## Polygons

### IMPORTANT POINTS

1. **Polygon**: A closed plane geometrical figure, bounded by at least three line segments, is called a polygon.

   The adjoining figure is a polygon as it is:

   ![Polygon](image)

   (i) Closed
   (ii) bounded by five line segments AB, BC, CD, DE and AE.

   Also, it is clear from the given polygon that:
   (i) the line segments AB, BC, CD, DE and AE intersect at their end points.
   (ii) two line segments, with a common vertex, are not collinear i.e. the angle at any vertex is not 180°.

   A polygon is named according to the number of sides (line-segments) in it:

<table>
<thead>
<tr>
<th>No. of sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of polygon</td>
<td>Triangle</td>
<td>Quadrilateral</td>
<td>Pentagon</td>
<td>Hexagon</td>
</tr>
</tbody>
</table>

2. **Sum of Interior Angles of a Polygon**

1. **Triangle**: Students already know that the sum of interior angles of a triangle is always 180°.

   i.e. for \( \triangle ABC \), \( \angle BAC + \angle ABC + \angle ACB = 180^\circ \)

   \( \Rightarrow \angle A + \angle B + \angle C = 180^\circ \)

   ![Triangle](image)

2. **Quadrilateral**: Consider a quadrilateral ABCD as shown alongside.

   If diagonal AC of the quadrilaterals drawn, the quadrilateral will be divided into two triangles ABC and ADC.

   Since, the sum of interior angles of a triangle is 180°.
∴ In $\triangle ABC$, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$
And, in $\triangle ADC \angle DAC + \angle ADC + \angle ACD = 180^\circ$
Adding we get:
$\angle ABC + \angle BAC + \angle ACB + \angle DAC + \angle ADC + \angle ACD = 180^\circ + 180^\circ$
$\Rightarrow (\angle BAC + \angle DAC) + \angle ABC + (\angle ACB + \angle ACD) + \angle ADC = 360^\circ$
$\Rightarrow \angle BAD + \angle ABC + \angle BCD + \angle ADC = 360^\circ$
$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$
Alternative method: On drawing the diagonal $AC$, the given quadrilateral is divided into two triangles. And, we know the sum of the interior angles of a triangle is $180^\circ$.

$\therefore$ **Sum of interior angles of the quadrilateral $ABCD$**

$= \text{Sum of interior angles of } \triangle ABC + \text{sum of interior angles of } \triangle ADC = 180^\circ + 180^\circ = 360^\circ$

3. **Pentagon**

Consider a pentagon $ABCDE$ as shown alongside.

On joining $CA$ and $CE$, the given pentagon is divided into three triangles $ABC$, $CDE$ and $ACE$.

Since, the sum of the interior angles of a triangle is $180^\circ$

Sum of the interior angles of the pentagon $ABCDE = \text{Sum of interior angles of } (\triangle ABC + \triangle CDE + \triangle ACE)$

$= 180^\circ + 180^\circ + 180^\circ = 540^\circ$

4. **Hexagon**

It is clear from the given figure that the sum of the interior angles of the hexagon $ABCDEF$. 
3. Using Formula: The sum of interior angles of a polygon can also be obtained by using the following formula:

Note: Sum of interior angles of a polygon = \((n - 2) \times 180°\)
where, \(n\) = number of sides of the polygon.

\(\therefore (i)\) For a triangle:

\[ n = 3 \text{ (a triangle has 3 sides)} \]
and, sum of interior angles  \(= (2n - 4) \times 90°\)
\[ = (6 - 4) \times 90° = 180° \]

\((ii)\) For a quadrilateral:

\[ n = 4 \]
and, sum of interior angles  \(= (2n - 4) \times 90°\)
\[ = (8 - 4) \times 90° = 360° \]

\((iii)\) For a pentagon:

\[ n = 5 \]
and, sum of interior angles  \(= (2n - 4) \times 90°\)
\[ = (10 - 4) \times 90° = 6 \times 90° = 540° \]

\((iv)\) For a hexagon:

\[ n = 6 \]
and, sum of interior angles  \(= (2n - 4) \times 90°\)
\[ = (12 - 4) \times 90° = 8 \times 90° = 720° \]

EXERCISE 28 (A)

Question 1.
State, which of the following are polygons:
Solution: Only figure (ii) and (iii) are polygons.

Question 2.
Find the sum of interior angles of a polygon with:
(i) 9 sides
(ii) 13 sides
(iii) 16 sides
Solution:
(i) 9 sides
No. of sides n = 9
\[ \text{Sum of interior angles of polygon} = (2n - 4) \times 90^\circ \]
\[ = (2 \times 9 - 4) \times 90^\circ \]
\[ = 14 \times 90^\circ = 1260^\circ \]
(ii) 13 sides
No. of sides n = 13
\[ \text{Sum of interior angles of polygon} = (2n - 4) \times 90^\circ = (2 \times 13 - 4) \times 90^\circ = 1980^\circ \]
(iii) 16 sides
No. of sides n = 16
\[ \text{Sum of interior angles of polygon} = (2n - 4) \times 90^\circ \]
\[ = (2 \times 16 - 4) \times 90^\circ \]
\[ = (32 - 4) \times 90^\circ = 28 \times 90^\circ \]
\[ = 2520^\circ \]

Question 3.
Find the number of sides of a polygon, if the sum of its interior angles is:
(i) 1440°
(ii) 1620°
Solution:
Question 4.
Is it possible to have a polygon, whose sum of interior angles is 1030°.
Solution:

(i) 1440°
Let no. of sides = n
∴ Sum of interior angles of polygon = 1440°
∴ (2n - 4) × 90° = 1440°

⇒ 2n - 4 = \frac{1440°}{90°}

⇒ 2(n - 2) = \frac{1440°}{90°}

⇒ n - 2 = \frac{1440°}{2 \times 90°}

⇒ n - 2 = 8
⇒ n = 8 + 2
⇒ n = 10

(ii) 1620°
Let no. of sides = n
∴ Sum of interior angles of polygon = 1620°
∴ (2n - 4) × 90° = 1620°

⇒ 2(n - 2) = \frac{1620°}{90°}

⇒ n - 2 = \frac{1620°}{2 \times 90°}

⇒ n - 2 = 9.
⇒ n = 9 + 2 ⇒ n = 11
Let no. of sides be = \( n \)

Sum of interior angles of polygon = 1030°

\[ (2n - 4) \times 90° = 1030° \]

\[ \Rightarrow 2(n - 2) = \frac{1030°}{90°} \]

\[ \Rightarrow (n - 2) = \frac{1030°}{2 \times 90°} \]

\[ \Rightarrow (n - 2) = \frac{103}{18} \]

\[ \Rightarrow n = \frac{103}{18} + 2 \]

\[ \Rightarrow n = \frac{139}{18} \]

Which is not a whole number. Hence it is not possible to have a polygon, the sum of whose interior angles is 1030°.

**Question 5.**

(i) If all the angles of a hexagon are equal, find the measure of each angle.
(ii) If all the angles of an octagon are equal, find the measure of each angle,
Solution:

(i) No. of sides of hexagon, \( n = 6 \)
Let each angle be \( x^\circ \)
\[ \therefore \text{Sum of angles} = 6x^\circ \]
\[ \therefore (2n - 4) \times 90^\circ = \text{Sum of angles} \]
\[ (2 \times 6 - 4) \times 90^\circ = 6x^\circ \]
\[ (12 - 4) \times 90^\circ = 6x^\circ \]
\[ \Rightarrow \frac{8 \times 90^\circ}{6} = x^\circ \]
\[ \Rightarrow x = 120^\circ \]
\[ \therefore \text{Each angle of hexagon} = 120^\circ \]
(ii) No. of sides of octagon \( n = 8 \)
Let each angle be \( x^\circ \)
\[ \therefore \text{Sum of angles} = 8x^\circ \]
\[ \therefore (2n - 4) \times 90^\circ = \text{Sum of angles} \]
\[ (2 \times 8 - 4) \times 90^\circ = 8x^\circ \]
\[ 12 \times 90^\circ = 8x^\circ \]
\[ \Rightarrow x^\circ = \frac{90^\circ \times 12^\circ}{8} \Rightarrow x^\circ = 135^\circ \]
\[ \therefore \text{Each angle of octagon} = 135^\circ \]

Question 6.
One angle of a quadrilateral is \( 90^\circ \) and all other angles are equal; find each equal angle.
Solution:

Let the angles of a quadrilateral be \( x^\circ \), \( x^\circ \), \( x^\circ \), and \( 90^\circ \)
\[ \therefore \text{Sum of interior angles of quadrilateral} = 360^\circ \]
\[ \Rightarrow x^\circ + x^\circ + x^\circ + 90^\circ = 360^\circ \]
\[ \Rightarrow 3x^\circ = 360^\circ - 90^\circ \]
\[ \Rightarrow x = \frac{270^\circ}{3} \]
\[ \Rightarrow x = 90^\circ \]

Question 7.
If angles of quadrilateral are in the ratio \( 4 : 5 : 3 : 6 \); find each angle of the quadrilateral.
Solution:

Let the angles of the quadrilateral be $4x$, $3x$, $3x$, and $6x$.

$4x + 5x + 3x + 6x = 360^\circ$

$18x = 360^\circ$

$x = \frac{360^\circ}{18} = 20^\circ$

$\therefore$ First angle $= 4x = 4 \times 20^\circ = 80^\circ$

Second angle $= 5x = 5 \times 20^\circ = 100^\circ$

Third angle $= 3x = 3 \times 20^\circ = 60^\circ$

Fourth angle $= 6x = 6 \times 20^\circ = 120^\circ$

Question 8.
If one angle of a pentagon is $120^\circ$ and each of the remaining four angles is $x^\circ$, find the magnitude of $x$.

Solution:
One angle of a pentagon = $120^\circ$
Let remaining four angles be $x$, $x$, $x$, and $x$
Their sum = $4x + 120^\circ$
But sum of all the interior angles of a pentagon = $(2n - 4) \times 90^\circ$
= $(2 \times 5 - 4) \times 90^\circ = 540^\circ$
= $3 \times 180^\circ = 540^\circ$
$\therefore$ $4x + 120^\circ = 540^\circ$
$4x = 540^\circ - 120^\circ$

$4x = 420$
$x = \frac{420}{4} \Rightarrow x = 105^\circ$

$\therefore$ Equal angles are $105^\circ$ (Each)

Question 9.
The angles of a pentagon are in the ratio $5 : 4 : 5 : 7 : 6$; find each angle of the pentagon.
Solution:

Let the angles of the pentagon be 5x, 4x, 5x, 7x, 6x

Their sum = 5x + 4x + 5x + 7x + 6x = 27x

Sum of interior angles of a polygon
= (2n - 4) × 90°
= (2 × 5 - 4) × 90° = 540°

∴ 27x = 540 \Rightarrow \frac{540}{27} \Rightarrow x = 20°

∴ Angles are 5 \times 20° = 100°
    4 \times 20° = 80
    5 \times 20° = 100°
    7 \times 20° = 140°
    6 \times 20° = 120°

Question 10.
Two angles of a hexagon are 90° and 110°. If the remaining four angles are equal, find each equal angle.

Solution:

Two angles of a hexagon are 90°, 110°
Let remaining four angles be x, x, x and x

Their sum = 4x + 200°
But sum of all the interior angles of a hexagon
= (2n - 4) × 90°
= (2 \times 6 - 4) × 90° = 8 \times 90° = 720°
∴ 4x + 200° = 720°
⇒ 4x = 720° - 200° = 520°
⇒ x = \frac{520°}{4} = 130°
∴ Equal angles are 130° (each)
Question 1.
Fill in the blanks:
In case of regular polygon, with

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Each exterior angle</th>
<th>Each interior angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 6</td>
<td>..........</td>
<td>..........</td>
</tr>
<tr>
<td>(ii) 8</td>
<td>..........</td>
<td>..........</td>
</tr>
<tr>
<td>(iii) ..........</td>
<td>36°</td>
<td>..........</td>
</tr>
<tr>
<td>(iv) ..........</td>
<td>20°</td>
<td>..........</td>
</tr>
<tr>
<td>(v) ..........</td>
<td>..........</td>
<td>135°</td>
</tr>
<tr>
<td>(vi) ..........</td>
<td>..........</td>
<td>165°</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Each exterior angle</th>
<th>Each interior angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 6</td>
<td>60°</td>
<td>120°</td>
</tr>
<tr>
<td>(ii) 8</td>
<td>45°</td>
<td>135°</td>
</tr>
<tr>
<td>(iii) 10</td>
<td>36°</td>
<td>144°</td>
</tr>
<tr>
<td>(iv) 18</td>
<td>20°</td>
<td>160°</td>
</tr>
<tr>
<td>(v) 8</td>
<td>45°</td>
<td>135°</td>
</tr>
<tr>
<td>(vi) 24</td>
<td>15°</td>
<td>165°</td>
</tr>
</tbody>
</table>
(i) Each exterior angle $= \frac{360^\circ}{6} = 60^\circ$
   Each interior angle $= 180^\circ - 60^\circ = 120^\circ$

(ii) Each exterior angle $= \frac{360^\circ}{8} = 45^\circ$
    Each interior angle $= 180^\circ - 45^\circ = 135^\circ$

(iii) Since each exterior angles $= 36^\circ$
    $\therefore$ Number of sides $= \frac{360^\circ}{36^\circ} = 10$
    Also, interior angle $= 180^\circ - 20^\circ = 160^\circ$

(iv) Since each exterior angles $= 20^\circ$
    $\therefore$ Number of sides $= \frac{360^\circ}{20^\circ} = 18$
    Also, interior angle $= 180^\circ - 20^\circ = 160^\circ$

(v) Since interior angle $= 135^\circ$
    $\therefore$ Exterior angle $= 180^\circ - 135^\circ$
    $\therefore$ Number of sides $= \frac{360^\circ}{45^\circ} = 8$

(vi) Since interior angle $= 165^\circ$
    $\therefore$ Exterior angle $= 180^\circ - 165^\circ = 15^\circ$
    $\therefore$ Number of sides $= \frac{360^\circ}{15^\circ} = 24$

**Question 2.**
Find the number of sides in a regular polygon, if its each interior angle is:
(i) $160^\circ$
(ii) $150^\circ$
Solution:

(i) 160°
Let no. of sides of regular polygon be \( n \)
Each interior angle = 160°

\[
\frac{(2n - 4) \times 90°}{n} = 160°
\]

\[
180n - 360° = 160n
\]
\[
180n - 160n = 360°
\]
\[
n = \frac{360°}{20}
\]
\[
n = 18
\]

(ii) 150°
Let no. of sides of regular polygon be \( n \)
Each interior angle = 150°

\[
\frac{(2n - 4) \times 90°}{n} = 150°
\]

\[
180n - 360° = 150n
\]
\[
180n - 150n = 360°
\]
\[
30n = 360°
\]
\[
n = \frac{360°}{30}
\]
\[
n = 12
\]

Question 3.
Find number of sides in a regular polygon, if its each exterior angle is:
(i) 30°
(ii) 36°
Solution:

(i) 30°
Let number of sides = \( n \)
\[ \therefore \quad \frac{360^\circ}{n} = 30^\circ \]

\[ n = \frac{360^\circ}{30^\circ} \]

\[ n = 12 \]

(ii) \[ 36^\circ \]

Let number of sides = \( n \)

\[ \therefore \quad \frac{360^\circ}{n} = 36^\circ \]

\[ n = \frac{360^\circ}{36^\circ} \]

\[ n = 10 \]

**Question 4.**

Is it possible to have a regular polygon whose each interior angle is:

(i) \( 135^\circ \)

(ii) \( 155^\circ \)

**Solution:**
Question 5.
Is it possible to have a regular polygon whose each exterior angle is:
(i) $100^\circ$
(ii) $36^\circ$

\begin{align*}
(i) & \quad 135^\circ \\
\text{No. of sides} & = n \\
\text{Each interior angle} & = 135^\circ \\
\therefore \quad \frac{(2n - 4) \times 90^\circ}{n} & = 135^\circ \\
180n - 360^\circ & = 135n \\
180n - 135n & = 360^\circ \\
\therefore \quad n & = \frac{360^\circ}{45^\circ} \\
n & = 8 \\
\text{Which is a whole number.} \\
\text{Hence, it is possible to have a regular polygon whose interior angle is 135}^\circ. \\

(ii) & \quad 155^\circ \\
\text{No. of sides} & = n \\
\text{Each interior angle} & = 155^\circ \\
\therefore \quad \frac{(2n - 4) \times 90^\circ}{n} & = 155^\circ \\
180n - 360^\circ & = 155n \\
180n - 155n & = 360^\circ \\
25n & = 360^\circ \\
\therefore \quad n & = \frac{360^\circ}{25^\circ} \\
n & = \frac{72^\circ}{5} \\
\text{Which is not a whole number.} \\
\text{Hence, it is not possible to have a regular polygon having interior angle is of 138}^\circ. \\
\end{align*}
(i) 100°
Let no. of sides = \( n \)
Each exterior angle = 100°

\[
= \frac{360°}{n} = 100°
\]

\( \therefore n = \frac{360°}{100°} \)

\( n = \frac{18}{5} \)

Which is not a whole number.
Hence, it is not possible to have a regular polygon whose each exterior angle is 100°.

(ii) 36°
Let number of sides = \( n \)
Each exterior angle = 36°

\[
= \frac{360°}{n} = 36°
\]

\( \therefore n = \frac{360°}{36°} \)

\( n = 10 \)
Which is a whole number.
Hence, it is possible to have a regular polygon whose each exterior angle is of 36°.

**Question 6.**
The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find:
(i) each exterior angle of this polygon.
(ii) number of sides in the polygon.
Solution:

(i) Interior angle : exterior angle = 2 : 1

\[ \therefore \text{Let interior angle} = 2x^\circ \]

and exterior angle = \( x^\circ \)

\[ \therefore \ 2x^\circ + x^\circ = 180^\circ \]

\[ 3x^\circ = 180^\circ = x = \frac{180^\circ}{3} = 60^\circ \]

(ii) \( x = 60 \)

\[ \therefore \text{Each exterior angle} = 60^\circ \]

\[ \therefore \frac{360^\circ}{n} = 60^\circ \]

\[ n = \frac{360^\circ}{60^\circ} = 6 \text{ sides} \]