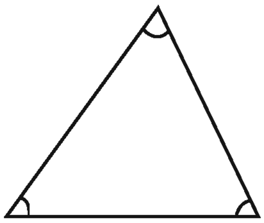
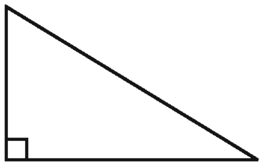
Chapter 8 Triangle and its Properties

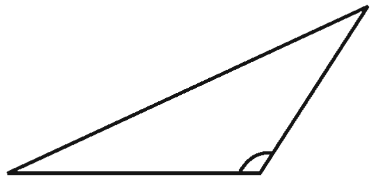
8.1 Triangle is a closed simple figure enclosed by three line segments, it has three sides, three angles and three vertices. Triangles are classified on the basis of sides and angles. Look at the triangles drawn hereunder.



(i)



(ii)



(iii)

What special do you find in these figures?

- All three angles of triangle (i) are acute, so it is called Acute Angled Triangle.
- In triangle (ii) one of the angles is right angle, so it is called Right Angled Triangle.
- In triangle (iii) one of the angles is obtuse angle, so it is called Obtuse Angled Triangle.

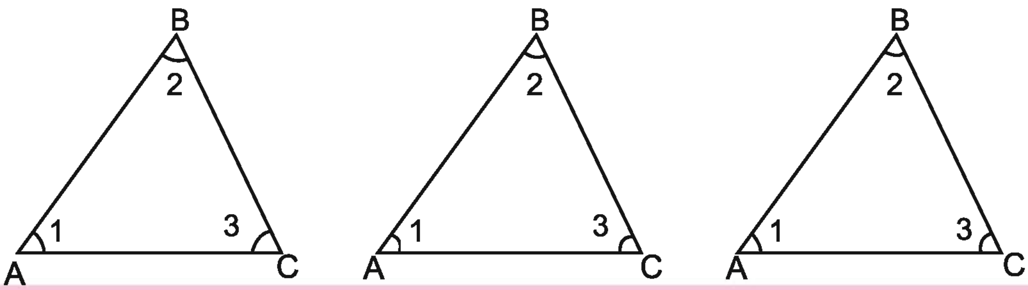
Does the measurement of other two angles change by changing the measurement of one of the three angles?

Try by drawing and testing various triangles and fill in the blanks.

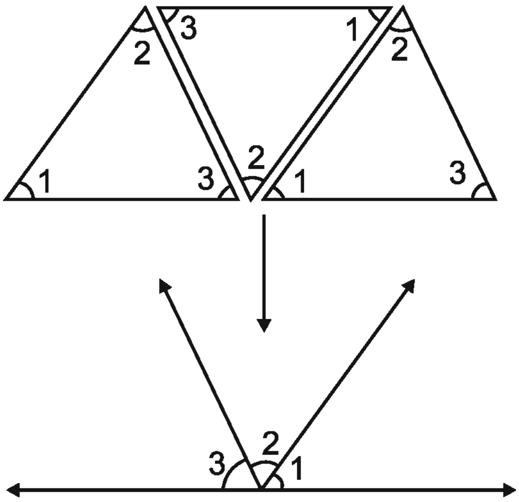
Name of Triangle	Angles
Δ ABC	$\angle A=50^0$, $\angle B=60^0$, $\angle C=70^0$
Δ ABC	$\angle A=30^0$, $\angle B=.....^0$, $\angle C=.....^0$
Δ ABC	$\angle A=100^0$, $\angle B=.....^0$, $\angle C=.....^0$

8.2 Property of Sum of Interior Angles of Triangle

1. Draw three triangles of same sides and angles and cut off the figures so drawn.



2. Arrange three triangles as given below



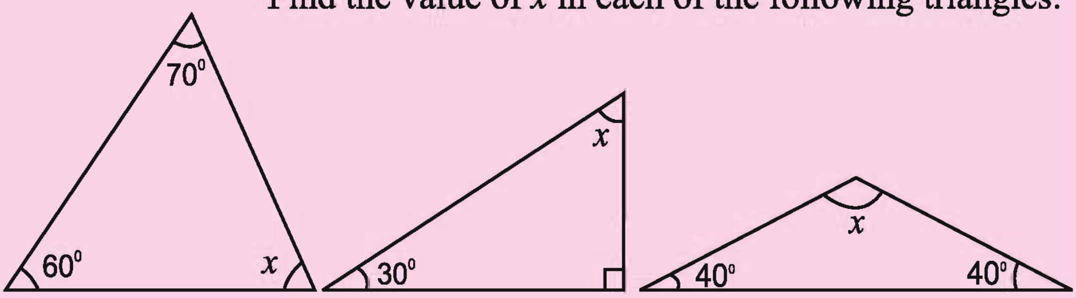
$\angle 1$, $\angle 2$, $\angle 3$ jointly form a straight angle so $\angle 1 + \angle 2 + \angle 3 = 180^\circ$.

Sum of three interior angles of a triangle is 180°.

Try this fact by drawing some more triangles.

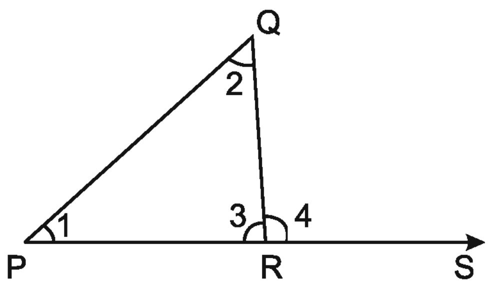
Do and learn

Find the value of x in each of the following triangles:

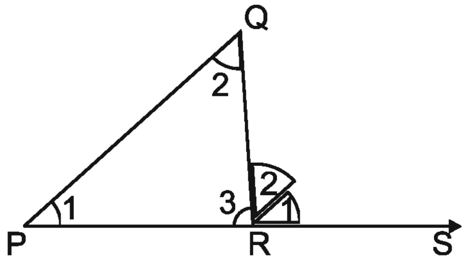


8.3 Exterior angles of a triangle and its properties

1. Construct a triangle PQR and extend the side PR.



2. Construct one more triangle similar to Δ PQR, cut off the angles ∠1 and ∠2 and put them on the exterior angle∠QRS of Δ PQR as given in figure given below.



We observe that the angles ∠1 and ∠2, covers the exterior angle ∠QRS of Δ PQR completely. So

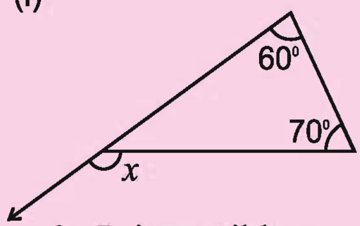
$$\angle QRS = \angle P + \angle Q.$$

Any exterior angle of a triangle is equal to the sum of two interior angles on the opposite sides.

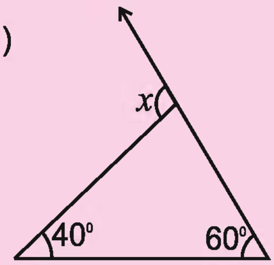
Do and learn:

1. Find the value of exterior angle x from the following figures:

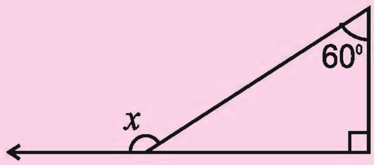
(i)



(ii)



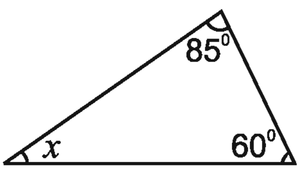
(iii)



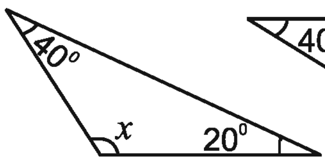
2. Is it possible to construct a triangle with two right angles?
3. Is it possible to construct a triangle whose all the three angles are greater than 60°?

Exercise 8.1

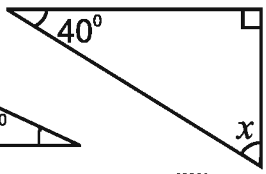
1. Find the value of unknown angle x for the following figures:



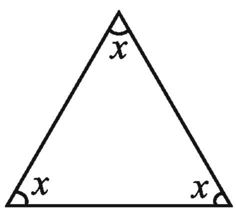
(i)



(ii)

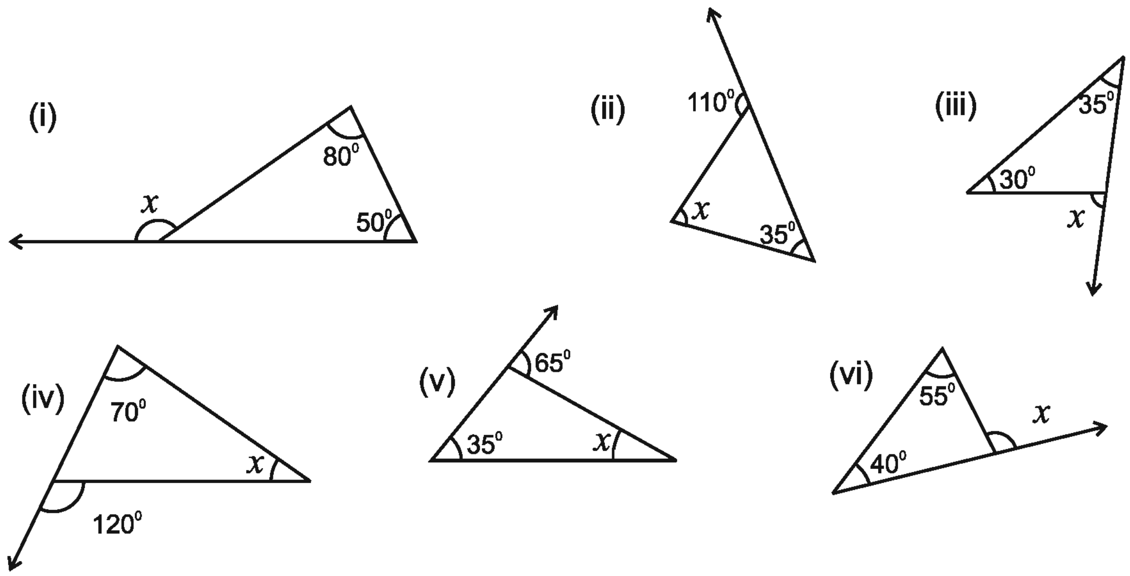


(iii)

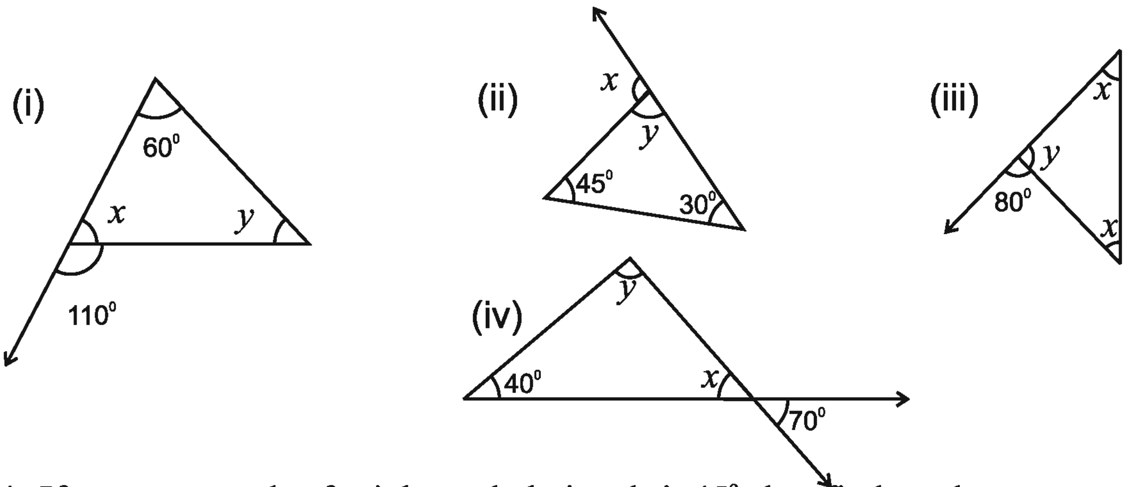


(iv)

2. Find the unknown angle x for the following figures:



3. Find the values of the unknown angles x and y in the following figures:



4. If one acute angle of a right angled triangle is 45°, then find another acute angle.
5. If two angles of a triangle are 50° each, then find the third angle.
6. If the angles of a triangle are in the ratio 1 : 2 : 3 then find each angle of the triangle.
7. Is it possible to construct a right angled triangle whose other two angles are 70° and 21°? If not, then why? Justify.
8. Given below are the triads. Which of the following triads represents the angles of a triangle?
   (i) 100°, 30°, 40° (ii) 30°, 59°, 91° (iii) 45°, 45°, 90° (iv) 120°, 30°, 50°

8.4 Relationship among sides of a triangle

8.4.1 Sum of the length of two sides of a triangle

Construct the triangles according to given measurement

1. Δ XYZ having sides 5 cm, 4 cm, 6 cm
2. Δ MNO having sides 6.5 cm, 4.5 cm, 3 cm
3. Δ PQR having sides 5 cm, 6 cm, 12 cm
4. Δ UVW having sides 2cm, 3cm, 5 cm

Were you able to construct triangles from all the given measurements? If not, why? Discuss. Write the measurement of the sides of triangles you constructed in the following table:

Name of Triangle	Length of Side	Sum of Two Sides	Relationship of Sides	Sum of Two Sides is greater than the third	Triangle Exists Yes/No
ΔXYZ	$x = 5$ $y = 4$ $z = 6$	$x + y = 5 + 4$ $y + z = 4 + 6$ $z + x = 6 + 5$	$x + y > z$ $9 > 6$ $y + z > x$ $10 > 5$ $z + x > y$ $11 > 4$	Yes Yes Yes	
ΔMNO	m = n = o =	m + n = n + o = o + m =			
ΔPQR	p = q = r =	p + q = q + r = r + p =			
ΔUVW	u = v = w =	u + v = v + w = w + u =			

We conclude from the above table that **the sum of any two sides of a triangle is always greater than the third side**.

8.4.2 Difference of length of two sides of triangle

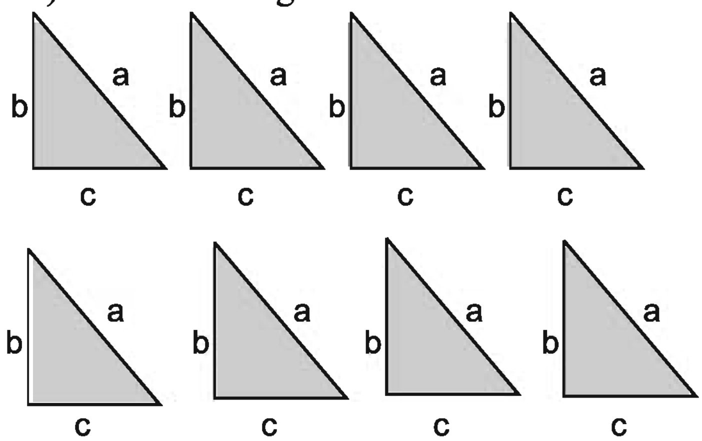
Consider the difference of length of two sides. What do you observe? Is the difference of length of any two sides of a triangle is less than, greater than or equal to third side. Observing few triangles you will find that the difference of length of any two sides of a triangle is less than the third side.

Do and learn:

1. Construct a triangle with side lengths 3.5 cm, 4.5 cm and 6 cm.
2. Is it possible to construct a triangle with sides having length 4 cm, 5 cm and 9 cm?

8.5 Bodhayan Theorem (Pythagoras Theorem)

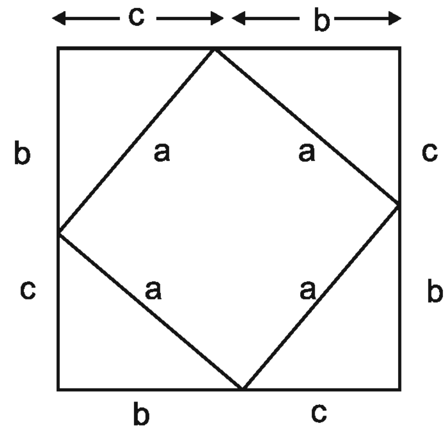
1. Construct a right angled triangle and copy it 8 times on a card sheet and cut them off. Suppose that the length of the side opposite to right angle (hypotenuse) is *a* and the length of other two sides are *b* and *c*.



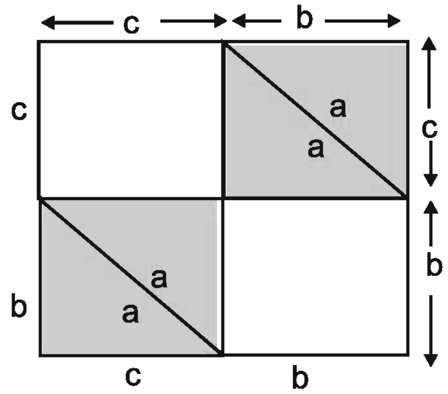
2. Now find the sum of sides *b* and *c* and construct two squares of side measuring the length equal to $(b+c)$ on another card sheet.

Square – I Square - II

3. Now arrange four triangles in Square-I and four triangles in Square-II as given in following figure

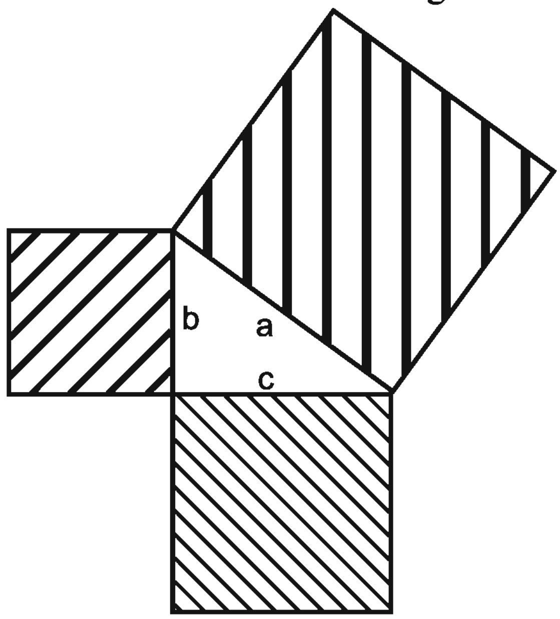


Square – I



Square – II

4. Both the squares are similar and all the 8 triangles are similar. So Area of the blank portion of Square – I = Area of blank portion of Square – II or Area of the square formed in blank portion of Square – I = the sum of areas of the squares formed in the blank space in Square – II.
i.e., $a^2 = b^2 + c^2$.
This relationship of right angled triangles is known as Pythagoras Theorem. This was first derived by the Indian Mathematician Bodhayan and later Pythagoras gave its systematic proof in modern mathematics. This theorem can also be explored in following way. We arrange the square of side a on the longest side (hypotenuse) of Δ abc and squares of side b and side c on the other sides b and c as given in the following figure:



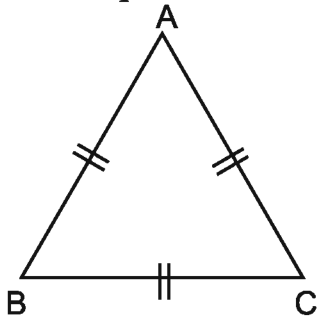
From this, we could say that

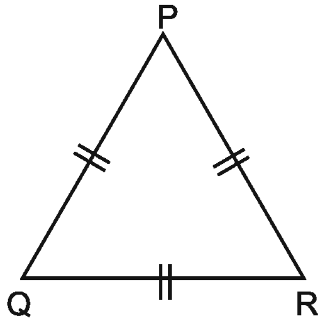
> In right angled triangle, the square of hypotenuse is the sum of squares of other two sides of the triangle. (i.e., base and perpendicular). Symbolically $a^2 = b^2 + c^2$.

8.6 Relation between sides and angles

Construct an equilateral triangle ΔABC and its replica ΔPQR (with help of a trace paper) and cut them off. Now, put the $\angle P$ of ΔPQR on all three angles of ΔABC one by one and observe:

When we put $\angle P$ on $\angle A$, then $\angle Q$ covers $\angle B$ and $\angle R$ covers $\angle C$.





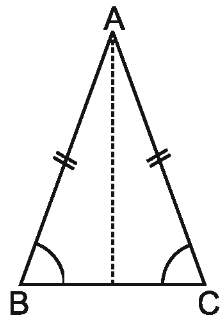
When we put equilateral triangles on each other they overlap completely. Thus, in an equilateral triangle all the sides as well as angles are equal.

Will two angles of a triangle be equal if its two sides are equal? If yes, then which one?

Construct an isosceles triangle ΔABC on a card sheet/paper. Fold the triangle such that equal sides coincide with each other.

Are the angles opposite to equal sides also equal?

You would find that the angles opposite to equal sides are also equal.

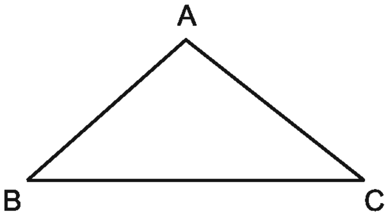


So, the sides AB and AC are called similar sides of the triangle and the angles $\angle B$ and $\angle C$ opposite to them are base angles and are mutually equal.

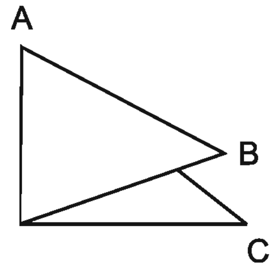
Therefore, in an isosceles triangle the angles opposite to the equal sides are equal.

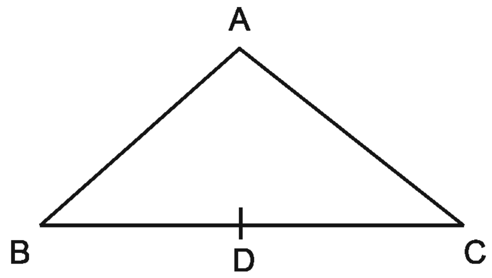
8.7 Medians of a triangle

1. Construct a triangle ΔABC on a card sheet and cut it off.

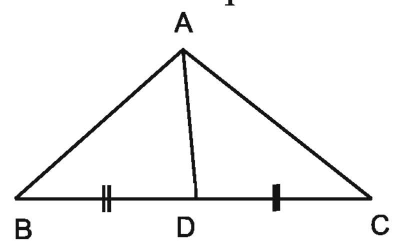


2. Fold the triangle so that the vertices B and C coincide. It will give middle point of BC. Name it D.



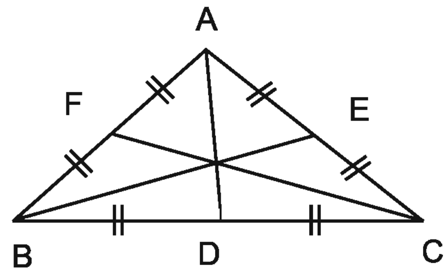


3. Now join the point A with the middle point of BC. AD is the median of the triangle ABC.



The line joining a vertex to the middle point of its opposite side of a triangle is known as median of the triangle.

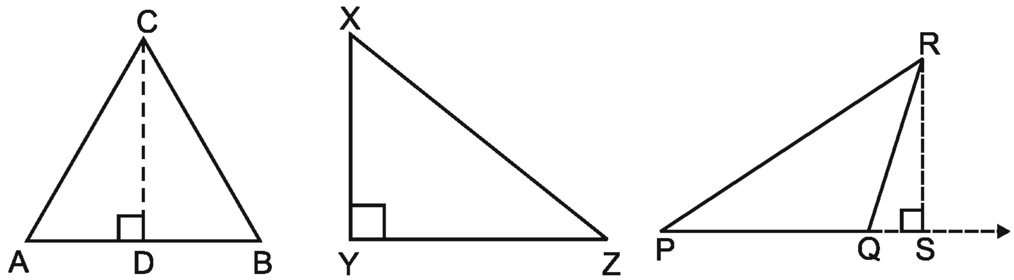
4. Similarly, median BE and CF can also be drawn as given below:



There can be maximum three medians in a triangle. The point of intersection of medians is called centroid of the triangle.

8.8 Altitude of triangle

An altitude of a triangle is a straight line drawn through the vertex and perpendicular to (i.e., forming a right angle) opposite side. Three such altitudes can be drawn for a triangle. The three altitudes intersect in a point called **Orthocentre** of the triangle. Identify the altitudes drawn in the following triangles:

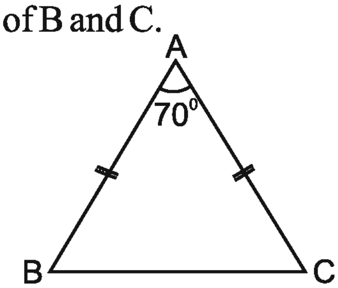


ΔABC is an acute angled triangle. All its altitudes lie inside triangle.
ΔXYZ is a right angled triangle. The two sides forming the right angle are themselves altitudes.
ΔPQR is an obtuse angled triangle. One of the altitudes lies outside the triangle.

The height of the triangle is equal to the altitude with base as the side on which it is drawn.

Exercise 8.2

1. Which of the following triads form a triangle:
   (i) 6, 5, 5 (ii) 2, 3, 5 (iii) 3, 4, 8 (iv) 3, 5, 6 (v) 4, 4, 8 (vi) 9, 2, 8
2. Find the value of all the angles of an equilateral triangle.
3. Fill in the blanks:
   (i) At least two angles in a triangle are...............
   (ii) One altitude of an .... Tringle lies outside the triangle.
   (iii) Sum of any two sides of a tringle is .... than the third side.
   (iv) Two angles of an .... triangle are equal.
   (v) The line joining the vertex to the middle point of opposite side of a triangle is called ....
   (vi) The point where three medians of a triangle meet is called ....
   (vii) All three .... of a triangle pass through the orthocentre of a triangle.
4. In ΔABC, $\angle A = 70^{\circ}$ and $AB = AC$. Find the value of B and C.



5. Construct a triangle and show in it one median and one altitude.
6. The length of two sides of a triangle are 3 cm and 6 cm. What could be the minimum and maximum value of its third side?
7. Draw two equilateral triangular traffic symbols (warning symbols) that attracts your attention towards possible danger on the road.

We Learnt

1. Three sides and three angles in a triangle are known as its elements.
2. Sum of all the three angles in a triangle is 180°.
3. An exterior angle of a triangle is formed by extending a side in only one direction. Extension of a side in both the directions give two exterior angles.
4. Exterior angle is the sum of two opposite interior angles.
5. Properties of sides of triangle:
   (i) Sum of length of any two sides of a triangle is greater than the length of third side.
   (ii) Difference of length of sides is less than the length of its third side.
   Both the properties are useful in the possible construction of a triangle when its sides are given.
6. In a right angled triangle the side opposite to right angle is called hypotenuse and other two are called legs. In right angled triangle: Square of hypotenuse = sum of square of legs.
7. The line segment joining the vertex to the middle point of the opposite side in a triangle is known as median of the triangle. There are three medians in a triangle. Point of intersection of medians is called centroid of the triangle.
8. A perpendicular drawn from a vertex of triangle on the opposite side is known as altitude. A triangle can have three altitudes. Point of intersection of altitudes is known as orthocentre of the triangle.