PARABOLA

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. Latus rectum of the parabola whose focus is (3, 4) and whose tangent at vertex has the equation $x + y = 7 + 5\sqrt{2}$ is -

(A) 5 (B) 10 (C) 20 (D) 15

2. Directrix of a parabola is $x + y = 2$. If it's focus is origin, then latus rectum of the parabola is equal to -

(A) $\sqrt{2}$ units (B) 2 units (C) $2\sqrt{2}$ units (D) 4 units

3. Which one of the following equations represents parametrically, parabolic profile ?

(A) $x = 3\cos t$; $y = 4\sin t$

(B) $x^2 - 2 = -\cos t$; $y = 4\cos^2\frac{t}{2}$

(C) $\sqrt{x} = \tan t$; $\sqrt{y} = \sec t$

(D) $x = \sqrt{1-\sin t}$; $y = \sin\frac{t}{2} + \cos\frac{t}{2}$

4. Let C be a circle and L a line on the same plane such that C and L do not intersect. Let P be a moving point such that the circle drawn with centre at P to touch L also touches C. Then the locus of P is -

(A) a straight line parallel to L not intersecting C

(B) a circle concentric with C

(C) a parabola whose focus is centre of C and whose directrix is L.

(D) a parabola whose focus is the centre of C and whose directrix is a straight line parallel to L.

5. If $(t^2, 2t)$ is one end of a focal chord of the parabola $y^2 = 4x$ then the length of the focal chord will be-

(A) $\left(t + \frac{1}{t}\right)^2$ (B) $\left(t + \frac{1}{t}\right)\sqrt{\left(t^2 + \frac{1}{t^2}\right)}$ (C) $\left(t - \frac{1}{t}\right)\sqrt{\left(t^2 + \frac{1}{t^2}\right)}$ (D) none

6. From the focus of the parabola $y^2 = 8x$ as centre, a circle is described so that a common chord of the curves is equidistant from the vertex and focus of the parabola. The equation of the circle is -

(A) $(x - 2)^2 + y^2 = 3$ (B) $(x - 2)^2 + y^2 = 9$ (C) $(x + 2)^2 + y^2 = 9$ (D) $x^2 + y^2 - 4x = 0$

7. The point of intersection of the curves whose parametric equations are $x = t^2+1$, $y = 2t$ and $x = 2s$, $y = 2/s$ is given by -

(A) (4, 1) (B) (2, 2) (C) (–2, 4) (D) (1, 2)

8. If M is the foot of the perpendicular from a point P of a parabola $y^2 = 4ax$ to its directrix and SPM is an equilateral triangle, where S is the focus, then SP is equal to -

(A) a (B) 2a (C) 3a (D) 4a

9. Through the vertex 'O' of the parabola $y^2 = 4ax$, variable chords OP and OQ are drawn at right angles. If the variable chord PQ intersects the axis of x at R, then distance OR :

(A) varies with different positions of P and Q (B) equals the semi latus rectum of the parabola

(C) equals latus rectum of the parabola (D) equals double the latus rectum of the parabola

10. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is -

(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

11. Point P lies on $y^2 = 4ax$ & N is foot of perpendicular from P on its axis. A straight line is drawn parallel to the axis to bisect NP and meets the curve in Q. NQ meets the tangent at the vertex in a point T such that AT = k NP, then the value of k is : (where A is the vertex)

(A) 3/2 (B) 2/3 (C) 1 (D) none

12. The tangents to the parabola $x = y^2 + c$ from origin are perpendicular then c is equal to -

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{1}{4}$

13. The locus of a point such that two tangents drawn from it to the parabola $y^2 = 4ax$ are such that the slope of one is double the other is -

(A) $y^2 = \frac{9}{2}ax$ (B) $y^2 = \frac{9}{4}ax$ (C) $y^2 = 9ax$ (D) $x^2 = 4ay$

14. T is a point on the tangent to a parabola $y^2 = 4ax$ at its point P. TL and TN are the perpendiculars on the focal radius SP and the directrix of the parabola respectively. Then -

(A) SL = 2 (TN) (B) 3 (SL) = 2 (TN) (C) SL = TN (D) 2 (SL) = 3 (TN)

15. The equation of the circle drawn with the focus of the parabola $(x - 1)^2 - 8y = 0$ as its centre and touching the parabola at its vertex is :

(A) $x^2 + y^2 - 4y = 0$ (B) $x^2 + y^2 - 4y + 1 = 0$

(C) $x^2 + y^2 - 2x - 4y = 0$ (D) $x^2 + y^2 - 2x - 4y + 1 = 0$

16. Length of the normal chord of the parabola, $y^2 = 4x$, which makes an angle of $\frac{\pi}{4}$ with the axis of x is-

(A) 8 (B) $8\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$

17. Tangents are drawn from the point (–1, 2) on the parabola $y^2 = 4x$. The length , these tangents will intercept on the line x = 2 :

(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none of these

18. Locus of the point of intersection of the perpendiculars tangent of the curve $y^2 + 4y - 6x - 2 = 0$ is :

(A) 2x – 1 = 0 (B) 2x + 3 = 0 (C) 2y + 3 = 0 (D) 2x + 5 = 0

19. Tangents are drawn from the points on the line x – y + 3 = 0 to parabola $y^2 = 8x$. Then the variable chords of contact pass through a fixed point whose coordinates are-

(A) (3, 2) (B) (2, 4) (C) (3, 4) (D) (4, 1)

20. The line 4x – 7y + 10 = 0 intersects the parabola, $y^2 = 4x$ at the points A & B. The co-ordinates of the point of intersection of the tangents drawn at the points A & B are :

(A) $\left(\frac{7}{2}, \frac{5}{2}\right)$ (B) $\left(-\frac{5}{2}, \frac{7}{2}\right)$ (C) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (D) $\left(-\frac{7}{2}, \frac{5}{2}\right)$

21. From the point (4, 6) a pair of tangent lines are drawn to the parabola, $y^2 = 8x$. The area of the triangle formed by these pair of tangent lines & the chord of contact of the point (4, 6) is

(A) 2 (B) 4 (C) 8 (D) none

22. TP & TQ are tangents to the parabola, $y^2=4ax$ at P & Q. If the chord PQ passes through the fixed point (-a, b) then the locus of T is -

(A) ay = 2b (x – b) (B) bx = 2a (y – a) (C) by = 2a (x – a) (D) ax = 2b (y – b)

23. If the tangent at the point P (x_1,y_1) to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q & R, then the mid point of QR is -

(A) $(x_1 + b, y_1 + b)$ (B) $(x_1 - b, y_1 - b)$ (C) (x_1, y_1) (D) $(x_1 + b, y_1)$

24. Let PSQ be the focal chord of the parabola, $y^2 = 8x$. If the length of SP=6 then, l(SQ) is equal to(where S is the focus) -

(A) 3 (B) 4 (C) 6 (D) none

25. Two parabolas $y^2 = 4a(x - l_1)$ and $x^2 = 4a(y - l_2)$ always touch one another , the quantities l_1 and l_2 are both variable. Locus of their point of contact has the equation -

(A) $xy = a^2$ (B) $xy = 2a^2$ (C) $xy = 4a^2$ (D) none

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

26. Equation $x^2 - 2x - 2y + 5 = 0$ represents -

(A) a parabola with vertex (1, 2) (B) a parabola with vertex (2, 1)

(C) a parabola with directrix $y = \frac{3}{2}$ (D) a parabola with directrix $y = \frac{2}{5}$

27. The normals to the parabola $y^2 = 4ax$ from the point (5a, 2a) are -

(A) $y = -3x + 33a$ (B) $x = -3y + 3a$ (C) $y = x - 3a$ (D) $y = -2x + 12a$

28. The equation of the lines joining the vertex of the parabola $y^2 = 6x$ to the points on it whose abscissa is 24, is -

(A) $2y + x + 1 = 0$ (B) $2y - x + 1 = 0$ (C) $x + 2y = 0$ (D) $x - 2y = 0$

29. The equation of the tangent to the parabola $y^2 = 9x$ which passes through the point (4, 10) is -

(A) $x + 4y + 1 = 0$ (B) $x - 4y + 36 = 0$ (C) $9x - 4y + 4 = 0$ (D) $9x + 4y + 4 = 0$

30. Consider the equation of a parabola $y^2 = 4ax$, $(a < 0)$ which of the following is false -

(A) tangent at the vertex is $x = 0$ (B) directrix of the parabola is $x = 0$

(C) vertex of the parabola is at the origin (D) focus of the parabola is at $(-a, 0)$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	B	D	A	B	B	D	C	C	B	D	A	C	D
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	B	D	C	C	A	C	C	A	C	A,C	C,D	C,D	B,C	B,D

EXTRA PRACTICE QUESTIONS ON PARABOLA

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is -

(A) $x^2 + 2y^2 - ax = 0$ (B) $2x^2 + y^2 - 2ax = 0$ (C) $2x^2 + 2y^2 - ay = 0$ (D) $2x^2 + y^2 - 2ay = 0$

2. Let A be the vertex and L the length of the latus rectum of parabola, $y^2 - 2y - 4x - 7 = 0$. The equation of the parabola with point A as vertex, 2L as the length of the latus rectum and the axis at right angles to that of the given curve is -

(A) $x^2 + 4x + 8y - 4 = 0$ (B) $x^2 + 4x - 8y + 12 = 0$

(C) $x^2 + 4x + 8y + 12 = 0$ (D) $x^2 + 8x - 4y + 8 = 0$

3. The parametric coordinates of any point on the parabola $y^2 = 4ax$ can be -

(A) $(at^2, 2at)$ (B) $(at^2, -2at)$ (C) $(asin^2t, 2asint)$ (D) $(asint, 2acost)$

4. PQ is a normal chord of the parabola $y^2 = 4ax$ at P, A being the vertex of the parabola. Through P a line is drawn parallel to AQ meeting the x-axis in R. Then the length of of AR is -

(A) equal to the length of the latus rectum

(B) equal to the focal distance of the point P.

(C) equal to twice the focal distance of the point P.

(D) equal to the distance of the point P from the directrix

5. The length of the chord of the parabola $y^2 = x$ which is bisected at the point (2, 1) is-

(A) $5\sqrt{2}$ (B) $4\sqrt{5}$ (C) $4\sqrt{50}$ (D) $2\sqrt{5}$

6. If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then -

(A) $x_1 = x_2$ (B) $x_1 = y_2$ (C) $y_1 = y_2$ (D) $x_2 = y_1$

7. Locus of the intersection of the tangents at the ends of the normal chords of the parabola $y^2 = 4ax$ is -

(A) $(2a + x)y^2 + 4a^3 = 0$ (B) $(x + 2a)y^2 + 4a^2 = 0$

(C) $(y + 2a)x^2 + 4a^3 = 0$ (D) none

8. The locus of the mid point of the focal radii of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

(A) latus rectum is half the latus rectum of the original parabola

(B) vertex is (a/2, 0)

(C) directrix is y-axis

(D) focus has the co-ordinates (a, 0)

9. The equation of a straight line passing through the point (3, 6) and cutting the curve $y = \sqrt{x}$ orthogonally is -

(A) $4x + y - 18 = 0$ (B) $x + y - 9 = 0$ (C) $4x - y - 6 = 0$ (D) none

10. The tangent and normal at P (t), for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through the points P, T and G is -

(A) $\cot^{-1}t$ (B) $\cot^{-1}t^2$ (C) $\tan^{-1}t$ (D) $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

11. A variable circle is described to passes through the point (1, 0) and tangent to the curve $y = \tan(\tan^{-1}x)$. The locus of the centre of the circle is a parabola whose -

(A) length of the latus rectum is $2\sqrt{2}$ (B) axis of symmetry has the equation $x + y = 1$

(C) vertex has the co-ordinates (3/4, 1/4) (D) none of these

12. AB, AC are tangents to a parabola $y^2 = 4ax$. p_1 p_2 and p_3 are the lengths of the perpendiculars from A, B and C respectively on any tangent to the curve, then p_2, p_1, p_3 are in-

(A) A.P. (B) G.P. (C) H.P. (D) none of these

13. Through the vertex O of the parabola, $y^2 = 4ax$ two chords OP and OQ are drawn and the circles on OP and OQ as diameter intersect in R. If θ_1, θ_2 and ϕ are the angles made with the axis by the tangent at P and Q on the parabola and by OR then the value of $\cot\theta_1 + \cot\theta_2 =$

(A) $-2\tan\phi$ (B) $-2\tan(\pi - \phi)$ (C) 0 (D) $2\cot\phi$

14. Two parabolas have the same focus. If their directrices are the x-axis & the y-axis respectively, then the slope of their common chord is -

(A) 1 (B) –1 (C) 4/3 (D) 3/4

15. Tangent to the parabola $y^2 = 4ax$ at point P meets the tangent at vertex A, at point B and the axis of parabola at T. Q is any point on this tangent and N is the foot of perpendicular from Q on SP, where S is focus. M is the foot of perpendicular from Q on the directrix then -

(A) B bisects PT (B) B trisects PT (C) QM = SN (D) QM = 2SN

16. If the distance between a tangent to the parabola $y^2 = 4x$ and a parallel normal to the same parabola is $2\sqrt{2}$, then possible values of gradient of either of them are -

(A) –1 (B) +1 (C) $-\sqrt{\sqrt{5}-2}$ (D) $+\sqrt{\sqrt{5}-2}$

17. If two distinct chords of a parabola $x^2 = 4ay$ passing through (2a, a) are bisected on the line $x + y = 1$, then length of latus rectum can be -

(A) 2 (B) 1 (C) 4 (D) 5

18. If PQ is a chord of parabola $x^2 = 4y$ which subtends right angle at vertex. Then locus of centroid of triangle PSQ (S is focus) is a parabola whose -

(A) vertex is (0, 3) (B) length of LR is 4/3

(C) axis is x = 0 (D) tangent at the vertex is x = 3

19. Identify the correct statement(s) -

(A) In a parabola vertex is the mid point of focus and foot of directrix.

(B) $P\left(at_1^2, 2at_1\right)$ & $Q\left(at_2^2, 2at_2\right)$ are two points on $y^2 = 4ax$ such that $t_1t_2 = -1$, then normals at P and Q are perpendicular.

(C) There doesn't exist any tangent of $y^2 = 4ax$ which is parallel to x-axis.

(D) At most two normals can be drawn to a parabola from any point on its plane.

20. For parabola $y^2 = 4ax$ consider three points A, B, C lying on it. If the centroid of ΔABC is (h_1, k_1) & centroid of triangle formed by the point of intersection of tangents at A, B, C has coordinates (h_2, k_2), then which of the following is always true -

(A) $2k_1 = k_2$ (B) $k_1 = k_2$ (C) $k_1^2 = \frac{4a}{3}\left(h_1 + 2h_2\right)$ (D) $k_1^2 = \frac{4a}{3}\left(2h_1 + h_2\right)$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A,B	A,B	C	D	C	A	A,B,C,D	A	C,D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B,C	B	A	A,B	A,C	A,B,C,D	A,B	A,B,C	A,B,C	B,C