DPP – 05

CLASS – 12^{th}

TOPIC – TYPES OF RELATION

Q.1 Show that the relation R defined by R = {(a, b): a – b is divisible by 3; a, b ∈ Z} is an equivalence relation.

Q.2 Show that the relation R on the set Z of integers, given by

R = {(a, b): 2 divides a – b}, is an equivalence relation.

Q.3 Prove that the relation R on Z defined by (a, b) ∈ R ⇔ a − b is divisible by 5 is an equivalence relation on Z.

Q.4 Let n be a fixed positive integer. Define a relation R on Z as follows:

(a, b) ∈ R ⇔ a − b is divisible by n.

Show that R is an equivalence relation on Z.

Q.5 Let Z be the set of integers. Show that the relation R = {(a, b): a, b ∈ Z and a + b is even} is an equivalence relation on Z.

Q.6 m is said to be related to n if m and n are integers and m − n is divisible by 13. Does this define an equivalence relation?

Q.7 Let R be a relation on the set A of ordered pair of integers defined by (x, y) R (u, v) if xv = y u. Show that R is an equivalence relation.

Q.8 Show that the relation R on the set A = {x ∈ Z; 0 ≤ x ≤ 12}, given by R = {(a, b): a = b}, is an equivalence relation. Find the set of all elements related to 1.

SOLUTION

(MATHS) **RELATIONS & FUNCTIONS**

DPP – 05

CLASS –12th

TOPIC – TYPES OF RELATION

Sol. 1 Given R = {(a, b): a − b is divisible by 3; a, b ∈ Z} is a relation

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Let us check these properties on R.

Reflexivity:

Let a be an arbitrary element of R.

Then, $a - a = 0 = 0 \times 3$

$\Rightarrow a - a$ is divisible by 3

$\Rightarrow (a, a) \in R$ for all $a \in Z$

So, R is reflexive on Z.

Symmetry:

Let $(a, b) \in R$

$\Rightarrow a - b$ is divisible by 3

$\Rightarrow a - b = 3p$ for some $p \in Z$

$\Rightarrow b - a = 3(-p)$

Here, $-p \in Z$

$\Rightarrow b - a$ is divisible by 3

$\Rightarrow (b, a) \in R$ for all $a, b \in Z$

So, R is symmetric on Z.

Transitivity:

Let (a, b) and $(b, c) \in R$

$\Rightarrow a - b$ and $b - c$ are divisible by 3

$\Rightarrow a - b = 3p$ for some $p \in Z$

And $b - c = 3q$ for some $q \in Z$

Adding the above two equations, we get

$a - b + b - c = 3p + 3q$

$\Rightarrow a - c = 3(p + q)$

Here, $p + q \in Z$

$\Rightarrow a - c$ is divisible by 3

$\Rightarrow (a, c) \in R$ for all $a, c \in Z$

So, R is transitive on Z.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on Z.

Sol. 2 Given R = {(a, b): 2 divides a – b} is a relation defined on Z.

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Let us check these properties on R.

Reflexivity:

Let a be an arbitrary element of the set Z.

Then, a ∈ R

⇒ a – a = 0 = 0 × 2

⇒ 2 divides a – a

⇒ (a, a) ∈ R for all a ∈ Z

So, R is reflexive on Z.

Symmetry:

Let (a, b) ∈ R

⇒ 2 divides a – b

⇒ (a-b)/2 = p for some p ∈ Z

⇒ (b-a)/2 = – p

Here, −p ∈ Z

⇒ 2 divides b – a

⇒ (b, a) ∈ R for all a, b ∈ Z

So, R is symmetric on Z

Transitivity:

Let (a, b) and (b, c) ∈ R

⇒ 2 divides a−b and 2 divides b−c

⇒ (a-b)/2 = p and (b-c)/2 = q for some p, q ∈ Z

Adding the above two equations, we get

(a – b)/2 + (b – c)/2 = p + q

⇒ (a – c)/2 = p +q

Here, p+ q ∈ Z

⇒ 2 divides a – c

⇒ (a, c) ∈ R for all a, c ∈ Z

So, R is transitive on Z.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on Z.

Sol.3 Given relation R on Z defined by (a, b) ∈ R ⇔ a – b is divisible by 5

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Let us check these properties on R.

Reflexivity:

Let a be an arbitrary element of R. Then,

$\Rightarrow a - a = 0 = 0 \times 5$

$\Rightarrow a - a$ is divisible by 5

$\Rightarrow (a, a) \in R$ for all $a \in Z$

So, R is reflexive on Z.

Symmetry:

Let $(a, b) \in R$

$\Rightarrow a - b$ is divisible by 5

$\Rightarrow a - b = 5p$ for some $p \in Z$

$\Rightarrow b - a = 5(-p)$

Here, $-p \in Z$ [Since $p \in Z$]

$\Rightarrow b - a$ is divisible by 5

$\Rightarrow (b, a) \in R$ for all $a, b \in Z$

So, R is symmetric on Z.

Transitivity:

Let (a, b) and $(b, c) \in R$

$\Rightarrow a - b$ is divisible by 5

$\Rightarrow a - b = 5p$ for some Z

Also, $b - c$ is divisible by 5

$\Rightarrow b - c = 5q$ for some Z

Adding the above two equations, we get

$a - b + b - c = 5p + 5q$

$\Rightarrow a - c = 5(p + q)$

$\Rightarrow a - c$ is divisible by 5

Here, $p + q \in Z$

$\Rightarrow (a, c) \in R$ for all $a, c \in Z$

So, R is transitive on Z.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on Z.

Sol. 4 Given $(a, b) \in R \Leftrightarrow a - b$ is divisible by n is a relation R defined on Z.

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Let us check these properties on R.

Reflexivity:

Let $a \in N$

Here, $a - a = 0 = 0 \times n$

$\Rightarrow a - a$ is divisible by n

$\Rightarrow (a, a) \in R$

$\Rightarrow (a, a) \in R$ for all $a \in Z$

So, R is reflexive on Z.

Symmetry:

Let $(a, b) \in R$

Here, $a - b$ is divisible by n

$\Rightarrow a - b = n\,p$ for some $p \in Z$

$\Rightarrow b - a = n\,(-p)$

$\Rightarrow b - a$ is divisible by n [$p \in Z \Rightarrow -p \in Z$]

$\Rightarrow (b, a) \in R$

So, R is symmetric on Z.

Transitivity:

Let (a, b) and $(b, c) \in R$

Here, $a - b$ is divisible by n and $b - c$ is divisible by n.

$\Rightarrow a - b = n\,p$ for some $p \in Z$

And $b - c = n\,q$ for some $q \in Z$

$a - b + b - c = n\,p + n\,q$

$\Rightarrow a - c = n\,(p + q)$

$\Rightarrow (a, c) \in R$ for all $a, c \in Z$

So, R is transitive on Z.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on Z.

Sol. 5 Given $R = \{(a, b): a, b \in Z$ and $a + b$ is even$\}$ is a relation defined on R.

Also given that Z be the set of integers

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Let us check these properties on R.

Reflexivity:

Let a be an arbitrary element of Z.

Then, $a \in R$

Clearly, $a + a = 2a$ is even for all $a \in Z$.

$\Rightarrow (a, a) \in R$ for all $a \in Z$

So, R is reflexive on Z.

Symmetry:

Let $(a, b) \in R$

$\Rightarrow a + b$ is even

$\Rightarrow b + a$ is even

$\Rightarrow (b, a) \in R$ for all $a, b \in Z$

So, R is symmetric on Z.

Transitivity:

Let (a, b) and $(b, c) \in R$

$\Rightarrow a + b$ and $b + c$ are even

Now, let $a + b = 2x$ for some $x \in Z$

And $b + c = 2y$ for some $y \in Z$

Adding the above two equations, we get

$A + 2b + c = 2x + 2y$

$\Rightarrow a + c = 2 (x + y - b)$, which is even for all $x, y, b \in Z$

Thus, $(a, c) \in R$

So, R is transitive on Z.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on Z

Sol. 6 Given that m is said to be related to n if m and n are integers and $m - n$ is divisible by 13

Now we have to check whether the given relation is equivalence or not.

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Let $R = \{(m, n): m, n \in Z : m - n \text{ is divisible by } 13\}$

Let us check these properties on R.

Reflexivity:

Let m be an arbitrary element of Z.

Then, $m \in R$

$\Rightarrow m - m = 0 = 0 \times 13$

$\Rightarrow m - m$ is divisible by 13

$\Rightarrow (m, m)$ is reflexive on Z.

Symmetry:

Let $(m, n) \in R$.

Then, $m - n$ is divisible by 13

$\Rightarrow m - n = 13p$

Here, $p \in Z$

$\Rightarrow n - m = 13 (-p)$

Here, $-p \in Z$

$\Rightarrow n - m$ is divisible by 13

$\Rightarrow (n, m) \in R$ for all $m, n \in Z$

So, R is symmetric on Z.

Transitivity:

Let (m, n) and (n, o) $\in$R

$\Rightarrow m - n$ and $n - o$ are divisible by 13

$\Rightarrow m - n = 13p$ and $n - o = 13q$ for some $p, q \in Z$

Adding the above two equations, we get

$m - n + n - o = 13p + 13q$

$\Rightarrow m-o = 13 (p + q)$

Here, $p + q \in Z$

$\Rightarrow m - o$ is divisible by 13

$\Rightarrow (m, o) \in R$ for all $m, o \in Z$

So, R is transitive on Z.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on Z.

Sol. 7 First let R be a relation on A

It is given that set A of ordered pair of integers defined by (x, y) R (u, v) if $xv = y u$

Now we have to check whether the given relation is equivalence or not.

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Reflexivity:

Let (a, b) be an arbitrary element of the set A.

Then, $(a, b) \in A$

$\Rightarrow a b = b a$

$\Rightarrow (a, b) R (a, b)$

Thus, R is reflexive on A.

Symmetry:

Let (x, y) and (u, v) $\in$A such that (x, y) R (u, v). Then,

$x v = y u$

$\Rightarrow v x = u y$

$\Rightarrow u y = v x$

$\Rightarrow (u, v) R (x, y)$

So, R is symmetric on A.

Transitivity:

Let (x, y), (u, v) and (p, q) ∈R such that (x, y) R (u, v) and (u, v) R (p, q)

⇒ x v = y u and u q = v p

Multiplying the corresponding sides, we get

x v × u q = y u × v p

⇒ x q = y p

⇒ (x, y) R (p, q)

So, R is transitive on A.

Therefore R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation on A.

Sol. 8 Given set A = {x ∈ Z; 0 ≤ x ≤ 12}

Also given that relation R = {(a, b): a = b} is defined on set A

Now we have to check whether the given relation is equivalence or not.

To prove an equivalence relation, it is necessary that the given relation should be reflexive, symmetric and transitive.

Reflexivity:

Let a be an arbitrary element of A.

Then, a ∈ R

⇒ a = a [Since, every element is equal to itself]

⇒ (a, a) ∈ R for all a ∈ A

So, R is reflexive on A.

Symmetry:

Let (a, b) ∈ R

⇒ a b

⇒ b = a

⇒ (b, a) ∈ R for all a, b ∈ A

So, R is symmetric on A.

Transitivity:

Let (a, b) and (b, c) ∈ R

⇒ a =b and b = c

⇒ a = b c

⇒ a = c

⇒ (a, c) ∈ R

So, R is transitive on A.

Hence, R is an equivalence relation on A.

Therefore R is reflexive, symmetric and transitive.

The set of all elements related to 1 is {1}.