CHAPTER 11

Fluid Mechanics

Pressure and Upthrust

- Upthrust $F = V_i \rho_l g_e$
- When a solid whose density is less than the density of liquid floats in it then, some fraction of solid remains immersed in the liquid. In this case,
  (i) Weight = upthrust
  (ii) Fraction of immersed volume, $f = \frac{\rho_s}{\rho_l}$
- When a solid whose density is more than the density of liquid is completely immersed in it, then upthrust acts on its 100% volume and apparent weight is less than its actual weight.

$$w_{\text{app}} = w - F$$

Here, F = Upthrust on 100% volume of solid.

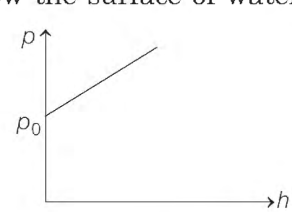
- Relative density (or specific gravity) of any substance

$$\text{RD} = \frac{\text{density of that substance}}{\text{density of water}} = \frac{\text{weight in air}}{\text{change in weight in water}}$$

- $1\,\text{Pa} = 1\,\text{Nm}^{-2}$, $1\,\text{Bar} = 10^5\,\text{Pa}$, $1\,\text{atm} = 1.013 \times 10^5\,\text{Pa}$

Gauge pressure = absolute pressure − atmospheric pressure

- Pressure at depth h below the surface of water,

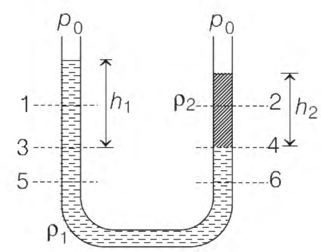


$$p = p_0 + \rho g h$$

Change in pressure per unit depth,

$$\frac{dp}{dh} = \rho g$$

- In the figure given below, liquid pressure will be same at all points at the same level (their provided speeds are same).



For example, in the figure

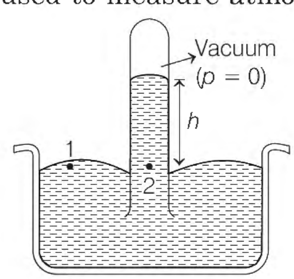
$$p_1 \neq p_2, \; p_3 = p_4 \text{ and } p_5 = p_6$$

Further, $\quad p_3 = p_4$

$\therefore \quad p_0 + \rho_1 g h_1 = p_0 + \rho_2 g h_2$

or $\quad \rho_1 h_1 = \rho_2 h_2 \quad$ or $\quad h \propto \frac{1}{\rho}$

- **Barometer** It is a device used to measure atmospheric pressure.



$\because \quad p_1 = p_2$

Here, $\quad p_1$ = atmospheric pressure (p_0)

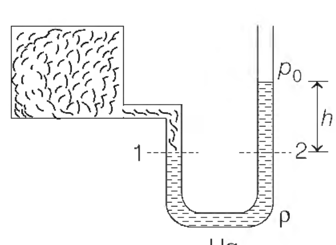
and $\quad p_2 = 0 + \rho g h = \rho g h$

Here, $\quad \rho$ = density of mercury

$\therefore \quad p_0 = \rho g h$

Thus, the mercury barometer reads the atmospheric pressure (p_0) directly from the height of the mercury column.

- **Manometer** It is a device used to measure the pressure of a gas inside a container.



The U-shaped tube often contains mercury

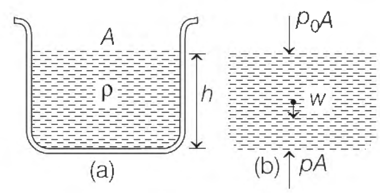
$$p_1 = p_2$$

Here, p_1 = pressure of the gas in the container (p)

and p_2 = atmospheric pressure ($p_0 + \rho g h$)

$\therefore$ $p_{gas} = p_1 = p_0 + h\rho g$

- **Free Body Diagram of a Liquid** The free body diagram of the liquid (showing the vertical forces only) is shown in Fig. (b). For the equilibrium of liquid,



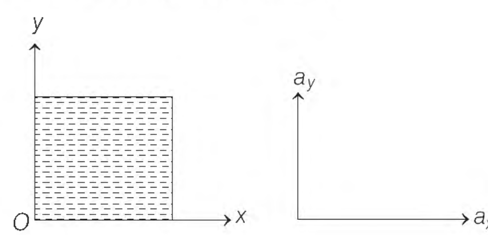
Net downward force = net upward force

$\therefore$ $$p_0A + W = pA$$

Here, $$W = \rho g h A$$

$\therefore$ $$p = p_0 + \rho g h$$

- **Change in Pressure in Accelerated Fluids**



If container has an acceleration component a_x in x-direction and a_y in y-direction.Then,

$$\frac{dp}{dx} = -\rho a_x$$

and $$\frac{dp}{dy} = -\rho (a_y + g)$$

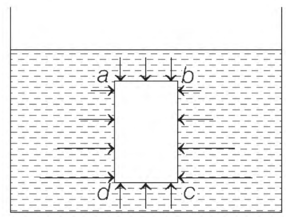
- **Pressure Difference in Rotating Fluids**

In a rotating fluid (also accelerating) pressure increases in moving away from the rotational axis. At a distance x from the rotational axis, the pressure difference is

$$\Delta p = \pm \frac{\rho \omega^2 x^2}{2}$$

Take, $\Delta p = +\frac{\rho \omega^2 x^2}{2}$ in moving away from the rotational axis, as pressure increases in this direction and take $\Delta p = -\frac{\rho \omega^2 x^2}{2}$ in moving towards the rotational axis.

- **Variation of Pressure with Depth** Consider a cylinder kept inside a liquid as shown in figure.



Pressure increases linearly with depth as

$$p = p_0 + \rho g h$$

Therefore, if h is same, then pressure is also same or we can say that on a horizontal surface pressure will be same.

For example, pressure at all points on horizontal surface (ab) will be same. Similarly, pressure at all points on horizontal surface (dc) is also same. But

$$p_{dc} > p_{ab} \qquad \text{(As } h_{dc} > h_{ab}\text{)}$$

But pressure on vertical faces ad and bc increases linearly with depth as shown in figure.

- **Pressure Force and its Torque**

$$p = \frac{F}{A} \Rightarrow F = pA$$

Let us call this force as the pressure force. Now, this force is calculated on a surface. If surface is horizontal then pressure is uniform at all points.

So, $F = pA$ can be applied directly. For calculation of torque, point of application of force is required. In the above case, point of application of force may be assumed at geometrical centre of the surface.

If the surface is vertical or inclined, pressure is non-uniform (it increases with depth) so pressure force and its torque can be obtained by integration. After finding force and torque by integration, we can also find point of application of this force by the relation,

$$r_{\perp} = \frac{\tau}{F} \qquad \text{(As } \tau = F \times r_1\text{)}$$

Note *For small heights, pressure in a gas (including the atmosphere) does not change much. So, atmospheric pressure P_0 is assumed almost constant unless height difference is large. So, pressure force due to P_0 will be P_0A and which can be found directly. The reason of constant pressure is,*

$$\Delta p = \rho g \Delta h$$

Since, ρ of a gas is negligible. Therefore, ΔP tends to zero or pressure is almost constant.

Ideal Fluid

- An ideal fluid is incompressible and non-viscous. An incompressible fluid cannot be pressed at all by applying pressure on it. Its bulk modulus of elasticity is infinite and compressibility is zero.

Since, its volume cannot be changed, so its density remains constant. A non-viscous fluid offers no internal friction. An object moving through this fluid does not experience a retarding force.

Normally, liquids have high viscosity and low compressibility (compared to gases). Gases have low viscosity but high compressibility. Bernoulli's equation is applicable for an ideal fluid and continuity equation is applicable for an incompressible fluid.

- **Streamlines** A streamline at any instant can be defined as an imaginary curve or line, so that tangent to the curve at any point represents the direction of the instantaneous velocity at that point.

In a laminar flow, the streamlines will be fixed. Every particle passing through P has velocity $\mathbf{v}_1$ and at Q it is $\mathbf{v}_2$.

- **Volume Flow Rate** $Q = Av$ or $\dfrac{dV}{dt} = Av$

Continuity Equation

$$Q_1 = Q_2 \quad \text{or} \quad \frac{dV_1}{dt} = \frac{dV_2}{dt}$$

or $$A_1 v_1 = A_2 v_2 \quad \text{or} \quad Av = \text{constant}$$

or $$v \propto \frac{1}{A}$$

Bernoulli's Equation

$$p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

or $$p_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

In Bernoulli's equation, there are three terms; p, $\frac{1}{2}\rho v^2$ and ρgh. Under following three cases, this equation reduces to a two term Bernoulli equation :

***Case* 1** If all points are open to atmosphere, then pressure at every point may be assumed to be constant ($= p_0$) and the Bernoulli equation can be written as,

$$\frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

At greater heights h, speed v will be less as ρ and g are constants.

***Case* 2** If the liquid is passing through a pipe of uniform cross-section, then from continuity equation (Av = constant), speed v is same at all points. Therefore, the Bernoulli equation becomes

$$p + \rho gh = \text{constant}$$

or $$p_1 + \rho gh_1 = p_2 + \rho gh_2$$

or $$p_1 - p_2 = \rho g(h_2 - h_1)$$

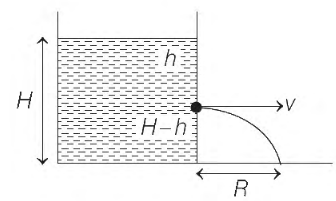
This is the pressure relation we have already discussed for a fluid at rest. Thus, pressure decreases with height of liquid and increases with depth of liquid.

***Case* 3** If a liquid is flowing in a horizontal pipe, then height h of the liquid at every point may be assumed to be constant. So, the two term Bernoulli equation becomes,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

From this equation, we may conclude that pressure decreases at a point where speed increases.

- (i) $v = \sqrt{2gh} = \sqrt{2gh_{top}}$



Here, h_{top} = distance of hole from top surface

(ii) $t = \sqrt{\frac{2(H-h)}{g}} = \sqrt{\frac{2h_{bottom}}{g}}$

Here, h_{bottom} = distance of hole from bottom

(iii) $R = vt = 2\sqrt{h(H-h)} = 2\sqrt{h_{top} \times h_{bottom}}$

(iv) $R_{max} = H$ at $h = \frac{H}{2}$

(v) Time taken to empty a tank if hole is made at bottom.

$$t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

Viscosity

- $F = -\eta A \frac{dv}{dy}$ $\left(\because \frac{dv}{dy} = \text{velocity gradient}\right)$
- For spherical ball, $F = 6\pi\eta rv$
- $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$ or $v_T \propto r^2$

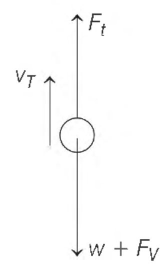
  Here, ρ = density of ball and σ = density of viscous medium in which ball is moving.
- Terminal velocity, $v_T \propto r^2$
- If the fluid is air, then its density σ is negligible compared to density of sphere. So, in that case upthrust will be zero and terminal velocity will be

$$v_T = \frac{2}{9}\frac{r^2 \rho g}{\eta}$$

In this case, weight is equal to the viscous force when terminal velocity is attained.

- If density of fluid is greater than density of sphere ($\sigma > \rho$), then terminal velocity comes out to be negative. So, in this case terminal velocity is upwards.

  In the beginning, upthrust is greater than the weight. Hence, viscous force in this case will be downwards.



$$v_T = \frac{2}{9}\frac{r^2(\sigma - \rho)g}{\eta}$$

When terminal velocity is attained, then

$$F_t = w + F_v$$

This is the reason why, air bubbles rise up in water.

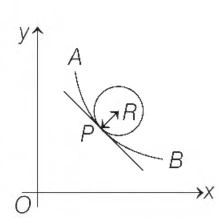
Surface Tension

- $T = \frac{F}{l} = \frac{\Delta w}{\Delta A}$ or $\Delta w = T \times \Delta A$
- $\Delta p = \frac{2T}{R}$ for single surface and $\frac{4T}{R}$ for double surface
- Capillary rise or fall,

$$h = \frac{2T}{R\rho g} = \frac{2T\cos\theta}{r\rho g} \qquad \left(\text{As } R = \frac{r}{\cos\theta}\right)$$

- **Radius of Curvature of a Curve**

  To describe the shape of a curved surface or interface, it is necessary to know the radii of curvature to a curve at some point. Consider the curve AB as shown in figure



Let P be a point on this curve. The radius of curvature R of AB at P is defined as the radius of the circle which is tangent to the curve at point P.

- **Principal Radii of Curvature of a Surface**
  An infinite set of pairs of radii is possible at any point on a surface. The maximum and minimum radii are called principal radii (denoted by R_1 and R_2).
- **Young-Laplace Equation**
  There exists a difference in pressure across a curved surface which is a consequence of surface tension. The pressure is greater on the concave side. The Laplace equation relates the pressure difference to the shape of the Young surface.
  This difference in pressure is given by

$$\Delta p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

For a spherical surface

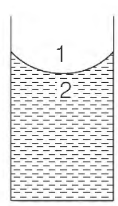
$$R_1 = R_2 = R \text{ (say), therefore } \Delta p = \frac{2T}{R}$$

For a cylindrical surface

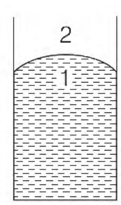
$$R_1 = R \text{ and } R_2 = \infty, \text{ therefore } \Delta p = \frac{T}{R}$$

For a planer surface

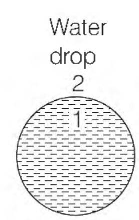
$$R_1 = R_2 = \infty \text{ , therefore } \Delta p = 0$$



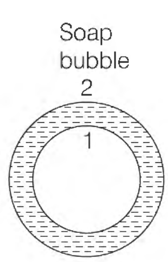
$$p_1 - p_2 = \frac{2T}{R}$$



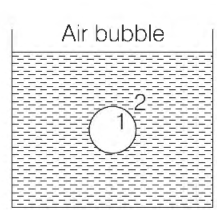
$$p_1 - p_2 = \frac{2T}{R}$$



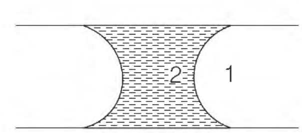
$$p_1 - p_2 = \frac{2T}{R}$$



$$p_1 - p_2 = \frac{4T}{R}$$



$$p_1 - p_2 = \frac{2T}{R}$$



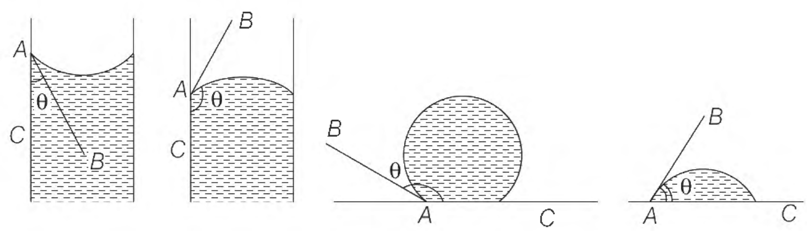
$$p_1 - p_2 = \frac{T}{R}$$

- In general, surface tension decreases with increase in temperature, because cohesive forces decrease with an increase of molecular thermal activity. Liquid molecules having large intermolecular force will have large surface tension.

- Surface tension of water will increase when highly soluble (like NaCl or sugar) impurities are added otherwise surface tension decreases.
- At critical temperature, surface tension becomes zero.
- **Why Hot Soup is Tastier than Cold Soup?**
  If surface tension is more, liquid surface will have lesser surface area. With increase in temperature, surface tension decreases, so the hot soup spreads over a larger area of tongue and the receptors get more taste than cold soup.

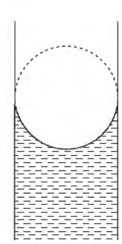
Contact angle (θ)

- The angle between the tangent at the liquid surface at the point of contact (of solid, liquid and gas) and the solid surface inside the liquid is called contact angle θ.

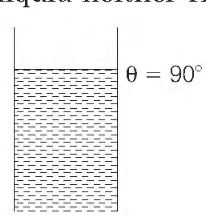


In the four figures shown above, θ is the angle between the lines AB and AC. Here, AB line is away from the solid surface and line AC is towards the liquid.

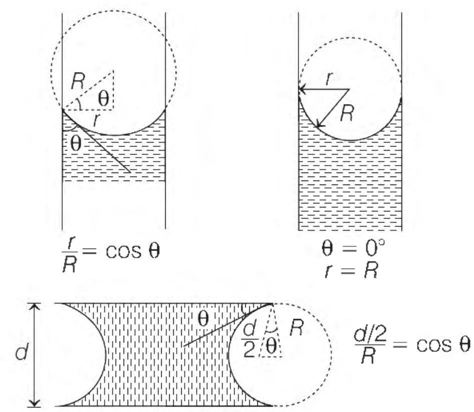
- For wetting liquids, adhesive force is greater than the cohesive force. Contact angle is acute and such liquids rise in the capillary tube.
- For non-wetting liquids, cohesive force is greater than the adhesive force, contact angle is obtuse and such liquids fall in the capillary tube.
- For pure water and glass, contact angle is 0° and the meniscus is as shown in figure below.



- If contact angle θ is 90°, then liquid neither rises nor falls in the capillary tube.



- Shapes of the liquid surface at different contact angles are as shown below.



Physical Significance of Reynolds Number

- Consider a narrow tube having a cross-sectional area A. Suppose a fluid flows through it with a velocity v for a time interval Δt.

  Length of the fluid = velocity × time = $v\Delta t$

  Volume of the fluid flowing through the tube in time $\Delta t = Av\Delta t$

  Mass of the fluid,

  $$\Delta m = \text{volume} \times \text{density} = Av\Delta t \times \rho$$

  Inertial force acting per unit area of the fluid

  $$= \frac{F}{A} = \frac{\text{rate of change of momentum}}{A}$$

  $$= \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av\Delta t\rho \times v}{\Delta t \times A} = \rho v^2$$

  Viscous force per unit area of the fluid = $\eta \times$ velocity gradient $= \eta \frac{v}{D}$

  $$\frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta v/D}$$

  $$= \frac{\rho v D}{\eta} = \text{Reynold's number } (K)$$

  Thus, Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

  When $0 < K < 2000$, the flow of liquid is streamlined.

  When $2000 < K < 3000$, the flow of liquid is variable between streamlined and turbulant.

  When $K > 3000$, the flow of liquid is turbulent.

  Reynold's number has no unit and dimension.