

Class: 9th

HERON'S FORMULA

(4)

Q1: → A traffic signal board including school ahead is an equilateral triangle with side 'a'. Find the area of the signal board using Heron's formula. If its perimeter is 180 cm. what will be the area of the signal board.

Ans: → Each side of triangle = a

Perimeter of triangle = $3a$

but given perimeter = 180 cm

$$\therefore 3a = 180 \text{ cm}$$

$$\Rightarrow a = 60 \text{ cm} \rightarrow a=b=c = 60 \text{ cm}$$

$$\text{Now Semi-perimeter } (S) = \frac{a+b+c}{2} = \frac{60+60+60}{2} = \frac{180}{2}$$
$$= 90 \text{ cm}$$

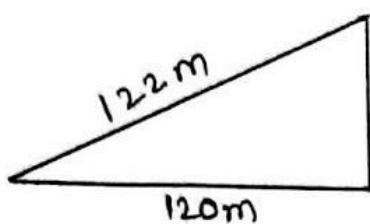
$$\begin{aligned}\therefore \text{Area of } \Delta \text{ by Heron's formula} &= \sqrt{S(S-a)(S-b)(S-c)} \\ &= \sqrt{90(90-60)(90-60)(90-60)} \\ &= \sqrt{90 \times 30 \times 30 \times 30} \\ &= \sqrt{3 \times 30 \times 30 \times 30 \times 30} \\ &= 30 \times 30 \sqrt{3} \\ &= 900 \sqrt{3} \text{ cm}^2 \text{ Ans,}\end{aligned}$$

Q2: → The triangular wall of flyover - - - How much did it pay?

Sol :→ Here $a = 122 \text{ m}$, $b = 120 \text{ m}$, $c = 22 \text{ m}$

$$S = \frac{a+b+c}{2} = \frac{122 \text{ m} + 120 \text{ m} + 22 \text{ m}}{2} = \frac{264}{2} = 132 \text{ m}$$

\therefore Area of the triangular side wall = $\sqrt{s(s-a)(s-b)(s-c)}$



$$\begin{aligned} &= \sqrt{132(132-122)(132-120)(132-22)} \\ &= \sqrt{132 \times 10 \times 12 \times 110} \text{ m}^2 \\ &= 1320 \text{ m}^2 \end{aligned}$$

Rent of one metre of wall for 1 year = Rs 5000

$$\text{''} \quad \text{''} \quad \text{''} \quad \text{''} \quad 1 \text{ month} = \frac{5000}{12}$$

$$\text{''} \quad \text{''} \quad \text{''} \quad \text{''} \quad 3 \text{ month} = \frac{5000}{12} \times 3$$

$$\begin{aligned} \text{Rent of } 1320 \text{ m}^2 \text{ of wall for 3 months} &= \frac{5000 \times 3 \times 1320}{12} \\ &= \text{Rs } 1650,000 \text{ Ans}, \end{aligned}$$

Q3 :→ There is a slide in a park ---- find the area painted in colour.

Sol: → Here $a = 15\text{m}$, $b = 11\text{m}$, $c = 6\text{m}$

$$s = \frac{a+b+c}{2} = \frac{15+11+6}{2} \text{ m} = \frac{32}{2} = 16 \text{ m}$$

$$\begin{aligned} \text{Area of A} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-15)(16-11)(16-6)} \\ &= \sqrt{16 \times 1 \times 5 \times 10} \\ &= \sqrt{\underbrace{4 \times 4}_{4} \times \underbrace{1 \times 5}_{2} \times \underbrace{2 \times 5}_{5}} \\ &= 4 \times 5 \sqrt{2} = 20\sqrt{2} \text{ m}^2 \text{ Ans}, \end{aligned}$$

Q4 → find the area of triangle two side of which are 18cm and 10cm and the perimeter is 42cm.

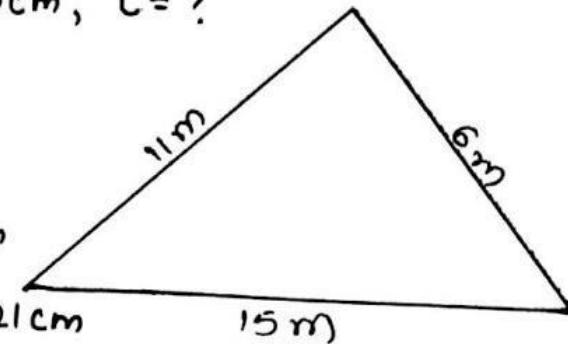
Sol: → Here $a = 18\text{cm}$, $b = 10\text{cm}$, $c = ?$

$$\text{Perimeter} = 42\text{cm}$$

$$\Rightarrow a + b + c = 42\text{ cm}$$

$$\Rightarrow 18 + 10 + c = 42\text{ cm}$$

$$\Rightarrow c = 42 - 18 - 10 = 21\text{ cm}$$



$$\text{Now } s = \frac{a+b+c}{2} = \frac{18+10+24}{2} = \frac{42}{2} = 21\text{ cm}$$

$$\begin{aligned}\text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-18)(21-10)(21-14)} \\ &= \sqrt{21 \times 3 \times 11 \times 7} \text{ cm}^2 \\ &= \sqrt{3 \times 7 \times 3 \times 11 \times 7} \\ &= 3 \times 7 \sqrt{11} \text{ cm}^2 \\ &= 21\sqrt{11} \text{ cm}^2\end{aligned}$$

Q3 → Side of a triangle are in ratio 12:17:25 and its perimeter is 540cm. find its area

Sol: → Let the sides of triangle be $12x$, $17x$ and $25x$

$$\therefore 12x + 17x + 25x = 540$$

$$\Rightarrow 54x = 540$$

$$\Rightarrow x = \frac{540}{54} = 10$$

$$\therefore a = 12x = 12 \times 10 = 120\text{ cm}$$

$$b = 17x = 17 \times 10 = 170\text{ cm}$$

$$c = 25x = 25 \times 10 = 250\text{ cm}$$

$$\text{Now } s = \frac{120+170+250}{2} = \frac{540}{2} = 270\text{ cm}$$

$$\begin{aligned}
 \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{270(270-120)(270-170)(270-250)} \\
 &= \sqrt{270 \times 150 \times 100 \times 20} \text{ cm}^2 \\
 &= \sqrt{81000000} = 9000 \text{ cm}^2
 \end{aligned}$$

Q6 :→ An isosceles triangle has perimeter 30cm and each of equal sides is 12cm . Find the area of triangle

Sol: → Here $a = b = 12\text{cm}$

$$\text{also } a+b+c = 30\text{ cm}$$

$$\therefore 12+12+c = 30\text{ cm}$$

$$c = 30 - 12 - 12 = 6\text{ cm}$$

$$\therefore s = \frac{12+12+6}{2} = \frac{30}{2} = 15\text{cm}$$

$$\begin{aligned}
 \text{Now Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{15(15-12)(15-12)(15-6)} \\
 &= \sqrt{15 \times 3 \times 3 \times 9} \\
 &= \sqrt{\underbrace{3 \times 5}_{3} \times \underbrace{3 \times 3}_{3} \times \underbrace{3 \times 3}_{3}} \\
 &= 3 \times 3 \sqrt{5 \times 3} \\
 &= 9\sqrt{15} \text{ cm}^2 \text{ Ans.}
 \end{aligned}$$

Exercise 12.2

(3)

Q1 A park in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$, AB = 9m, BC = 12m, CD = 5m, AD = 8m, How much area does it occupy?

Sol : → As ABCD is a park

Join BD

Now In $\triangle DBC$ we have $\angle C = 90^\circ$

$$\therefore DB^2 = BC^2 + CD^2 \quad (\text{By pyth theorem})$$

$$\Rightarrow DB^2 = (12)^2 + 5^2$$

$$\Rightarrow DB = \sqrt{144 + 25} = \sqrt{169}$$

$$\Rightarrow DB = 13m$$

$$\begin{aligned}\therefore \text{Area of } \triangle DBC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 12 \times 5 = 30m^2\end{aligned}$$

In $\triangle ABD$ a = 9m, b = 8m, c = 13m

$$\therefore s = \frac{a+b+c}{2} = \frac{9+8+13}{2} = 15m$$

$$\therefore \text{Area of } \triangle ABD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-9)(15-8)(15-13)} m^2$$

$$= \sqrt{15 \times 6 \times 7 \times 2} m^2$$

$$= \sqrt{1260} = 35.5m^2 \text{ (Approx.)}$$

$$\therefore \text{Area of park} = (30 + 35.5) = 65.5 \text{ m}^2 \text{ Ans},$$

Q2 find the area of quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm

Sol : → In $\triangle ABC$ we have

$$AB^2 + BC^2 = AC^2$$

$$(3)^2 + (4)^2 = AC^2$$

$$\Rightarrow 9 + 16 = (5)^2$$

$$\Rightarrow 25 = 25$$

Hence By inverse of Pythagoras theorem
 $\triangle ABC$ is a right angled triangle

$$\begin{aligned}\therefore \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{Alt} \\ &= \frac{1}{2} \times 3 \times 4 \\ &= 6 \text{ cm}^2\end{aligned}$$

In $\triangle ACD$, a = 5 cm, b = 4 cm c = 5 cm

$$\therefore s = \frac{5+4+5}{2} = \frac{14}{2} = 7 \text{ cm}$$

$$\begin{aligned}\text{Area of } \triangle ACD &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{7(7-5)(7-4)(7-5)} \\ &= \sqrt{7 \times 2 \times 3 \times 2} \\ &= \sqrt{84} \text{ cm}^2 = 9.2 \text{ cm}^2 (\text{approx.})\end{aligned}$$

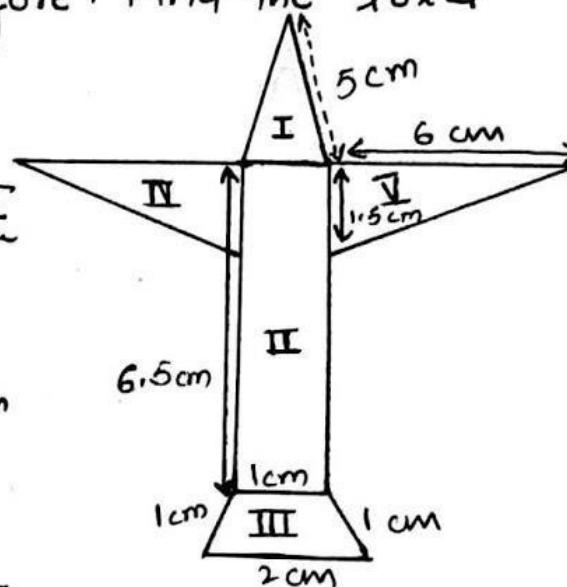
$$\therefore \text{Area of quad} = (6 + 9.2) \text{ cm} = 15.2 \text{ cm}^2 \text{ Ans},$$

③ Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used

Sol: → For the triangle marked I

$$a = 5 \text{ cm}, b = 5 \text{ cm}, c = 1 \text{ cm}$$

$$\therefore S = \frac{s(s-a)(s-b)(s-c)}{2} = \frac{11}{2} = 5.5 \text{ cm}$$



$$\therefore \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$$

$$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}$$

$$= \sqrt{6.1875} \text{ cm}^2 = 2.5 \text{ cm}^2$$

For the rectangle marked II : →

$$\text{Length} = 6.5 \text{ cm}$$

$$\text{breadth} = 1 \text{ cm}$$

$$\therefore \text{Area of rectangle} = L \times b$$

$$= 6.5 \times 1 \text{ cm} = 6.5 \text{ cm}^2$$

For the trapezium marked III : →

Draw AF || DC and AE ⊥ BC

$$AD = FC = 1 \text{ cm}, DC = AF = 1 \text{ cm}$$

$$\therefore BF = BC - FC = (2-1) = 1 \text{ cm}$$

Hence $\triangle ABF$ is equilateral

Also, E is mid-point of BF

$$\therefore BE = \frac{1}{2} \text{ cm} = 0.5 \text{ cm}$$

$$AB^2 = AE^2 + BE^2 \quad (\text{By Pythagoras theorem})$$

$$\Rightarrow AE^2 = 1^2 - (0.5)^2 = 0.75$$

$$AE = 0.9 \text{ cm} (\text{approx})$$

$$\therefore \text{Area of trapezium} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (BC + AD) \times AE$$

$$= \frac{1}{2} \times (2+1) \times 0.9$$

$$= \frac{1}{2} \times 3 \times \frac{9}{10}$$

$$= 1.4 \text{ cm}^2$$

For the triangle marked IV : →

It is a right-triangle

$$\therefore \text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 6 \times 1.5 = 4.5 \text{ cm}^2$$

For the triangle marked V : →

This triangle is congruent to the triangle marked IV

$$\text{Hence area of triangle V} = 4.5 \text{ cm}^2$$

$$\therefore \text{Total area of paper used} = (2.5 + 6.5 + 1.4 + 4.5 + 4.5)$$

$$= 19.4 \text{ cm}^2 \text{ Ans.}$$

Q4 : → A triangle and parallelogram ----- find
the height of the parallelogram

(5)

Sol: → In the figure ABCD is a ||gram and ABE is the triangle which stands on Base AB

For ΔABE $a = 30\text{cm}$, $b = 28\text{cm}$

$$c = 26\text{cm}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{30+28+26}{2} = \frac{84}{2} = 42\text{cm}$$

$$\begin{aligned}\therefore \text{Area of } \Delta ABE &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{42(42-30)(42-28)(42-26)} \\ &= \sqrt{42 \times 12 \times 14 \times 16} \\ &= \sqrt{112896} \text{ cm}^2 \\ &= 336 \text{ cm}^2\end{aligned}$$

Now area of ||gram = base \times height

$$\Rightarrow 336 = 28 \times \text{height}$$

$$\Rightarrow \text{height} = \frac{336}{28} = 12\text{cm} \text{ Ans/}$$

Q5 → A rhombus shaped field --- area of grass field which each cow be getting?

Sol: → Clearly the diagonal AC of the rhombus divides it into two congruent triangles
for ΔABC

$$a = b = 30\text{m}, c = 48\text{m}$$

$$\begin{aligned}\therefore s &= \frac{a+b+c}{2} = \frac{30+30+48}{2} \\ &= 54\text{m}\end{aligned}$$

$$\begin{aligned}
 \text{Now area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\
 &= \sqrt{54(54-30)(54-30)(54-48)} \\
 &= \sqrt{54 \times 24 \times 24 \times 6} = 432 \text{ m}^2 \\
 \Rightarrow \text{Area of rhombus} &= 2 \times 432 \text{ m}^2 \\
 &= 864 \text{ m}^2
 \end{aligned}$$

Number of cows = 18

Hence area of grass field which each cow gets $= \frac{864}{18} \text{ m}^2 = 48 \text{ m}^2$ Ans,,

Q6 An Umbrella is made by --- required for the umbrella?

Sol :→ firstly we find area of triangular piece

Here $a = b = 50\text{cm}$, $c = 20\text{cm}$

$$\therefore s = \frac{a+b+c}{2} = \frac{50+50+20}{2} = \frac{120}{2} = 60\text{cm}$$

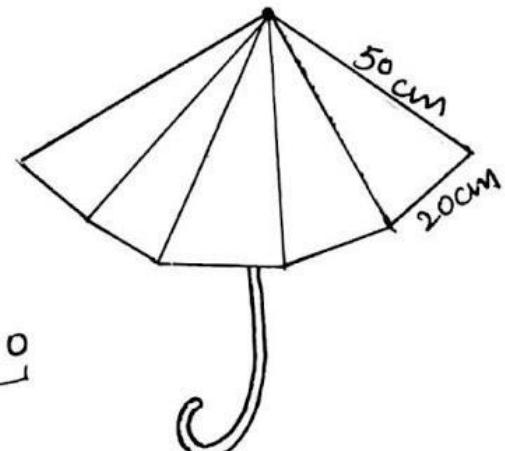
$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{60(60-50)(60-50)(60-20)}$$

$$= \sqrt{60 \times 10 \times 10 \times 40}$$

$$= \sqrt{6 \times 10 \times 10 \times 10 \times 2 \times 2 \times 10}$$

$$= 200\sqrt{6} \text{ cm}^2$$



$$\text{Area of 10 such pieces} = 10 \times 200\sqrt{6} = 2000\sqrt{6} \text{ cm}^2$$

$$\text{Hence cloth required for each colour} = \frac{2000\sqrt{6}}{2}$$

$$= 1000\sqrt{6} \text{ cm}^2 \text{ Ans,,}$$

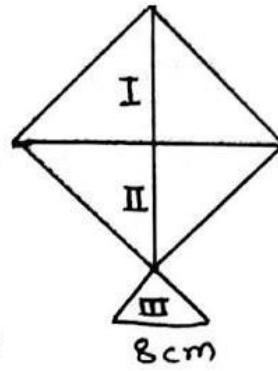
Q7 A kite is in the shape --- each shade has been used in it?

Sol: →

ABCD is a Square

$$\text{So } AO = OC, OB = OD$$

and $\angle AOB = 90^\circ$ {Diagonal of a square bisect each other at right angles}



$$BD = 32 \text{ cm} \text{ Given}$$

$$\Rightarrow OA = \frac{32}{2} = 16 \text{ cm}$$

, $\triangle ABD$ is a right triangle

$$\begin{aligned} \text{So, area of } \triangle ABD &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 32 \times 16 = 256 \text{ cm}^2 \end{aligned}$$

$$\text{Thus area of triangle } \triangle BCD = 256 \text{ cm}^2$$

For triangle CEF, a = b = 6 cm, c = 8 cm

$$\therefore s = \frac{a+b+c}{2} = \frac{6+6+8}{2} \text{ cm} = 10 \text{ cm}$$

$$\therefore \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{10(10-6)(10-6)(10-8)}$$

$$= \sqrt{10 \times 4 \times 2} = \sqrt{320} \text{ cm}^2$$

$$= 17.92 \text{ cm}^2$$

Hence paper needed for shade I = 256 cm^2

for shade II = 256 cm^2 and for shade III = 17.92 cm^2

Q8: → A floral design on a floor is --- polishing the tiles at the rate of 50 p per cm^2

Sol : \rightarrow we have lengths of the sides of triangular tiles are

$$a = 35\text{ cm}, b = 28\text{ cm}, c = 9\text{ cm}$$

$$\therefore s = \frac{a+b+c}{2} = \frac{35+28+9}{2}$$

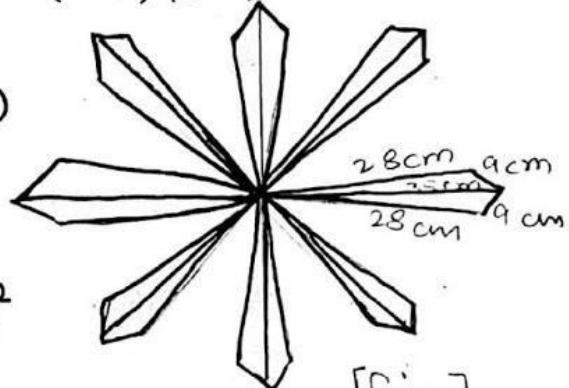
$$= \frac{72}{2} = 36\text{ cm}$$

$$\therefore \text{Area of 1 triangular tile} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(36-35)(36-28)(36-9)}$$

$$= \sqrt{36 \times 1 \times 8 \times 27}$$

$$= \sqrt{7776}\text{ cm}^2 = 88.2\text{ cm}^2$$



[Fig.]

$$\therefore \text{Area of 16 Such tiles} = 16 \times 88.2\text{ cm}^2$$

$$\text{Cost of polishing } 1\text{ cm}^2 = \text{Rs } 0.50$$

\therefore Total cost of polishing the floral design

$$= 16 \times 88.2 \times 0.50$$

$$= \text{Rs } 705.60 \text{ Ans.,}$$

Q9 : \rightarrow A field is in the shape of trapezium --- area of the field.

Sol : \rightarrow In the figure ABCD is a field

$$DC = AF = 10\text{ cm}$$

$$AD = FC = 13\text{ m}$$

For $\triangle BCF$, $a = 15\text{ m}$, $b = 14\text{ m}$, $c = 13\text{ m}$

$$\therefore s = \frac{15+14+13}{2} = \frac{42}{2} = 21\text{ m}$$

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$$\therefore \text{Area of } \triangle BCF = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-15)(21-14)(21-13)} \text{ m}^2$$

$$= \sqrt{21 \times 6 \times 7 \times 8} \text{ m}^2$$

$$= \sqrt{7056} \text{ m}^2$$

$$= 84 \text{ m}^2$$

Also area of $\triangle BCF = \frac{1}{2} b \times A$

$$= \frac{1}{2} \times BF \times CG$$

$$\Rightarrow 84 = \frac{1}{2} \times 15 \times CG$$

$$\Rightarrow CG = \frac{84 \times 2}{15} = 11.2 \text{ m}$$

$$\therefore \text{Area of trapezium} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{Alt}$$

$$= \frac{1}{2} \times (25+10) \times 11.2 \text{ m}^2$$

$$= 196 \text{ m}^2$$

Hence area of the field = 196 m^2 Ans,,

