Chapter 12. Mid-point and Its Converse [ Including Intercept Theorem]

Exercise 12(A)

Solution 1:

The triangle is shown below,

Since \(M\) is the midpoint of \(AB\) and \(MN\parallel BC\) hence \(N\) is the midpoint of \(AC\). Therefore

\[MN = \frac{1}{2} BC = \frac{1}{2} \times 7 = 3.5\text{cm}\]

And \[MN = \frac{1}{2} AC = \frac{1}{2} \times 5 = 2.5\text{cm}\]

Solution 2:

The figure is shown below,

Let \(ABCD\) be a rectangle where \(PQ, RS\) are the midpoint of \(AB, BC, CD, DA\). We need to show that \(PQRS\) is a rhombus. For help we draw two diagonal \(BD\) and \(AC\) as shown in figure. Where \(BD=AC\) (Since diagonal of rectangle are equal).

Proof:

From \(\triangle ABD\) and \(\triangle BCD\)

\[PS = \frac{1}{2} BD = QR \text{ and } PS \parallel BD \parallel QR\]

\[2PS = 2QR = BD \text{ and } PS \parallel QR \quad \ldots (1)\]

Similarly \(2PQ = 2SR = AC\) and \(PQ \parallel SR \quad \ldots (2)\)

From (1) and (2) we get

\[PQ = QR = RS = PS\]

Therefore \(PQRS\) is a rhombus.

Hence proved.
Solution 3:
The figure is shown below

Given that \( \triangle ABC \) is an isosceles triangle where \( AB = AC \).
Since \( D, E, F \) are midpoints of \( AB, BC, CA \), therefore
\( 2DE = AC \) and \( 2EF = AB \) this means \( DE = EF \).
Therefore \( \triangle DEF \) is an isosceles triangle and \( DE = EF \).
Hence proved

Solution 4:
Here from triangle \( \triangle ABD \) \( P \) is the midpoint of \( AD \) and \( PR \parallel AB \), therefore \( Q \) is the midpoint of \( BD \)
Similarly \( R \) is the midpoint of \( BC \) as \( PR \parallel CD \parallel AB \)
From triangle \( \triangle ABD \) \( 2PQ = AB \) .....(1)
From triangle \( \triangle BCD \) \( 2QR = CD \) .....(2)
Now (1)+(2) =>
\[
2(PQ+QR) = AB + CD
\]
\[
PR = \frac{1}{2}(AB + CD)
\]
Hence proved
Solution 5:

Let we draw a diagonal AC as shown in the figure below,

(i) Given that $AB=11\text{cm}, CD=8\text{cm}$
From triangle $ABC$

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 11 = 5.5\text{cm}$$

From triangle $ACD$

$$OM = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4\text{cm}$$

Hence $MN=OM+ON=(4+5.5)=9.5\text{cm}$

(ii) Given that $CD=20\text{cm}, MN=27\text{cm}$
From triangle $ACD$

$$OM = \frac{1}{2} CD = \frac{1}{2} \times 20 = 10\text{cm}$$

Therefore $ON=27-10=17\text{cm}$
From triangle $ABC$

$$AB = 2ON = 2 \times 17 = 34\text{cm}$$

(iii) Given that $AB=23\text{cm}, MN=15\text{cm}$
From triangle $ABC$

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 23 = 11.5\text{cm}$$

Therefore $OM=15-11.5=3.5\text{cm}$
From triangle $ACD$

$$CD = 2OM = 2 \times 3.5 = 7\text{cm}$$
**Solution 6:**

The figure is shown below.

Let $ABCD$ be a quadrilateral where $P, Q, R, S$ are the midpoint of $AB, BC, CD, DA$. Diagonal $AC$ and $BD$ intersects at right angle at point $O$. We need to show that $PQRS$ is a rectangle.

Proof:

From $\triangle ABC$ and $\triangle ADC$

1. $2PQ = AC$ and $PQ \parallel AC$ ....(1)
2. $2RS = AC$ and $RS \parallel AC$ ....(2)

From (1) and (2) we get.

$PQ = RS$ and $PQ \parallel RS$

Similarly we can show that $PS = RQ$ and $PS \parallel RQ$

Therefore $PQRS$ is a parallelogram.

Now $PQ \parallel AC$, therefore $\angle AOD = \angle PXO = 90^\circ$ [Corresponding angle]

Again $BD \parallel RQ$, therefore $\angle PXO = \angle RQX = 90^\circ$ [Corresponding angle]

Similarly $\angle QRS = \angle RSP = \angle SPQ = 90^\circ$

Therefore $PQRS$ is a rectangle.

Hence proved.

**Solution 7:**

The required figure is shown below.

From figure,

BL$=DM$ and BL$\parallel DM$ and BLMD is a parallelogram, therefore BM$\parallel DL$

From triangle $ABY$

L is the midpoint of $AB$ and XL$\parallel BY$, therefore X is the midpoint of $AY$, i.e $AX = XY$ ....(1)

Similarly for triangle $CDX$

CY$=XY$ ....(2)

From (1) and (2)

$AX = XY = CY$ and $AC = AX + XY + CY$

Hence proved.
Solution 8:
Given that $AD = BC$ ....(1)
From the figure,
For triangle $ADC$ and triangle $ABD$
$2GH = AD$ and $2EF = AD$, therefore $2GH = 2EF = AD$ ....(2)
For triangle $BCD$ and triangle $ABC$
$2GF = BC$ and $2EH = BC$, therefore $2GF = 2EH = BC$ ....(3)
From (1), (2), (3) we get,
$2GH = 2EF = 2GF = 2EH$
$GH = EF = GF = EH$
Therefore $EFGH$ is a rhombus.
Hence proved

Solution 9:
For help we draw the diagonal $BD$ as shown below

The diagonal $AC$ and $BD$ cuts at point $X$.
We know that the diagonal of a parallelogram intersects equally each other. Therefore $AX = CX$ and $BX = DX$.

Given,
\[ CQ = \frac{1}{4} AC \]
\[ CQ = \frac{1}{4} \times 2CX \]
\[ CQ = \frac{1}{2} CX \]
Therefore $Q$ is the midpoint of $CX$.

(i) For triangle $CDX$ $PQ || DX$ or $PR || BD$.

Since for triangle $CDX$
$Q$ is the midpoint of $CX$ and $QR || BX$. Therefore $R$ is the midpoint of $BC$.

(ii) For triangle $BCD$.
As $P$ and $R$ are the midpoint of $CD$ and $BC$, therefore $\frac{FR}{DB} = \frac{1}{2}$.
Solution 10:
The required figure is shown below

For triangle ABC and OBC
2DE=BC and 2PQ=BC, therefore DE=PQ....(1)
For triangle ABO and ACO
2PD=AO and 2FO=AO, therefore PD=FO....(2)
From (1), (2) we get that PQFD is a parallelogram.
Hence proved.

Solution 11:
The required figure is shown below

From the figure it is seen that P is the midpoint of BC and PQ || AC and QR || BC
Therefore Q is the midpoint of AB and R is the midpoint of AP
(i) Therefore AP = 2AR
(ii) Here we increase QR so that it cuts AC at S as shown in the figure.
(iii) From triangle PQR and triangle ARS
\[ \angle PQR = \angle ARS \quad \text{(Opposite angle)} \]
\[ PR = AR \]
\[ PQ = AS \quad \text{[ } PQ = AS = \frac{1}{2} AC \text{] } \]
\[ \triangle PQR \cong \triangle ARS \quad \text{(SAS Postulate)} \]
Therefore QR = RS
Now
\[ BC = 2QS \]
\[ BC = 2 \times 2QR \]
\[ BC = 4QR \]
Hence proved.
Solution 12:
The required figure is shown below.

(i)
From $\triangle PBD$ and $\triangle ABP$
- $PD = AP$ [P is the midpoint of AD]
- $\angle DPE = \angle AFB$ [Opposite angle]
- $\angle PBD = \angle PBA$ [$AB\parallel CB$]

$\therefore \triangle PDE \cong \triangle ABP$ [ASA postulate]

$BP = BP$

(ii) For triangle ECB PQ||CE
Again CE||AB
Therefore PQ||AB
Hence proved

Solution 13:
The required figure is shown below.

For help we draw a line DG||BF
Now from triangle ADG, DG||BF and E is the midpoint of AD
Therefore F is the midpoint of AG, i.e. $AF = GF$ .... (1)
From triangle BCF, DG||BF and D is the midpoint of BC
Therefore G is the midpoint of CF, i.e. $GF = CF$ .... (2)

$AC = AF + GF + CF$
AC = 3AF (From (1) and (2))
Hence proved
Solution 14:
The required figure is shown below

(i) Since F is the midpoint and EF || AB.
Therefore E is the midpoint of BC.
So \( BE = \frac{1}{2} BC \) and \( EF = \frac{1}{2} AB \) ....(1)

Since D and F are the midpoint of AB and AC.
Therefore DE || BC.
So \( DF = \frac{1}{2} BC \) and \( DB = \frac{1}{2} AB \) ....(2)

From (1),(2) we get
BE = DF and BD = EF.
Hence BDEF is a parallelogram.

(ii) Since
\[ AB = 2 \times EF \]
\[ = 2 \times 4.8 \]
\[ = 9.6 \text{ cm} \]

Solution 15:

In \( \triangle ABC \),
AD is the median of BC.
\( \Rightarrow D \) is the midpoint of BC.
Given that DE \parallel PA.
By the converse of the midpoint theorem,
\( \Rightarrow \) DE bisects AC.
\( \Rightarrow E \) is the midpoint of AC.
\( \Rightarrow BE \) is the median of AC.
that is BE is also a median.
Solution 16:
Construction : Draw \(DY \parallel EQ\)
In \(\triangle ABC\) and \(\triangle DCY\),
\(\angle BCQ = \angle DCY\) (Common)
\(\angle BQC = \angle DYC\) (Corresponding angles)
So, \(\triangle ABC \sim \triangle DCY\) (AA Similarity criterion)
\[
\frac{EQ}{DY} = \frac{EC}{CY} \quad \text{(Corresponding sides are proportional)}
\]
\[
\frac{EQ}{DY} = \frac{2CD}{CD} \quad \text{(D is the mid-point of BC)}
\]
\[
\frac{EQ}{DY} = 2 \quad \text{(i)}
\]
Similarly, \(\triangle AEQ \sim \triangle ADY\)
\[
\frac{EQ}{DY} = \frac{AE}{ED} = \frac{1}{2} \quad \text{(E is the mid-point of AD)}
\]
that is \(\frac{EQ}{DY} = \frac{1}{2} \quad \text{(ii)}\)
Dividing (i) by (ii), we get
\[
\frac{EQ}{EQ} = 4
\]
\[
BE + EQ = 4EQ
\]
\[
BE = 3EQ
\]
\[
BE = \frac{3}{1}
\]

Solution 17:
In \(\triangle EDF\),
\(M\) is the mid-point of \(AB\) and \(N\) is the mid-point of \(DE\).
\[
\Rightarrow MN = \frac{1}{2}EF \quad \text{(Mid-point theorem)}
\]
\[
\Rightarrow EF = 2MN \quad \text{(i)}
\]
In \(\triangle ABC\),
\(M\) is the mid-point of \(AB\) and \(N\) is the mid-point of \(BC\).
\[
\Rightarrow MN = \frac{1}{2}AC \quad \text{(Mid-point theorem)}
\]
\[
\Rightarrow AC = 2MN \quad \text{(ii)}
\]
From (i) and (ii), we get
\[
\Rightarrow EF = AC
\]

Exercise 12(B)

Solution 1:
According to equal intercept theorem since \(CD = DE\)
Therefore \(AB = BC\) and \(EF = GF\)
(i) \(BC = AB = 7.2\text{cm}\)
(ii) \(GE = EF + GF = 2EF = 2 \times 4 = 8\text{cm}\)
Since \(BDF\) are the midpoint and \(AE \parallel BF \parallel CG\)
Therefore \(AE = 2BD\) and \(CG = 2DF\)
(iii) \(AE = 2BD = 2 \times 4.1 = 8.2\)
(iv) \(DF = \frac{1}{2}CG = \frac{1}{2} \times 11 = 5.5\text{cm}\)
Solution 2:

Given that $AD=AP=PB$ as $2AD=AB$ and $p$ is the midpoint of $AB$.

(i) From triangle DPR, A and Q are the midpoint of DP and DR.

Therefore $AQ || PR$.

Since PR || BS, hence AQ || BS.

(ii) From triangle ABC, P is the midpoint and PR || BS

Therefore R is the midpoint of BC.

From $\triangle BPS$ and $\triangle QRC$

$\angle BRS = \angle QRC$

$BR = RC$

$\angle BRS = \angle CRC$

$\therefore \triangle BRS \cong \triangle QRC$

$\therefore QR = RS$

DS = DQ + QR + RS = QR + QR + RS = 3RS

Solution 3:

Consider the figure:

Here D is the midpoint of BC and DP is parallel to AB, therefore P is the midpoint of AC and $PD = \frac{1}{2} AB$.

(i)

Again from the triangle AEF we have $AE || PD || CR$ and $AP = \frac{1}{3} AE$.

Therefore $DF = \frac{1}{3} EF$ or we can say that $3DF = EF$.

Hence it is shown.

(ii)

From the triangle PED we have PD || CR and C is the midpoint of PE therefore $CR = \frac{1}{2} PD$.

Now

$PD = \frac{1}{2} AB$

$\frac{1}{2} PD = \frac{1}{4} AB$

$CR = \frac{1}{4} AB$

$4CR = AB$

Hence it is shown.
Solution 4:
The figure is shown below

From triangle BPC and triangle APN
\[ \angle BPC = \angle APN \quad \text{[Opposite angle]} \]
\[ BP = AP \]
\[ PC = PN \]
\[ \therefore \triangle BPC \cong \triangle APN \quad \text{[SAS postulate]} \]
\[ \therefore \angle PBC = \angle PAN \quad \text{.....(1)} \]

And BC=AN ......(3)

Similarly \[ \angle QCB = \angle QAN \quad \text{.....(2)} \]

And BC=AM ......(4)

Now
\[ \angle ABC + \angle ACB + \angle BAC = 180^\circ \]
\[ \angle PAN + \angle QAM + \angle EAC = 180^\circ \quad \text{[(1),(2) we get]} \]

Therefore M,A,N are collinear

(ii) From (3) and (4) MA=NA

Hence A is the midpoint of MN
Solution 5:

The figure is shown below.

From the figure EF \parallel AB and E is the midpoint of BC.

Therefore F is the midpoint of AC.

Here EF \parallel BD, EF = BD as D is the midpoint of AB.

BE \parallel DF, BE = DF as E is the midpoint of BC.

Therefore BEFD is a parallelogram.
Solution 6:
The figure is shown below

(i)
From $\triangle HBB$ and $\triangle FHC$
$BB = FC$
$\angle EHB = \angle FHC$ \quad [\text{Opposite angle}]
$\angle HBB = \angle HFC$
$\therefore \triangle HBB \cong \triangle FHC$
$\therefore EH = CH, BH = FH$

(ii)
Similarly AG = GF and EG = DG \quad (1)

For triangle ECD, F and H are the midpoint of CD and EC.

Therefore $HF \parallel DE$ and $HF = \frac{1}{2} DB$ \quad (2)

(1),(2) we get, $HF = EG$ and $HF \parallel EG$

Similarly we can show that $EH = GF$ and $EH \parallel GF$

Therefore GEHF is a parallelogram.
**Solution 7:**
The figure is shown below

For triangle AEG
D is the midpoint of AE and DF $\parallel$ EG $\parallel$ BC
Therefore F is the midpoint of AG.
$AF=GF ....(1)$
Again DF $\parallel$ EG $\parallel$ BC DE=BE, therefore GF=GC ....(2)
(1),(2) we get AF=GF=GC.
Similarly Since GN $\parallel$ FM $\parallel$ AB and AF $\parallel$ GF, therefore BM=MN=NC
Hence proved

**Solution 8:**
The figure is shown below

Since M and N are the midpoint of AB and AC, MN $\parallel$ BC
According to intercept theorem Since MN $\parallel$ BC and AM=BM,
Therefore AX=DX. Hence proved
Solution 9:
The figure is shown below

Let ABCD be a quadrilateral where PQR,RS are the midpoint of AB,BC,CD,DA,PQRS is a rectangle. Diagonal AC and BD intersect at point O. We need to show that AC and BD intersect at right angle.

Proof:
PQ||AC, therefore \( \angle AOD = \angle PXO \) [Corresponding angle] \( \ldots (1) \)

\( \angle AOB = \angle RQX = 90^\circ \) [Corresponding angle and angle of rectangle] \( \ldots (2) \)

From (1) and (2) we get,
\( \angle AOD = 90^\circ \)

Similarly \( \angle BOC = \angle DOC = 90^\circ \)

Therefore diagonals AC and BD intersect at right angle

Hence proved

Solution 10:
The figure is shown below

From figure since E is the midpoint of AC and EF||AB

Therefore F is the midpoint of BC and 2DE=BC or DE=BF

Again D and E are midpoint, therefore DE||BF and EF=BD

Hence BDEF is a parallelogram.

Now

\( BD = BF = \frac{1}{2} AB = \frac{1}{2} \times 1.6 = 0.8 \text{ cm} \)

\( BF = DE = \frac{1}{2} BC = \frac{1}{2} \times 18 = 9 \text{ cm} \)

Therefore perimeter of BDEF=2(BF+EF)=2(9+8)=34 cm
Solution 11:
Given AD and CE are medians and DF || CE.
We know that from the midpoint theorem, if two lines are parallel and the starting point of segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.
Consider triangle BEC. Given DF || CE and D is midpoint of BC.
So F must be the midpoint of BE.

So \( FB = \frac{1}{2} BE \) but \( BE = \frac{1}{2} AB \)

Substitute value of \( BE \) in first equation, we get

\( FB = \frac{1}{4} AB \)

Hence Proved

Solution 12:
Given ABCD is parallelogram, so \( AD = BC, AB = CD \).
Consider triangle APB, given EC is parallel to AP and E is midpoint of side AB. So by midpoint theorem, C has to be the midpoint of BP.
So BP = 2BC, but BC = AD as ABCD is a parallelogram.
Hence \( BP = 2AD \)
Consider triangle APB, AB || OC as ABCD is a parallelogram. So by midpoint theorem, O has to be the midpoint of AP.
Hence Proved

Solution 13:
Consider trapezium ABCD.
Given E and F are midpoints on sides AD and BC, respectively.

![Trapezium ABCD with midpoints E and F](image)

We know that \( AB = GH = IJ \)

From midpoint theorem, \( EG = \frac{1}{2} DI, HF = \frac{1}{2} JC \)

Consider LHS,
\( AB + CD = AB + CJ + JI + ID = AB + 2HF + AB + 2EG \)
So \( AB + CD = 2(AB + HF + EG) = 2(EG + GH + HF) = 2EF \)
\( AB + CD = 2EF \)
Hence Proved

Solution 14:
Given \( \Delta ABC \)
AD is the median. So D is the midpoint of side BC.
Given DE || AB. By the midpoint theorem, E has to be midpoint of AC.
So line joining the vertex and midpoint of the opposite side is always known as median. So BE is also median of \( \Delta ABC \).