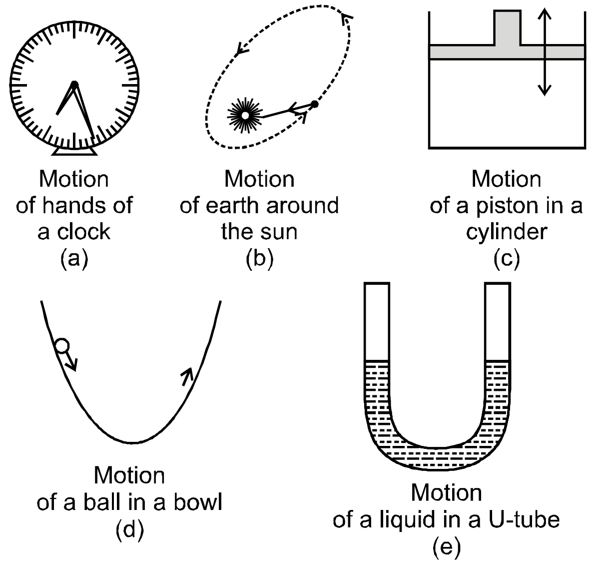
Simple Harmonic Motion

1. Periodic Motion

- A motion which repeats itself after regular interval of time is called periodic motion and the time interval after which the motion is repeated is called **time period**.
- A periodic motion can be either rectilinear or closed or open curvilinear (Fig.)



- In case of periodic motion, force is always directed towards a fixed point.

2. Oscillatory Motion

- The motion of body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after regular interval of time.
- The fixed point about which the body oscillates is called **mean position** or equilibrium position.

Examples

(i) Vibration of the wire of 'Sitar'.
(ii) Oscillation of the mass suspended from spring.

Note :-

- Every oscillatory motion is periodic but every periodic motion is not oscillatory.
- Oscillatory (or vibratory) motion is a constrained periodic motion between certain precisely fixed limits.
- For any body to undergo oscillation, the body must experience a force which is always directed towards mean position of the body. **(Restoring force)**
  It is given by $F = -kx$.

3. Simple Harmonic Motion

If in case of oscillatory motion a particle moves back and forth (or up and down) about a fixed point (called equilibrium position) through a force or torque (called restoring force or torque) which is directly proportional to the displacement (or angular displacement) but directed opposite to it, the motion is called simple harmonic.

i.e. $F \propto -y$ or $\tau \propto -\theta$

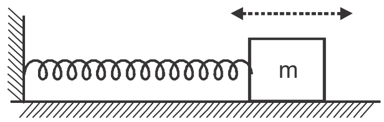
Simple harmonic motion is the simplest form of vibratory or oscillatory motion.

3.1 Types of SHM

(a) Linear S.H.M.

When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line, then its motion is called linear simple harmonic motion.

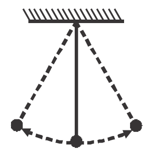
Example : Motion of a mass connected to spring.



(b) Angular S.H.M.

When a system oscillates angularly with respect to a fixed axis, then its motion is called angular simple harmonic motion.

Example :- Motion of a bob of simple pendulum.



3.2 Necessary Condition to Execute S.H.M.

- Motion of particle should be oscillatory.
- Total mechanical energy of particle should be conserved (Kinetic energy + Potential energy = constant)
- extreme position should be well defined.
- **In linear S.H.M.**

  The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

  $\therefore \quad F \propto -y \qquad \text{or} \qquad a \propto -y$

  Negative sign shows that direction of force and acceleration is towards equilibrium position and y is displacement of particle from equilibrium position.

3.3 Equation of SHM

The necessary and sufficient condition for a motion to be simple harmonic (linear) is

$F = -ky,$

i.e. $\quad m\dfrac{d^2y}{dt^2} = -ky \quad \left[\text{as}\, F = ma = m\dfrac{d^2y}{dt^2}\right]$

or $\quad \dfrac{d^2y}{dt^2} = -\omega^2 y$ with $\omega^2 = \dfrac{k}{m}$ ...(i)

So $\quad \dfrac{d^2y}{dt^2} + \dfrac{k}{m}\,y = 0$

is termed as **differential equation** of linear SHM

on solving this differential equation, we get

$y = A\sin(\omega t + \phi)$

This is the equation of linear SHM

Example 1:

If equation of displacement of a particle is $y = A \sin Qt + B \cos Qt$ then find the nature of the motion of particle.

Solution:

$y = A\sin Qt + B\cos Qt$

Differentiate with respect to t

$$\frac{dy}{dt} = AQ \cos Qt - BQ \sin Qt$$

Again differentiating with respect to t

$$\frac{d^2y}{dt^2} = -Q^2A \sin Qt - Q^2 B \cos Qt$$

$$\frac{d^2y}{dt^2} = -Q^2 (A\sin Qt + B\cos Qt)$$

$$\Rightarrow \frac{d^2y}{dt^2} = -Q^2y$$

$$\Rightarrow \frac{d^2y}{dt^2} + Q^2y = 0$$

It is differential equation of linear S.H.M. So motion of the particle is simple harmonic.

Example 2:

Which of the following functions are

(a) aperiodic (nonperiodic)

(b) periodic but not simple harmonic

(c) simple harmonic :

(i) $\sin 2\omega t$

(ii) $1 + \cos 2\omega t$

(iii) $a \sin \omega t + b \cos \omega t$,

(iv) $\sin\omega t + \sin 2\omega t + \cos 2\omega t$

(v) $\sin^3\omega t$

(vi) $\log(1+ \omega t)$

(vii) $\exp(-\omega t)$?

Solution:

(a) Functions (vi) $\log(1 + \omega t)$ and (vii) $\exp(-\omega t)$ increase (or decrease) continuously with time and can never repeat themselves so are aperiodic.

(b) Functions (iv) $\sin\omega t + \sin 2\omega t + \cos 2\omega t$ and (v) $\sin^3\omega t$ are periodic [i.e., $f(t + T) = f(t)$] with periodicity (π/ω), $(2\pi/\omega)$ & $(2\pi/\omega)$ respectively but not simple harmonic as for these functions (d^2y/dt^2) is not $\propto -y$.

(c) Functions (i) $\sin 2\omega t$, (ii) $(1 + \cos 2\omega t)$ and (iii) $a\sin\omega t + b\cos\omega t$,

i.e., $(a^2 + b^2)^{1/2} \sin[\omega t + \tan^{-1}(b/a)]$ are simple harmonic [with time period (π/ω), (π/ω) and $(2\pi/\omega)$ respectively] as for these $(d^2y/dt^2) \propto -y$.

3.4 Comparison Between Linear & Angular SHM

Linear S.H.M	Angular S.H.M
• $F \propto -x$ • $F = -kx$ Where k is the restoring force constant • $a = -\frac{k}{m}x$ • $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ It is known as differential equation of linear S.H.M. • $x = A\sin\omega t$ • $a = -\omega^2 X$ where ω is the angular frequency • $\omega^2 = \frac{k}{m}$ • $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi n$ where T is time period and n is frequency • $T = 2\pi\sqrt{\frac{m}{k}}$ • $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ This concept is valid for all types of linear S.H.M.	• $\tau \propto -\theta$ • $\tau = -C\theta$ Where C is the restoring torque constant • $\alpha = -\frac{C}{I}\theta$ • $\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$ It is known as differential equation of angular S.H.M. • $\theta = \theta_0\sin\omega t$ • $a = \omega^2\theta$ • $\omega^2 = \frac{C}{I}$ • $\omega = \sqrt{\frac{C}{I}} = \frac{2\pi}{T} = 2\pi n$ • $T = 2\pi\sqrt{\frac{I}{C}}$ • $n = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$ This concept is valid for all types of angular S.H.M.

3.5 Characteristics of SHM

Mean Position

The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.

Restoring Force

- The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.
- This force is always directed towards the mean position.
- Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- It is given by $F = -kx$ and has dimension $[MLT^{-2}]$.

Amplitude

The maximum displacement of particle from mean position is defined as amplitude.

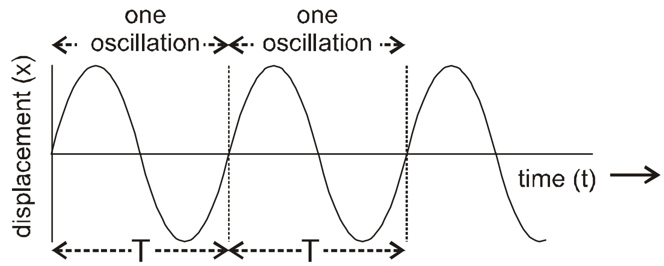
Time period (T)

- The time after which the particle keeps on repeating its motion is known as time period.
- The smallest time taken to complete one oscillation or vibration is also defined as time period.
- It is given by $T = \frac{2\pi}{\omega}$, $T = \frac{1}{n}$

where ω is angular frequency and n is frequency.

Oscillation or Vibration :

When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.

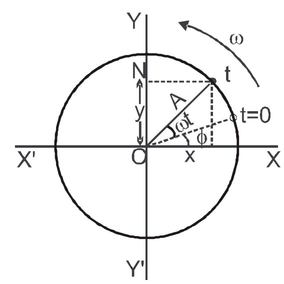


Frequency (n or f)

- The number of oscillations per second is defined as frequency.
- It is given by $n = \frac{1}{T}$, $n = \frac{\omega}{2\pi}$
- **SI unit :** hertz (Hz)
  1 hertz = 1 cycle per second (cycle is a number not a dimensional quantity).
- **Dimension :** $M^0 L^0 T^{-1}$.

Phase

- Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.
- Consider a particle in UCM. Taking projection of particle's position on Y-axis & X-axis.
  $y = A \sin(\omega t + \phi)$ or $x = A \cos(\omega t + \phi)$
  The quantity $(\omega t + \phi)$ represents the phase angle at that instant.



- The phase angle at time t = 0 is known as **initial phase** or **epoch.**
- The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.
- If the phase angles of two particles executing S.H.M. are $(\omega t + \phi_1)$ and $(\omega t + \phi_2)$ respectively, then the phase difference between two particles is given by
  $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1)$ or $\Delta\phi = \phi_2 - \phi_1$
- Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π,
  i.e., $\Delta\phi = 2n\pi \Rightarrow$ Same phase.
- Two vibrating particles are said to be in opposite phase if the phase difference between them is an odd multiple of π
  i.e., $\Delta\phi = (2n + 1)\pi \Rightarrow$ opposite phase.

Angular Frequency (ω)

- The rate of change of phase angle of a particle with respect to time is defined as its angular frequency.
- **SI Unit :** radian/second **Dimension :** $M^0 L^0 T^{-1}$
- $\omega = \sqrt{\frac{k}{m}}$

Example 3:

A particle's motion is represented as $y = 0.5 \sin(6\pi t + \frac{\pi}{4})$ then find out

(A) Amplitude (B) Time period (C) frequency (D) Initial phase

(E) Initial position (F) Location of particle at $t = \frac{1}{6}$ sec.

Solution:

On comparing with general equation

$y = A \sin(\omega t + \phi)$

(A) $A = 0.5$

(B) $\omega = 6\pi$ So, $T = \frac{2\pi}{\omega} = \frac{1}{3}$ sec.

(C) $f = \frac{\omega}{2\pi} = 3Hz.$

(D) Initial phase $\phi = \frac{\pi}{4}$

(E) at $t = 0$, $y = 0.5 \sin \frac{\pi}{4}$ $\Rightarrow y = \frac{0.5}{\sqrt{2}} \cong 0.35$

(F) at $t = \frac{1}{6}$; $y = 0.5 \sin\left(\pi + \frac{\pi}{4}\right)$ $\Rightarrow y = -0.5 \sin \frac{\pi}{4}$ or $y = -0.35$

Concept Builder-1

Q.1 In simple harmonic motion, the particle is :

(1) always accelerated (2) always retarded

(3) alternately accelerated and retarded (4) neither accelerated nor retarded

Q.2 Which of the following functions represent SHM?

(1) $\sin 2\omega t$ (2) $\sin^2 \omega t$ (3) $\sin\omega t + 2\cos\omega t$ (4) $\sin\omega t + \cos 2\omega t$

Q.3 Which one of the following is SHM ?

(1) Motion of a particle in a wave moving through a string fixed at both ends

(2) Earth spinning about its own axis

(3) Ball bouncing between two rigid vertical walls

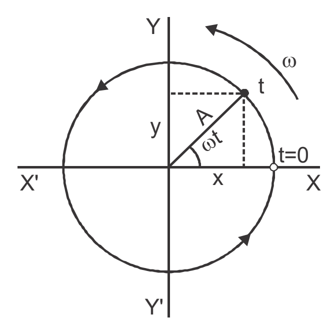
(4) Particle moving in a circle with uniform speed

3.6 Geometrical Meaning of S.H.M. (Phasor Diagrams)

If a particle is moving with uniform speed along the circumference of a circle, then the straight line motion of the foot of perpendicular drawn from the particle on the diameter of the circle is called **S.H.M**.

Description of S.H.M. Based on Circular Motion

- Draw a circle, having radius equal to amplitude (A) of S.H.M
- Suppose particle is moving with uniform speed with angular velocity ω along the circumference of the circle.
- Shadow (foot of the perpendicular from particle's position) of particle performs S.H.M. on vertical and horizontal diameter of circle.



- Position of particle shadow can be represented on diameter at t = 0 or any instant and position of particle performing circular motion can be determined by direction of velocity.
- By joining centre of circle to particle position, angle θ is determined from horizontal or vertical diameter. After time t radius vector will turn ωt. so $\theta = \omega t$.

Example 4:

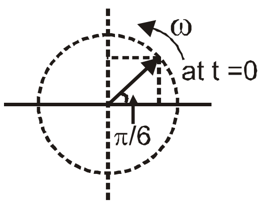
Represent the equation in phasor form:

$$y = A \sin (\omega t + \frac{\pi}{6})$$

Solution:

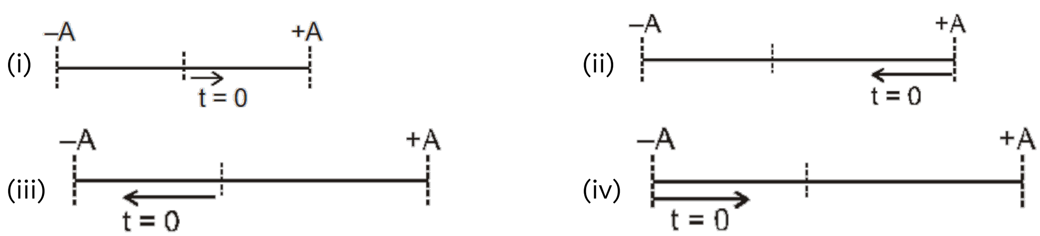
The phasors can be drawn in different manners depending upon the standards :

Let us assume, the projection being taken on vertical diameter & ACW direction to be positive then



Example 5:

What will be the equation of displacement in the following different conditions? If the angular frequency of SHM is ω.



Solution:

(i) $x = A \sin\omega t$ (ii) $x = A\sin\left[\omega t + \frac{\pi}{2}\right]$ (ii) $x = A \sin (\omega t + \pi)$ (iv) $x = A \sin\left[\omega t + \frac{3\pi}{2}\right]$

Example 6:

How long after the beginning of motion is the displacement of a harmonically oscillating point equal to one half of its amplitude if the period is 24 sec. and initial phase is zero ?

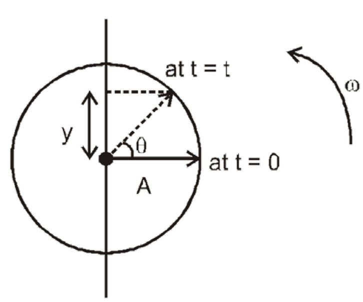
(1) 12 sec (2) 2 sec (3) 4 sec (4) 6 sec

Solution:

Using the phasor diagram

$$y = A \sin \theta = \frac{A}{2} \Rightarrow \theta = 30° = \frac{\pi}{6}$$

$$\omega t = \frac{\pi}{6}; \qquad t = \frac{\pi}{6\omega} = \frac{T}{12} = 2 \text{ sec.}$$



Concept Builder-2

Q.1 Two particles are executing SHMs. The equations of their motions are:

$$y_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right) \text{ and } y_2 = 5 \sin\left(\omega t + \frac{\sqrt{3}\pi}{4}\right)$$

What is the ratio of their amplitudes ?

(1) 1 : 1 (2) 2 : 1 (3) $\sqrt{2} : 1$ (4) None of these

Q.2 The phase (at a time t) of a particle in simple harmonic motion tells :

(1) only the position of the particle at time t

(2) only the direction of motion of the particle at time t

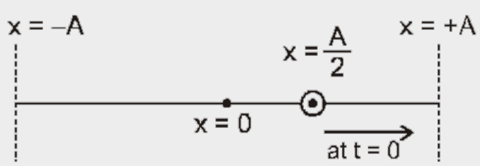
(3) both the position and direction of motion of the particle at time t

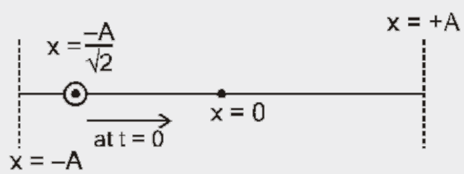
(4) neither the position of the particle nor its direction of motion at time t

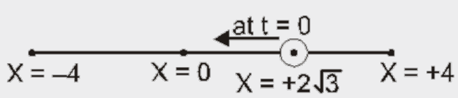
Q.3 Two particles are executing simple harmonic motion. At an instant of time t, their displacements are $y_1 = a \cos(\omega t)$ and $y_2 = b \sin(\omega t)$. Then the phase difference between y_1 and y_2 is:

(1) 120° (2) 90° (3) 180° (4) zero

Q.4 Write the equation of following SHM, All the motions are having a period of 2 sec.

(a)


(b)


(c)


Q.5 A simple harmonic motion has an amplitude A and time period T. The time required by it to travel from x = A to x = A/2 is

(1) T/6 (2) T/4 (3) T/3 (4) T/2

Q.6 A particle executes S.H.M. from extreme position and covers a distance equal to half of its amplitude in 1 s. Determine the time period of motion.

(1) 10 s (2) 16 s (3) 6 s (4) 12 s

4. Kinematic Parameters of SHM

4.1 Displacement

- Change in position vector of the particle with respect to its mean position is the displacement of the particle.
- Equation for displacement is given by
  $x = A \sin \omega t$

Note : (i) The direction of displacement is always away from the mean position whether the particle is moving away from or coming towards the mean position.
(ii) In linear S.H.M. the length of shm path = 2A
(iii) In. S.H.M. total work done in one complete oscillation is zero but total covered length is 4A

4.2 Velocity of the Particle in SHM

- It is defined as the time rate of change of the displacement of the particle at the given instant.
- Velocity in S.H.M. is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}(A \sin \omega t) \Rightarrow v = A\omega \cos \omega t$$

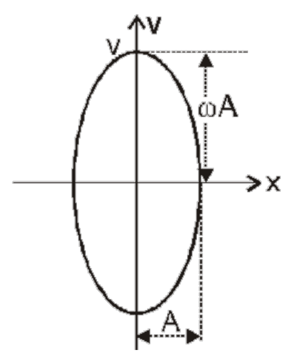
$$v = \pm A\omega\sqrt{1 - \sin^2 \omega t} \Rightarrow v = \pm A\omega\sqrt{1 - \frac{x^2}{A^2}} \qquad v = \pm \omega\sqrt{(A^2 - x^2)}$$

Squaring both the sides $v^2 = \omega^2 (A^2 - x^2)$

$$\Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2 \qquad \Rightarrow \frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \qquad \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.

- The graph between velocity and displacement is shown in figure.
  If particle oscillates with unit angular frequency ($\omega = 1$), then curve between v and x will be circular.



Note:

(i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position.
(ii) At mean position (x = 0), velocity is maximum (=Aω) and at extreme position (x = ± A), the velocity of particle executing S.H.M. is zero.

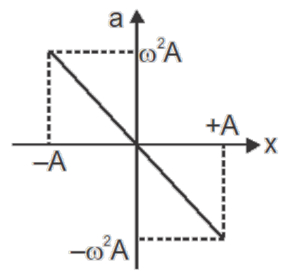
4.3 Acceleration of the Particle in SHM

- It is defined as the time rate of change of the velocity of the particle at given instant.
- Acceleration in S.H.M. is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t) = -\omega^2 A \sin \omega t$$

$$\Rightarrow \quad a = -\omega^2 x$$

- The graph between acceleration and displacement is as shown in figure



Note:

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The magnitude of acceleration of the particle executing S.H.M. is maximum at extreme position (= $\omega^2 A$) and minimum at mean position (= zero)

Example 7:

An object performs S.H.M. of amplitude 5 cm and time period 4 s. If timing is started when the object is at the centre of the oscillation i.e., x = 0 then calculate.

(i) Frequency of oscillation

(ii) The displacement at 0.5 sec.

(iii) The maximum acceleration of the object.

(iv) The velocity at a displacement of 3 cm.

Solution:

(i) Frequency $f = \frac{1}{T} = \frac{1}{4} = 0.25$ Hz

(ii) The displacement equation of object, $x = A \sin \omega t$

so at t = 0.5 s

$x = 5\sin(2\pi \times 0.25 \times 0.5)$

$= 5\sin\frac{\pi}{4} = \frac{5}{\sqrt{2}}$ cm

(iii) Maximum acceleration

$a_{max} = \omega^2 A = (0.5\pi)^2 \times 5 = 12.3$ cm/s^2

(iv) Velocity at x = 3 cm is

$v = \pm\omega\sqrt{A^2 - x^2} = \pm 0.5\pi\sqrt{5^2 - 3^2} = \pm 6.28$ cm/s

Example 8:

Amplitude of a harmonic oscillator is A, when velocity of particle is half of maximum velocity, then determine position of particle.

Solution:

$v = \omega\sqrt{A^2 - x^2}$ but $v = \frac{v_{max}}{2} = \frac{A\omega}{2}$

$\frac{A\omega}{2} = \omega\sqrt{A^2 - x^2}$ $\Rightarrow$ $A^2 = 4[A^2 - x^2]$

$\Rightarrow x^2 = \frac{4A^2 - A^2}{4}$ $\Rightarrow$ $x = \frac{\sqrt{3}A}{2}$

Example 9:

The velocity of a particle in S.H.M. at position x_1 and x_2 are v_1 and v_2 respectively. Determine value of time period and amplitude.

Solution:

$v = \omega\sqrt{A^2 - x^2}$ $\Rightarrow$ $v^2 = \omega^2 (A^2 - x^2)$

At position x_1 velocity $v_1^2 = \omega^2 (A^2 - x_1^2)$ ... (i)

At position x_2 velocity $v_2^2 = \omega^2 (A^2 - x_2^2)$ ... (ii)

Subtracting (ii) from (i), $v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$

$$\Rightarrow \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

Time period $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

Dividing (i) by (ii), $\frac{v_1^2}{v_2^2} = \frac{A^2 - x_1^2}{A^2 - x_2^2}$

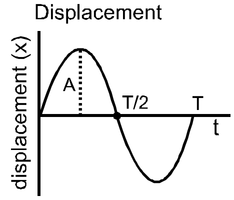
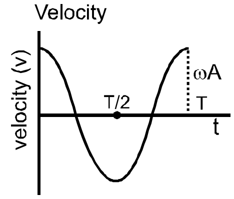
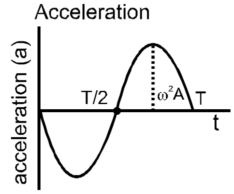
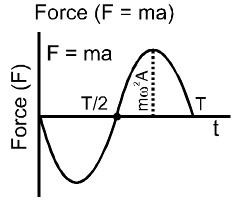
$$\Rightarrow v_1^2A^2 - v_1^2x_2^2 = v_2^2A^2 - v_2^2x_1^2$$

So $A^2(v_1^2 - v_2^2) = v_1^2x_2^2 - v_2^2x_1^2$

$$\Rightarrow A = \sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}$$

5. Graphical Representation of SHM

Graphical study of displacement, velocity, acceleration and force in S.H.M.

S.N	Graph	In terms of time	In terms of x	Maximum value
1	Displacement 	$x = A\sin\omega t$	$x = x$	$x = \pm A$
2	Velocity 	$v = A\omega\cos\omega t$	$v = \pm\omega\sqrt{A^2 - x^2}$	$v = \pm\omega A$
3	Acceleration 	$a = -\omega^2 A\sin\omega t$	$a = -\omega^2 x$	$a = \pm\omega^2 A$
4	Force (F = ma) 	$F = -m\omega^2 A\sin\omega t$	$F = -m\omega^2 x$	$F = \pm m\omega^2 A$

Concept Builder-3

Q.1 The displacement of a particle executing SHM is given by y =0.25 sin 200t cm. The maximum speed of the particle is :

(1) 200 cms^{-1} (2) 100 cms^{-1} (3) 50 cms^{-1} (4) 5.25 cms^{-1}

Q.2 When a particle executes SHM, there is always a constant ratio between its displacement and

(1) velocity (2) acceleration (3) mass (4) time period

Q.3 If a particle is executing simple harmonic motion then acceleration of particle :

(1) is uniform (2) varies linearly with time

(3) is non-uniform (4) Both (2) and (3)

Q.4 The maximum acceleration of a body moving in SHM is a_0 and maximum velocity is v_0. The amplitude is given by :

(1) $\frac{v_0^2}{a_0}$ (2) a_0v_0 (3) $\frac{a_0^2}{v_0}$ (4) $\frac{1}{a_0v_0}$

Q.5 A body executing SHM has its velocity 10 cm/sec and 7 cm/sec when its displacement from the mean position are 3 cm and 4 cm respectively. Calculate the length of the parth.

Q.6 The displacement of a particle executing simple harmonic motion is given by $y = 10\sin(6t + \frac{\pi}{3})$. Here y is in metre and t is in second. Find initial displacement & velocity of the particle.

6. Energy of Particle in SHM

6.1 Potential Energy (U or PE)

(a) In Terms of Displacement

The potential energy is related to force by the relation

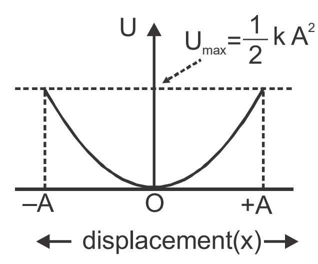
$$F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F dx$$

$$\int dU = -\int(-kx)dx = \int kx\,dx \Rightarrow U = \frac{1}{2}kx^2 + C$$

At x = 0, $U = U_0$ $\Rightarrow$ $C = U_0$

So $U = \frac{1}{2}kx^2 + U_0$

the potential energy at equilibrium position = U_0



When $U_0 = 0$ then $U = \frac{1}{2}kx^2$

(b) In Terms of Time

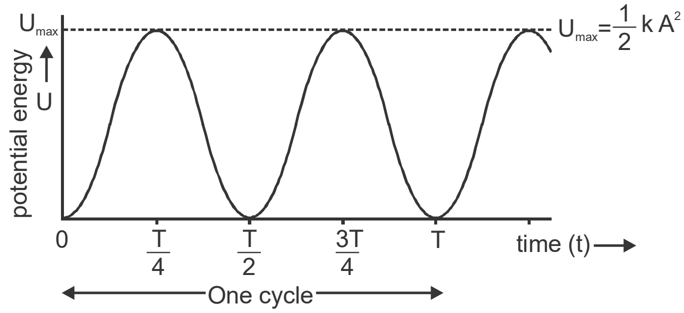
Since $x = A \sin(\omega t + \phi)$

P. E. $U = \frac{1}{2} kA^2 \sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero

$U = \frac{1}{2} kA^2 \sin^2 \omega t$ (as $k = \omega^2 m$)

$\Rightarrow$ $U = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$



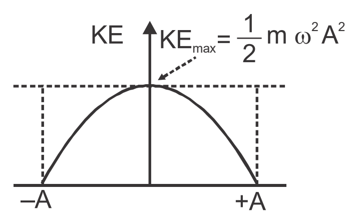
Note :

In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position (x = 0) and maximum at extreme position (x = ±A)

6.2 Kinetic Energy (K. E.)

(a) In Terms of Displacement

If mass of the particle executing S.H.M. is m and Its velocity is v then kinetic energy at any instant.



$K.E. = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2)$

$K.E. = \frac{1}{2} k(A^2 - x^2)$

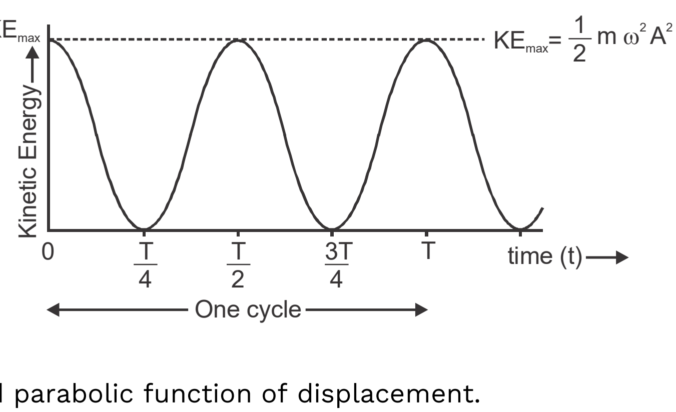
(b) In Terms of Time

$v = A\omega \cos(\omega t + \phi)$

$K. E. = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)$

If initial phase ϕ is zero

$K. E. = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$



Note:

In S.H.M. the kinetic energy is an inverted parabolic function of displacement.

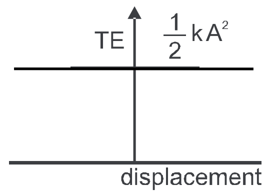
The kinetic energy is maximum ($\frac{1}{2} kA^2$) at mean position (x = 0) and minimum (zero) at extreme position (x = ±A)

6.3 Total Energy (E)

Total energy in S.H.M. is given by ; E = P. E. + K. E.

(a) In Terms of Displacement

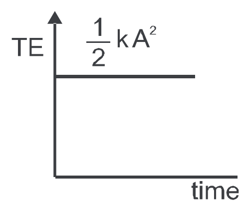
$$E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \quad \Rightarrow E = \frac{1}{2}kA^2$$



(b) In Terms of Time

$$E = \frac{1}{2}m\omega^2A^2 \sin^2 \omega t + \frac{1}{2}m\omega^2A^2 \cos^2 \omega t$$

$$E = \frac{1}{2}m\omega^2A^2 \quad \Rightarrow E = \frac{1}{2}kA^2$$



Note:

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

Average Energy in S.H.M.

- The **time average** of P.E. and K.E. over one cycle is

(a) $\langle K.E.\rangle_t = \frac{1}{4}kA^2$

(b) $\langle P.E.\rangle_t = \frac{1}{4}kA^2 + U_0$

(c) $\langle T.E.\rangle_t = \frac{1}{2}kA^2 + U_0$

Key Points

- Both K. E. and P. E. vary periodically but the variation is not simple harmonic.
- The frequency of oscillation of P. E. and K. E. is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
- Frequency of total energy is zero

Example 10:

In case of simple harmonic motion –

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude.

(b) At what displacement the kinetic and potential energies are equal.

Solution:

In S.H.M.

K. E. = $\frac{1}{2}k(A^2 - x^2)$ P. E. = $\frac{1}{2}kx^2$ T.E. = $\frac{1}{2}kA^2$

(a) $f_{K.E.} = \frac{K.E.}{T.E.} = \frac{A^2 - x^2}{A^2}$

$f_{P.E.} = \frac{P.E.}{T.E} = \frac{x^2}{A^2}$

at $x = \frac{A}{2}$ $f_{K.E.} = \frac{A^2 - A^2/4}{A^2} = \frac{3}{4}$

and $f_{P.E.} = \frac{A^2/4}{A^2} = \frac{1}{4}$

(b) K. E. = P. E.

$\Rightarrow \quad \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$

$\Rightarrow \quad 2x^2 = A^2$

$\Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$

Example 11:

The potential energy of a particle oscillating on x-axis is $U = 20 + (x - 2)^2$. Here U is in joules and x in meters. Total mechanical energy of the particle is 36 J

(a) State whether the motion of the particle is simple harmonic or not.

(b) Find the mean position.

(c) Find the maximum kinetic energy of the particle.

Solution:

(a) $F = -\frac{dU}{dx} = -2(x - 2)$ By assuming $x - 2 = X$, we have $F = -2X$

Since, $F \propto -X$ The motion of the particle is simple harmonic

(b) The mean position of the particle is $X = 0$

$\Rightarrow x - 2 = 0$, which gives $x = 2m$

(c) Maximum kinetic energy of the particle is,

$K_{max} = E - U_{min} = 36 - 20 = 16$ J

Note : U_{min} is 20 J at mean position or at x = 2m

Example 12:

The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of the amplitude. Then determine total energy of particle.

Solution:

$$\text{P.E.} = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}k\frac{A^2}{4} = 2.5$$

$$\Rightarrow \text{total energy} = \frac{1}{2}kA^2 = 2.5 \times 4 = 10 \text{ J}$$

Example 13:

A harmonic oscillator of force constant 4×10^6 Nm and amplitude 0.01 m has total energy 240 J. What is maximum kinetic energy and minimum potential energy ?

Solution:

$k = 4 \times 10^6$ N/m, a = 0.01 m, T.E. = 240 J,

As $\omega^2 = \frac{k}{m}$

Maximum kinetic energy

$$= \frac{1}{2}m\omega^2a^2$$

$$= \frac{1}{2}ka^2 = \frac{1}{2} \times 4 \times 10^6 \times (0.01)^2 = 200 \text{ J}$$

Minimum potential energy

= T.E. – maximum kinetic energy = 40 J

Concept Builder-4

Q.1 A particle executes SHM on a line 8 cm long. Its KE and PE will be equal when its distance from the mean position is :

(1) 4 cm (2) 2 cm (3) $2\sqrt{2}$ cm (4) $\sqrt{2}$ cm

Q.2 The value of total mechanical energy of a particle in SHM is

(1) Always constant (2) Depend on time

(3) $\frac{1}{2}kA^2\cos^2(\omega t + \phi)$ (4) $\frac{1}{2}mA^2\cos^2(\omega t + \phi)$

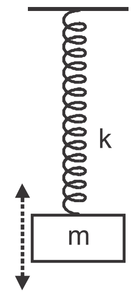
Q.3 A particle of mass 10g is placed in potential field given by V = $(50x^2 + 100)$ erg/g. What will be frequency of oscillation? (where 'x' is in cm)

Q.4 The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively:

(1) K_0 and K_0 (2) 0 and $2K_0$ (3) $\frac{K_0}{2}$ and K_0 (4) K_0 and $2K_0$

8. Spring Pendulum

- When a small mass is suspended from a mass less spring then this arrangement is known as spring pendulum.



For small linear displacement, the motion of spring pendulum is simple harmonic.

- For a spring pendulum

$F = -kx \quad \Rightarrow \quad m\frac{d^2x}{dt^2} = -kx \qquad [\because F = ma = m\frac{d^2x}{dt^2}]$

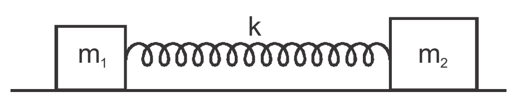
or $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ or $\frac{d^2x}{dt^2}\ \omega^2 x$ with $\omega^2 = \frac{k}{m}$

This is standard equation of linear S.H.M.

Time period $T = \frac{2\pi}{\omega} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

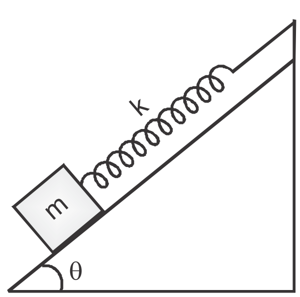
- Time period of a spring pendulum is independent of acceleration due to gravity. This is why a clock based on oscillation of spring pendulum will keep proper time everywhere on a hill or moon or in a satellite or different places of earth.
- By increasing the mass, time period of spring pendulum increases $(T \propto \sqrt{m})$, but by increasing the force constant of spring (k), its time period decreases $\left[T \propto \frac{1}{\sqrt{k}}\right]$ whereas frequency increases $(n \propto \sqrt{k})$
- If two masses m_1 and m_2 are connected by a spring and made to oscillate then time period



$T = 2\pi\sqrt{\frac{\mu}{k}}$

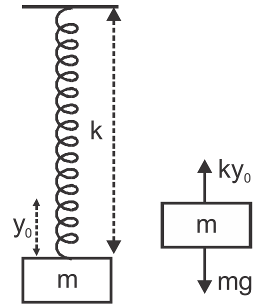
Here, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

- If a spring pendulum oscillating in a vertical plane is made to oscillate on a horizontal surface or on an inclined plane, then time period will remain unchanged.



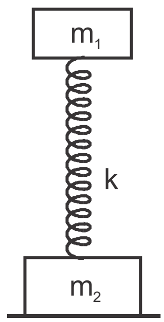
- If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass m, $ky_0 = mg$

  still, time period, $T = 2\pi\sqrt{\frac{m}{k}}$



  but remember time period of spring pendulum is independent of acceleration due to gravity.

- If two particles are attached with spring in which only one is oscillating

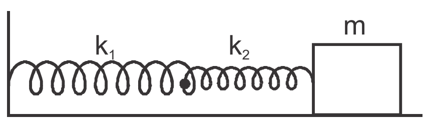


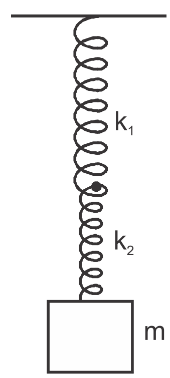
  Time period = $\frac{2\pi}{\omega}$; $T = 2\pi\sqrt{\frac{m_1}{k}}$

8.1 Various Spring Arrangements

- **Series Combination of Springs**

Total displacement $x = x_1 + x_2$





Force acting on both springs

$F = -k_1x_1 = -k_2x_2$

$\because x_1 = -\frac{F}{k_1}$ and $x_2 = -\frac{F}{k_2}$

$\therefore x = -\left[\frac{F}{k_1} + \frac{F}{k_2}\right]$ ....(i)

If equivalent force constant is k_s then $F = -k_s x$

so by equation (i) $-\frac{F}{k_s} = -\frac{F}{k_1} - \frac{F}{k_2}$ $\Rightarrow \frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$ $\Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$

Time period $T = 2\pi\sqrt{\frac{m}{k_s}}$

$= 2\pi\sqrt{\frac{m}{k_s}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k_s}{m}}$,

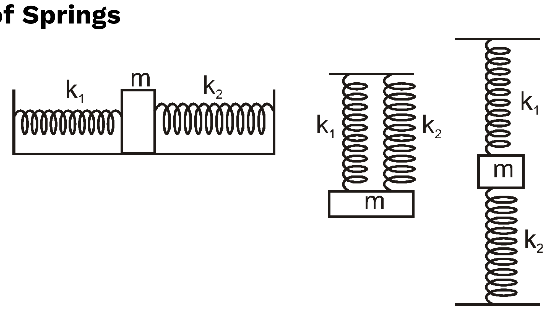
Angular frequency $\omega = \sqrt{\frac{k_s}{m}}$

Key Points

- In series combination, same force exerts in all springs but extension will be different.
- In series combination, extension of spring will be reciprocal of its spring constant.
- Spring constant of spring is directly proportional to reciprocal of its length

$\because k \propto \frac{1}{\ell}$ $\therefore k_1\ell_1 = k_2\ell_2 = k_3\ell_3$

Parallel Combination of Springs



In this arrangement displacement on each spring is same but restoring force is different.

Force acting on the system

$F = F_1 + F_2$ $\Rightarrow$ $F = -k_1x - k_2x$ ...(i)

If equivalent force constant is k_p then,

$F = -k_px$

so by equation (i) $-k_px = -k_1x - k_2x$ $\Rightarrow$ $k_p = k_1 + k_2$

Time period $T = 2\pi\sqrt{\frac{m}{k_P}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k_P}{m}}$

Angular frequency $\omega = \sqrt{\frac{k_1 + k_2}{m}}$

Key Points

- In parallel combination, different forces exert in all springs but extension will be same.
- In parallel combination, forces on spring will be proportional of its spring constant.
- If the length of the spring is made n times then effective force constant becomes $\frac{1}{n}$ times and the time period becomes $\sqrt{n}$ times.
- If a spring of spring constant k is divided into n equal parts, the spring constant of each part becomes nk and time period becomes $\frac{1}{\sqrt{n}}$ times.
- In case of a loaded spring the time period comes out to be the same in both horizontal and vertical arrangement of spring system
- The force constant k of a stiffer spring is higher than that of a soft spring. So the time period of a stiffer spring is less than that of a soft spring.

Example 14:

A body of mass m is attached to a spring which is oscillating with time period 4 seconds. If the mass of the body is increased by 4 kg, its time period increases by 2 sec. Determine value of initial mass m.

Solution:

$T = 2\pi\sqrt{\frac{m}{k}}$ 1^{st} case $4 = 2\pi\sqrt{\frac{m}{k}}$ ...(i)

and 2^{nd} case $6 = 2\pi\sqrt{\frac{m+4}{k}}$ ...(ii)

divide (i) by (ii) $\frac{4}{6} = \sqrt{\frac{m}{m+4}}$

$\Rightarrow \frac{16}{36} = \frac{m}{m+4}$ $\Rightarrow m = 3.2$ kg

Example 15:

One body is suspended from a spring of length ℓ, spring constant k and has time period T. Now if spring is divided in two equal parts which are joined in parallel and the same body is suspended from this arrangement then determine new time period.

Solution:

Spring constant in parallel combination

k' = 2k + 2k = 4k

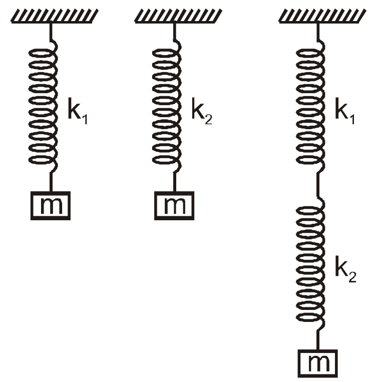
$\therefore T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{4k}}$

and $T' = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{4}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$

Example 16:

Periodic time of oscillation T_1 is obtained when a mass is suspended from a spring. If another spring is used with same mass then periodic time of oscillation is T_2. Now if the mass is suspended from series combination of above springs, then calculate the time period.

Solution:



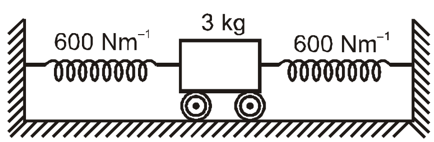
$T \propto \frac{1}{\sqrt{k}}$ or $T^2 \propto \frac{1}{k}$

For spring combination

$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$; $T'^2 = T_1^{\,2} + T_2^{\,2}$; $T' = \sqrt{T_1^2 + T_2^2}$

Example 17:

A trolley of mass 3 kg as shown in figure is connected to two springs, each of spring constant 600 Nm^{-1}. If the trolley is displaced from its equilibrium position by 5.0 cm and released, what is



(a) the period of ensuing oscillations, and

(b) the maximum speed of the trolley ?

(c) How much energy is dissipated as heat by the time the trolley comes to rest due to damping forces?

Solution:

(a) $K_{eff} = K_1 + K_2 = 1200 \text{ Nm}^{-1}$

$$T = 2\pi\sqrt{\frac{m}{K_{eff}}} = 2\pi\sqrt{\frac{3}{1200}}$$

$$T = \frac{\pi}{10} = 0.314 \text{ sec.}$$

(b) Applying energy conservation

$$\frac{1}{2}K_1x^2 + \frac{1}{2}K_2x^2 = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow 1200\,(5.0 \times 10^{-2})^2 = (3)\,(v)^2$$

$$v = 1 \text{ ms}^{-1}$$

(c) Total dissipated Energy $= \frac{1}{2}mv^2_{max}$

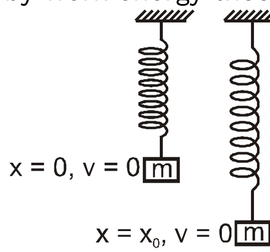
$$= \frac{1}{2}\,(3)(1)^2 = 1.5 \text{ J}$$

Example 18:

A block of mass m is suspended from a spring of spring constant k. If the mass is released with spring at natural length, find the amplitude of SHM

Solution:

Let amplitude of SHM be A then by work energy theorem W = ΔKE



$$mgx_0 - \frac{1}{2}kx_0^2 = 0 \Rightarrow x_0 = \frac{2mg}{k}$$

So amplitude $A = \frac{x_0}{2} = \frac{mg}{k}$

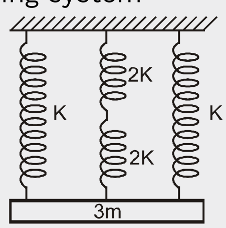
Concept Builder-5

Q.1 A body of mass 'm' suspended from a vertical spring of length 'L' executes SHM with a time period T. Find the time period if

(a) Length of spring is made half.

(b) Mass of body is made half.

Q.2 Calculate the time period of following system



Q.3 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the approximate weight of the body?

Q.4 When a mass of 1 kg is suspended from a vertical spring, its length increases by 0.98 m. If this mass is pulled downwards and then released, what will be periodic time of vibration of spring? ($g = 9.8\ m/s^2$)

Q.5 A spring having a spring constant 1200 Nm^{-1} is mounted on a horizontal table. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled to a distance of 2.0 cm and released. Determine Maximum acceleration of mass.

Angular S.H.M.

The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position.

$\therefore \tau \propto -0$ or $\alpha \propto -\theta$

Note : Similarly for angular SHM, necessary and sufficient condition is $\tau = -c\theta$ and as $\tau = I\alpha = Id^2\theta/dt^2$,

The equation of angular SHM will be

$$\frac{d^2\theta}{dt^2} = -\frac{c}{I}\theta$$

i.e., $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \frac{c}{I}$

which on integration gives

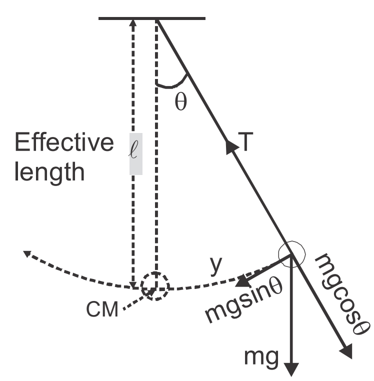
$= \frac{d\theta}{dt} = \omega\sqrt{\theta_0^2 - \theta^2}$ and $\theta = \theta_0 \sin(\omega t + \phi)$

7. Simple Pendulum

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum

Expression for Time Period

For small angular displacement, $\sin\theta \approx \theta$,
so that



$F = -mg\sin\theta = -mg\,\theta = -\left(\frac{mg}{\ell}\right)y = -ky$

(because $y = \ell\theta$),

Thus, the time period of the simple pendulum is

$T = 2\pi\sqrt{\frac{m}{k}}$ or $T = 2\pi\sqrt{\frac{\ell}{g}}$

The period is independent of mass of the pendulum.

Second's Pendulum

If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

For second's pendulum, time period

$T = 2 = 2\pi\sqrt{\frac{\ell}{g}}$

At the surface of earth $g = 9.8\ m/s^2 \approx \pi^2\ m/s^2$,

So length of second's pendulum at the surface of earth, $\ell = 1$ metre

Example 19:

The angle made by the string of a simple pendulum with the vertical depends upon time as $\theta = \frac{\pi}{90}\sin\pi t$. Find the length of the pendulum if $g = \pi^2\ ms^{-2}$.

Solution:

Here $\omega = \pi$ so

$T = \frac{2\pi}{\omega} = 2$ sec. So it is a second's pendulum, its length will be 1m

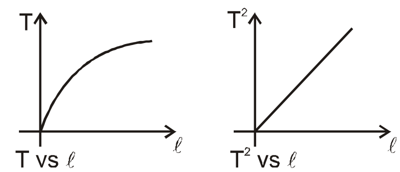
7.1 Factors influencing Time Period

(A) Mass & Amplitude

- Time period of simple pendulum is independent of amplitude as long as its motion is simple harmonic.
- Time period of simple pendulum is also independent of mass of the bob.

(B) Effective Length

- Time period depends on L as $T \propto \sqrt{L}$, i.e. $T^2 \propto L$. So the graph between T and L will be a parabola while between T^2 and L will be a straight line. Here L is the distance between point of suspension and centre of mass of the bob and is called 'effective length' :



Some Examples of Effective Length Variation

(a) When a sitting girl on a swinging swing stands up, her centre of mass will go up and so L and hence T will decrease.

(b) If a hole is made in the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the centre of mass will be at the centre of the sphere. However as water drains off the sphere, the centre of mass of the system will first move down and then will come up. Due to this L and hence T first increases, reaches a maximum and then decreases to its initial value.

(c) If the bob is suspended by a wire, due to change in temperature, length L will change and so the time period. If $\Delta\theta$ is the increase in temperature then as

$L = L_0(1 + \alpha\Delta\theta)$

$$\frac{T}{T_0} = \sqrt{\frac{L}{L_0}} = (1 + \alpha\Delta\theta)^{1/2} \simeq \left[1 + \frac{1}{2}\alpha\Delta\theta\right]$$

or $\frac{T}{T_0} - 1 \simeq \frac{1}{2}\alpha\Delta\theta$ i.e., $\frac{\Delta T}{T} \simeq \frac{1}{2}\alpha\Delta\theta$

- If $\Delta\ell$ is change in length and Δg is the change in acceleration, then for small variation (up to 5%) change in time period (ΔT) will be

$$\frac{\Delta T}{T} \times 100 = \left[\frac{1}{2}\frac{\Delta\ell}{\ell} - \frac{1}{2}\frac{\Delta g}{g}\right] \times 100$$

- $T = 2\pi\sqrt{\frac{\ell}{g}}$ is valid when length of simple pendulum (ℓ) is negligible as compared to radius of earth (ℓ <<R) but if ℓ is comparable to radius of earth then time period

$$T = 2\pi\sqrt{\frac{R}{\left[1 + \frac{R}{\ell}\right]g}} = 2\pi\sqrt{\frac{\ell}{g\left[1 + \frac{\ell}{R}\right]}} = 2\pi\sqrt{\frac{1}{\left[\frac{1}{\ell} + \frac{1}{R}\right]g}}$$

- The time period of oscillation of simple pendulum of infinite length

$$T = 2\pi\sqrt{\frac{R}{g}} \simeq 84.6 \text{ minute}$$

It is maximum time period.

Example 20:

If length of a simple pendulum is increased by 4%. Then determine percentage change in time period.

Solution:

Percentage change in time period

$\frac{\Delta T}{T} \times 100\% = \frac{1}{2}\frac{\Delta \ell}{\ell} \times 100$ [$\because$ $\Delta g = 0$]

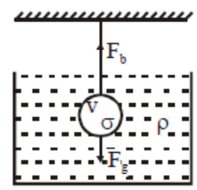
According to question $\frac{\Delta \ell}{\ell} \times 100 = 4\%$

$\therefore$ $\frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times 4\% = 2\%$

(C) Effective Gravity

- Time period of a simple pendulum also depends on acceleration due to gravity and as $T \propto \left(\frac{1}{\sqrt{g}}\right)$, with increase in g, T will decrease and *vice-versa*.
- If time period of clock based on simple pendulum increases then clock will be slow, if time period decreases then clock will be fast.
- Due to change in shape of earth (not spherical but elliptical) gravitational acceleration is different at different places. So time period of simple pendulum varies with variation of g.
- If a simple pendulum of density ρ is made to oscillate in a liquid of density σ then its time period will increase as compared to that in air and is given by

$$T = 2\pi\sqrt{\frac{\ell}{\left[1-\frac{\sigma}{\rho}\right]g}}$$

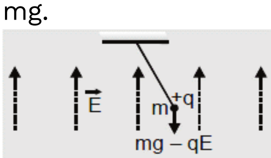
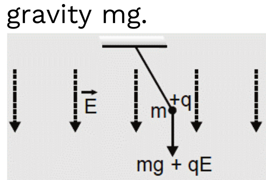


- If a simple pendulum is in a carriage which is accelerating

Upwards	Downwards	Horizontally
Then as $g' = g + a$ $T = 2\pi\sqrt{\frac{L}{(g+a)}}$ i.e., T will decrease	Then as $g' = g - a$ $T = 2\pi\sqrt{\frac{L}{(g-a)}}$ i.e., T will increase	Then as $g' = \sqrt{g^2+a^2}$ $T = 2\pi\sqrt{\frac{L}{\sqrt{(g^2+a^2)}}}$ i.e., T will decrease

For Student's Practice Only (Optional)

- If the simple pendulum has charge q and is oscillating in a uniform electric field E which is:

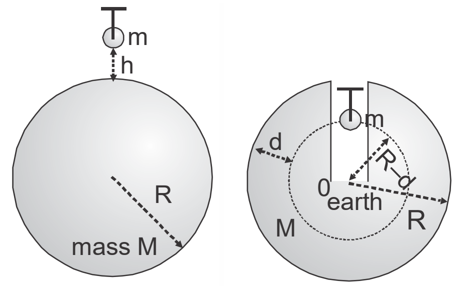
Opposite to g	In the direction of g	Perpendicular to g
Electric force qE will be Opposite to the force of gravity mg.  i.e., $g' = g - \frac{qE}{m}$ So, T will increase	Electric force qE will be in the direction of force of gravity mg.  i.e.$g' = g + \frac{qE}{m}$ So, T will decrease	Electric force qE will be perpendicular to force of gravity mg. i.e., $g' = \sqrt{g^2 + (qE/m)^2}$ So, T will decreases.

- If a simple pendulum is made to oscillate in a freely falling lift or a satellite,
  $g' = g - a = 0$ [as a = g in free fall]
  So, $T = 2\pi\sqrt{L/0} = \infty$.
  This implies, that the pendulum will not oscillate and will remain where left, as there will be no restoring force.
- If simple pendulum is taken above or below the surface of earth, then value of gravitational acceleration decreases and time period increases.

  $T \propto \frac{1}{\sqrt{g}}$

  At height h $g_h = g_s\left(1 - \frac{2h}{R}\right)$ (h << R)

  At depth d $g_d = g_s\left(1 - \frac{d}{R}\right)$ (d << R)



Example 21:

A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T. With what acceleration should lift be accelerated upwards in order to reduce its time period to $\frac{T}{2}$

Solution:

In stationary lift $T = 2\pi\sqrt{\frac{\ell}{g}}$ ...(i)

In accelerated lift $\frac{T}{2} = T' = 2\pi\sqrt{\frac{\ell}{g+a}}$ ...(ii)

Divide (i) by (ii)

$2 = \sqrt{\frac{g+a}{g}}$

or $g + a = 4g$

or $a = 3g$

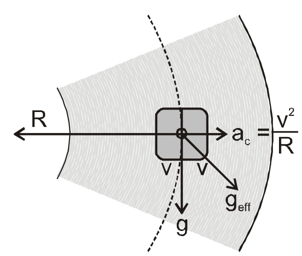
Example 22:

A simple pendulum of length L and mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes oscillation in a radial direction about its equilibrium position, then calculate its time period.

Solution:

Centripetal acceleration $a_c = \frac{v^2}{R}$ & Acceleration due to gravity = g

So $g_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$



$\Rightarrow$ Time period $T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$

Example 23:

The length of a second's pendulum at the surface of earth is 1m. Determine the length of second's pendulum at the surface of moon.

Solution:

For second's pendulum at the surface of earth

$$T = 2\pi\sqrt{\frac{\ell_e}{g_e}} \qquad ...(i)$$

For second's pendulum at the surface of moon

$$T = 2\pi\sqrt{\frac{\ell_m}{g_m}} \qquad ...(ii)$$

From (i) and (ii)

$$\frac{\ell_e}{g_e} = \frac{\ell_m}{g_m} \Rightarrow \ell_m = \left[\frac{g_m}{g_e}\right]\ell_e$$

$$\Rightarrow \ell_m = \frac{\ell_e}{6} \quad \left[\because \quad g_m = \frac{g_e}{6}\right]$$

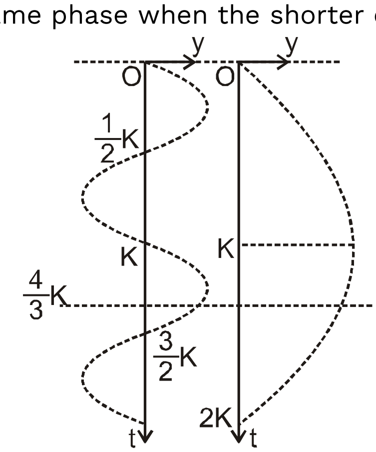
Example 24:

Two simple pendulums of length 1m and 16m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed n oscillations. Calculate n.

Solution:

In case of simple pendulum as

$T = 2\pi\sqrt{(L/g)}$, i.e., $T = K\sqrt{L}$, so the time period of shorter pendulum will be small, i.e., it will complete more oscillations in the same time than the longer pendulum. So if for the first time the two pendulums are in same phase when the shorter one has completed n oscillations,



$nT_S = (n - 1)\,T_L$, $nK\sqrt{L_S}$ i.e., $= (n - 1)\,K\sqrt{L_L}$

or $\quad n\sqrt{1} = (n - 1)\,\sqrt{16} \qquad$ i.e., $3n = 4$

or $\quad n = (4/3)$,

i.e., the two pendulums will be in the same phase for the first time when the shorter pendulum has completed (4/3) oscillations. (See figure)

Concept Builder-6

Q.1 The length of a simple pendulum is increased four times of its initial value, its time period with respect to its previous value will :

(1) become twice (2) be $\frac{1}{\sqrt{2}}$ times (3) be halved (4) be $\sqrt{2}$ times

Q.2 If the metal bob of a simple pendulum is replaced by a wooden bob, then its time period will :

(1) increase (2) decrease
(3) remain the same (4) may increase or decrease

Q.3 By What percentage time period of a simple pendulum change if length of pendulum is increased by

(a) 2.4% (b) 44 %

Q.4 A cabin is falling freely under gravity, what is the time period of a pendulum attached to its ceiling?

(1) zero (2) ∞ (3) 1 s (4) 2 s

Q.5 A simple pendulum has time period T_1 when it is on earth surface and T_2 when it is taken at height 2R above the earth surface. Calculate $\frac{T_2}{T_1}$. (R is the radius of earth)

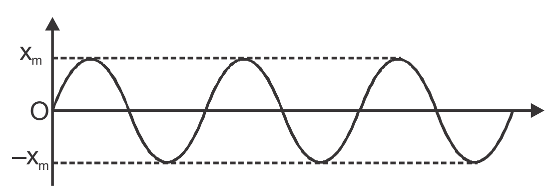
Q.6 If a simple pendulum is taken to a place where g decreases by 2%, then time period :

(1) increase by 5% (2) increase by 1% (3) increase by 2% (4) decrease by 5%

9. Different Type of Oscillations

(a) Free Oscillation

- The oscillations of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations.



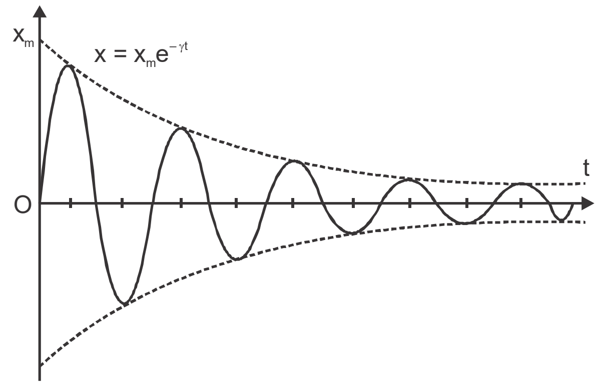
- The amplitude, frequency and energy of oscillations remain constant.
- The oscillator which keeps on oscillating with constant amplitude for infinite time is known as free oscillator.
- The equation of free SHM is

$$m\frac{d^2(x)}{dt^2}+k(x)=0 \quad \text{and}$$

Displacement $x = A_0 \sin \omega_0 t$

(b) Damped Oscillations

- The oscillations of a body whose amplitude goes on decreasing with time are defined as damped oscillations.



- In these oscillations the amplitude of oscillations decreases exponentially due to damping forces like frictional force, viscous force etc.
- The damping force may be written as
  F = −bv
  Here b is damping force constant & v is velocity.
- The equation of damped SHM is
  $m\frac{d^2(x)}{dt^2} + b\frac{d(x)}{dt} + k(x) = 0$ and
  Displacement $x = A_1 \sin(\omega_1 t + \delta)$
  Here, $A_1 = A_0 e^{-\frac{bt}{2m}}$ & $\omega_1 = \sqrt{\omega_0{}^2 - (b/2m)^2}$
- If initial amplitude is X_m then amplitude after time t will be $X = X_m e^{-\frac{b}{2m}t}$
- If initial energy is E_m then energy after time t will be $E = E_m e^{-\frac{b}{m}t}$

(c) Forced Oscillations

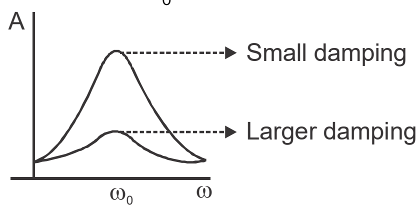
- The oscillations in which a body oscillates under the influence of an external periodic force (driver) are known as forced oscillations.
- The driven body does not oscillate with its natural frequency rather it oscillates with the frequency of the driver.
- The amplitude of oscillator decreases due to damping forces but on account of the energy gained from the external source (driver) it remains constant.
- The amplitude of forced vibration is determined by the difference between the frequency of the applied force and the natural frequency.
- If the difference between these two frequencies is small, then the amplitude will be large.
- The equation of forced SHM is

$$m\frac{d^2(x)}{dt^2} + b\frac{d(x)}{dt} + k(x) = F_0 \sin\omega t$$

  and
  Displacement $x = A\sin(\omega t + \phi)$
  Here, $A = \frac{F_0/m}{\sqrt{(\omega_D^2 - \omega_0^2)^2 + (b\omega_D/m)^2}}$

(d) Resonance

- When the frequency of external force (driver) is equal to the natural frequency of the oscillator (driven), then this state of the driver and the driven is known as the state of resonance.
- In the state of resonance, there occurs maximum transfer of energy from the driver to the driven. Hence the amplitude of motion becomes maximum.
- In the state of resonance the frequency of the driver (ω) is known as the **resonant frequency**.
- The amplitude is larger if damping is small, So the resonance is sharp. The amplitude rapidly falls if ω is different from ω_0.



Example 25:

The amplitude of a damped oscillator becomes half in one minute. The amplitude after 3 minutes will be 1/x times of the original. Determine value of x.

Solution:

Amplitude of damped oscillation is $A = A_0 e^{-\gamma t}$ [from $x = x_m e^{-\gamma t}$]

at $t = 1$ min $A = \frac{A_0}{2}$

so $\frac{A_0}{2} = A_0 e^{-\gamma}$ or $e^{\gamma} = 2$

After 3 minutes $A = \frac{A_0}{x}$ $\left[\gamma = \frac{b}{2m}\right]$

so $\frac{A_0}{x} = A_0 e^{-\gamma \times 3}$ or $x = e^{3\gamma} = (e^{\gamma})^3 = 2^3 = 8$

Example 26:

The amplitude of a damped oscillator becomes half in one minute. The amplitude after t minutes will be $\frac{1}{32}$ times of the original. Determine value of t.

Solution:

$\frac{A_0}{2} = A_0 e^{-r[1]}$

$\frac{1}{2} = e^{-r[1]}$

$\frac{A_0}{32} = A_0 e^{-r[t]}$

$\left(\frac{1}{2}\right)^5 = [e^{-r}]^t$

t = 5 minutes

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. (3) **2.** (1), (2) and (3)

3. (1)

CONCEPT BUILDER-2

1. (2) **2.** (3)

3. (2)

4. (a) $x = A \sin(\pi t + \pi/6)$

(b) $x = A \sin(\pi t - \pi/4)$

or $x = A \sin(\pi t + 7\pi/4)$

(c) $x = 4 \sin(\pi t + 2\pi/3)$

5. (1) **6.** (3)

CONCEPT BUILDER-3

1. (3) **2.** (2)

3. (3) **4.** (1)

5. 9.52 m **6.** $5\sqrt{3}$ m, 30 m/s

CONCEPT BUILDER-4

1. (3) **2.** (1)

3. $\frac{5}{\pi}$ Hz **4.** (1)

CONCEPT BUILDER-5

1. (a) $T' = \frac{T}{\sqrt{2}}$ (b) $T' = \frac{T}{\sqrt{2}}$

2. $2\pi\sqrt{\frac{m}{K}}$ **3.** $\approx$ 225 N

4. 2s **5.** 8 m/s

CONCEPT BUILDER-6

1. (1) **2.** (3)

3. (a) Time period will increase by 1.2%

(b) Time period increase by 20%

4. (2) **5.** 3

6. (2)

Exercise - I

Periodic Motion, Oscillations & SHM, Equation Representing SHM, Characteristics of SHM

1. A particle executing S.H.M. completes a distance (taking friction as negligible) in one complete time period, equal to:
(1) Four times the amplitude
(2) Two times the amplitude
(3) One times the amplitude
(4) Eight times the amplitude

2. A particle of mass m is executing S.H.M. If amplitude is a and frequency n, the value of its force constant will be:
(1) mn^2 (2) $4mn^2a^2$
(3) ma^2 (4) $4\pi^2mn^2$

3. The equation of motion of a particle executing S.H.M. is :
(1) $\frac{d^2x}{dt^2} = -\frac{k}{m}x$
(2) $\frac{d^2x}{dt^2} = +\omega^2x$
(3) $\frac{d^2x}{dt^2} = -\omega^2x^2$
(4) $\frac{d^2x}{dt^2} = -kmx$

4. The phase of a particle in SHM at time t is $\pi/6$. The following inference is drawn from this:
(1) The particle is at x = a/2 and moving in +X-direction
(2) The particle is at x = a/2 and moving in –X-direction
(3) The particle is at x = – a/2 and moving in +X-direction
(4) The particle is at x = – a/2 and moving in –X-direction

5. The value of phase at maximum distance from the mean position of a particle in S.H.M. is:
(1) $\pi / 2$ (2) π
(3) Zero (4) 2π

6. Two particles execute S.H.M. along the same line at the same frequency. They move in opposite direction at the mean position. The phase difference will be:
(1) 2π (2) $2\pi/3$
(3) π (4) $\pi/2$

7. The time period of an oscillator is 8 sec. The phase difference from t = 2 sec to t = 4 sec will be:
(1) π (2) $\frac{\pi}{2}$
(3) $\frac{\pi}{4}$ (4) 2π

8. Which of the following equation does not represent a simple harmonic motion?
(1) $y = a \sin \omega t$
(2) $y = b \cos \omega t$
(3) $y = a \sin \omega t + b \cos \omega t$
(4) $y = a \tan \omega t$

9. The equation of a simple harmonic motion is x = 0.34 cos (3000t + 0.74). Where x and t are in mm and sec. respectively. The frequency of the motion is:
(1) 3000 (2) $3000/2\pi$
(3) $0.74/2\pi$ (4) $3000/\pi$

10. The displacement of a particle in SHM is indicated by equation $y = 10 \sin(20t + \pi/3)$ where y is in metre. The value of time period of vibration will be (in seconds):
(1) $10/\pi$ (2) $\pi/10$
(3) $2\pi/10$ (4) $10/2\pi$

11. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium positions is-
(1) T/2 (2) T/4
(3) T/8 (4) T/12

12. Which one of the following equations of motion represents simple harmonic motion–

(1) Acceleration = kx

(2) Acceleration = $-k_0x + k_1 x^2$

(3) Acceleration = – k(x + a)

(4) Acceleration = k(x + a)

Where k, k_0, k_1 and a are all positive

15. A particle executes SHM of type $x = a\sin\omega t$. It takes time t_1 from x = 0 to x = a / 2 and t_2 from x = a / 2 to x = a. The ratio of $t_1 : t_2$ will be:

(1) 1 : 1 (2) 1 : 2

(3) 1 : 3 (4) 2 :1

16. The time taken by a particle in SHM for maximum displacement from mean position is :

(1) T/8 (2) T/6

(3) T/2 (4) T/4

Velocity & Acceleration in SHM

15. The acceleration of a particle executing S.H.M. is :

(1) Always directed towards the equilibrium position

(2) Always towards the one end

(3) Continuously changing in direction

(4) Maximum at the mean position

16. The phase of a particle in S.H.M. is $\pi/2$, then:

(1) Its velocity will be maximum

(2) Its acceleration will be minimum

(3) Restoring force on it will be minimum

(4) Its displacement will be maximum.

17. The phase difference between the displacement and acceleration of particle executing S.H.M. in radian is:

(1) $\pi/4$ (2) $\pi/2$

(3) π (4) 2π

18. The phase difference in radians between displacement and velocity in S.H.M. is

(1) $\pi/4$ (2) $\pi/2$

(3) π (4) 2π

19. If the maximum velocity of a particle in SHM is v_0 then its velocity at half the amplitude from position of rest will be:

(1) $v_0/2$ (2) v_0

(3) $v_0\sqrt{3/2}$ (4) $v_0\sqrt{3}/2$

20. The acceleration of a particle in SHM at 5 cm from its mean position is 20 cm/sec^2. The value of angular frequency in radian/sec will be:

(1) 2 (2) 4

(3) 10 (4) 14

21. The amplitude of a particle in SHM is 5 cms and its time period is π. At a displacement of 3 cms from its mean position the velocity in cms/sec will be

(1) 8 (2) 12

(3) 2 (4) 16

22. The maximum velocity and acceleration of a particle in S.H.M. are 100 cm/sec and 157 cm/sec^2 respectively. The time period in seconds will be:

(1) 4 (2) 1.57

(3) 0.25 (4) 1

Paragraph for (Q.23 Q.25)

If the displacement, velocity and acceleration of a particle in SHM are 1 cm, 1cm/sec, 1 cm/sec^2 respectively then

23. Its time period will be (in seconds):

(1) π (2) 0.5π

(3) 2π (4) 1.5π

24. Amplitude of SHM will be –

(1) $\sqrt{2}$ cm (2) 2 cm

(3) 1/2 cm (4) 1.5 cm

25. What will be the maximum velocity and maximum acceleration of the particle ?

(1) 2 cm/s, 2 cm/s^2

(2) $\sqrt{2}$ cm/s, $\sqrt{2}$ cm/s^2

(3) 1.5 cm/s, 1.5 cm/s^2

(4) None of these

26. The particle is executing S.H.M. on a line 4 cms long. If its velocity at mean position is
12 cm/sec, its frequency in Hertz will be:

(1) $2\pi/3$ (2) $3/2\pi$

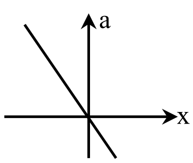
(3) $\pi/3$ (4) $3/\pi$

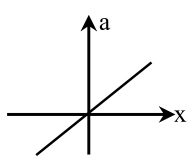
27. If the amplitude of a simple pendulum is doubled, how many times will the value of its maximum velocity be that of the maximum velocity in initial case:

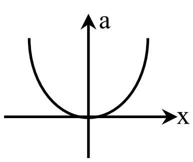
(1) 1/2 (2) 2

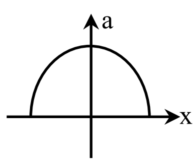
(3) 4 (4) 1/4

28. The variation of acceleration (a) and displacement (x) of the particle executing SHM is indicated by the following curve:

(1) 

(2) 

(3) 

(4) 

29. The time period of an oscillating body executing SHM is 0.05 sec and its amplitude is 40 cm. The maximum velocity of particle is:

(1) 16π ms^{-1} (2) $2\pi ms^{-1}$

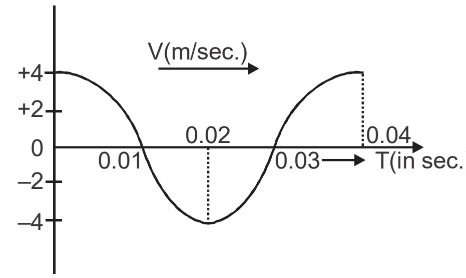
(3) 3.1 ms^{-1} (4) 4π ms^{-1}

30. A body of mass 5 gm is executing S.H.M. about a point with amplitude 10 cm. Its maximum velocity is 100 cm/sec. Its velocity will be 50 cm/sec at a distance :

(1) 5 cm (2) $5\sqrt{2}$ cm

(3) $5\sqrt{3}$ cm (4) $10\sqrt{2}$ cm

31. The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is:



(1) 25 Hz (2) 50 Hz

(3) 12.25 Hz (4) 33.3 Hz

32. Displacement of a particle is x = 3 sin 2t + 4 cos 2t, the amplitude and the max. velocity will be:

(1) 5, 10 (2) 3, 2

(3) 4, 2 (4) 3, 8

33. In SHM velocity is maximum:

(1) At extreme position

(2) When displacement is half of amplitude

(3) At the central position

(4) When displacement is $\frac{1}{\sqrt{2}}$ of amplitude

34. The maximum velocity of simple harmonic motion represented by $y = 3\sin\left(100t + \frac{\pi}{6}\right)$ is given by:

(1) 300 (2) $\frac{3\pi}{6}$

(3) 100 (4) $\frac{\pi}{6}$

35. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 m is 4.4 m/s. The period of oscillation is:

(1) 100 s (2) 0.01 s

(3) 10 s (4) 0.1 s

36. Which one of the following statements is true for the speed 'v' and the acceleration 'a' of a particle executing simple harmonic motion

(1) Value of 'a' is zero, whatever may be the value of 'v'

(2) When 'v' is zero, 'a' is zero

(3) When 'v' is maximum, 'a' is zero

(4) When 'v' is maximum, 'a' is maximum

37. For a particle executing simple harmonic motion which of the following statement is not correct:

(1) The total energy of particle always remains the same

(2) The restoring force is always directed towards a fix point.

(3) The restoring force is maximum at the extreme positions

(4) The acceleration of particle is maximum at the equilibrium positions.

38. A body oscillates with SHM according to the equation $x = 5.0 \cos(2\pi t + \pi)$. At time $t = 1.5$ s, its displacement, speed and acceleration respectively is:

(1) $0, -10\pi, +20\pi^2$

(2) $5, 0, -20\pi^2$

(3) $2.5, +20\pi, 0$

(4) $-5.0, +5\pi, -10\pi^2$

39. Two simple Harmonic Motions of angular frequency 100 and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is:

(1) $1 : 10^3$ (2) $1 : 10^4$

(3) $1 : 10$ (4) $1 : 10^2$

40. A simple pendulum performs simple harmonic motion about x = 0 with an amplitude a and time period T. The speed of the pendulum at x = a/2 will be:

(1) $\frac{\pi a\sqrt{3}}{T}$ (2) $\frac{\pi a\sqrt{3}}{2T}$

(3) $\frac{\pi a}{T}$ (4) $\frac{3\pi^2 a}{T}$

Energy in SHM

41. Which of the following statement is incorrect for an object executing S.H.M.

(1) The value of acceleration is maximum at the extreme points

(2) The total work done for completing one oscillation is zero

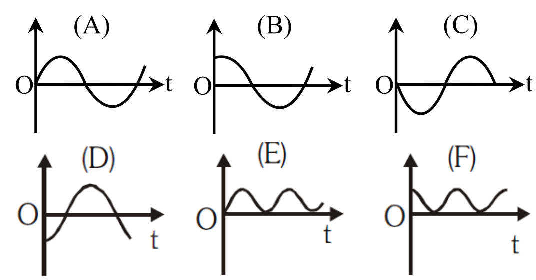
(3) The energy changes from one form to another

(4) The velocity at the mean position is zero

Paragraph for (Q.42 to Q.46)

The displacement of a particle in S.H.M. is $x = a \sin\omega t$. Referring the given graphs answer the following questions.

42. Which graph between displacement and time is correct ?



(1) A (2) B

(3) C (4) D

43. Which of the graph between velocity and time is correct?

(1) A (2) B

(3) C (4) D

44. Which of the graph between kinetic energy and time is correct?

(1) A (2) B

(3) E (4) F

45. Which of the graph between potential energy and time is correct?

(1) A (2) B

(3) E (4) F

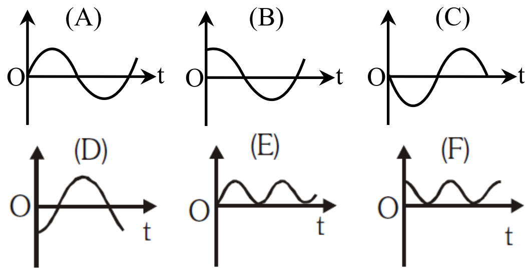
46. Which of the graph between acceleration and time is correct?

(1) A (2) B

(3) C (4) D

Paragraph for (Q.47 to Q.51)

The displacement of a particle in S.H.M. is $x = a \cos\omega t$. Referring the given graphs answer the following questions.



47. Which of the graph between displacement and time is correct ?

(1) A (2) B

(3) C (4) D

48. Which of the graph between velocity and time is correct?

(1) A (2) B

(3) C (4) D

49. Which of the graph between acceleration and time is correct?

(1) A (2) B

(3) C (4) D

50. Which of the graph between K.E. and time is correct?

(1) A (2) B

(3) E (4) F

51. Which of the graph between P.E. and time is correct?

(1) A (2) B

(3) E (4) F

52. The energy at the mean position of a pendulum will be: (If PE at mean position is zero)

(1) Zero

(2) Partial P.E. and partial K.E.

(3) Totally K.E.

(4) Totally P.E.

53. The total energy of a particle executing SHM is directly proportional to the square of the following quantity.

(1) Acceleration (2) Amplitude

(3) Time period (4) Mass

54. The total energy of a vibrating particle in SHM is E. If its amplitude and time period are doubled, its total energy will be:

(1) 16E (2) 8E

(3) 4E (4) E

55. The total vibrational energy of a particle in S.H.M. is E. Its kinetic energy at half the amplitude from mean position will be:

(1) E/2 (2) E/3

(3) 3E/2 (4) 3E/4

56. If total energy of a particle in SHM is E, then the potential energy of the particle at half the amplitude will be-

(1) E/2 (2) E/4

(3) 3E/4 (4) E/8

57. The potential energy of a simple harmonic oscillator at mean position is 3 joules. If its mean K.E. is 4 joules, its total energy will be:

(1) 7J (2) 8J

(3) 10 J (4) 11 J

58. The total energy of a harmonic oscillator of mass 2 kg is 9 joules. If its potential energy at mean position is 5 joules, its K.E. at the mean position will be :

(1) 9J (2) 14J

(3) 4J (4) 11J

59. The average P.E. of a body executing S.H.M. is:

(1) $\frac{1}{2}ka^2$ (2) $\frac{1}{4}ka^2$

(3) ka^2 (4) Zero

60. A particle is describing SHM with amplitude 'a'. When the potential energy of particle is one fourth of the maximum energy during oscillation, then its displacement from mean position will be:

(1) $\frac{a}{4}$ (2) $\frac{a}{3}$

(3) $\frac{a}{2}$ (4) $\frac{2a}{3}$

61. The ratio of K.E. of the particle at mean position to the point when distance is half of amplitude will be:

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $\frac{4}{3}$ (4) $\frac{3}{2}$

62. A particle is executing S.H.M., If its P.E. & K.E. is equal then the ratio of displacement & amplitude will be :

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

63. If < E > and < U > denotes the average kinetic and average potential energies respectively of mass describing a simple harmonic motion over one period then the correct relation is:

(1) $< E > = < U >$ (2) $< E > = 2 < U >$

(3) $< E > = -2 < U >$ (4) $< E > = - < U >$

64. Which of the following is constant during SHM:

(1) Velocity

(2) Acceleration

(3) Total energy

(4) Phase

65. A particle executes SHM with a frequency f. The frequency of its P.E. will be:

(1) f/2 (2) f

(3) 2f (4) 4f

66. The potential energy of a long spring when stretched by 2cm is U. If the spring is stretched by 8cm the potential energy stored in it is:-

(1) 4 U (2) 8 U

(3) 16 U (4) $\frac{U}{4}$

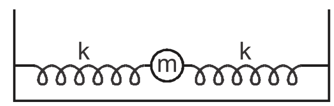
Spring System, Other Oscillatory System

67. The maximum K.E. of a oscillating spring is
5 joules and its amplitude 10 cms. The force constant of the spring is:

(1) 100 Newton/m (2) 1000 Newton-m

(3) 1000 Newton/m (4) 1000 watts

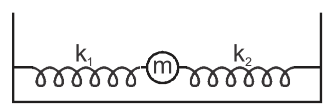
68. On suspending a mass m from a spring of force constant k, frequency of vibration f is obtained. If a second spring as shown in the figure, is arranged then the frequency will be:



(1) $f\sqrt{2}$ (2) $f/\sqrt{2}$

(3) 2f (4) f

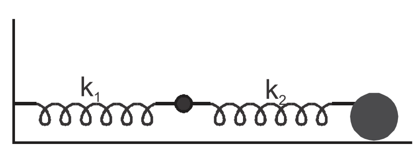
69. In the adjoining figure the frequency of oscillation for a mass m will be proportional to:



(1) k_1k_2 (2) $k_1 + k_2$

(3) $\sqrt{k_1+k_2}$ (4) $\sqrt{1/(k_1+k_2)}$

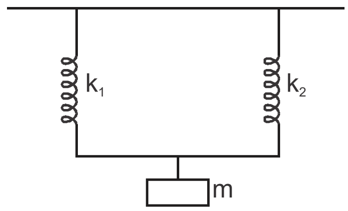
70. A shown in the figure, two light springs of force constant k_1 and k_2 oscillate a block of mass m. Its effective force constant will be:



(1) k_1k_2 (2) $k_1 + k_2$

(3) $\frac{1}{k_1}+\frac{1}{k_2}$ (4) $\frac{k_1k_2}{k_1+k_2}$

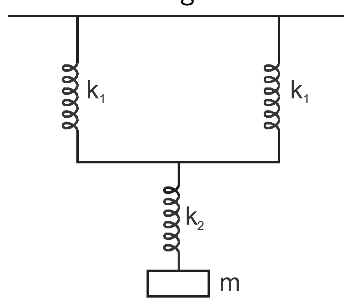
71. The spring constant of two springs of same length are k_1 and k_2 as shown in figure. If an object of mass m is suspended and set vibration, the time period will be:



(1) $2\pi\sqrt{\dfrac{mk_1}{k_2}}$ (2) $2\pi\sqrt{\dfrac{m}{k_1k_2}}$

(3) $2\pi\sqrt{\dfrac{m}{k_1-k_2}}$ (4) $2\pi\sqrt{\dfrac{m}{k_1+k_2}}$

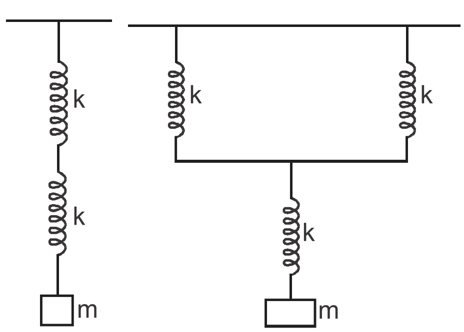
72. The total spring constant of the system as shown in the figure will be:



(1) $\dfrac{k_1}{2}+k_2$ (2) $\left[\dfrac{1}{2k_1}+\dfrac{1}{k_2}\right]^{-1}$

(3) $\dfrac{1}{2k_1}+\dfrac{1}{k_2}$ (4) $\left[\dfrac{2}{k_1}+\dfrac{1}{k_2}\right]^{-1}$

73. Some springs are combined in series and parallel arrangement as shown in the figure and a mass m is suspended from them. The ratio of their frequencies will be:



(1) 1 : 1 (2) 2 : 1

(3) $\sqrt{3}$: 2 (4) 4 : 1

74. On loading a spring with bob, its period of oscillation in a vertical plane is T. If this spring pendulum is tied with one end to a frictionless table and made to oscillate in a horizontal plane, its period of oscillation will be-

(1) T

(2) 2T

(3) T/2

(4) will not execute S.H.M.

75. In a winding (spring) watch, the energy is stored in the form of :

(1) Kinetic energy (2) Potential energy

(3) Electrical energy (4) None of these

76. An object of mass m is suspended from a spring and it executes S.H.M. with frequency ν. If the mass is increased 4 times, the new frequency will be:

(1) 2ν (2) $\nu/2$

(3) ν (4) $\nu/4$

77. Force constant of a spring is K. One fourth part is detached then force constant of remaining spring will be:

(1) $\dfrac{3}{4}K$ (2) $\dfrac{4}{3}K$

(3) K (4) 4K

78. The spring constant of a spring is K. When it is divided into n equal parts, then what is the spring constant of one piece:

(1) nK (2) K / n

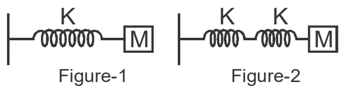
(3) $\dfrac{nK}{(n+1)}$ (4) $\dfrac{(n+1)K}{n}$

79. Mass 'm' is suspended from a spring of force constant K. Spring is cut into two equal parts and same mass is suspended from it, then new frequency will be:

(1) 2 v (2) $\sqrt{2}$ v

(3) v (4) $\dfrac{v}{2}$

80. In figure-1 if the time period is T then calculate time period for figure-2 in which two springs are connected in series with same mass M:-



(1) $\sqrt{2}T$ (2) $\frac{T}{\sqrt{2}}$

(3) 2T (4) $\frac{T}{2}$

81. A mass of 10g is connected to a massless spring then time period of small oscillation is 10 seconds. If 10 g mass is replaced by 40 g mass in same spring, then its time period will be:-

(1) 5s (2) 10s

(3) 20s (4) 40s

82. In an artificial satellite, the object used is:

(1) Spring watch

(2) Pendulum watch

(3) Watches of both spring and pendulum

(4) None of these

83. The time period of a spring pendulum on earth is T. If it is taken on the moon, and made to oscillate, the period of vibration will be :

(1) Less than T (2) Equal to T

(3) More than T (4) None of these

Simple Pendulum, Physical Pendulum

84. The length of a simple pendulum is $39.2/\pi^2$ m. If g = 9.8 m/sec^2, the value of time period is:

(1) 4 sec (2) 8 sec

(3) 2 sec (4) 3 sec

85. The length of a simple pendulum is increased four times of its initial value, its time period with respect to its previous value will:

(1) Become twice (2) Not be different

(3) Be halved (4) Be $\sqrt{2}$ times

86. Water is filled in a hollow metallic sphere and it is suspended from a long string. A fine hole is made at the bottom of the sphere through which water tickles. The sphere is set into oscillations. Its period of oscillation will:

(1) Remain constant

(2) Decrease continuously

(3) Increase continuously

(4) First increase then decrease

87. The time taken for a second pendulum from one extreme point to another is:

(1) 1 s (2) 2 s

(3) 1/2 s (4) 4 s

88. The length of a seconds pendulum is (approximately):

(1) 1 m (2) 1 cm

(3) 2 m (4) 2 cm

89. The acceleration due to gravity at height R above the surface of the earth is g/4. The periodic time of a simple pendulum in an artificial satellite at this height will be:

(1) $T = 4\pi\sqrt{2\ell / g}$ (2) $T = 2\pi\sqrt{2\ell / g}$

(3) Zero (4) Infinity

90. In an artificial satellite, the use of a pendulum watch is discarded, because :

(1) The satellite is in a constant state of motion

(2) The value of effective g becomes zero in the earth satellite

(3) The periodic time of the pendulum watch is reduced

(4) None of these

91. The length of a simple pendulum is made equal to the radius of the earth. Its period of oscillation will be:

(1) 84.6 min. (2) 59.8 min.

(3) 42.3 min. (4) 21.15 min.

92. The maximum time period of oscillation of a simple pendulum is: (If its length varies g = 9.8 m/sec^2)

(1) Infinity (2) 24 hours

(3) 12 hours (4) 1½ hours

93. In a simple oscillating pendulum, the work done by the string in one oscillation will be:
(1) Equal to the total energy of the pendulum
(2) Equal to the K.E. of the pendulum
(3) Equal to the P.E. of the pendulum
(4) Zero

94. A simple pendulum performs simple harmonic motion about X = 0 with an amplitude 4 cm and time period 2 sec. The speed of the pendulum at X = A / 2 will be (in cm/s):
(1) $2\sqrt{3}\pi$ (2) 2π
(3) $\sqrt{3}\pi$ (4) $6\pi^2$

95. A simple pendulum is suspended from the ceiling of a vehicle, its time period is T. Vehicle is moving with constant velocity, then time period of simple pendulum will be :
(1) Less than T (2) Equal to T
(3) More than T (4) Cannot predict

96. Time period of a simple pendulum is T, if its mass is doubled, then its times period will be:
(1) Unchanged (2) 2 times
(3) $\sqrt{2}$ times (4) 4 times

97. A child swinging on a swing in sitting position, stands up, then the period of the swing will be:
(1) Increase
(2) Decrease
(3) Remain same
(4) Increase if child is long and decrease if child is short

Damped Oscillation, Resonance

98. An oscillating pendulum stops, because its energy
(1) Changes into kinetic energy
(2) Change into potential energy
(3) Change into heat energy
(4) Is destroyed

99. Amplitude of vibrations does not remains constant in case of :
(1) Free vibrations
(2) Damped vibrations
(3) Maintained vibrations
(4) Forced vibrations

100. For forced vibrations which statement is incorrect:
(1) Displacement varies with the frequency of force
(2) Displacement is not in phase with force
(3) Amplitude decreases exponentially with time
(4) Amplitude becomes ∞ if the frequency of force coincides with that of body if there is zero damping

101. The amplitude of a SHM reduces to 1/3 in first 20 second then in first 40 second its amplitude becomes:
(1) $\frac{1}{3}$ (2) $\frac{1}{9}$
(3) $\frac{1}{27}$ (4) $\frac{1}{\sqrt{3}}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	1	4	1	1	1	3	2	4	2	2	4	3	2	4	1	4	4	2	4	1	1	1	3	1	2
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	4	2	1	1	3	1	1	3	1	3	3	4	2	4	1	4	1	2	4	3	3	2	3	4	3
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	4	3	2	4	4	2	4	3	2	3	3	1	1	3	3	3	3	1	3	4	4	2	3	1	2
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	2	2	1	2	1	3	1	2	1	1	4	1	1	4	2	2	4	4	1	2	1	2	3	2	3
Que.	101																								
Ans.	2																								

Exercise - II

1. A simple pendulum oscillation in air has period T. The bob of the pendulum is completely immersed in a non–viscous liquid.

The density of the liquid is $(1/16)^{th}$ of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $4T\sqrt{\frac{1}{15}}$ (2) $2T\sqrt{\frac{1}{10}}$

(3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{14}}$

2. A particle executes simple haromic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity of SI units is equal to that of its acceleration. Then, its periodic time in seconds is:

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$

(3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

3. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then–

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$

(3) $K_2 = 2K_1$ (4) $K_2 = K_1$

4. A simple harmonic motion is represented by:

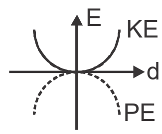
$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

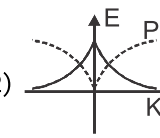
The amplitude and time period of the motion are:

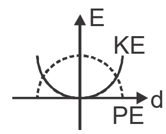
(1) 5 cm, $\frac{3}{2}$s (2) 5 cm, $\frac{2}{3}$s

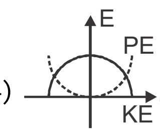
(3) 10 cm, $\frac{3}{2}$s (4) 10 cm, $\frac{2}{3}$s

5. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale)

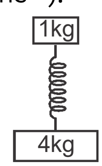
(1) 

(2) 

(3) 

(4) 

6. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take $g = 10\ ms^{-2}$).



(1) 20 N (2) 10 N

(3) 60 N (4) 40 N

7. A body is in simple harmonic motion with time period half second (T = 0.5 s) and amplitude one cm (A = 1 cm). Find the average velocity in the interval in which it moves from equilibrium position to half of its amplitude.

(1) 4 cm/s (2) 6 cm/s

(3) 12 cm/s (4) 16 cm/s

8. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5 sec. In another 10 sec it will decrease to 'a' times its original magnitude , where 'a' equals

(1) 0.7 (2) 0.81

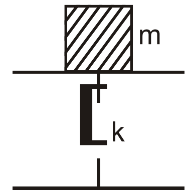
(3) 0.729 (4) 0.6

9. Two simple pendulum of length 1m and 4m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed number of oscillations equal to :

(1) 3 (2) 7

(3) 5 (4) 2

10. A mass=1.0 kg is put on a flat pan attached to a vertical spring fixed on the ground. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes simple harmonic motion. The spring constant is 500 N/m. What is the amplitude A of the motion, so that the mass m tends to get detached from the pan ? The spring of stiff enough so that it does not get distorted during the motion.



(1) A = 2.0 cm

(2) A < 2.0 cm

(3) A = 1.5 cm

(4) A > 2.0 cm

ANSWER KEY										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	4	2	4	4	3	3	3	4	4

Exercise – III (Previous Year Question)

1. The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be–

[AIPMT Pre-2010]

(1) T (2) $T/\sqrt{2}$

(3) 2T (4) $\sqrt{2}\,T$

2. Out of the following functions representing motion of a particle which represents SHM : **[AIPMT Pre-2011]**

(A) $y = \sin\omega t - \cos\omega t$

(B) $y = \sin^3 \omega t$

(C) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

(D) $y = 1 + \omega t + \omega^2 t^2$

(1) Only (A)

(2) Only (D) does not represent SHM

(3) Only (A) and (C)

(4) Only (A) and (B)

3. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other moving in opposite directions when their displacement is half of the amplitude. The mean position of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is :-

[AIPMT Mains-2011]

(1) $\frac{\pi}{6}$ (2) 0

(3) $\frac{2\pi}{3}$ (4) π

4. The oscillation of a body on a smooth horizontal surface is represented by the equation,

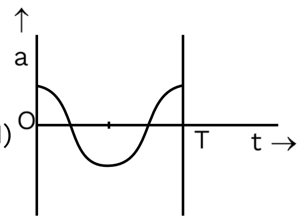
$X = A\cos(\omega t)$

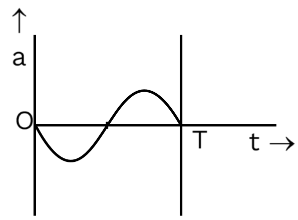
where X = displacement at time t

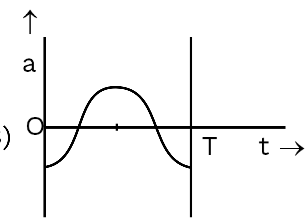
w = frequency of oscillation

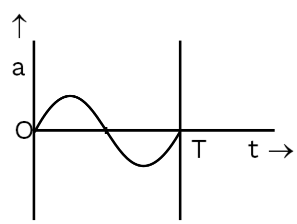
Which one of the following graphs shows currently the variation 'a' with 't'?

[AIPMT (Pre)-2014]

(1) 

(2) 

(3) 

(4) 

5. When two displacements represented by $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are superimposed the motion is :

[AIPMT - 2015]

(1) simple harmonic with amplitude $\frac{a}{b}$

(2) simple harmonic with amplitude $\sqrt{a^2 + b^2}$

(3) simple harmonic with amplitude $\frac{(a+b)}{2}$

(4) not a simple harmonic

6. A particle is executing SHM along a straight line. Its velocities at distance x_1 and x_2 from the mean position are V_1 and V_2, respectively. Its time period is : **[AIPMT - 2015]**

(1) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$ (2) $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$

(3) $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$ (4) $2\pi\sqrt{\frac{x_1^2 + x_2^2}{V_1^2 + V_2^2}}$

7. Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. The are stretched, first by the same amount (case a,) then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively : **[AIPMT - 2015]**

(1) $W_P = W_Q$; $W_P = W_Q$

(2) $W_P > W_Q$; $W_Q > W_P$

(3) $W_P < W_Q$; $W_Q < W_P$

(4) $W_P = W_Q$; $W_P > W_Q$

8. A particle is executing a simple harmonic motion. Its maximum acceleration is a and maximum velocity is b. Then its time period of vibration will be:- **[AIPMT - 2016]**

(1) $\frac{2\pi\beta}{\alpha}$ (2) $\frac{\beta^2}{\alpha^2}$

(3) $\frac{\alpha}{\beta}$ (4) $\frac{\beta^2}{\alpha}$

9. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3s. When the mass m is increased by 1 kg. the time period of oscillations becomes 5 s. The value of m in kg is:- **[AIPMT - 2016]**

(1) $\frac{16}{9}$ (2) $\frac{9}{16}$

(3) $\frac{3}{4}$ (4) $\frac{4}{3}$

10. A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k'. Then they are connected in parallel and force constant is k". Then k' : k" is: **[NEET - 2017]**

(1) 1 : 9 (2) 1 : 11

(3) 1 : 14 (4) 1 : 6

11. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is: **[NEET - 2017]**

(1) $\frac{\sqrt{5}}{\pi}$ (2) $\frac{\sqrt{5}}{2\pi}$

(3) $\frac{4\pi}{\sqrt{5}}$ (4) $\frac{2\pi}{\sqrt{3}}$

12. A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m/s^2 at a distance of 5 m from the mean position. The time period of oscillation is **[NEET- 2018]**

(1) 2 π sec (2) π sec

(3) 2 sec (4) 1 sec

13. The displacement of a particle executing simple harmonic motion is given by

$y = A_0 + A \sin\omega t + B \cos\omega t$

Then the amplitude of its oscillation is given by: **[NEET- 2019]**

(1) $\sqrt{A_0^2 + (A + B)^2}$ (2) A + B

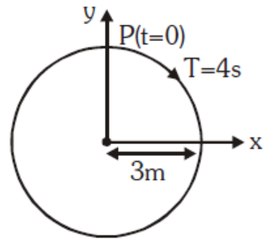
(3) $A_0 + \sqrt{A^2 + B^2}$ (4) $\sqrt{A^2 + B^2}$

14. Average velocity of a particle executing SHM in one complete vibration is: **[NEET- 2019]**

(1) $\frac{A\omega^2}{2}$ (2) zero

(3) $\frac{A\omega}{2}$ (4) $A\omega$

15. The radius of circle the period of revolution initial position and sense of revolution are indicated in the fig.



y-projection of the radius vector of rotating particle P is : **[NEET- 2019]**

(1) $y(t) = -3\cos 2\pi t$, where y in m

(2) $y(t) = 4\sin\left(\frac{\pi t}{2}\right)$, where y in m

(3) $y(t) = 3\cos\left(\frac{3\pi t}{2}\right)$, where y in m

(4) $y(t) = 3\cos\left(\frac{\pi t}{2}\right)$, where y in m

16. The distance covered by a particle undergoing SHM in one time period is (amplitude = A) - **[NEET- 2019(Odisha)]**

(1) zero (2) A

(3) 2A (4) 4A

17. A mass falls from a height 'h' and its time of fall 't' is recorded in terms of time period T of a simple pendulum. On the surface of earth it is found that t = 2T. The entire set up is taken on the surface of another planet whose mass is half of that of earth and radius the same. Same experiment is repeated and corresponding times noted as t' and T', then: **[NEET- 2019(Odisha)]**

(1) $t' = \sqrt{2}T'$ (2) t' > 2T'

(3) t' < 2T' (4) t' = 2T'

18. The phase difference between displacement and acceleration of a particle in a simple harmonic motion is: **[NEET- 2020]**

(1) zero (2) π rad

(3) $\frac{3\pi}{2}$ rad (4) $\frac{\pi}{2}$ rad

19. Identify the function which represents a periodic motion **[NEET (Covid)- 2020]**

(1) $e^{\omega t}$

(2) $\log_e(\omega t)$

(3) $\sin \omega t + \cos \omega t$

(4) $e^{-\omega t}$

20. A body is executing simple harmonic motion with frequency 'n', the frequency of its potential energy is:- **[NEET-2021]**

(1) n (2) 2n

(3) 3n (4) 4n

21. A spring is stretched by 5 cm by a force 10 N. The time period of the oscillations when a mass of 2 kg is suspended by it is: **[NEET-2021]**

(1) 0.0628 (2) 6.285

(3) 3.145 (4) 0.6285

22. Two pendulums of length 121 cm and 100 cm start vibrating in phase. At some instant, the two are at their mean position in the same phase. The minimum number of vibrations of the shorter pendulum after which the two are again in phase at the mean position is: **[NEET-2022]**

(1) 11 (2) 9

(3) 10 (4) 8

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Ans.	4	3	3	3	2	1	2	1	2	2	3	2	4	2	4	4	4	2	3	2	4	1