides the medians in the ratio 2:1
That is $AG:GD = 2:1$

Since $BG$ divides $AD$ in the ratio 2:1, we have,

\[
\frac{\text{Area}(\triangle AGB)}{\text{Area}(\triangle BGD)} = \frac{2}{1}
\]

\[
\Rightarrow \text{Area}(\triangle AGB) = 2 \times \text{Area}(\triangle BGD)
\]

From the figure, it is clear that,

\[
\text{Area}(\triangle ABD) = \text{Area}(\triangle AGB) + \text{Area}(\triangle BGD)
\]

\[
\Rightarrow \text{Area}(\triangle ABD) = 2 \times \text{Area}(\triangle BGD) + \text{Area}(\triangle BGD)
\]

\[
\Rightarrow \text{Area}(\triangle ABD) = 3 \times \text{Area}(\triangle BGD)\ldots(1)
\]

(ii)

Medians intersect at centroid.
Given that $G$ is the point of intersection of medians and hence $G$ is the centroid of the triangle $ABC$.
Centroid divides the medians in the ratio 2:1
That is $AC:CD = 2:1$

Similarly $CG$ divides $AD$ in the ratio 2:1, we have,

\[
\frac{\text{Area}(\triangle AGC)}{\text{Area}(\triangle CGD)} = \frac{2}{1}
\]

\[
\Rightarrow \text{Area}(\triangle AGC) = 2 \times \text{Area}(\triangle CGD)
\]

From the figure, it is clear that,

\[
\text{Area}(\triangle ACD) = \text{Area}(\triangle AGC) + \text{Area}(\triangle CGD)
\]

\[
\Rightarrow \text{Area}(\triangle ACD) = 2 \times \text{Area}(\triangle CGD) + \text{Area}(\triangle CGD)
\]

\[
\Rightarrow \text{Area}(\triangle ACD) = 3 \times \text{Area}(\triangle CGD)\ldots(2)
\]

(iii)

Adding equations (1) and (2), we have,

\[
\text{Area}(\triangle ABD) + \text{Area}(\triangle ACD) = 3 \times \text{Area}(\triangle BGD) + 3 \times \text{Area}(\triangle CGD)
\]

\[
\Rightarrow \text{Area}(\triangle ABC) = 3 \times [\text{Area}(\triangle BGD) + \text{Area}(\triangle CGD)]
\]

\[
\Rightarrow \text{Area}(\triangle ABC) = 3 \times [\text{Area}(\triangle BGC)]
\]

\[
\Rightarrow \frac{\text{Area}(\triangle ABC)}{3} = [\text{Area}(\triangle BGC)]
\]

\[
\Rightarrow \text{Area}(\triangle BGC) = \frac{1}{3} \times \text{Area}(\triangle ABC)
\]
Solution 6:
Consider that the sides be \(x\) cm, \(y\) cm and \((37-x-y)\) cm. Also, consider that the lengths of altitudes be \(6a\) cm, \(5a\) cm and \(4a\) cm.

\[ \text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{altitude} \]

\[ \frac{1}{2} \times 6a \times 5a = \frac{1}{2} \times y \times 5a = \frac{1}{2} \times (37 - x - y) \times 4a \]

\[ 6x = 5y = 148 - 4x - 4y \]

\[ 6x = 5y \text{ and } 6x = 148 - 4x - 4y \]

\[ 6x = 5y = 0 \text{ and } 10x + 4y = 148 \]

Solving both the equations, we have

\(x = 10\) cm, \(y = 12\) cm and \((37-x-y)\) cm = 15 cm

Solution 7:
(i)

Consider the triangles \(\triangle AFE\) and \(\triangle DFC\).

\(\angle AFE = \angle DEC\) \([\text{Vertically opposite angles}]\)

\(\angle FAE = \angle DCF\) \([\text{AB and DC are parallel lines, AC is a transversal, alternate interior angles are equal}]\)

Thus, by Angle – Angle similarity, we have,

\(\triangle AFE \sim \triangle DFC\)

Therefore, we have,

\[ \frac{DF}{FE} = \frac{DC}{AE} = \frac{CF}{AF} = \frac{2}{1} \]

\[ \Rightarrow DF:FE = 2:1 \]
Since from part(i) we have DF:FE = 2:1, therefore,
Area(△DCF) = 4Area(△AFE) ...(1)
Also we know that,
Area(△ADF) + Area(△AFE) = Area(△ADE)
→ 60 + Area(△AFE) = Area(△ADE) [Area(△ADF) = 60 cm²]
→ 2Area(△ADE) = 2(60 + Area(△AFE)]
Median divides the triangle into two equal areas of triangle.
Therefore, 2Area(△ADE) = Area(△ABD)
→ Area(△ABD) = 2(60 + Area(△AFE)]
→ Area(△ABD) = 120 + 2Area(△AFE) ...(2)
Triangles with equal bases and between the parallels are of equal area.
Area(△ABD) = Area(△ACD)
Thus, Equation (2), becomes,
Area(△ACD) = 120 + 2Area(△AFE) ...(3)
From the figure, it is clear that,
Area(△ACD) = Area(△DCF) + Area(△ADF)
→ Area(△ACD) = Area(△DCF) + 60
→ Area(△ACD) = 4Area(△AEF) + 60 ...(4)
Equating equations (3) and (4), we have,
120 + 2Area(△AFE) = 4Area(△AEF) + 60

→ 2Area(△AFE) = 60
→ Area(△AFE) = \frac{60}{2}
→ Area(△AFE) = 30
→ Area(△ADE) = Area(△ADF) + Area(△AFE)
→ Area(△ADE) = 60 + 30
→ Area(△ADE) = 90 cm²

(iii)
Median of a triangle divides it into two equal areas of triangle.
Area(△ADB) = 2Area(△ADE)
→ Area(△ADB) = 2Area(△ADE)
→ Area(△ADB) = 2 × 90 cm²
→ Area(△ADB) = 180 cm²

(iv)
Since DB divides the parallelogram ABCD into two equal triangles, therefore Area of △DBC = Area of △ADB
= 180 cm²
Thus the area of the parallelogram ABCD = Area of △ADB + Area of △DBC
= 180 cm² + 180 cm²
= 360 cm²
Solution 8:
Here BCED is a parallelogram, since BD = CE and BD || CE.
\[ \text{ar}(\triangle DBC) = \text{ar}(\triangle EBC) \]
(Since they have the same base and are between the same parallels)
In \triangle ABC,
BE is the median,
So, \[ \text{ar}(\triangle EBC) = \frac{1}{2} \text{ar}(\triangle ABC) \]
Now, \[ \text{ar}(\triangle ABC) = \text{ar}(\triangle EBC) + \text{ar}(\triangle ABE) \]
Also, \[ \text{ar}(\triangle ABC) = 2 \times \text{ar}(\triangle EBC) \]
\Rightarrow \[ \text{ar}(\triangle ABC) = 2 \times \text{ar}(\triangle DBC) \]

Solution 9:
Given:
\[ \triangle CAD = 140 \text{ cm}^2 \]
\[ \triangle ODC = 172 \text{ cm}^2 \]
AB \parallel CD
As \triangle DBC and \triangle CAD have same base CD and between the same parallel lines
Hence,
\[ \text{Area of } \triangle DBC = \text{Area of } \triangle CAD = 140 \text{ cm}^2 \]
\[ \text{Area of } \triangle OAC = \text{Area of } \triangle CAD + \text{Area of } \triangle ODC = 140 \text{ cm}^2 + 172 \text{ cm}^2 = 312 \text{ cm}^2 \]
\[ \text{Area of } \triangle CDB = \text{Area of } \triangle DBC + \text{Area of } \triangle ODC = 140 \text{ cm}^2 + 172 \text{ cm}^2 = 312 \text{ cm}^2 \]