

- Q.1** The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1 , then the denominator is eight times the numerator. Find the fraction.
- Q.2** A fraction becomes $\frac{9}{11}$ if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.
- Q.3** A fraction becomes $\frac{1}{3}$ if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes $\frac{1}{2}$. Find the fraction.
- Q.4** If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1 . It also becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?
- Q.5** The sum of the numerator and denominator of a fraction is 12 . If the denominator is increased by 3 , the fraction becomes $\frac{1}{2}$. Find the fraction.

Sol.1 Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y .

So, the required fraction is x/y .

From the question it's given as,

The numerator of the fraction is 4 less the denominator.

Thus, the equation so formed is,

$$x = y - 4$$

$$\Rightarrow x - y = -4 \quad \dots(i)$$

And also it's given in the question as,

If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is 8 times the numerator.

Putting the above condition in an equation, we get

$$y + 1 = 8(x - 2) \Rightarrow y + 1 = 8x - 16 \Rightarrow 8x - y = 1 + 16$$

$$\Rightarrow 8x - y = 17 \quad \dots(ii)$$

Solving (i) and (ii),

Subtracting the equation (ii) from (i), we get

$$(x - y) - (8x - y) = -4 - 17$$

$$\Rightarrow x - y - 8x + y = -21 \Rightarrow -7x = -21$$

$$\Rightarrow x = 21/7 \Rightarrow x = 3$$

Substituting the value of $x = 3$ in the equation (i), we find y

$$3 - y = -4 \Rightarrow y = 3 + 4 \Rightarrow y = 7$$

Therefore, the fraction is $3/7$.

Sol.2 Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y .

So, the required fraction is x/y .

From the question it's given as, If 2 is added to both numerator and the denominator, the fraction becomes $9/11$. Thus, the equation so formed is,

$$\begin{aligned}x + 2y + 2 &= 9/11 \Rightarrow 11(x + 2) = 9(y + 2) \\ \Rightarrow 11x + 22 &= 9y + 18 \Rightarrow 11x - 9y = 18 - 22 \\ \Rightarrow 11x - 9y + 4 &= 0 \quad \dots(i)\end{aligned}$$

And also it's given in the question as,

If 3 is added to both numerator and the denominator, the fraction becomes $5/6$, Expressing the above condition in an equation, we have

$$\begin{aligned}x + 3y + 3 &= 5/6 \\ \Rightarrow 6(x + 3) &= 5(y + 3) \\ \Rightarrow 6x + 18 &= 5y + 15 \\ \Rightarrow 6x - 5y &= 15 - 18 \\ \Rightarrow 6x - 5y + 3 &= 0 \quad \dots(ii)\end{aligned}$$

Solving (i) and (ii), to find the fraction

By using cross-multiplication method, we have

$$\begin{aligned}\frac{x}{-9 * 3 - (-5) * 4} &= \frac{y}{11 * 3 - 6 * 4} = \frac{1}{11 * (-5) - 6 * (-9)} \\ \Rightarrow \frac{x}{-27 + 20} &\Rightarrow \frac{-y}{33 - 24} = \frac{1}{-55 + 54} \\ \Rightarrow \frac{x}{-7} &= \frac{-y}{9} = \frac{1}{-1} \\ \Rightarrow \frac{x}{7} &= \frac{y}{9} = 1\end{aligned}$$

$$x = 7, y = 9$$

Hence, the required fraction is $7/9$.

Sol.3 Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y .

So, the required fraction is x/y .

From the question, it's given that:

If 1 is subtracted from both the numerator and the denominator, the fraction becomes $1/3$.

Thus, the equation so formed is,

$$\frac{x-1}{y-1} = \frac{1}{3}$$

$$\Rightarrow 3(x-1) = (y-1) \Rightarrow 3x-3 = y-1$$

$$\Rightarrow 3x - y - 2 = 0 \quad \dots(i)$$

And also, it's given in the question as,

If 1 is added to both the numerator and the denominator, the fraction becomes $1/2$. Expressing the above condition in an equation, we have

$$(x+1)/(y+1) = 1/2$$

$$\Rightarrow 2(x+1) = (y+1)$$

$$\Rightarrow 2x + 2 = y + 1$$

$$\Rightarrow 2x - y + 1 = 0 \quad \dots(ii)$$

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\frac{x}{(-1) * 1 - (-1) * (-2)} = \frac{-y}{3 * 1 - 2 * (-2)} = \frac{1}{3 * (-1) - 2 * (-1)}$$

$$\Rightarrow \frac{x}{-1-2} \Rightarrow \frac{-y}{3+4} = \frac{1}{-3+2}$$

$$\Rightarrow \frac{x}{-3} = \frac{-y}{7} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{7} = 1 \Rightarrow x = 3, y = 7$$

Hence, the required fraction is $3/7$.

Sol.4 Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y .

So, the required fraction is x/y .

From the question, it's given that:

If 1 is added to the numerator and 1 is subtracted from the denominator, the fraction becomes 1.

Thus, the equation so formed is,

$$\frac{x+1}{y-1} = 1$$

$$\Rightarrow (x+1) = (y-1)$$

$$\Rightarrow x+1-y+1=0$$

$$\Rightarrow x-y+2=0 \quad \dots(i)$$

And also, it's given in the question as,

If 1 is added to the denominator, the fraction becomes $1/2$.

Expressing the above condition in an equation, we have

$$x/(y+1) = 1/2 \Rightarrow 2x = (y+1)$$

$$\Rightarrow 2x - y - 1 = 0 \quad \dots(ii)$$

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\frac{x}{(-1) * (-1) - (-1) * 2} = \frac{y}{1 * (-1) - 2 * 2} = \frac{1}{1 * (-1) - 2 * (-1)}$$

$$\Rightarrow \frac{x}{1+2} = \frac{-y}{-1-4} = \frac{1}{-1+2}$$

$$\Rightarrow \frac{x}{3} = \frac{-y}{-5} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{5} = 1$$

$$\Rightarrow x=3, y=5$$

Hence, the required fraction is $3/5$.

Sol.5 Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y .

So, the required fraction is x/y .

From the question, it's given that:

The sum of the numerator and denominator of the fraction is 12 .

Thus, the equation so formed is,

$$x + y = 12 \Rightarrow x + y - 12 = 0$$

And also, it's given in the question as,

If the denominator is increased by 3 , the fraction becomes $1/2$.

Putting this as an equation, we get

$$x/(y + 3) = 1/2$$

$$\Rightarrow 2x = (y + 3)$$

$$\Rightarrow 2x - y - 3 = 0$$

The two equations are,

$$x + y - 12 = 0 \quad \dots(i)$$

$$2x - y - 3 = 0 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$x + y - 12 + (2x - y - 3) = 0$$

$$\Rightarrow 3x - 15 = 0 \Rightarrow x = 5$$

Using $x = 5$ in (i), we find y

$$5 + y - 12 = 0$$

$$\Rightarrow y = 7$$

Therefore, the required fraction is $5/7$.