TEST – VI

JEE (Main)-2022

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 Questions.

Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

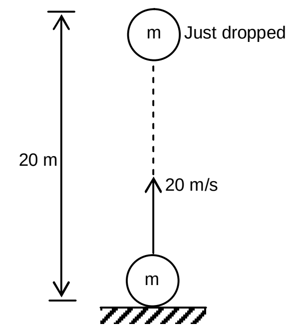
Physics

PART – A

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. The two masses collide in air and get stick together. After how much time combined mass will fall to the ground (calculate the time from the starting when the motion was started) ($g = 10$ m/s^2)



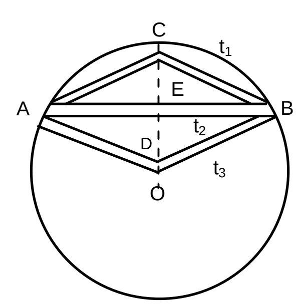
(A) $\left(1+\sqrt{2}\right)$s
(B) $2\sqrt{2}$s
(C) $\left(2+\sqrt{2}\right)$s
(D) $\left(\sqrt{3}+1\right)$s

2. A boat having length of 3 metres and breadth 2 metres is floating on a lake. The boat sinks by one cm when a man gets on it. The mass of the man is
(A) 60 kg
(B) 62 kg
(C) 72 kg
(D) 128 kg

3. Two sound waves of slightly different frequently have amplitude ratio 11/9. What is the different of sound levels in decibels of maximum and minimum intensities head at a point?
(A) 100
(B) 10
(C) 16
(D) 20

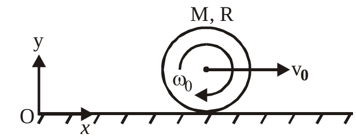
4. Kundt's tube can be used to
(A) Produce standing wave.
(B) Determine viscosity of water.
(C) Produce Doppler's effect in sound.
(D) Determine velocity of the source of the disturbance.

5. A particle is dropped in three tunnels ACB, AEB & ADB through earth and takes time to reach at point B respectively t_1, t_2 and t_3. Then which of the following relation is correct: (D is not the centre of the earth)



(A) $t_1 > t_2 > t_3$
(B) $t_1 < t_2 > t_3$
(C) $t_3 > t_2 > t_1$
(D) $t_2 < t_1 < t_3$

6. A sphere of mass M and radius R rolls on a horizontal surface with velocity v_0 as shown in figure. Its angular momentum about a fixed point O on the surface is



(A) Mv_0R

(B) $\frac{2}{5}MR^2\omega_0$

(C) $Mv_0R + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega_0$

(D) $Mv_0R + \frac{2}{5}MR^2\omega_0$

7. A sphere of mass m moving with a constant velocity u hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of final velocity of the sphere moving initially to the final velocity of the sphere initially at rest is:

(A) $\left(\frac{1-e}{1+e}\right)$

(B) $\left(\frac{1+e}{1-e}\right)$

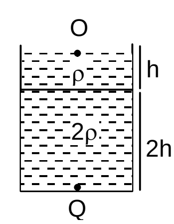
(C) $\left(\frac{e+1}{e+1}\right)$

(D) $\left(\frac{e-1}{e+1}\right)$

8. A projectile is throw horizontally from a big tower with a speed of 20 ms^{-1}. If g = 10 ms^{-2}, the speed of the projectile after 5 second will be nearly,

(A) 0.5 ms^{-1}
(B) 5 ms^{-1}
(C) 54 ms^{-1}
(D) 500 ms^{-1}

9. A container having two immiscible liquids of density ρ and 2ρ moves with an upwards acceleration a = g. The value of $P_Q - P_O$ is (P_0 = atm pressure) :



(A) 5 ρgh
(B) 10 ρgh
(C) 2.5 ρgh
(D) ρgh

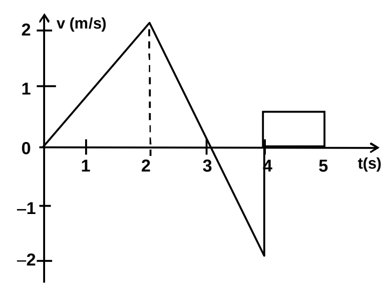
10. If error in the measurement of mass is 0.8 of and in volume it is 0.4% then error in the measurement of density is

(A) 1.2 %
(B) 0.4 %
(C) 0.8 %
(D) 1 %

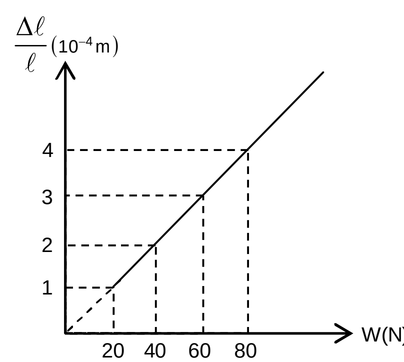
11. A moving body in straight line is covering the distance directly proportional to the square of time. The acceleration of the body is:
(A) Increasing
(B) decreasing
(C) zero
(D) constant

12. An open pipe is closed at one end. As a result the frequency of the third harmonic of the closed pipe is found to be higher than the fundamental frequency of the open pipe by 100 Hz. What is the fundamental frequency of the open pipe?
(A) 50 Hz
(B) 100 Hz
(C) 200 Hz
(D) 300 Hz

13. The velocity v versus t graph of body in a straight line is as shown in figure then displacement in 6 sec will be
(A) 2 m
(B) 3 m
(C) 4 m
(D) 5 m



14. The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire
(A) 2×10^7 N/m^2
(B) 2×10^9 N/m^2
(C) 2×10^{11} N/m^2
(D) 2×10^{13} N/m^2



15. Two racing cars of masses m_1 and m_2 are moving in two circular paths of radii R_1 and R_2 with same speed. The ratio of respective coefficient of friction so that the cars just take the safe turn is
(A) R_1/R_2
(B) R_1^2/R_2^2
(C) R_2/R_1
(D) R_2^2/R_1^2

16. A body of mass m is situated in a potential field $U_x = U_0 (1 - \cos\alpha x)$ when U_0 and α are constants. The time period of small oscillations of the mass is

(A) $\pi\sqrt{\frac{m}{u_o\alpha^2}}$

(B) $2\pi\sqrt{\frac{m}{u_o\alpha^2}}$

(C) $2\pi\sqrt{\frac{2m}{u_o\alpha^2}}$

(D) $2\pi\sqrt{\frac{3m}{u_o\alpha^2}}$

17. One half mole of a monoatomic ideal gas absorbs 1200 J of heat energy while performing 2196 J of work. The temperature of the gas changes by

(A) 160^0C
(B) 80^0C
(C) 40^0C
(D) -160^0C

18. In a tug of war, the team that exerts a larger tangential force on the ground wins. Consider the period in which a team is dragging the opposite team by applying a larger tangential force on the ground. Which of the following works are negative?

(A) work by the loosing team on the winning team.
(B) work by the ground on the winning team.
(C) work by the ground on the loosing team.
(D) None of these

19. P–V diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be

(A) 4R
(B) 25R
(C) 3R
(D) 4R/3

20. The escape velocity of a body on an imaginary planet which is thrice the radius of the earth and double the mass of earth is (v_0 is the escape velocity of earth)

(A) $\sqrt{2/3}v_0$

(B) $\sqrt{3/2}v_0$

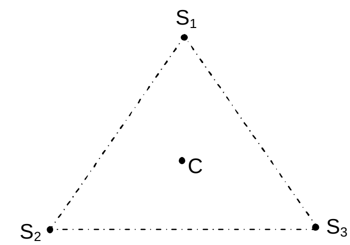
(C) $\sqrt{2}/3v_0$

(D) $2/\sqrt{3}v_0$

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

21. Three identical sources S_1, S_2 and S_3 are placed at the vertices of an equilateral triangle. If they have intensity I_0 each at centroid c of triangle. The resulting intensity of sound at c will be …. Times I_0

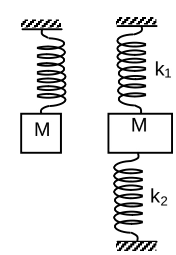


22. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of Earth is R, the radius of planet would be $\frac{R}{x}$, where 'x' is

23. The time period of the SHM represented by $\frac{d^2x}{dt^2} + 4\pi^2 x = 0,$ where x is the displacement at time t, in seconds is

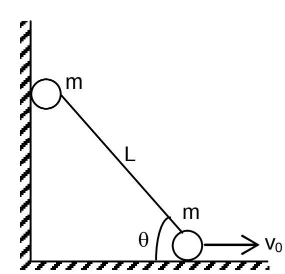
24. A moving car encounters air resistance which is proportional to the square of the speed of the car. Find the value of $\frac{2 \times \text{Power required at 40 kmph}}{\text{Power required at 80 kmph}}$

25. A mass M is oscillating with frequency f_o if hung with a spring of stiffness k. Now it is cut in two parts in the ratio of 2 : 1 and connected as shown. The new frequency is 'x' times of $\frac{1}{\pi}\sqrt{\frac{k}{2M}}$. Find the value of 'x'.

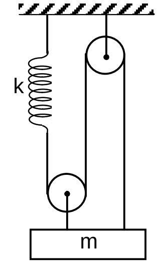


26. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation. $y = Kt^2$. ($K = 1 m/s^2$) where y is vertical displacement. The time period now become T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is ($g = 10\ m/s^2$)

27. In the shown diagram two point masses m are joined by a massless rod of length 5.5 m. The lower end of the mass system is dragged with constant velocity v_0 rightwards. The magnitude of velocity of centre of mass when $\theta = 37^0$, as multiples of v0 will be



28. For a system in equilibrium as shown in figure elongation in spring will be (n mg) /k, then n is



29. Velocity time equation of a particle moving in a straight line is $V = t^2 - 5t + 6$. The distance travelled by the particle in the time interval from t = 0 to t = 4 sec

30. A hollow sphere and a solid sphere have equal mass and equal moment of inertia about the respective diameter. Find the ratio of their square of radii is given by

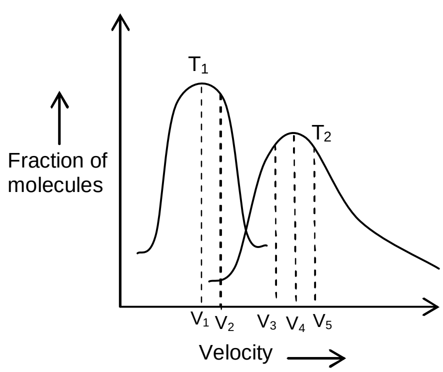
Chemistry PART – B

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

31. Which quantum number is different for the unpaired electrons of carbon atom?
    (A) Principal quantum number
    (B) Azimuthal quantum number
    (C) Magnetic quantum number
    (D) Spin quantum number

32. Which of the following molecule has the highest value of dipole moment?
    (A) PF_2Cl_3
    (B) PBr_3Cl_2
    (C) PCl_5
    (D) PCl_2F_3

33.


Which of the following velocity is expressed correctly?

(A) $V_1 = \sqrt{\frac{3RT_1}{M}}$

(B) $V_4 = \sqrt{\frac{2RT_2}{M}}$

(C) $V_2 = \sqrt{\frac{2RT_1}{M}}$

(D) $V_3 = \sqrt{\frac{3RT_2}{M}}$

34. Which of the following substance absorbs CO_2 and evolves O_2 gas in a single reaction?
    (A) CaO
    (B) KO_2
    (C) Na_2O
    (D) Li_2O

35. $2Mg(s) + O_2(g) \rightleftharpoons 2MgO(s)$

Above reaction attains equilibrium at 80^oC and 0.5 atm pressure. The equilibrium constant K_p of the reaction is:

(A) $1.5\ atm^{-1}$
(B) $0.5\ atm^{-1}$
(C) $2\ atm^{-1}$
(D) $2.5\ atm^{-1}$

36. Which of the following solution mixture does not behave as buffer?

(A) $CH_3COOH + NaOH$ (2 : 1 molar ratio)
(B) $NH_4OH + HCl$ (1 : 2 molar ratio)
(C) $CH_3COOH + NH_4OH$ (1 : 1 molar ratio)
(D) $CH_3COONa + CH_3COOH$ (8 : 1 molar ratio)

37. What is the overall order of the following elementary reaction?

$2X(g) + Y_2(g) \longrightarrow X_2Y_2(g)$

(A) 2
(B) 3
(C) 4
(D) 1

38. The entropy of a reversible process at equilibrium is $150\ J\ K^{-1}\ mol^{-1}$ at 25^oC. What would be the entropy of the surrounding?

(A) $75\ J\ K^{-1}\ mol^{-1}$
(B) $-150\ J\ K^{-1}\ mol^{-1}$
(C) $150\ J\ K^{-1}\ mol^{-1}$
(D) $-75\ J\ K^{-1}\ mol^{-1}$

39. Hydrolysis of borax produces

(A) $B(OH)_3$, Na^+, $H_2B_4O_7$
(B) Na^+, BO_3^-, $B(OH)_4^-$
(C) HBO_2, Na^+, $B_4O_4^{2-}$
(D) $B(OH)_3$, Na^+, $[B(OH)_4]^-$

40. The most acidic compound out of the following is:

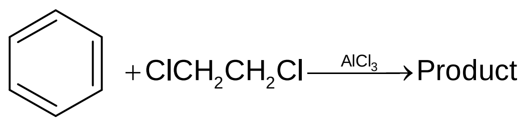
(A) CH_3CH_2COOH
(B) $CH_3CH(CH_3)COOH$
(C) $ClCH_2COOH$
(D) O_2NCH_2COOH

41. Silicones are polymers of

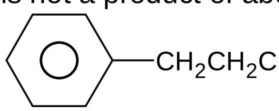
(A) silicon chloride
(B) silanes
(C) alkyl silicon hydroxides
(D) alkyl silanes

42. The acyclic functional isomer of 2-butyne contains
(A) one triple bond
(B) one double bond
(C) two double bonds
(D) one triple bond and one double bond

43.



Which is not a product of above reaction?

(A)


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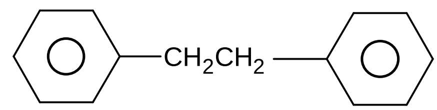

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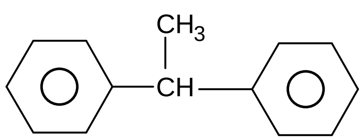
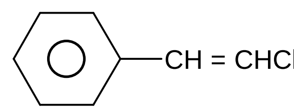


(D)


44. Which of the following reaction does not form CH_4?

(A) $CH_3COONa \xrightarrow[\Delta]{\text{Sodalime}}$

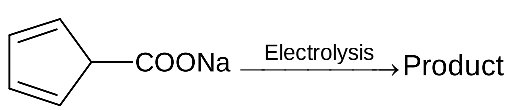
(B) $C_2H_5Cl \xrightarrow{\text{Alc.KOH}}$

(C) $CH_3MgBr + CH_3OH \longrightarrow$

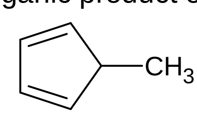
(D) $CH_3Cl \xrightarrow{LiAlH_4}$

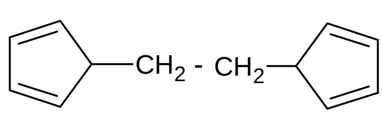
45. Which of the following substance on ozonolysis does not form CH_3CHO?
(A) 2-butene
(B) 2-pentene
(C) 2-hexene
(D) 3-hexene

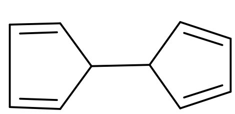
46.

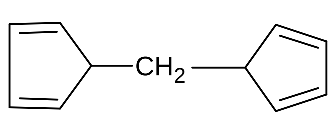


The organic product of above reaction?

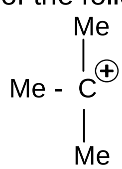
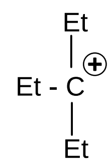
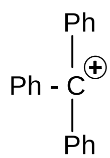
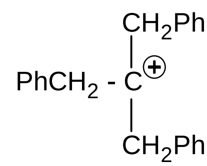
(A)


(B)


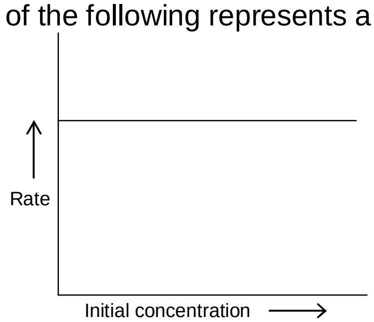
(C)


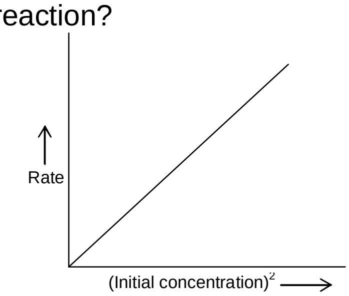
(D)


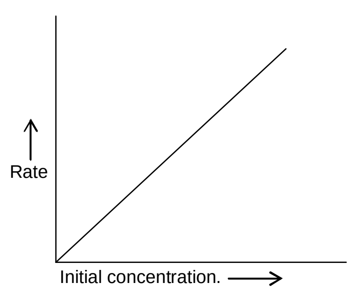
47. Which does not form precipitate with ammonical solution of $AgNO_3$?
(A) $CH_3CH_2CH_2C \equiv CH$
(B) $CH_3 - C \equiv C - CH_3$
(C) $HC \equiv CH$
(D) $CH_3 - C \equiv CH$

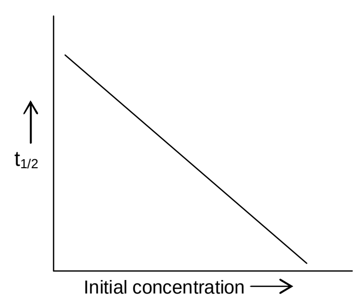
48. Which of the following carbocation is most stable?
(A)

(B)

(C)

(D)


49. Which of the following gases deplete ozone layer in the upper atmosphere?
(A) CHF_3 and NO_2
(B) CF_2Cl_2 and NO
(C) $CFCl_3$ and N_2O
(D) CH_2Cl_2 and NO

50. Which of the following represents a first order reaction?
(A)


(B)


(C)


(D)


SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

51. What is the pH of 0.1 M $NaHCO_3$ solution?
$[H_2CO_3$: $K_{a_1} = 10^{-7}$ & $K_{a_2} = 10^{-10}]$

52. $CH_2 = CH - CH = CH - CH = CH - CH = CH_2$
How many geometrical isomer(s) is/are possible for the above compound?

53. The atomic number of iron is 26. How many maximum number of electron(s) of an iron atom has/have s = +½?

54. What is the translational kinetic energy of one mole of an diatomic ideal gas at 480 K in calorie unit?

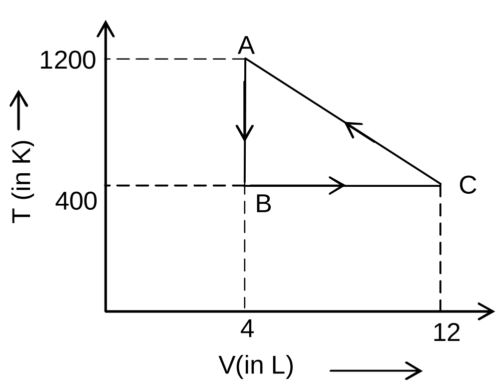
55. What volume in mL of 0.4 M acidified $KMnO_4$ solution can completely oxidize 0.9612 moles of Fe^{2+} into Fe^{3+} ions?

56. A container contains 240 mL of 0.2 M HCl solution. 360 mL of 0.2 M H_2SO_4 solution was added to it. What volume in litre of 0.16 M NaOH solution can completely neutralize the acid solution?

57. $A(g) \longrightarrow B(g) + C(g)$
In above first order reaction, gas A was taken in a container at 142 mm of Hg. What will be the partial pressure of A(g) in the container in mm of Hg unit after three half-lives?

58. An acyclic compound(A) contains six carbon atoms and the required number of hydrogen atoms. It can absorb one molecules of H_2 in presence of metal catalyst. It forms one monochloro substituted product when reacts with Cl_2 in presence of light. What is the mass of all CH_3 groups present in (A) in g mol^{-1} unit?

59. What is the mass of one mole of beryllium pyrosilicate in g unit?
[At. mass of Be = 9, Si = 28, O = 16]

60.


One mole of an ideal gas is subjected to the above thermodynamic process. What is the pressure of the system in atm unit?

Mathematics PART – C

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. Let $S = \frac{8}{5} + \frac{16}{65} + \frac{24}{325} + ... + \frac{128}{2^{18}+1}$, then

(A) $S = \frac{1088}{545}$

(B) $S = \frac{545}{1088}$

(C) $S = \frac{1056}{545}$

(D) $S = \frac{545}{1056}$

62. If the curve y = x^2 + bx + c touches the straight line y = x at the point (1, 1), then b and c are given by

(A) –1, 1
(B) –1, 2
(C) 2, 1
(D) 1, 1

63. The number of pairs of integer (x, y) that satisfy the following two equations

$$\begin{cases} \cos(xy) = x \\ \tan(xy) = y \end{cases}$$

(A) 1
(B) 2
(C) 4
(D) 6

64. The total number of solutions of $\ln|\sin x| = -x^2 + 2x$ in $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is equal to

(A) 1
(B) 2
(C) 4
(D) None of these

65. In triangle ABC, base BC and area of triangle Δ are fixed. Locus of the centroid of triangle ABC is a straight line that is

(A) parallel to BC
(B) right bisector of side BC
(C) right angle of BC
(D) inclined at an angle $\sin^{-1}\left(\frac{\sqrt{\Delta}}{BC}\right)$ to side BC

66. Let AD be a median of the ΔABC. If AE and AF are medians of the triangle ABD and ADC, respectively. And $AD = m_1$, $AE = m_2$, $AF = m_3$, then $a^2/8$ is equal to

(A) $m_2^2 + m_3^2 - 2m_1^2$

(B) $m_1^2 + m_2^2 - 2m_3^2$

(C) $m_1^2 + m_3^2 - 2m_2^2$

(D) none of these

67. The value of $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^{k}C_{r-1}\right)$ (where r, k, n $\in$ N) is equal to

(A) $2^{n+1} - 2$

(B) $2^{n+1} - 1$

(C) 2^{n+1}

(D) None of these

68. If $b_{n+1} = \dfrac{1}{1-b_n}$ for $n \ge 1$ and $b_1 = b_3$, then $\sum_{r=1}^{2001} b_r^{\,2001}$ is equal to

(A) 2001

(B) –2001

(C) 0

(D) None of these

69. The number of points (p, q) such that $p, q \in \{1, 2, 3, 4\}$ and the equation $px^2 + qx + 1 = 0$ has real roots is

(A) 7

(B) 8

(C) 9

(D) none of these

70. On the portion of the straight line $x + y = 2$ which is intercepted between the axes, a square is constructed, away from the origin, with this portion as one of its side. If p denotes the perpendicular distance of a side of this square from the origin, then the maximum value of p is

(A) $2\sqrt{3}$

(B) $3\sqrt{2}$

(C) $\dfrac{2}{\sqrt{3}}$

(D) $\dfrac{3}{\sqrt{2}}$

71. If common chord of the circle C with centre at (2, 1) and radius r and the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a diameter of the second circle, then value of r is

(A) 3

(B) 2

(C) 3/2

(D) 1

72. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity e of the hyperbola, satisfies

(A) $1<e<\frac{2}{\sqrt{3}}$

(B) $e=\frac{2}{\sqrt{3}}$

(C) $e=\frac{\sqrt{3}}{2}$

(D) $e>\frac{2}{\sqrt{3}}$

73. The number of positive integral solutions of the equation $x_1 x_2 x_3 x_4 x_5 = 1050$ is

(A) 1800
(B) 1600
(C) 1400
(D) None of these

74. Reflection of the line $\bar{a}z+a\bar{z}=0$ in the real axis is

(A) $\bar{a}\bar{z}+az=0$

(B) $\frac{\bar{a}}{a}=\frac{\bar{z}}{z}$

(C) $\left(a+\bar{a}\right)\left(z+\bar{z}\right)=0$

(D) None of these

75. If $a,b,c \in R$ and $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has

(A) at least one root in [0, 1]
(B) at least one root in [1, 2]
(C) at least one root in $\left[\frac{3}{2}, 2\right]$
(D) None of these

76. Let a and b be non – zero real numbers. Then the equation
$\left(ax^2+by^2+c\right)\left(x^2-5xy+6y^2\right)=0$ represents

(A) four straight lines, when c = 0 and a, b are of the same sign.
(B) two straight lines and a circle, when a = b and c is sign opposite to that of a.
(C) two straight lines and a circle, when a = b and c is of sign opposite to that of a
(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

77. If x=9 is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is
(A) $9x^2 - 8y^2 + 18x - 9 = 0$
(B) $9x^2 - 8y^2 - 18x + 9 = 0$
(C) $9x^2 - 8y^2 - 18x - 9 = 0$
(D) $9x^2 - 8y^2 + 18x + 9 = 0$

78. If laturs recturm of the ellipse $x^2 \tan^2\alpha + y^2 \sec^2\alpha = 1$ is ½, then α ($0 < \alpha < \pi$) is equal to
(A) $\pi/12$
(B) $\pi/6$
(C) $7\pi/12$
(D) None of these

79. If the circle $x^2 + y^2 = 1$ cuts the rectangular hyperbola xy = 1 in four points (x_i, y_i) i = 1, 2, 3, 4, then
(A) $x_1x_2x_3x_4 = 1$
(B) $y_1y_2y_3y_4 = 1$
(C) $x_1 + x_2 + x_3 + x_4 = 0$
(D) All of the above

80. Let z_1, z_2 be two complex numbers represented by points on the circle $|z| = 1$ and $|z| = 2$ respectively, then
(A) max $|2z_1 + z_2| = 4$
(B) min $|z_1 - z_2| = 1$
(C) $\left|z_2 + \frac{1}{z_1}\right| \le 3$
(D) All of the above

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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81. If number of roots of the equation $2^{\tan\left(x-\frac{\pi}{4}\right)} - 2(0.25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}} + 1 = 0$ are p then (p+1)/4 is

82. If ΔAEX, T is the midpoint of XE, and P is the midpoint of ET. If ΔAPE is equilateral of side length equal to unity, then the value of $(AX)^2 / 2$ is

83. The remainder when $\left(\sum_{k=1}^{5} {}^{20}C_{2k-1}\right)^6$ is divided by 11, is k then k/9 is

84. If $4\sin 27^{o} = \sqrt{\alpha} - \sqrt{\beta}$, then the value of $\frac{\alpha+\beta-\alpha\beta+2}{\sqrt{5}}$ is

85. If $\cos 4x = a_0 + a_1\cos^2 x + a_2\cos^4 x$ is true for all values of $x \in R$, then the value of $5a_0 + a_1 + a_2$

86. If A = tan 6° tan 42° and B = cot 66° cot 78°, then 3A / 2B is

87. If the point (α, β) lies on the line $2x+3y=6$, the smallest value of $\alpha^2+\beta^2$ is

88 If tangent at (1, 2) to the circle $x^2+y^2=5$ intersects the circle $x^2+y^2=9$ at P and Q and tangent at P and Q to the second circle meet at point R, then the X co – ordinates of R is

89. If the intersection point of the straight lines represented by $x^2+2xy+4y^2-2ax+8y+c=0$ is $\left(1, -\frac{5}{4}\right)$, then c - 3a is

90. The number of solutions of $3\sec\theta - 5 = 4\tan\theta$ in $[0, 4\pi]$ is k then k/5 is

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. D

Sol. Collision will occur after 1 sec.
Just before collision velocity of each object will be 10 m/sec.
Just after collision velocity of combined system will be zero.
So, time taken to reach ground = $(\sqrt{3}+1)$ sec.

2. A

Sol. $mg = V\rho g$
$= 3 \times 2 \times (0.01 \text{ m}) \times (10^3 \text{ kg/m3}) \times 10$
$Mg = 600$
$\Rightarrow$ m = 60 kg

3. D

Sol. Apply the loudness in decibels formula.

4. A

Sol. Fact based.

5. A

Sol. Acceleration will be least in part ACB as its distance from the centre of earth is maximum and acceleration will be maximum in part ADB as its distance from centre of earth is minimum.

6. D

Sol. $I_{\omega}\vec{w} + \vec{r} \times M\vec{v}_{\omega}$ = Combined Angular momentum.

7. A

Sol. mu = $mv_1 + mv_2$; $v_1 + v_2 = u$

$-e = \frac{v_2 - v_1}{0 - u}$

$v_2 - v_1 = eu$

$v_1 = \frac{u}{2}(1 - e)$; $v_2 = \frac{u}{2}(1 + e)$

8. C

Sol. $v_x = v = 20\ ms^{-1}$

$v_y = u_y + a_y t = 0 + gt = 10 \times 5 = 50\ ms^{-1}$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 50^2} \approx 54 ms^{-1}$.

9. B

Sol. $P_Q = P_O = \rho g_1 h + 2\rho g_1 2h = 5\rho(g + a)h = 10\rho gh$

10. A

Sol. By $\rho = m/v$, so percentage error can find.

11. D

Sol. $s \propto t^2$

$s = kt^2$

$\frac{ds}{dt} = 2kt$

$\frac{d^2s}{dt^2} = 2k$ = acceleration

12. C

Sol. $\frac{3}{4L}V = \frac{V}{2L} + 100$ Hz

$\Rightarrow \frac{1}{4}\frac{V}{L} = 100$ Hz

$\Rightarrow \frac{V}{2L} = 200$ Hz

13. B

Sol. Area under the curve gives the displacement with proper sign

$d = \frac{1}{2} \times 3 \times 2 - \frac{1}{2} \times 1 \times 2 + 1 \times 1$

$= 3 - 1 + 1 = 3$ m

14. C

Sol. $y = \dfrac{l}{A \times \text{slope of Curve}} = 2 \times 10^{11}$

15. C

Sol. $\therefore\ F_1 = \dfrac{m_1 v^2}{R_1}$ and $F_2 = \dfrac{m_2 v^2}{R_2}$

$\therefore\ \dfrac{\mu_1 m_1 g}{\mu_2 m_2 g} = \left[\dfrac{m_1 v^2}{R_1} \Big/ \dfrac{m_2 v^2}{R_2}\right]$

$\therefore\ \dfrac{\mu_1}{\mu_2} = \dfrac{R_2}{R_1}$

16. B

Sol. We know that $F = \dfrac{-dU}{dx} = \dfrac{-d}{dx}\left(U_0 - U_0 \cos \alpha x\right)$

Or $F = -U_0\, \alpha \sin \alpha x \approx -U_0\, \alpha \times \alpha x$

[$\because$ For small oscillations angular displacement is small so $\sin \alpha x \approx \alpha x$]

$\therefore\ F = -U_0 \alpha^2 x$ . . . (i)

As $F \,\alpha\, x$ and –ve sign shows that the force F is directed towards the equilibrium position, so if the body is left free, it will execute S.H.M.

Comparing (i) with equation, $F = -ky$ . . . (ii)

Spring factor, $k = U_0 \alpha^2$

Inertia factor = mass of body = m

As time period, $T = 2\pi \sqrt{\dfrac{\text{inertia factor}}{\text{spring factor}}} = 2\pi \sqrt{\dfrac{m}{U_0 \alpha^2}}$

Get $\tan \theta = M \Rightarrow \theta = 30°$

17. D

Sol. $1200 = \dfrac{1}{2} \times \dfrac{1}{2} \times 3 \times 8.3 \times \Delta T + 2196$

18. C

Sol. Displacement of the winning team is zero. Hence work done by loosing team on the winning team is zero. Ground applies a frictional force on the losing team in a direction opposite to its displacement hence work done by ground on losing team is zero.

19. C

Sol. P =KV

$\therefore\ PV^{-1}$ = constant (Polytrophic process with x = –1)

20. A

Sol. $v = \sqrt{\dfrac{2Gm}{R}}$

SECTION – B

21. 00009.00

Sol. $I_r = \left(\sqrt{I_o} + \sqrt{I_o} + \sqrt{I_o}\right)^2 = 9I_o$

22. 00002.00

Sol. $g = \dfrac{G\left(\dfrac{4\pi}{3}R^3\rho\right)}{R^2} = \dfrac{4\pi}{3}R\rho G \Rightarrow \rho R = 2\rho R_2 \Rightarrow R_2 = \dfrac{R}{2}$

23. 00001.00

Sol. $\omega^2 = 4\pi^2$

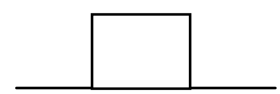
24. 00000.25

Range: 0.21 to 0.28

Sol. $P = Fv = kv^2v$

$\Rightarrow \quad P \propto v^3$

$\Rightarrow$ Ratio = 1 : 8



25. 00001.50

Range: 00001.40 to 00001.70

Sol. $k_1 = \dfrac{3}{2}k \quad ; \quad k_2 = 3k$

$k_{eff} = k_1 + k_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{9k}{2m}} = \dfrac{9}{2}k$

26. 00001.20

Range: 00001.10 to 00001.30

Sol. $a = \dfrac{d^2y}{dt^2} = 2k$, $T_1 = 2\pi\sqrt{\dfrac{\ell}{g}}$ & $T_2 = 2\pi\sqrt{\dfrac{\ell}{g+a}}$

$\dfrac{T_1^2}{T_2^2} = \dfrac{g+a}{g} = \dfrac{12}{10} = \dfrac{6}{5}$

27. 00000.83

Range: 00000.80 to 00000.90

Sol. $x^2 + y^2 = \ell^2$

$\dfrac{dy}{dt} = -\dfrac{x}{y}\left(\dfrac{dx}{dt}\right)$

If $\theta = 37°$

$\dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt} = -\dfrac{4}{3}v_o$

$\vec{v}_{cm} = \dfrac{mv_o\hat{i} - m\dfrac{4v_o}{3}\hat{y}}{2m}$

$$\left|\vec{V}_{cm}\right| = \frac{5v_o}{6}$$

28. 00000.33

Range: 00000.31 to 00000.35

Sol. $Kx = \frac{Mg}{3}$; $x = \frac{Mg}{3k}$

29. 00005.67

Range: 00005.60 to 00005.70

Sol. $\text{distance} = \int_0^4 |t^2 - 5t + 6|\, dt$

30. 00000.60

Range: 00000.50 to 00000.70

Sol. $\frac{2}{3}MR_1^2 = \frac{2}{5}MR_2^2$

Chemistry PART – B

SECTION – A

31. C

Sol. For the unpaired electrons of carbon n = 2, ℓ =1, m = 0, ±1, s = ±½

32. D

Sol. In PCl_2F_3, F atoms are present in axial as well as equatorial positions. So it is unsymmetrical and the dipole vectors are not cancelled. Therefore it's dipole moment is non zero. Other molecules have zero dipole moments.

33. B

Sol. The highest velocity is Crms which is $\sqrt{\frac{3RT}{M}}$ and the velocity possessed by the highest fraction of molecule is Cmp which is $\sqrt{\frac{2RT}{M}}$

34. B

Sol. $4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + 3O_2$

35. C

Sol. $p_{O_2} = 0.5$ atm

$$K_P = \frac{1}{p_{o_2}} = \frac{1}{0.5} = 2$$

36. B

Sol. Moles of HCl is greater than NH_4OH. So it is an acidic solution.

37. B

Sol. Rate = $K[X]^2[Y_2]^1$

$\therefore$ overall order = 2 + 1 = 3

38. B

Sol. For a reversible reaction

ΔS(system) = -ΔS(surrounding)

$\therefore \Delta S$(total) = zero

39. D

Sol. $Na_2B_4O_7 \xrightarrow{H_2O} B(OH)_3 + Na^+[B(OH)_4]^-$

40. D

Sol. The electron-withdrawing effect of $NO_2$is higher than chlorine. Stronger the electron withdrawing group, higher is the acidic strength.

41. C

Sol. $$n\left(HO-\underset{\underset{R}{|}}{\overset{\overset{R}{|}}{Si}}-OH\right)\xrightarrow{-H_2O}\left(O-\underset{\underset{R}{|}}{\overset{\overset{R}{|}}{Si}}-O\right)_n$$

42. C

Sol. The acyclic functional isomers of 2-butyne are
$CH_2 = CH - CH = CH_2$ and $CH_3 - CH = C = CH_2$

43. D

Sol. The electrophiles that can attack a benzene ring are

$\overset{\oplus}{C}H_2\,CH_2Cl$, $PhCH_2CH_2^{\oplus}$, $Ph\overset{\overset{CH_3}{|}}{C}H^{\oplus}$

44. B

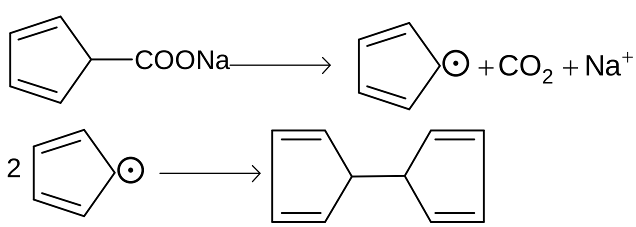
Sol. $C_2H_5Cl \xrightarrow{\text{Alc.KOH}} C_2H_4$

45. D

Sol. $CH_3CH_2CH = CHCH_2CH_3 \xrightarrow[Zn/H_2O]{O_3} 2CH_3CH_2CHO$

46. C

Sol.



47. B

Sol. 2-butyne does not contain terminal alkyne group. So it does not form precipitate with ammonical $AgNO_3$ solution.

48. C

Sol. $Ph-\underset{\underset{Ph}{|}}{\overset{\overset{Ph}{|}}{C^{\oplus}}}$ is most stable due to it’s largest number of resonating structure

49. B

Sol. CF_2Cl_2 and NO can form free radical in presence of radiation.

50. C

Sol. Rate = k[Concentration]

or Rate α [Concentration]

SECTION – B

51. 00008.50

Sol. Since it is an amphiprotic salt

$$pH = \frac{1}{2}\left[p^{K_{a_1}} + p^{K_{a_2}}\right] = \frac{1}{2}(7+10) = 8.5$$

52. 00003.00

Sol. Due to identical terminal groups, the number of isomer is three.

53. 00015.00

Sol. Number of s-electrons with s = +½ = 4

Number of p-electrons with s = +½ = 6

Number of d-electrons with s = +½ = 5

54. 01440.00

Sol. $E_K = \frac{3}{2}nRT = \frac{3}{2}(1)(2)(480) = 1440$ cal

55. 00480.60

Sol. M_{eq} of MnO_4^- = M_{eq} of Fe^{2+}

or, $V \times N$ = mole $\times$ 1000

or, $V \times n \times M = 0.9612 \times 1000$

or, $V \times 5 \times 0.4 = 961.2$

or, $V = \frac{961.2}{2} = 480.6$ mL

56. 00001.20

Sol. Moles of H^+ = $\frac{240 \times 0.2}{1000} + \frac{360 \times 0.2 \times 2}{1000}$ = 0.192

$\therefore$ Moles of NaOH required = 0.192

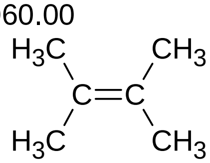
$\therefore 0.192 = \frac{V \times 0.16}{1000}$

or, V = 1200 mL = 1.2 L

57. 00017.75

Sol. $P_t = \left(\frac{1}{2}\right)^n P_o = \left(\frac{1}{2}\right)^3 P_o = \frac{142}{8} = 17.75$

58. 00060.00

Sol. A is 

59. 00195.00

Sol. Formula of beryllium pyrosilicate is $Be_3Si_2O_7$.

60. 00008.21

Sol. $$P = \frac{nRT}{V} = \frac{(R)400}{4} = 100\ R = 100(0.0821) = 8.21$$

Mathematics PART – C

SECTION – A

61. A

Sol. $\because \quad S = \sum_{r=1}^{16} \frac{8r}{\left(4r^4+1\right)}$

$= \sum_{r=1}^{16} \frac{8r}{\left(2r^2-2r+1\right)\left(2r^2+2r+1\right)}$

$= 2\sum_{r=1}^{16} \left(\frac{1}{2r^2-2r+1} - \frac{1}{2r^2+2r+1}\right)$

$= 2\left\{\frac{1}{1} - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \frac{1}{13} + ... + \frac{1}{481} - \frac{1}{545}\right\}$

$= 2\left(1 - \frac{1}{545}\right) = \frac{1088}{545}$

62. A

Sol. y = x touches y = x^2 + bx + c

1 = 1 + b + c $\Rightarrow$ b + c = 0 ……(1)

$\frac{dy}{dx} = 2x + b\Big|_{(1,\ 1)} = 2 + b = 1 \Rightarrow b = -1,\ c = 1.$

(b, c) = (–1, 1)

63. A

Sol. $\frac{\sin(xy)}{\cos(xy)} = y$

$\Rightarrow \sin(xy) = xy$

$\Rightarrow xy = 0$

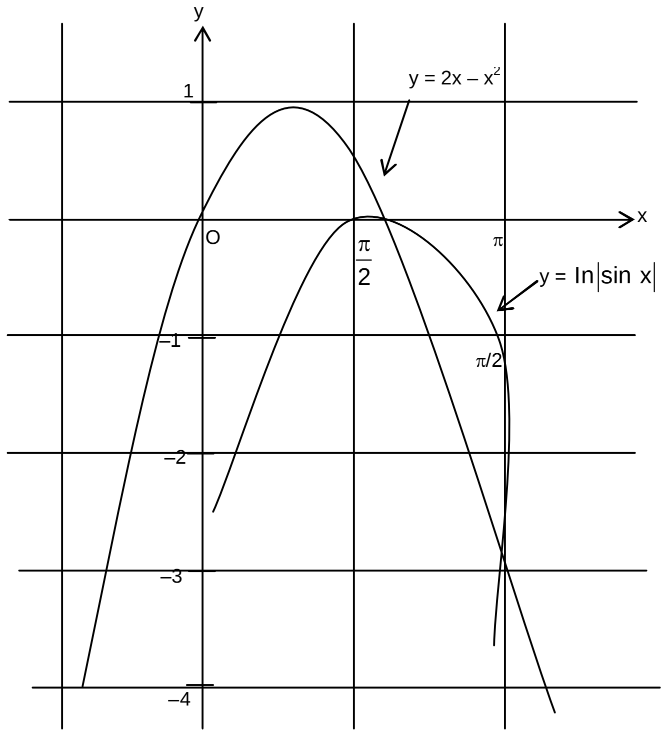
$\Rightarrow x = 0$ or y = 0

But x = 0 is not possible

$\therefore$ y = 0 and x = 1, i.e. (1, 0)

64. B

Sol.



$\ln|\sin x| = -x(x-2)$

Graphs of $y = \ln|\sin x|$ and $y = -x(x-2)$ meet exactly two times in $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

65. A

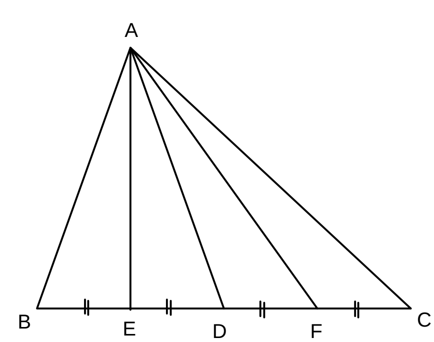
Sol. $\Delta = \frac{1}{2}(BC)h$, where h is the distance of vertex A from side BC

$\Delta_{GBC} = \frac{\Delta}{3} = \frac{(BC)h}{6}$, where G is the centroid $\Rightarrow$ $h = \frac{2\Delta}{BC} =$ constant

Thus, distance of vertex A from side is fixed. This, in turn, implies that distance of centroid from side BC will be fixed, hence locus of G will be a line parallel to BC.

66. C

Sol. In ΔABC, $AD^2 = m_1^2 = \frac{c^2+b^2}{2} = \frac{a^2}{4}$



In ΔABD, $AE^2 = m_2^2 = \frac{AD^2+c^2}{2} - \frac{\left(\frac{a}{2}\right)^2}{4}$

[Apollonius Theorem]

In ΔADC, $AF^2 = m_3^2 = \frac{AD^2+b^2}{2} - \frac{\left(\frac{a}{2}\right)^2}{4}$

$\therefore m_2^2 + m_3^2 = AD^2 + \frac{b^2+c^2}{2} - \frac{a^2}{8} = m_1^2 + \frac{a^2}{4} - \frac{a^2}{8} = 2m_1^2 + \frac{a^2}{8}$

$$\Rightarrow \quad m_2^2 + m_3^2 - 2m_1^2 = \frac{a^2}{8}$$

67. A

Sol. $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^{k}C_{r-1}\right)$

$= \sum_{r=1}^{n+1}\left(\sum_{k=1}^{n}\left({}^{k+1}C_r - {}^{k}C_r\right)\right)$

$= \sum_{r=1}^{n+1}\left({}^{n+1}C_r - {}^{1}C_r\right)$

$= 2^{n+1} - 2$

68. B

Sol. $b_2 = \frac{1}{1-b_1}$

$b_3 = \frac{1}{1-b_2} = \frac{1}{1-\frac{1}{1-b_1}} = \frac{1-b_1}{-b_1} = \frac{b_1-1}{b_1}$

$b_1 = b_3 \Rightarrow b_1^2 - b_1 + 1 = 0$

$\Rightarrow b_1 = -\omega \quad \text{or} \quad \omega^2 \Rightarrow b_2 = \frac{1}{1+\omega} = -\omega \text{ or } \omega^2$

$\sum_{r=1}^{2001} b_r^{2001} = \sum_{r=1}^{2001}(-\omega)^{2001}$

$= \sum_{r=1}^{2001} 1$

= –2001

69. A

Sol. $px^2 + qx + 1 = 0$ has real roots

If $q^2 - 4p \ge 0$ or $q^2 \ge 4p$

Since, $p, q \in \{1, 2, 3, 4\}$

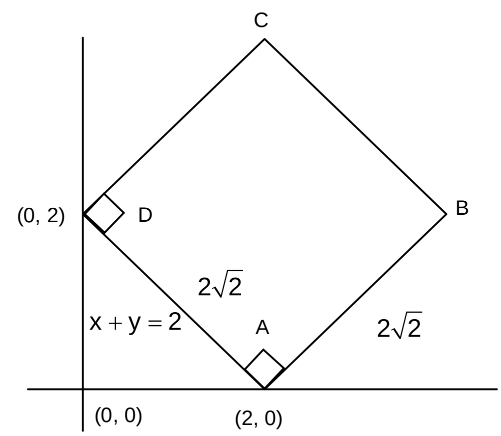
The required points are $(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (4, 4)$

So the required number is 7.

70. B

Sol. Clearly p = perpendicular distance from (0, 0) to AD + side of the square (Figure)

$= \frac{2}{\sqrt{2}} + 2\sqrt{2} = 3\sqrt{2}$



71. A

Sol. Equation of the circle C is $(x-2)^2 + (y-1)^2 = r^2$

$\Rightarrow \quad x^2 + y^2 - 4x - 2y + 5 - r^2 = 0$

Equation of the common chord is

$(x^2 + y^2 - 4x - 2y + 5 - r^2) - (x^2 + y^2 - 2x - 6y + 6) = 0$

$\Rightarrow 2x - 4y + r^2 + 1 = 0$

If it is a diameter of the second circle, it passes through the centre (1, 3) of the circle

So $2 - 4 \times 3 + r^2 + 1 = 0 \Rightarrow r^2 = 9 \Rightarrow r = 3$

72. D

Sol. Let the coordinates of P be (α, β).

Then $PQ = 2\beta$ and $OP = \sqrt{\alpha^2 + \beta^2}$

Since OPQ is an equilateral triangle OP = PQ

$\Rightarrow \quad \alpha^2 + \beta^2 = 4\beta^2 \Rightarrow \alpha^2 = 3\beta^2$

$\Rightarrow \quad \alpha = \pm\sqrt{3}\beta$

Also, since (α, β) lies on the given hyperbola,

$\frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1$

$\Rightarrow \quad \frac{3\beta^2}{a^2} - \frac{\beta^2}{b^2} = 1 \Rightarrow \frac{3}{a^2} - \frac{1}{b^2} = \frac{1}{\beta^2} > 0 \Rightarrow \frac{b^2}{a^2} > \frac{1}{3}$

$\Rightarrow \quad e^2 - 1 > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$

73. D

Sol. Using prime factorization of 1050, we can write the given equation as

$x_1 x_2 x_3 x_4 x_5 = 2 \times 3 \times 5^2 \times 7$

We can assign 2, 3 or 7 to any of 5 variables. We can assign entire 5^2 to just one variable in 5 ways or can assign $5^2 = 5 \times 5$ to two variable in ${}^5C_2$ ways. Thus, 5^2 can be assigned in ${}^5C_1 + {}^5C_2 = 5 + 10 = 15$ ways

Hence, required number of solutions $= 5 \times 5 \times 5 \times 15 = 1875$

74. A

Sol. Let $a = \alpha + i\beta$ and $z = x + iy$, then $\bar{a}z + a\bar{z} = 0$ becomes $\alpha x + \beta y = 0$ or $y = \left(\frac{-\alpha}{\beta}\right)x$.

It reflection in the x – axis is $y = \frac{\alpha}{\beta}x$ or $\alpha x - \beta y = 0$

or $\left(\frac{a+\bar{a}}{2}\right)\left(\frac{z+\bar{z}}{2}\right) - \left(\frac{a+\bar{a}}{2i}\right)\left(\frac{z+\bar{z}}{2i}\right) = 0$

or $az + \bar{a}\bar{z} = 0$

75. A

Sol. Let $f(x) = ax^3 + bx^2 + cx$. Note that f is continuous and derivable on R. Also $f(0) = 0$ and $f(1) = a + b + c = 0$. By the Rolle's theorem, there exists at least one $\alpha \in (0,1)$ such that $f'(\alpha) = 0 \Rightarrow 3a\alpha^2 + 2b\alpha + c = 0$

Thus, $3ax^2 + 2bx + c = 0$ has at least one root in $[0, 1]$.

76. B

Sol. Given equation represents

$ax^2 + by^2 + c = 0,\ x - 2y = 0,\ x - 3y = 0$

If $a = b$ and c and a are of opposite sign, the $ax^2 + by^2 + c = 0 \Rightarrow x^2 + y^2 - c/a$ which represents a circle as $-c/a > 0$.

Note : When $c = 0,\ ax^2 + by^2 = 0$ do not represent a pair of straight lines as a and b are of same sign.

Next, when a and b are of the same sign and c is sign opposite to that of a.

$ax^2 + by^2 = -c \Rightarrow \frac{x^2}{-c/a} + \frac{y^2}{-c/b} = 1$

$-c/a > 0,\ -c/b > 0,$ so it represents an ellipse.

77. B

Sol. Let R (h, k) be the point of intersection f the tangent to H at the extremities of the chord L : x = 9 then equation of L is $hx - ky = 9 \Rightarrow h = 1, k = 0$

$\therefore$ coordinates of R are (1, 0).

Equation of the pair of tangents from R to H is

$(x^2 - y^2 - 9)(1 - 9) = (x - 9)^2\ (SS_1 = T^2)$

$\Rightarrow\ 9x^2 - 8y^2 - 18x + 9 = 0$

78. A

Sol. $x^2 \tan^2\alpha + y^2 \sec^2\alpha = 1$

$\Rightarrow \quad \frac{x^2}{\cot^2\alpha} + \frac{y^2}{\cos^2\alpha} = 1$

$\because \quad \cos^2\alpha = \cot^2\alpha\,(1 - e^2)$

$\Rightarrow \quad \sin^2\alpha = (1 - e^2)$

$\therefore \quad e^2 = \cos^2\alpha\ (\alpha \neq 90^o)$

$e = \cos\alpha$

$\because$ Latus rectum = 1/2 = $2b^2/a$

$\Rightarrow$ $a = 4b^2$,

$\Rightarrow$ $\cot\alpha = 4\cos^2\alpha$

$\Rightarrow$ $\dfrac{1}{\sin\alpha} = 4\cos\alpha$

$\Rightarrow$ $\sin 2\alpha = \dfrac{1}{2}$

$2\alpha = n\pi + (-1)^n \dfrac{\pi}{6}$

$\Rightarrow$ $\alpha = \dfrac{n\pi}{2} + (-1)^n \dfrac{\pi}{12}$

79. D

Sol. Let $(x_i, y_i) = (t_i, 1/t_i); = 1, 2, 3, 4$

Any point on the rectangular hyperbola xy = 1 is (t, 1/t) which lies on the circle $x^2 + y^2 = 1$, if $t^2 + \dfrac{1}{t^2} = 1 \Rightarrow t^4 - t^2 + 1 = 0$

The roots of the equation are t_1, t_2, t_3, t_4 where $t_1 + t_2 + t_3 + t_4 = 0 \Rightarrow x_1 + x_2 + x_3 + x_4 = 0$

$\sum t_1t_2 = -1 \Rightarrow \sum t_1t_2t_3 = 0$

$t_1t_2t_3t_4 = 1 \Rightarrow x_1x_2x_3\, x_4 = y_1y_2y_3y_4 = 0$ and $y_1 + y_2 + y_3 + y_4 = \dfrac{1}{t_1} + \dfrac{1}{t_2} + \dfrac{1}{t_3} + \dfrac{1}{t_4}$

$= \dfrac{\sum t_1t_2t_3}{t_1t_2t_3t_4} = 0$

80. D

Sol. Since z_1 and z_2 lie on $|z| = 1$ and $|z| = 2$,

then $|z_1| = 1$ and $|z_2| = 2$

Alternate (a) :

$|2z_1 + z_2| \le 2|z_1| + |z_2| = 2.1 + 2 \le 4$

max $|2z_1 + z_2| = 4$

Alternate (b) :

$|z_1 - z_2| \ge ||z_1| - |z_2||$

$= |1 - 2|$

= 1

$\therefore$ $|z_1 - z_2| \ge 1$

min $|z_1 - z_2| = 1$

Alternate (c):

$\left|z_2 + \dfrac{1}{z_1}\right| \le |z_2| + \left|\dfrac{1}{z_1}\right|$

$= |z_2| + \dfrac{1}{|z_1|}$

$= 2 + 1 = 3$

$\therefore \left|z_2 + \frac{1}{z_1}\right| \le 3$

SECTION – B

81. 00000.25

Sol. $\frac{\sin^2\left(x - \frac{\pi}{4}\right)}{\cos 2x} = \frac{\frac{1}{2}(\sin x - \cos x)^2}{\cos^2 x - \sin^2 x} = \frac{-\frac{1}{2}(\sin x - \cos x)}{\cos x + \sin x} = -\frac{1}{2}\tan\left(x - \frac{\pi}{4}\right)$

Given equation reduces to $2^{\tan\left(x-\frac{\pi}{4}\right)} - 2(0.25)^{\frac{1}{2}\tan\left(x-\frac{\pi}{4}\right)} + 1 = 0$

$\Rightarrow 2^{\tan\left(x-\frac{\pi}{4}\right)} = 1$

$\Rightarrow x = \pi / 4$ which is not possible as $\cos 2x = 0$ for this value of x, which is not defining the original equation.

82. 00006.50

Sol. We have AE = EP = AP = 1

$\Rightarrow AP = PT = 1$

$\Rightarrow \Delta APT$ is isosceles

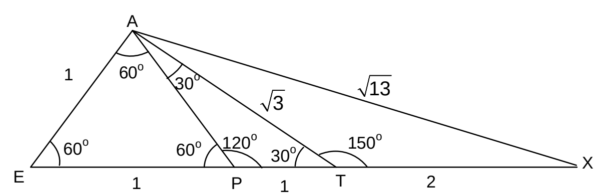
$\Rightarrow \angle EAT = 90^o$

$\Rightarrow AT = \sqrt{3}$ and $\angle ATX = 150^o$

Since TX = 2, by applying Cosine rule in $\Delta ATX,$ we get

$(AX)^2 = 3 + 4 - 4\sqrt{3}\cos 150^o = 7 + 6 = 13$

$\Rightarrow AX = \sqrt{13}$



83. 00000.33

Sol. $\left({}^{20}C_1 + {}^{20}C_3 + {}^{20}C_5 + {}^{20}C_7 + {}^{20}C_9\right)^6$

$= (\lambda)^6$

Sum of all odd coefficients $= 2\lambda = 2^{19}$

$= \lambda = 2^{18}$

$= (\lambda)^6 = 2^{108}$

$= 8 \times \left(2^5\right)^{21}$

$= 8 \times (33 - 1)^{21} = 33\lambda - 8$

$= 33\lambda - 8 - 3 + 3$

$\therefore$ Remainder = 3

84. 00002.00

Sol. $\left(\cos 27^o + \sin 27^o\right)^2 = 1 + \sin 54^o = 1 + \cos 36^o$

$\Rightarrow \quad \cos 27^o + \sin 27^o = \sqrt{1 + \cos 36^o} \qquad (\because LHS > 0)$

Also, $\cos 27^{o} - \sin 27^{o} = \sqrt{1-\cos 36^{o}}$ $\left(\because \cos 27^{o} > \sin 27^{o}\right)$

$\therefore\ 2\sin 27^{o} = \sqrt{1+\cos 36^{o}} - \sqrt{1-\cos 36^{o}} = \sqrt{1+\frac{\sqrt{5}+1}{4}} - \sqrt{1-\frac{\sqrt{5}+1}{4}}$

$\therefore\ 4\sin 27^{o} = \sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}$

On comparing, we get $\alpha = 5+\sqrt{5}, \beta = 3-\sqrt{5}$.

85. 00005.00

Sol. $\cos 4x = 2\cos^2 2x - 1$

$= 2\left(2\cos^2 x - 1\right)^2 - 1$

$= 2\left(4\cos^4 x + 1 - 4\cos^2 x\right) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$

$\therefore a_0 = 1, a_1 = -8, a_2 = 8$

$\therefore 5a_0 + a_1 + a_2 = 5$

86. 00001.50

Sol. $\frac{A}{B} = \tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ$

$= \frac{\tan 6^\circ \tan\left(60^\circ - 6^\circ\right)\tan\left(60^\circ + 6^\circ\right)}{\tan 54^\circ} \times \frac{\tan 18^\circ \tan\left(60^\circ - 18^\circ\right)\tan\left(60^\circ + 18^\circ\right)}{\tan 18^\circ}$

$= \frac{\tan\left(3\times 6^\circ\right)}{\tan 54^\circ} \times \frac{\tan\left(3\times 18^\circ\right)}{\tan 18^\circ} = 1$

87. 00001.77

Sol. The smallest value of $\alpha^2 + \beta^2$ is the square of the perpendicular distance of the origin from the line.

88. 00001.80

Sol. The equation of tangent at (1, 2) is $x + 2y - 5 = 0$ ……..(i)

Let $R = \left(\alpha, \beta\right)$

Now, this tangent will be a chord of contact of this point R w.r.t. to the second circle and its equation will be $x\alpha + y\beta - 9 = 0$ ……..(ii)

$\because$ Equation (i) and (ii) are identical

$\therefore\ \frac{1}{\alpha} = \frac{2}{\beta} = \frac{5}{9} \Rightarrow \left(\alpha, \beta\right) = \left(\frac{9}{5}, \frac{18}{5}\right)$

89. 00005.50

Sol. We have $\begin{vmatrix} 1 & 1 & -a \\ 1 & 4 & 4 \\ -a & 4 & c \end{vmatrix} = 0$ and $1.1 + 1.\left(-\frac{5}{4}\right) - a = 0, 1.1 + 4.\left(-\frac{5}{4}\right) + 4 = 0$

$\Rightarrow a = -\frac{1}{4}, \left(4c - 16\right) - 1\left(c + 4a\right) - a\left(4 + 4a\right) = 0$

$\Rightarrow c=\frac{19}{4}$

90. 00001.60

Sol. $3-5\cos\theta=4\sin\theta$

Put $t=\tan\frac{\theta}{2}$

$\Rightarrow 3-5\frac{\left(1-t^2\right)}{1+t^2}=\frac{4\left(2t\right)}{1+t^2}\Rightarrow 4t^2-4t-1=0$

$\Rightarrow t=\frac{1}{2}\pm\frac{1}{\sqrt{2}}\Rightarrow \tan\frac{\theta}{2}=\frac{1}{2}\pm\frac{1}{\sqrt{2}}$

$\Rightarrow \tan\theta=\frac{4\left(1+\sqrt{2}\right)}{1-2\sqrt{2}},\frac{4\left(1-\sqrt{2}\right)}{1+2\sqrt{2}}.$

Thus 8 solutions on $\left[0,\ 4\pi\right]$