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Circle

CIRCLE

A circle is a locus of a point whose distance from a fixed point (called centre) is always constant (called radius).

1. EQUATION OF A CIRCLE IN VARIOUS FORM

(a) The circle with centre as origin & radius 'r' has the equation; $\mathbf{x^2 + y^2 = r^2}$.

(b) The Circle with centre (h, k) & radius 'r' has the equation; $\mathbf{(x - h)^2 + (y - k)^2 = r^2}$

(c) The general equation of a circle is

$\mathbf{x^2 + y^2 + 2gx + 2fy + c = 0}$ with centre as (–g, –f) and

radius = $\sqrt{g^2 + f^2 - c}$. If :

$g^2 + f^2 - c > 0 \Rightarrow$ real circle.

$g^2 + f^2 - c = 0 \Rightarrow$ point circle

$g^2 + f^2 - c < 0 \Rightarrow$ imaginary circle, with real centre, that is (–g, –f)

Note that every second degree equation in x & y, in which coefficient of x^2 is equal to coefficient of y^2 & the coefficient of xy is zero, always represents a circle.

(d) The equation of circle with (x_1, y_1) & (x_2, y_2) as extremities of its diameter is :

$$\mathbf{(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.}$$

Note that this will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

2. INTERCEPTS MADE BY A CIRCLE ON THE AXES

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co-ordinate axes are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$ respectively. If :

$g^2 - c > 0 \Rightarrow$ circle cuts the x axis at two distinct points.

$g^2 - c = 0 \Rightarrow$ circle touches the x–axis

$g^2 - c < 0 \Rightarrow$ circle lies completely above or below the x–axis.

3. PARAMETRIC EQUATIONS OF A CIRCLE

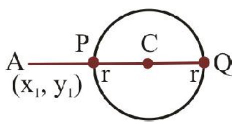
The parametric equations of $(x - h)^2 + (y - k)^2 = r^2$ are :

$$x = h + r\cos\theta\ ;\ \ y = k + r\sin\theta\ ;\ -\pi < \theta \le \pi$$

where (h, k) is the centre, r is the radius & θ is a parameter.

4. POSITION OF A POINT WITH RESPECT TO A CIRCLE

The point (x_1, y_1) is inside, on or outside the circle $S \equiv x_1^2 + y_1^2 + 2gx + 2fy + c = 0$.



according as $\mathbf{S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < = or > 0.}$

Note...

The greatest & the least distance of a point A from a circle with centre C & radius r is AC + r & AC – r respectively.

5. LINE AND A CIRCLE

Let L = 0 be a line & S = 0 be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then:

(i) $p > r \Leftrightarrow$ the line does not meet the circle i.e. passes outside the circle.

(ii) $p = r \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)

(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.

(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

Also, if y = mx + c is line and $x^2 + y^2 = a^2$ is circle then.

(i) $c^2 < a^2(1 + m^2) \Leftrightarrow$ the line is a secant of the circle.

(ii) $c^2 = a^2(1 + m^2) \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)

(iii) $c^2 > a^2(1 + m^2) \Leftrightarrow$ the line does not meet the circle i.e. passes out side the circle.

6. TANGENT

(a) Slope form :

$y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.

Hence, equation of tangent is $\mathbf{y = mx \pm a\sqrt{1+m^2}}$ and the point of contact is $\left(-\frac{a^2m}{c}, \frac{a^2}{c}\right)$, where $c = \pm a\sqrt{1+m^2}$.

(b) Point form :

(i) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is, $\mathbf{x x_1 + y y_1 = a^2}$.

(ii) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is :

$\mathbf{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.}$

Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$;

$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$

Note...

In general the equation of tangent to any second degree curve at point (x_1, y_1) on it can be obtained by replacing x^2 by $x x_1$, y^2 by yy_1, x by $\frac{x+x_1}{2}$, y by $\frac{y+y_1}{2}$, xy by $\frac{x_1y+xy_1}{2}$ and c remains as c.

(c) Parametric form :

The equation of a tangent to circle $x^2 + y^2 = a^2$ at $(a\cos\alpha, a\sin\alpha)$ is : $\mathbf{x\cos\alpha + y\sin\alpha = a.}$

Note...

The point of intersection of the tangents at the points $P(\alpha)$ & $Q(\beta)$ is :

$$\left(\frac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \frac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$$

7. FAMILY OF CIRCLES

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ and $S_2 = 0$ is : $\mathbf{S_1 + K\,S_2 = 0}$

($K \neq -1$, provided the co-efficient of x^2 & y^2 in S_1 & S_2 are same)

(b) The equation of the family of circles passing through the point of intersection of a circle $S = 0$ & a line $L = 0$ is given by $\mathbf{S + KL = 0.}$

(c) The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form :

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

where K is a parameter.

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is

$(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$ where K is a parameter.

Note...

★ Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ and $L_3 = 0$ is given by ; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided co-efficient of $xy = 0$ and co-efficient of x^2 = co-efficient of y^2.

★ Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ are $\mu L_1L_3 + \lambda L_2L_4 = 0$ where value of μ and λ can be found out by using condition that co-efficient of x^2 = co-efficient of y^2 and co-efficient of $xy = 0$.

8. NORMAL

If a line is normal /orthogonal to a circle then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is ;

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$$

9. PAIR OF TANGENTS FROM A POINT

The equation of a pair of tangents drawn from the point A (x_1, y_1) to the circle.

$x^2 + y^2 + 2gx + 2fy + c = 0$ is : $\mathbf{SS_1 = T^2}$

10. LENGTH OF A TANGENT AND POWER OF A POINT

The length of a tangent from an external point (x_1, y_1) to the circle :

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by

$$\mathbf{L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}}$$

Square of length of the tangent from the point P is also called the power of point w.r.t. a circle. Power of a point w.r.t a circle remains constant.

Power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

11. DIRECTOR CIRCLE

The locus of the point of intersection of two perpendicular tangents is called the director circle of the given circle. The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the original circle.

12. CHORD OF CONTACT

If two tangents PT_1 & PT_2 are drawn from the point P (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is :

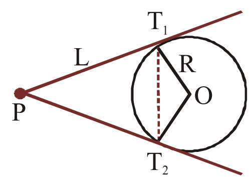
$\mathbf{xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.}$ T = 0

Note...

Here R = radius, L = Length of tangent.

(a) Chord of contact exists only if the point 'P' is not inside.

(b) Length of chord of contact $T_1T_2 = \dfrac{2LR}{\sqrt{R^2 + L^2}}$



(c) Area of the triangle formed by the pair of the tangents & its chord of contact $= \dfrac{RL^3}{R^2 + L^2}$

(d) Tangent of the angle between the pair of tangents from $(x_1, y_1) = \left(\dfrac{2RL}{L^2 - R^2}\right)$

(e) Equation of the circle circumscribing the triangle PT_1T_2 is :

$(x - x_1)(x + g) + (y - y_1)(y + f) = 0.$

13. POLE AND POLAR (NOT IN SYLLABUS)

(i) If through a point P in the plane of the circle there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q and R is called the Polar of the point P; also P is called the Pole of the Polar.

(ii) The equation of the polar of a point P $(x_1\ y_1)$ w.r.t. the circle $x^2 + y^2 = a^2$ is given by $xx_1 + yy_1 = a^2$ and if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ i.e. T = 0. Note that if the point $(x_1\ y_1)$ be on the circle then the tangent & polar will be represented by the same equation. Similary if the point $(x_1\ y_1)$ be outside the circle then the chord of contact and polar will be represented by the same equation.

(iii) Pole of a given line $Ax + By + C = 0$ w.r.t. circle

$x^2 + y^2 = a^2$ is $\left(-\dfrac{Aa^2}{C}, -\dfrac{Ba^2}{C}\right)$

(iv) If the polar of a point P pass through a point Q then the polar of Q passes through P.

(v) Two lines L_1 & L_2 are conjugate of each other if Pole of L_1 lies on L_2 and vice versa. Similarly two points P and Q are said to be conjuagate of each other if the polar of P passes through Q and vice-versa.

14. EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $\mathbf{T = S_1}$.

Note...

(i) The shortest chord of a circle passing through a point 'M' inside the circle is one chord whose middle point is M.

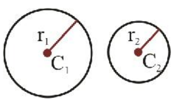
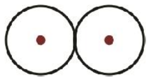
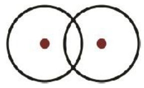
(ii) The chord passing through a point 'M' inside the circle and which is at a maximum distance from the centre is a chord with middle point M.

15. EQUATION OF THE CHORD JOINING TWO POINTS OF CIRCLE

The equation of chord PQ of the circle $x^2 + y^2 = a^2$ joining two points $P(\alpha)$ and $P(\beta)$ on it is given by. The equation of a straight line joining two point α and β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

16. COMMON TANGENTS TO TWO CIRCLES

CASE	NUMBER OF TANGENTS	CONDITION
(i) 	4 common tangents (2 direct and 2 transverse)	$r_1 + r_2 < c_1c_2$.
(ii) 	3 common tangents	$r_1 + r_2 = c_1c_2$.
(iii) 	2 common tangents	$\lvert r_1 - r_2\rvert < c_1c_2 < r_1 + r_2$
(iv) 	1 common tangents	$\lvert r_1 - r_2\rvert = c_1c_2$.
(v) 	No common tangent.	$c_1c_2 < \lvert r_1 - r_2\rvert$

(Here c_1c_2 is distance between centres of two circles.)

Note...

(i) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.

Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

(ii) Length of an external (or direct) common tangent & internal (or transverse) common tangent to the two circles are given by : $L_{ext.} = \sqrt{d^2 - (r_1 - r_2)^2}$

and $L_{int.} = \sqrt{d^2 - (r_1 + r_2)^2}$, where d = distance between the centres of the two circles and r_1, r_2 are the radii of the two circles. Note that length of internal common tangent is always less than the length of the external or direct common tangent.

17. ORTHOGONALITY OF TWO CIRCLES

Two circles $S_1 = 0$ & $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is : $\mathbf{2g_1g_2 + 2f_1f_2 = c_1 + c_2}$.

Note...

(a) The centre of a variable circle orthogonal to two fixed circles lies on the radical axis of two circles.

(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$, $S_2 = 0$ & $S_3 = 0$ are concurrent is a circle which is orthogonal to all the three circles. (Not in syllabus).

(c) The centre of a circle which is orthogonal to three given circles is the radical centre provided the radical centre lies outside all the three circles.

18. RADICAL AXIS AND RADICAL CENTRE

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ & $S_2 = 0$ is given by :

$S_1 - S_2 = 0$

i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$

The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles. Note that the length of tangents from radical centre to the three circles are equal.

Note...

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis will pass through the mid point of the line joining the centres of the two circles only if the two circles have equal radii.

(e) Radical axis bisects a common tangent between the two circles.

(f) A system of circles, every two of which have the same radical axis, is called a coaxal system.

(g) Pairs of circles which do not have radical axis are concentric.

SOLVED EXAMPLES

Example – 1

In each of the following questions, find the centre and radius of the circles

(i) $(x + 5)^2 + (y - 3)^2 = 36$

(ii) $x^2 + y^2 - 4x - 8y - 45 = 0$

(iii) $x^2 + y^2 - 8x + 10y - 12 = 0$

(iv) $2x^2 + 2y^2 - x = 0$

Sol. $(x + 5)^2 + (y - 3)^2 = 36$

or $(x - (-5))^2 + (y - 3)^2 = 6^2$

$h = -5, k = 3$ and $r = 6$

Therefore, the given circle has centre at (–5, 3) and radius 6.

(ii) The given equation is

$x^2 + y^2 - 4x - 8y - 45 = 0$

or $(x^2 - 4x) + (y^2 - 5y) = 45$

Now completing the squares with in the parenthesis, we get

$(x^2 - 4x + 4) + (y^2 - 8y + 16) = 4 + 16 + 45$

or $(x-2)^2 + (y-4)^2 = 65$

Therefore, the given circle has centre at (2, 4) and radius $\sqrt{65}$

(iii) The given equation is

$x^2 + y^2 - 8x + 10y - 12 = 0$

or $(x^2 - 8x) + (y^2 + 10) = 12$

or $(x^2 - 8x + 16) + (y^2 + 10y + 25) = 12 + 16 + 25$

or $(x - 4)^2 + (y + 5)^2 = 53$

Therefore, the given circle has centre at (4, –5) and radius $\sqrt{53}$.

(iv) The given equation is

$2x^2 + 2y^2 - x = 0$

or $x^2 + y^2 - \frac{x}{2} = 0$

or $\left(x^2 - \frac{x}{2}\right) + y^2 = 0$

or $\left(x^2 - \frac{x}{2} + \frac{1}{16}\right) + y^2 = \frac{1}{16}$

or $\left(x - \frac{1}{4}\right)^2 + y^2 = \frac{1}{16}$

Therefore, the given circle has centre at $\left(\frac{1}{4}, 0\right)$ and has radius $\frac{1}{4}$.

Example – 2

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line 4x + y=16

Sol. Let the equations of the circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through (4, 1) and (6, 5), we have

$(4 - h)^2 + (1 - k)^2 = r^2$ ... (i)

and $(6 - h)^2 + (5 - k)^2 = r^2$ ... (ii)

Also, since the centre lies on the line $4x + y = 16$, we have

$4h + k = 16$ ... (iii)

Simplifying the equation (i), we get

$16 - 8h + h^2 + 1 - 2k + k^2 = r^2$

or $17 - 8h + h^2 - 2k + k^2 = r^2$

from equation (iii), we get

$36 - 12h + h^2 - 25 - 10k + k^2 = r^2$

or $61 - 12h + h^2 - 10k + k^2 = r^2$

Now,

$17 - 61 - 8h + 12h + h^2 - 2k + 10k + k^2 - k^2 = 0$

(eliminating square terms)

or $-44 + 4h + 8k = 0$

or $4h + 8k = 44$ ... (iv)

Solving equations (iii) and (iv) we get

$4h + k = 16$

$4h + 8k = 44$

\- \- \-

$-7k = -28 \Rightarrow k = 4$

Substituting k = 4 in equation (iii) we get

$4h + 4 = 16$

or $4h = 12$

or $h = 3$

Substituting the value of h = 3, k = 4 in equation (i) we get

$(4 - 3)^2 + (1 - 4)^2 = r^2$

or $1 + 9 = r^2$

or $10 = r^2$

Hence, the required equation of the circle is

$(x - 3)^2 + (y - 4)^2 = 10$

or $x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

or $x^2 + y^2 - 6x - 8y + 15 = 0$

Example – 3

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Sol. Let the equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through, (2, 3) and its radius is 5

$(2 - h)^2 + (3 - k)^2 = 25$

or $4 - 4h + h^2 + 9 - 6k + k^2 = 25$

or $-12 - 4h + h^2 - 6k + k^2 = 0$ ... (i)

Also since the centre lies on the x-axis, we have

$k = 0$ ... (ii)

Putting k = 0 in equation (i) we have

$-12 - 4h + h^2 = 0$

or $h^2 - 4h - 12 = 0$

or $h^2 - 6h + 2h - 12 = 0$

or $h(h - 6) + 2(h - 6) = 0$

or $(h - 6)(h + 2) = 0$

or $h = 6$ or $h = -2$

Hence, the required equations of the circle are

(a) For h = 6

$(x - 6)^2 + (y - 0)^2 = (5)^2$

or $x^2 - 12x + 36 + y^2 = 25$

or $x^2 + y^2 - 12x + 11 = 0$

(b) For h = –2

$(x - (-2))^2 + (y - 0)^2 = (5)^2$

or $(x + 2)^2 + y^2 = 25$

or $x^2 + 4x + 4 + y^2 = 25$

or $x^2 + y^2 + 4x - 21 = 0$

Example – 4

Find the equation of a circle with centre (2, 2) which passes through the point (4, 5) ?

Sol. Since the centre is (2, 2)

We have $(x - 2)^2 + (y - 2)^2 = r^2$

This circle passes through the point (4, 5)

$(4 - 2)^2 + (5 - 2)^2 = r^2$

or $4 + 9 = r^2$

or $r^2 = 13$

The required equation of the circle is

$(x - 2)^2 + (y - 2)^2 = 13$

or $x^2 - 4x + 4 + y^2 - 4y + 4 = 13$

or $x^2 + y^2 - 4x - 4y - 5 = 0$

Example – 5

A circle has radius equal to 3 units and its centre lies on the line y = x – 1. Find the equation of the circle if it passes through (7, 3).

Sol. Let the centre of the circle be (α, β). It lies on the line $y = x - 1$

$\Rightarrow$ $\beta = \alpha - 1$. Hence the centre is $(\alpha, \alpha - 1)$.

$\Rightarrow$ Then equation of the circle is $(x - \alpha)^2 + (y - \alpha + 1)^2 = 9$.

It passes through (7, 3)

$\Rightarrow$ $(7 - \alpha)^2 + (4 - \alpha)^2 = 9$

$\Rightarrow$ $\alpha^2 - 11\alpha + 28 = 0$

$\Rightarrow$ $(\alpha - 4)(\alpha - 7) = 0$ $\Rightarrow \alpha = 4, 7$.

Hence the required equations are

$x^2 + y^2 - 8x - 6y + 16 = 0$ and

$x^2 + y^2 - 14x - 12y + 76 = 0$.

Example – 6

Find the equation of the circle that passes through the points (1, 0), (–1, 0) and (0, 1).

Sol. Let the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

It passes through (1, 0), (–1, 0) and (0, 1). Therefore, on substituting the coordinates of three points successively in equation (i), we get

$1 + 2g + c = 0$ ... (ii),

$1 - 2g + c = 0$ ... (iii),

$1 + 2f + c = 0$ ... (iv)

Subtracting (iii) from (ii), we get

$4g = 0 \Rightarrow g = 0$

Putting $g = 0$ in (ii), we obtain $c = -1$

Now putting $c = -1$ in (iv), we get $f = 0$

Substituting the values of g, f and c in (i), we obtain the equation of the required circle as $x^2 + y^2 = 1$

Example – 7

Find the equation of the circle which passes through the points (5, –8), (2, –9) and (2, 1). Find also the coordinates of its centre and radius.

Sol. Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

It passes through(5, –8), (2, –9) and (2, 1). Therefore,

$\therefore$ $89 + 10g - 16f + c = 0$ ... (ii)

$85 + 4g - 18f + c = 0$ ... (iii)

$5 + 4g + 2f + c = 0$ ... (iv)

Subtracting (iii) from (ii), we obtain

$4 + 6g + 2f = 0 \Rightarrow 2 + 3g + f = 0$ ... (v)

Subtracting (iv) from (iii), we get

$80 + 0\,g - 20f = 0 \Rightarrow f = 4$

Putting $f = 4$ in (v), we get $g = -2$

Putting $f = 4$, $g = -2$ in (iv), we get

$5 - 8 + 8 + c = 0 \Rightarrow c = -5$

Substituting the values of g, f, c in (i), we obtain the equation of the required circle as

$x^2 + y^2 - 4x + 8y - 5 = 0$

The coordinates of the centre are (–g, –f) i.e., (2, –4)

and, Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{4 + 16 + 5} = 5$

Example – 8

The straight line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the coordinate axes at A and B. Find the equation of the circle passing through O (0, 0), A and B

Sol. The straight line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the coordinate axes at A (a, 0) and B (0, b).

Let $x^2 + y^2 + 2gx + 2gy + c = 0$ be the circle passing through O, A and B. Then, ... (i)

$0 + 0 + c = 0$ ... (ii)

$a^2 + 2ga + c = 0$ ... (iii)

$b^2 + 2fb + c = 0$ ... (iv)

Solving (ii), (iii) and (iv), we obtain

$$g = -\frac{a}{2}, f = -\frac{b}{2} \text{ and } c = 0$$

Substituting these values in (i), we obtain the equation of the required circle as

$x^2 + y^2 - ax - by = 0$

Example – 9

Find the equation of the circle passing through (1, 0) and (0, 1) and having the smallest possible radius.

Sol. Let the equation of the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

This passes through A (1, 0) and B (0, 1).

$\therefore$ $1 + 2g + c = 0$ and, $1 + 2f + c = 0$

$$\Rightarrow \quad g = -\left(\frac{c+1}{2}\right) \text{ and, } f = -\left(\frac{c+1}{2}\right)$$

Let r be the radius of circle (i). Then,

$$r = \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \quad r = \sqrt{\left(\frac{c+1}{2}\right)^2 + \left(\frac{c+1}{2}\right)^2 - c}$$

$$\Rightarrow \quad r = \sqrt{\left(\frac{c^2+1}{2}\right)}$$

$\Rightarrow \quad r^2 = \frac{1}{2}(c^2+1)$

Clearly, r is minimum when c = 0 and the minimum value of r is $\frac{1}{\sqrt{2}}$.

For c = 0, we have

$g = -\frac{1}{2}$ and $f = -\frac{1}{2}$

Substituting the values of g, f and c in (i), we get

$x^2 + y^2 - x - y = 0$

as the equation of the required circle.

Example – 10

Show that the points (9, 1), (7, 9) (–2, 12) and (6, 10) are concyclic.

Sol. Let the equation of the circle passing through (9, 1), (7, 9) and (–2, 12) be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

Then, $82 + 18g + 2f + c = 0$ ... (ii)

$130 + 14g + 18f + c = 0$ ... (iii)

$148 - 4g + 24f + c = 0$ ... (iv)

Subtracting (ii) from (iii), we get

$48 - 4g + 16f = 0 \Rightarrow 12 - g + 4f = 0$ ... (v)

Subtracting (iii) from (iv), we get

$18 - 18g + 6f = 0 \Rightarrow 3 - 3g + f = 0$ ... (vi)

Solving (v) and (vi) as simultaneous linear equations in g and f, we get

$f = -3, g = 0$

Putting f = –3, g = 0 in (ii), we get

$82 + 0 - 6 + c = 0 \Rightarrow c = -76$

Substituting the values of g, f, c in (i), we get

$x^2 + y^2 - 6y - 76 = 0$

as the equation of the circle passing through (9, 1), (7, 9) and (–2, 12).

Clearly, point (6, 10) satisfies this equation. Hence, the given points are concyclic.

Example – 11

Find the equation of the circle which passes through the points (1, –2) and (4, –3) and has its centre on the line 3x+4y=7.

Sol. Let the equation of the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

It passes through (1, –2) and (4, –3) ... (ii)

$\therefore \quad 5 + 2g - 4f + c = 0$ ... (iii)

and, $25 + 8g - 6f + c = 0$

The centre (–g, –f) of (i) lies on 3x + 4y = 7. Therefore,

$-3g - 4f = 7$ ... (iv)

Subtracting (ii) from (iii), we get

$20 + 6g - 2f = 0 \Rightarrow 10 + 3g - f = 0$ ... (v)

Solving (iv) and (v) as simultaneous equations, we get

$g = -\frac{47}{15}$ and $f = \frac{3}{5}$

Substituting the values of g and f in (ii), we get

$5 - \frac{94}{15} - \frac{12}{5} + c = 0 \Rightarrow c = \frac{55}{15} = \frac{11}{3}$

Substituting the values of g, f and c in (i) we obtain the required equation of the circle as

$x^2 + y^2 - \frac{94}{15}x + \frac{6}{5}y + \frac{11}{3} = 0$

$\Rightarrow 15(x^2 + y^2) - 94x + 18y + 55 = 0$

Example – 12

Find the equation of the circle circumscribing the triangle formed by the lines x + y = 6, 2x + y = 4 and x + 2y = 5.

Sol. Let the equations of sides AB, BC and CA of Δ ABC are respectively

$x + y = 6$ ... (i) $\quad 2x + y = 4$ ... (ii)

and $x + 2y = 5$ ... (iii)

Solving (i) and (iii), (i) and (ii); (ii) and (iii) we get the coordinates of A, B and C. The coordinates A, B and C are (7, –1), (–2, 8) and (1, 2) respectively.

Let the equation of the circumcircle of ΔABC be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (iv)

It passes through A (7, –1), B (–2, 8) and C (1, 2). Therefore,

$50 + 14g - 2f + c = 0$ ... (v)

$68 - 4g + 16f + c = 0$ ... (vi)

$5 + 2g + 4f + c = 0$ ... (vii)

Subtracting (v) from (vi), we get

$18 - 18g + 18f = 0 \Rightarrow 1 - g + f = 0$ ... (viii)

Subtracting (v) from (vii), we get $-45 - 12g + 6f = 0$... (ix)

Solving (viii) and (ix), we get $g = -17/2$, $f = -19/2$.

Putting the values of g and f in (v), we get c = 50.

Substituting the values of g, f and c in (iv), the equation of the required circumcircle is

$x^2 + y^2 - 17x - 19y + 50 = 0$

Example – 13

Find the radius of the circle

$(x \cos \alpha + y \sin \alpha - a)^2 + (x \sin \alpha - y \cos \alpha - b)^2 = k^2$,

if α varies, the locus of its centre is again a circle. Also, find its centre and radius.

Sol. The given equation is

$(x \cos \alpha + y \sin \alpha - a)^2 + (x \sin \alpha - y \cos \alpha - b)^2 = k^2$

$\Rightarrow$ $x^2(\cos^2\alpha + \sin^2\alpha) + y^2(\sin^2\alpha + \cos^2\alpha) - 2(a \cos \alpha + b \sin \alpha)\, x - 2(a \sin \alpha - b \cos \alpha)\, y + a^2 + b^2 - k^2 = 0$

$\Rightarrow$ $x^2 + y^2 - 2x(a \cos \alpha + b \sin \alpha) - 2y(a \sin \alpha - b \cos \alpha) + a^2 + b^2 - k^2 = 0$

The coordinates of the centre of this circle are

$(a \cos \alpha + b \sin \alpha,\ a \sin \alpha - b \cos \alpha)$

and, Radius =

$$\sqrt{(a\cos\alpha + b\sin\alpha)^2 + (a\sin\alpha - b\cos\alpha)^2 - (a^2 + b^2 - k^2)}$$

$\Rightarrow$ Radius =

$$\sqrt{a^2(\cos^2\alpha + \sin^2\alpha) + b^2(\sin^2\alpha + \cos^2\alpha) - (a^2 + b^2 - k^2)}$$

$\Rightarrow$ Radius = $\sqrt{a^2 + b^2 - a^2 - b^2 + k^2} = k$

Let (p, q) be the coordinates of the centre of the given circle. Then,

$p = a \cos \alpha + b \sin \alpha$ and $q = a \sin \alpha - b \cos \alpha$

To find the locus of (p, q) we have to eliminate α. Squaring and adding these two, we get

$p^2 + q^2 = (a \cos \alpha + b \sin \alpha)^2 + (a \sin \alpha - b \cos \alpha)^2$

$\Rightarrow$ $p^2 + q^2 = a^2(\cos^2\alpha + \sin^2\alpha) + b^2(\sin^2\alpha + \cos^2\alpha)\ h^2 + k^2$

$\Rightarrow$ $p^2 + q^2 = a^2 + b^2$

Hence, the locus of (p, q) is $x^2 + y^2 = a^2 + b^2$

This is a circle having centre at (0, 0) and radius equal to $\sqrt{a^2 + b^2}$.

Example – 14

Find the area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Sol. Let ABC be an equilateral triangle inscribed in the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

Let O (–g, –f) be the centre of the circle. Then,

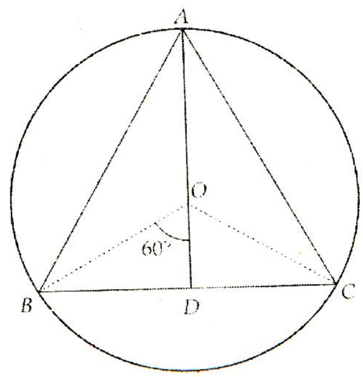
$$OA = OB = OC = \sqrt{g^2 + f^2 - c}$$

In Δ OBD, we have

$$\sin 60° = \frac{BD}{OB}$$

$\Rightarrow$ $BD = \frac{\sqrt{3}}{2}\sqrt{g^2 + f^2 - c}$

$\Rightarrow$ BC = 2 BD



$\Rightarrow$ $BC = \sqrt{3}\sqrt{g^2 + f^2 - c}$

$\therefore$ Area of Δ ABC = $\frac{\sqrt{3}}{4}(\text{Side})^2$

$\Rightarrow$ Area of Δ ABC = $\frac{\sqrt{3}}{4} \times 3\,(g^2 + f^2 - c)$ sq. units = $\frac{3\sqrt{3}}{4}$ $(g^2 + f^2 - c)$ sq. units.

Example – 15

Find the equation of the circle whose diameter is the line joining the points (–4, 3) and (12, –1). Find also the intercept made by it on y-axis.

Sol. Equation of circle having (–4, 3) and (12, –1) as the ends of a diameter is

$(x + 4)(x - 12) + (y - 3)(y + 1) = 0$

$\Rightarrow \quad x^2 + y^2 - 8x - 2y - 51 = 0 \qquad \ldots (1)$

Comparing (1) with standard equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

then $g = -4, f = -1, c = -51$

$\therefore$ Intercept on y-axis $= 2\sqrt{f^2 - c}$

$= 2\sqrt{1+51} = 4\sqrt{13}$.

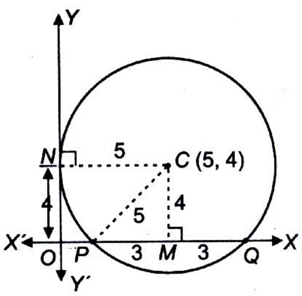
Example – 16

Find the equation of the circle which touches the axis of y at a distance of 4 units from the origin and cuts the intercept of 6 units from the axis of x.

Sol. $\because$ CM = NO = 4

In Δ PCM, $(PC)^2 = (3)^2 + (4)^2$

$\therefore$ PC = 5



radius of circle = 5

$\therefore$ NC = 5

centre of circle is (5, 4)

$\therefore$ Equation of circle, if centre in I quadrant

$(x - 5)^2 + (y - 4)^2 = 25$

If centre in II, III and IV quadrant, then equations are

$(x + 5)^2 + (y - 4)^2 = 25,$

$(x + 5)^2 + (y + 4)^2 = 25$

and $(x - 5)^2 + (y + 4)^2 = 25$

Hence, there are 4 circles which satisfy the given conditions. They are

$(x \pm 5)^2 + (y \pm 4)^2 = 25$

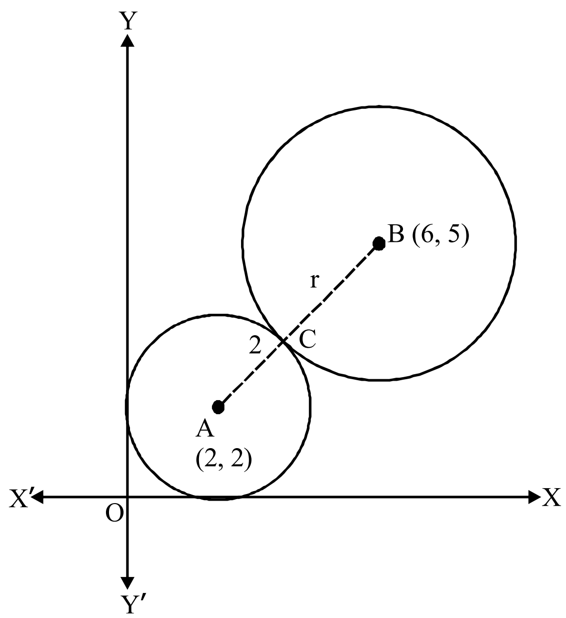
or $x^2 + y^2 \pm 10x \pm 8y + 16 = 0$

Example – 17

A circle of radius 2 lies in the first quadrant and touches both the axes of co-ordinates. Find the equation of the circle with centre at (6, 5) and touching the above circle externally.

Sol. Given, AC = 2 units

and $A \equiv (2, 2), B \equiv (6, 5)$



then $AB = \sqrt{(2-6)^2 + (2-5)^2} = \sqrt{16+9} = 5$

Since AC + CB = AB

$\therefore$ 2 + CB = 5

$\therefore$ CB = 3

Hence equation of required circle with centre at (6, 5) and radius 3 is

$(x - 6)^2 + (y - 5)^2 = 3^2$

or $x^2 + y^2 - 12x - 10y + 52 = 0$

Example – 18

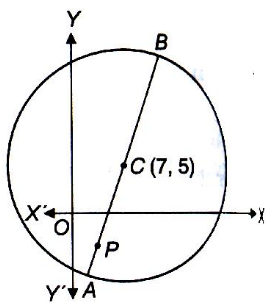
Find the shortest and largest distance from the point (2, –7) to the circle

$x^2 + y^2 - 14x - 10y - 151 = 0$

Sol. Let $S \equiv x^2 + y^2 - 14x - 10y - 151 = 0$

$\therefore$ $S_1 = (2)^2 + (-7)^2 - 14(2) - 10(-7) - 151$

$= -56 < 0$



$\therefore$ P (2, –7) inside the circle

radius of the circle, $r = \sqrt{(-7)^2 + (-5)^2 + 151} = 15$

$\because$ Centre of circle C $\equiv$ (7, 5)

$\therefore$ $CP = \sqrt{(7-2)^2 + (5+7)^2} = 13$

$\therefore$ Shortest distance = PA = r – CP = 15 – 13 = 2

and Largest distance = PB = r + CP = 15 + 13 = 28

Example – 19

Find the equation of the circle passing through (1, 1) and the points of inter-section of the circles

$x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$

Sol. The given circles are

$x^2 + y^2 + 13x - 3y = 0$ ... (1)

and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$

or $x^2 + y^2 + 2x - \frac{7}{2}y - \frac{25}{2} = 0$ ... (2)

Equation of any circle passing through the point of intersection of the circles (1) and (2) is

$$(x^2 + y^2 + 13x - 3y) + \lambda\left(x^2 + y^2 + 2x - \frac{7}{2}y - \frac{25}{2}\right) = 0 \quad ...(3)$$

Its passes through (1, 1) then

$$(1 + 1 + 13 - 3) + \lambda\left(1 + 1 + 2 - \frac{7}{2} - \frac{25}{2}\right) = 0$$

$\Rightarrow$ $12 + \lambda(-12) = 0$

$\therefore$ $\lambda = 1$

Substituting the value of λ in (3), the required equation is

$$x^2 + y^2 + 13x - 3y + x^2 + y^2 + 2x - \frac{7}{2}y - \frac{25}{2} = 0$$

$\Rightarrow$ $2x^2 + 2y^2 + 15x - \frac{13}{2}y - \frac{25}{2} = 0$

$\Rightarrow$ $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

Example – 20

Find the equation of the circle passing through the point of intersection of the circles $x^2 + y^2 - 6x + 2y + 4 = 0$, $x^2 + y^2 + 2x - 4y - 6 = 0$ and with its centre on the line $y = x$

Sol. Equation of any circle through the points of intersection of given circles is

$(x^2 + y^2 - 6x + 2y + 4) + \lambda(x^2 + y^2 + 2x - 4y - 6) = 0$

$\Rightarrow$ $x^2(1 + \lambda) + y^2(1 + \lambda) - 2x(3 - \lambda) + 2y(1 - 2\lambda) + (4 - 6\lambda) = 0$

or $$x^2 + y^2 - \frac{2x(3-\lambda)}{(1+\lambda)} + \frac{2y(1-2\lambda)}{(1+\lambda)} + \frac{(4-6\lambda)}{(1+\lambda)} = 0 \quad ... (1)$$

Its centre $\left\{\frac{3-\lambda}{1+\lambda}, \frac{2\lambda-1}{1+\lambda}\right\}$ lies on the line y = x

then $\frac{2\lambda-1}{1+\lambda} = \frac{3-\lambda}{1+\lambda}$

$\Rightarrow$ $\lambda \neq -1$

$\therefore$ $2\lambda - 1 = 3 - \lambda$

or $3\lambda = 4$

$\therefore$ $\lambda = 4/3$

$\therefore$ Substituting the value of $\lambda = 4/3$ in (1), we get the required equation is

$7x^2 + 7y^2 - 10x - 10y - 12 = 0$

Example – 21

Find the equation of the circle through points of intersection of the circle $x^2 + y^2 - 2x - 4y + 4 = 0$ and the line $x + 2y = 4$ which touches the line $x + 2y = 0$.

Sol. Equation of any circle through points of intersection of the given circle and the line is

$(x^2 + y^2 - 2x - 4y + 4) + \lambda (x + 2y - 4) = 0$

or $x^2 + y^2 + (\lambda - 2)x + (2\lambda - 4) y + 4 (1 - \lambda) = 0$ ... (1)

It will touch the line $x + 2y = 0$ if solution of equation (1) and $x = -2y$ be unique. Hence the roots of the equation

$(-2y)^2 + y^2 + (\lambda - 2)(-2y) + (2\lambda - 4) y + 4 (1 - \lambda) = 0$

or $5y^2 + 4 (1 - \lambda) = 0$

must be equal.

Then $0 - 4.5.4 (1 - \lambda) = 0$ or $1 - \lambda = 0$ or $\lambda = 1$

From (1), the required circle is $x^2 + y^2 - x - 2y = 0$

Example – 22

Find the co–ordinates of the point from which tang-ents are drawn to the circle $x^2 + y^2 - 6x - 4y + 3 = 0$ such that the mid point of its chord of contact is (1, 1).

Sol. Let the required point be $P(x_1, y_1)$. The equation of the chord of contact of P with respect to the given circle is

$xx_1 + yy_1 - 3(x + x_1) - 2(y + y_1) + 3 = 0.$ ...(1)

Then equation of the chord with mid–point (1, 1) is

$x + y - 3(x + 1) - 2(y + 1) + 3 = 1 + 1 - 6 - 4 + 3$

$\Rightarrow$ $2x + y = 3$.

Equating the ratios of the coefficients of x, y and the constant terms and solving for x_1, y_1 we get $x_1 = -1, y_1 = 0$

Example – 23

Find the equation of the circle described on the common chord of the circles $x^2 + y^2 - 4x - 5 = 0$ and $x^2 + y^2 + 8y + 7 = 0$ as diameter.

Sol. Equation of the common chord is $S_1 - S_2 = 0$

$\Rightarrow$ $x + 2y + 3 = 0$.

Equation of the circle through the two circles is $S_1 + \lambda S_2 = 0$

$$\Rightarrow x^2 + y^2 - \frac{4}{1+\lambda}x + \frac{8\lambda y}{1+\lambda} + \frac{7\lambda - 5}{1+\lambda} = 0.$$

Its centre $\left(\frac{2}{1+\lambda}, -\frac{4\lambda}{1+\lambda}\right)$ lies on $x + 2y + 3 = 0$

$$\Rightarrow \frac{2}{1+\lambda} - \frac{8\lambda}{1+\lambda} + 3 = 0 \Rightarrow 2 - 8\lambda + 3 + 3\lambda = 0 \Rightarrow \lambda = 1.$$

Hence the required circle is $x^2 + y^2 - 2x + 4y + 1 = 0$.

Example – 24

Find all the common tangents to the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$.

Sol. The centres and the radii of the circles are

C_1 (1, 3) and $r_1 = \sqrt{1+9-9} = 1$, $C_2(-3, 1)$ and $r_2 = \sqrt{9+1-1} = 3$, $C_1C_2 = \sqrt{20}$, $r_1 + r_2 = 4 = \sqrt{16}$ and $C_1C_2 > r_1 + r_2$.

Since the circles are non–intersecting. Thus there will be four common tangents.

Transverse common tangents are tangents drawn from the point P which divides C_1C_2 intemally in the ratio of radii 1 : 3.

Direct common tangents are tangents drawn from the poin Q which divides C_1 C_2 internally in the ratio of radii 1 : 3.

Coordinates of P are

$$\left(\frac{1(-3)+3.1}{1+3}, \frac{1.1+3.3}{1+3}\right) \text{ i.e. } \left(0, \frac{5}{2}\right)$$

and coordinates of Q are (3, 4).

Transverse tangents are tangents through the point $\left(0, \frac{5}{2}\right)$.

Any line through $\left(0, \frac{5}{2}\right)$ is

$$y - \frac{5}{2} = mx \quad ...(1)$$

$$\Rightarrow \quad mx - y + \frac{5}{2} = 0$$

Applying the usual condition of tangency to any of the circle, we get

$$\frac{m.1 - 3 + \frac{5}{2}}{\sqrt{(m^2+1)}} = 1 \Rightarrow \left(m - \frac{1}{2}\right)^2 \quad m^2 + 1$$

$\Rightarrow \quad -m - \frac{3}{4} = 0$ or $0.m^2 - m - \frac{3}{4} = 0$

$\Rightarrow \quad m = -\frac{3}{4}$ and ∞ as coefficient of m^2 is zero.

Therefore from (1),

$$\frac{y - \frac{5}{2}}{x} = m = \infty \text{ and } -\frac{3}{4}$$

$\Rightarrow$ x = 0 is a tangent and

3x + 4y – 10 = 0 is another tangent.

Direct tangents are tangents drawn from the point Q(3, 4).

Now proceeding as for transverse tangents their equations are y = 4, 4x – 3y = 0.

Example – 25

A tangent is drawn to each of the circles $x^2 + y^2 = a^2$; $x^2 + y^2 = b^2$. Show that if the two tangents are perpendicular to each other, the locus of their point of intersection is a circle concentric with the given circles.

Sol. Let $P \equiv (x_1, y_1)$ be the point of intersection of the tangents PA and PB where A, B are points of contact with the two circles respectively.

As PA perpendicular to PB. the corresponding radii OA and OB are also perpendicular.

Let $\angle AOX = \theta$

$\Rightarrow \angle BOX = \theta + 90°$

Using the parametric form of the circles we can take :

$A \equiv (a\cos\theta, a\sin\theta)$

$B \equiv [b\cos(\theta + 90°), b\sin(\theta + 90°)]$

$B \equiv (-b\sin\theta, b\cos\theta)$

The equation of PA is : $x(a\cos\theta) + y(a\sin\theta) = a^2$

$\Rightarrow x\cos\theta + y\sin\theta = a$

The equation of PB is :

$x(-b\sin\theta) + y(b\cos\theta) = b^2$

$\Rightarrow y\cos\theta - x\sin\theta = b$

$\Rightarrow p \equiv (x_1, y_1)$ lies on PA and PB both.

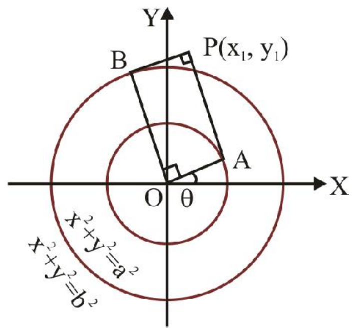
$\Rightarrow x_1\cos\theta + y_1\sin\theta = a$

and $y_1\cos\theta - x_1\sin\theta = b$

As θ is changing quantity (different for different positions of P), we will eliminate.

Squaring and adding, we get : $x_1^2 + y_1^2 = a^2 + b^2$

$\Rightarrow$ the locus of P is $x^2 + y^2 = a^2 + b^2$ which is concentric with the given circles.

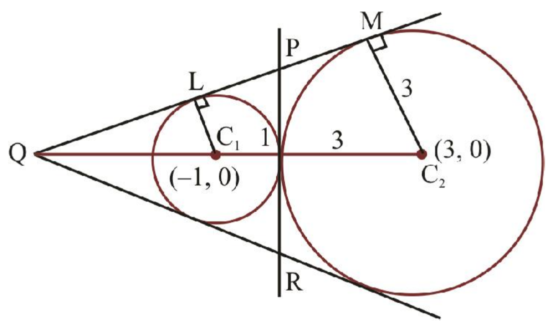


Example – 26

Show that the common tangents to the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ form are equilateral triangle.

Sol. For the circle $x^2 + y^2 - 6x = 0$ centre $C_1 \equiv (3, 0)$ and $r_1 = 3$

And for the circle $x^2 + y^2 + 2x = 0$ centre $C_2 \equiv (-1, 0)$ and $r_2 = 1$



Now $C_1C_2 = \sqrt{[3-(-1)]^2 + 0} = 4$ and $r_1 + r_2 = 4$

$\therefore \quad C_1C_2 = r_1 + r_2$

Hence the two circles touch each other externally, therefore, there will be three common tangents.

Equation of the common tangent at the point of contact is $S_1 - S_2 = 0$

$\Rightarrow$ $(x^2 + y^2 - 6x) - (x^2 + y^2 + 2x) = 0$

$\Rightarrow$ $-8x = 0$

$\therefore$ $x = 0$

Let the point of intersection of direct common tangent be Q (h, k). Then Q divides $C_1 C_2$ externally in the ratio of 1 : 3.

$\therefore$ $h = \dfrac{1.(3) - 3.(-1)}{1-3} = -3$ and $k = 0$

$\Rightarrow$ $Q \equiv (-3, 0)$

Let the equation of direct common tangent be :

$y - 0 = m(x + 3)$ or $mx - y + 3m = 0$ ...(i)

Apply condition of tangency with second circle to get :

$\Rightarrow$ $\dfrac{|-m - 0 + 3m|}{\sqrt{m^2+1}} = 1$ $\Rightarrow$ $4m^2 = m^2 + 1$

$\Rightarrow$ $3m^2 = 1$ $\Rightarrow$ $m = \pm\dfrac{1}{\sqrt{3}}$

From(i), direct common tangents are :

$$y = \frac{x}{\sqrt{3}} + \sqrt{3} \text{ and } y = -\frac{x}{\sqrt{3}} - \sqrt{3}$$

Hence all common tangents are

$x = 0$ ...(ii)

$y = \dfrac{x}{\sqrt{3}} + \sqrt{3}$ ...(iii)

and $y = -\dfrac{x}{\sqrt{3}} - \sqrt{3}$ ....(iv)

Let P, Q, R be the point of intersection of these lines.

On solving, we get :

$P \equiv (0, \sqrt{3})$; $Q \equiv (-3, 0)$ and $R \equiv (0, -\sqrt{3})$

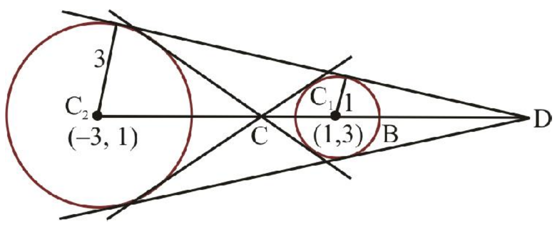
Now $PQ = QR = RP = 2\sqrt{3}$

Hence ΔPQR is an equilateral triangle. Thus common tangents form an equilateral triangle.

Example – 27

Find equation of all the common tangents to the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

Sol. For the circle $x^2 + y^2 - 2x - 6y + 9 = 0$ centre $C_1 \equiv (1, 3)$ and $r_1 = 1$ and for the circle $x^2 + y^2 + 6x - 2y + 1 = 0$ centre $C_2 \equiv (-3, 1)$ and $r_2 = 3$



Now $C_1 C_2 = \sqrt{(16+4)} = 2\sqrt{5}$ and $r_1 + r_2 = 4$

$\Rightarrow$ $C_1 C_2 > r_1 + r_2$

Hence the circles do not intersect to each other.

So there will be four common tangents between them. Applying the working rule given in previous section, we get :

Step 1 : The direct common tangents meet AB produced at D, then point D will divide. $C_2 C_1$ in the ratio 3 : 1 (externally). And the point C divide $C_2 C_1$ in the ratio 3 : 1 (internally)

$\Rightarrow$ Co–ordinates of D are $\left(\dfrac{3(1) - 1(-3)}{3-1}, \dfrac{3(3) - 1(1)}{3-1}\right)$

or (3, 4)

and co–ordinates of C are $\left(\dfrac{3(1) + 1(-3)}{3+1}, \dfrac{3(3) + 1(1)}{3+1}\right)$

or (0, 5/2)

Step 2 : (Equation of transverse common tangent)

Equation of line through $C \equiv (0, 5/2)$ is $y - 5/2 = mx$

or $mx - y + 5/2 = 0$ ...(i)

Setp (i) Apply the usual condition of tangency using first circle to get :

$\therefore$ $\dfrac{m \cdot 1 - 3 + 5/2}{\sqrt{m^2+1}} = \pm 1$

$\Rightarrow$ $m^2 + \dfrac{1}{4} - m = m^2 + 1$

$\Rightarrow$ $0.\,m^2 - m - \frac{3}{4} = 0.$

$\Rightarrow$ $m = \infty$ and $m = -3/4$

Step (ii) Hence equations of transverse tangents are

$x - 0$ and $3x + 4y - 10 = 0$.

Step 3 : (Equation of direct common tangent)

Equation of any line through (3, 4) is $y - 4 = m(x - 3)$

$\Rightarrow$ $mx - y + 4 - 3m = 0$

Step (iii) Apply the usual condition of tangency using first circle to get :

$$\frac{m - 3 + 4 - 3m}{\sqrt{m^2 + 1}} = \pm 1$$

$\Rightarrow$ $(-2m + 1)^2 = m^2 + 1$

$\Rightarrow$ $3m^2 - 4m = 0$

$\therefore$ $m = 0,\ m = 4/3$

Step (iv) Equations of Direct common tangents are

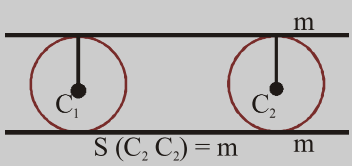
$y = 4$ and $4x - 3y = 0$.

Note...

When two S_1 and S_2 are of same radii then

(i) For Indirect tangents the approach is same as given above.

(ii) For Direct tangent we can use following approach.



Let the slope of direct common tangent be m from figure we have slope of C_1C_2 = m then equation of direct common tangent be $y = mx + c$, where 'c' is the unknown variable parameter. Apply condition tangency to find the value of c.

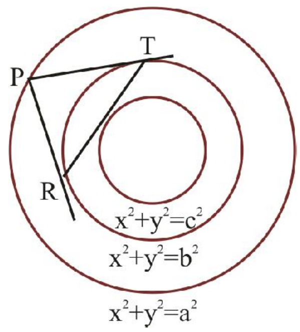
Example – 28

The chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in G.P.

Sol. Let P $(a\cos\theta, a\sin\theta)$ be a point on the circle $x^2 + y^2 = a^2$.

Then equation of chord of contact of tangents drawn from

P $(a\cos\theta, a\sin\theta)$ to the circle $x^2 + y^2 = b^2$ is



$ax\cos\theta + ay\sin\theta = b^2$ ...(i)

This touches the circle $x^2 + y^2 = c^2$ ...(ii)

$\therefore$ Length of perpendicular from (0, 0) to (i) = radius of (ii)

$\therefore$ $$\frac{|0 + 0 - b^2|}{\sqrt{(a^2\cos^2\theta + a^2\sin^2\theta)}} = c$$

$\Rightarrow$ $b^2 = ac$ $\Rightarrow$ a, b, c are in G.P.

Example – 29

Find the condition that chord of contact of any external point (h, k) to the circle $x^2 + y^2 = a^2$ should subtend right angle at the centre of the circle.

Sol. Equation of chord of contact AB is

$hx + ky = a^2$ ...(i)

Now we will find equation of pair of lines OA and OB by homogenising $x^2 + y^2 = a^2$ with equation of chord of contact AB. Homogenising means making degree of every term same.

From (i) $hx + ky = a^2 \cdot 1$

$\Rightarrow$ $$\frac{hx + ky}{a^2} = 1$$ ...(ii)

To homogenise circle $x^2 + y^2 = a^2$ with equation of chord of contact AB, we can write circle as $x^2 + y^2 = a^2 . 1^2$ and replace value of (i) from (ii).

$\Rightarrow \quad x^2 + y^2 = a^2\left(\frac{hx+ky}{a^2}\right)^2$

$\Rightarrow \quad a^2(x^2 + y^2) = (hx + ky)^2$

$\Rightarrow \quad x^2(a^2 - h^2) - 2hkxy + y^2(a^2 - k^2) = 0$

The equation represents pair of straight lines.

It is given that AB substends angle $\pi/2$ at O. It implies

$\angle AOB = \frac{\pi}{2}$

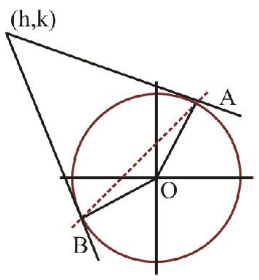
$\Rightarrow$ angle between pair of lines OA and OB is $\pi/2$.

Angle between pair of lines is 90° if

coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad a^2 - h^2 + a^2 - k^2 = 0$

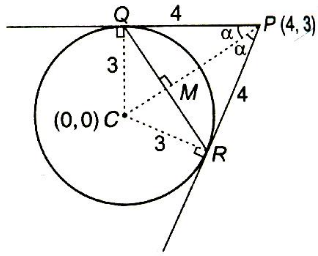
$\Rightarrow \quad h^2 + k^2 = 2a^2$



Example – 30

Show that the area of the triangle formed by tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line segment joining their points of contact is $7\frac{17}{25}$ square units in length.

Sol. Since $PQ = PR = \sqrt{4^2 + 3^2 - 9} = 4$ units



$\therefore \quad \angle CPQ = \angle CPR = \alpha$ (let)

$\therefore \quad PC = \sqrt{(4-0)^2 + (3-0)^2} = 5$ units

$\therefore$ In Δ PQC, $\tan\alpha = \frac{3}{4}$,

$\therefore \quad \sin\alpha = \frac{3}{5}$

and $\quad \cos\alpha = \frac{4}{5}$

In Δ PMQ, $\cos\alpha = \frac{PM}{4} = \frac{4}{5}$

$\therefore \quad PM = \frac{16}{5}$

and $\quad \sin\alpha = \frac{QM}{4} = \frac{3}{5}$

$\therefore \quad QM = \frac{12}{5}$

$\therefore \quad$ Area of $\Delta PQR = \frac{1}{2}.QR.PM$

$= \frac{1}{2}(2QM).PM = (QM)(PM)$

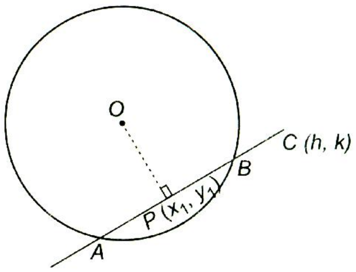
$= \left(\frac{12}{5}\right)\left(\frac{16}{5}\right) = \frac{192}{25}$

$= 7\frac{17}{25}$ sq.units

Example – 31

Through a fixed point (h, k), secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of mid point of the portions of secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

Sol. Let P (x_1, y_1) be the middle point of any chord AB, which passes through the point C (h, k).



Equation of chord AB is $T = S_1$

$\therefore$ $xx_1 + yy_1 - r^2 = x_1^2 + y_1^2 - r^2$

or $x_1^2 + y_1^2 = xx_1 + yy_1$

But since AB passes through C (h, k) then

$x_1^2 + y_1^2 = hx_1 + ky_1$

$\therefore$ Locus of P (x_1, y_1) is $x^2 + y^2 = hx + ky$

Example – 32

Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other, if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Sol. Given circles are

$x^2 + y^2 + 2ax + c^2 = 0$ ... (1)

and $x^2 + y^2 + 2by + c^2 = 0$ ... (2)

Let C_1 and C_2 be the centres of circles (1) and (2), respectively and r_1 and r_2 be their radii, then

$C_1 = (-a, 0)$, $C_2 = (0, -b)$, $r_1 = \sqrt{a^2 - c^2}, r_2 = \sqrt{b^2 - c^2}$

Here we do not find the two circles touch each other internally or externally.

For touch, $|C_1 C_2| = |r_1 \pm r_2|$

or $\sqrt{(a^2 + b^2)} = |\sqrt{(a^2 - c^2)} \pm \sqrt{(b^2 - c^2)}\,|$

On squaring $a^2 + b^2 = a^2 - c^2 + b^2 - c^2 \pm 2\sqrt{(a^2 - c^2)}\sqrt{(b^2 - c^2)}$

or $c^2 = \pm\sqrt{a^2b^2 - c^2(a^2 + b^2) + c^4}$

Again squaring, $c^4 = a^2b^2 - c^2(a^2 + b^2) + c^4$

or $c^2(a^2 + b^2) = a^2b^2$

or $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Example – 33

Find the equation of the circle which cuts the circle $x^2 + y^2 + 5x + 7y - 4 = 0$ orthogonally, has its centre on the line x = 2 and passes through the point (4, –1).

Sol. Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (1)

Since (4, –1) lie on (1) then

$17 + 8g - 2f + c = 0$ ... (2)

Centre of (1) is (–g, –f)

Since centre lie on x = 2

then $-g = 2$

$\therefore$ $g = -2$ ... (3)

From (2), $1 - 2f + c = 0$ ... (4)

and given circle is $x^2 + y^2 + 5x + 7y - 4 = 0$ ... (5)

Given the circles (1) and (5) cut each other orthogonally,

$\therefore$ $2g \times \frac{5}{2} + 2f \times \frac{7}{2} = c - 4$

or $5g + 7f = c - 4$

$-10 + 7f = c - 4$ (from (3))

or $-6 + 7f - c = 0$ ... (6)

Solving (4) and (6), we get

f = 1 and c = 1

Substituting the values of g, f, c in (1), we get

$x^2 + y^2 - 4x + 2y + 1 = 0$

Example – 34

If two tangents are drawn from a point on the circle $x^2 + y^2 = 50$ to the circle $x^2 + y^2 = 25$ then find the angle between the tangents.

Sol. $\because$ $x^2 + y^2 = 50$ is the director circle of $x^2 + y^2 = 25$

Hence angle between tangents = 90°

Example – 35

Examine if the two circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ touch each other externally or internally.

Sol. Given circles are

$$x^2 + y^2 - 2x - 4y = 0 \quad \text{... (1)}$$

and $$x^2 + y^2 - 8y - 4 = 0 \quad \text{... (2)}$$

Let centres and radii of circles (1) and (2) are represented by C_1, r_1 and C_2, r_2 respectively.

$\therefore$ $C_1 \equiv (1, 2), r_1 = \sqrt{1+4}$

or $r_1 = \sqrt{5}$

and $C_2 \equiv (0, 4), r_2 = \sqrt{0+16+4}$

or $r_2 = 2\sqrt{5}$

Now $C_1C_2 = \sqrt{(1-0)^2 + (2-4)^2}$

$C_1C_2 = \sqrt{5} = r_2 - r_1$

Hence the two circles touch each other internally.

Example – 36

Prove that the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other, if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

Sol. Given circles are

$$x^2 + y^2 + 2ax + c^2 = 0 \quad \text{... (1)}$$

and $$x^2 + y^2 + 2by + c^2 = 0 \quad \text{... (2)}$$

Let C_1 and C_2 be the centres of circles (1) and (2), respectively and r_1 and r_2 be their radii, then

$C_1 = (-a, 0)$, $C_2 = (0, -b)$, $r_1 = \sqrt{a^2 - c^2}$, $r_2 = \sqrt{b^2 - c^2}$

Here we do not find the two circles touch each other internally or externally.

For touch, $|C_1C_2| = |r_1 \pm r_2|$

or $\sqrt{(a^2 + b^2)} = |\sqrt{(a^2 - c^2)} \pm \sqrt{(b^2 - c^2)}|$

On squaring $a^2 + b^2 = a^2 - c^2 + b^2 - c^2 \pm 2\sqrt{(a^2 - c^2)}\sqrt{(b^2 - c^2)}$

or $c^2 = \pm\sqrt{a^2b^2 - c^2(a^2 + b^2) + c^4}$

Again squaring, $c^4 = a^2b^2 - c^2(a^2 + b^2) + c^4$

or $c^2(a^2 + b^2) = a^2b^2$

or $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Example – 37

Find the equation of the circle passing through the point of intersection of the circles $x^2 + y^2 - 6x + 2y + 4 = 0$, $x^2 + y^2 + 2x - 4y - 6 = 0$ and with its centre on the line $y = x$.

Sol. Equation of any circle through the points of intersection of given circles is

$(x^2 + y^2 - 6x + 2y + 4) + \lambda(x^2 + y^2 + 2x - 4y - 6) = 0$

$\Rightarrow$ $x^2(1 + \lambda) + y^2(1 + \lambda) - 2x(3 - \lambda) + 2y(1 - 2\lambda) + (4 - 6\lambda) = 0$

or $$x^2 + y^2 - \frac{2x(3-\lambda)}{(1+\lambda)} + \frac{2y(1-2\lambda)}{(1+\lambda)} + \frac{(4-6\lambda)}{(1+\lambda)} = 0 \quad \text{... (1)}$$

Its centre $\left\{\frac{3-\lambda}{1+\lambda}, \frac{2\lambda-1}{1+\lambda}\right\}$ lies on the line $y = x$

then $\frac{2\lambda-1}{1+\lambda} = \frac{3-\lambda}{1+\lambda}$

$\Rightarrow$ $\lambda \neq -1$

$\therefore$ $2\lambda - 1 = 3 - \lambda$

or $3\lambda = 4$

$\therefore$ $\lambda = 4/3$

$\therefore$ Substituting the value of $\lambda = 4/3$ in (1), we get the required equation is

$7x^2 + 7y^2 - 10x - 10y - 12 = 0$

Example – 38

Find the equations of the tangents to the circle $x^2 + y^2 = 16$ drawn from the point (1, 4).

Sol. Given circle is

$$x^2 + y^2 = 16 \quad \text{... (1)}$$

Any tangent of (1) in terms of slope is

$$y = mx + 4\sqrt{(1+m^2)} \quad \text{... (2)}$$

which passes through (1, 4)

then $4 = m + 4\sqrt{(1+m^2)}$

$\Rightarrow$ $(4-m)^2 = 16(1 + m^2)$

$\Rightarrow$ $15m^2 + 8m = 0 \therefore m = 0, -\frac{8}{15}$

From (2), equations of tangents drawn from (1, 4) are

$$y = 4 \text{ and } y = -\frac{8}{15}x + 4\sqrt{1+\frac{64}{225}}$$

or $8x + 15y = 68$ respectively.

Example – 39

Find the length of tangents drawn from the point (3, –4) to the circle $2x^2 + 2y^2 - 7x - 9y - 13 = 0$.

Sol. The equation of the given circle is

$2x^2 + 2y^2 - 7x - 9y - 13 = 0$

Re-writing the given equation of the circle in standard form

i.e., $x^2 + y^2 - \frac{7}{2}x - \frac{9}{2}y - \frac{13}{2} = 0$

Let $S = x^2 + y^2 - \frac{7}{2}x - \frac{9}{2}y - \frac{13}{2}$

$\therefore$ $S_1 = (3)^2 + (-4)^2 - \frac{7}{2}\times 3 - \frac{9}{2}\times(-4) - \frac{13}{2}$

$= 25 - \frac{21}{2} + 18 - \frac{13}{2}$

$= 43 - 17 = 26$

$\therefore$ Length of tangent $= \sqrt{S_1} = \sqrt{26}$

Example – 40

If the length of tangent from (f, g) to the circle $x^2 + y^2 = 6$ be twice the length of the tangent from (f, g) to circle $x^2 + y^2 + 3x + 3y = 0$ then will $f^2 + g^2 + 4f + 4g + 2 = 0$?

Sol. According to the question

$$\sqrt{(g^2 + f^2 - 6)} = 2\sqrt{(f^2 + g^2 + 3f + 3g)}$$

On squaring $g^2 + f^2 - 6 = 4f^2 + 4g^2 + 12f + 12g$

or $3f^2 + 3g^2 + 12f + 12g + 6 = 0$

or $f^2 + g^2 + 4f + 4g + 2 = 0$

which is true $\therefore$ yes.

Example – 41

Find the power of point (2, 4) with respect to the circle

$$x^2 + y^2 - 6x + 4y - 8 = 0$$

Sol. The power of the point (2, 4) with respect to the circle

$$x^2 + y^2 - 6x + 4y - 8 = 0 \text{ is } (\sqrt{S_1})^2 \text{ or } S_1$$

where $S = x^2 + y^2 - 6x + 4y - 8$

$\therefore$ $S_1 = (2)^2 + (4)^2 - 6 \times 2 + 4 \times 4 - 8$

$= 4 + 16 - 12 + 16 - 8 = 16$

[Hence (2, 4) is outside from the circle $x^2 + y^2 - 6x + 4y - 8 = 0$]

Example – 42

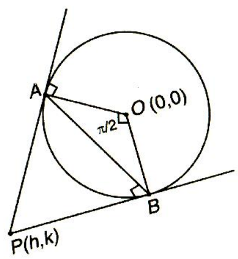
Find the condition that chord of contact of any external point (h, k) to the circle $x^2 + y^2 = a^2$ should subtend right angle at the centre of the circle.

Sol. Equation of chord of contact AB is

$$hx + ky = a^2 \quad \text{... (1)}$$

For equation of pair of tangents of OA and OB, make homogeneous $x^2 + y^2 = a^2$ with the help of $hx + ky = a^2$ or

$$\frac{hx + ky}{a^2} = 1$$



then $x^2 + y^2 = a^2\left(\frac{hx+ky}{a^2}\right)^2$

or $a^2(x^2 + y^2) = (hx + ky)^2$

or $x^2(a^2 - h^2) - 2hkxy + y^2(a^2 - k^2) = 0$

but $\angle AOB = \frac{\pi}{2}$

∴ Coefficient of x^2 + coefficient of $y^2 = 0$

⇒ $a^2 - h^2 + a^2 - k^2 = 0$

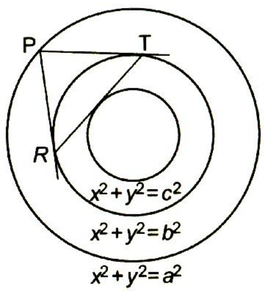
or $h^2 + k^2 = 2a^2$

Example – 43

The chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in GP.

Sol. Let P (a cos θ, a sin θ) be a point on the circle $x^2 + y^2 = a^2$.

Then equation of chord of contact of tangents drawn from P (a cos θ, a sin θ) to the circle $x^2 + y^2 = b^2$ is



$ax\cos\theta + ay\sin\theta = b^2$ ... (1)

This touches the circle $x^2 + y^2 = c^2$ ... (2)

∴ Length of perpendicular from (0, 0) to (1) = radius of (2)

∴ $\frac{|0+0-b^2|}{\sqrt{(a^2\cos^2\theta + a^2\sin^2\theta)}} = c$

or $b^2 = ac \Rightarrow$ a, b, c are in G.P.

Example – 44

Find the equation of the chord of $x^2 + y^2 - 6x + 10y - 9 = 0$ which is bisected at (–2, 4).

Sol. The equation of the required chord is

$-2x + 4y - 3(x - 2) + 5(y + 4) - 9 = 4 + 16 + 12 + 40 - 9$

⇒ $-5x + 9y - 46 = 0$

or $5x - 9y + 46 = 0$

Example – 45

Find the equations of the tangents from the point A(3, 2) to the circle $x^2 + y^2 + 4x + 6y + 8 = 0$.

Sol. Combined equation of the pair of tangents drawn from A(3, 2) to the given circle $x^2 + y^2 + 4x + 6y + 8 = 0$ can be written in the usual notation.

$T^2 = SS_1$ namely

⇒ $[3x + 2y + 2(x + 3) + 3(y + 2) + 8]^2$

$= [x^2 + y^2 + 4x + 6y + 8]\,[9 + 4 + 12 + 12 + 8]$

⇒ $(5x + 5y + 20)^2 = 45(x^2 + y^2 + 4x + 6y + 8)$

⇒ $5(x + y + 4)^2 = 9(x^2 + y^2 + 4x + 6y + 8)$

⇒ $5(x^2 + y^2 + 2xy + 8x + 8y + 16) = 9(x^2 + y^2 + 4x + 6y + 8)$

⇒ $4x^2 + 4y^2 - 10xy - 4x + 14y - 8 = 0$

or $2x^2 + 2y^2 - 5xy - 2x + 7y - 4 = 0$

or $(2x - y - 4)(x - 2y + 1) = 0$

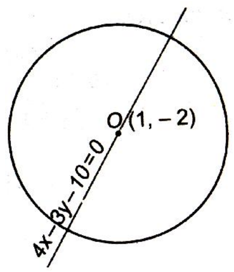
Hence the required tangents to the circle from A (3, 2) are

$2x - y - 4 = 0$ and $x - 2y + 1 = 0$

Example – 46

Find the length of the intercept on the straight line $4x - 3y - 10 = 0$ by the circle $x^2 + y^2 - 2x + 4y - 20 = 0$.

Sol. Centre and radius of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are $(1, -2)$ and $\sqrt{1+4+20} = 5$ respectively.



Let OM be the perpendicular from O on the line $4x - 3y - 10 = 0$

then $OM = \dfrac{|4\times1-3\times(-2)-10|}{\sqrt{4^2+(-3)^2}} = 0$

Hence line $4x - 3y - 10 = 0$ passes through the centre of the circle.

hence intercepted length = drameter of the circle

$= 2 \times 5 = 10$

Example – 47

Prove that the tangents to the circle $x^2 + y^2 = 25$ at $(3, 4)$ and $(4, -3)$ are perpendicular to each other.

Sol. The equations of tangents to $x^2 + y^2 = 25$ at $(3, 4)$ and $(4,-3)$ are

$3x + 4y = 25$ ... (1)

and $4x - 3y = 25$ ... (2)

respectively,

Now slope of (1) $= -\dfrac{3}{4} = m_1$ (say)

and slope of (2) $= \dfrac{4}{3} = m_2$ (say)

Clearly, $m_1 m_2 = -1$

Hence (1) and (2) are perpendicular to each other.

Example – 48

Show that the circles $x^2 + y^2 - 4x + 6y + 8 = 0$ and $x^2 + y^2 - 10x - 6y + 14 = 0$ touch at $(3, -1)$.

Sol. Equation of tangent at $(3, -1)$ of the circle $x^2 + y^2 - 4x + 6y + 8 = 0$ is

$3x + (-1)\,y - 2\,(x + 3) + 3\,(y - 1) + 8 = 0$

or $x + 2y - 1 = 0$ ... (1)

and equation of tangent at $(3, -1)$ of the circle $x^2 + y^2 - 10x - 6y + 14 = 0$ is

$3\,.\,x + (-1)\,.\,y - 5\,(x + 3) - 3\,(y - 1) + 14 = 0$

or $-2x - 4y + 2 = 0$

or $x + 2y - 1 = 0$ ... (2)

which is the same as (1).

Hence the given circles touch at $(3, -1)$.

Example – 49

Find the equations of the tangents to the circle $x^2 + y^2 = 9$, which

(i) are parallel to the line $3x + 4y - 5 = 0$

(ii) are perpendicular to the line $2x + 3y + 7 = 0$

(iii) make an angle of 60° with the x-axis

Sol. (i) Slope of $3x + 4y - 5 = 0$ is $-\dfrac{3}{4}$

Let $m = -\frac{3}{4}$

and equation of circle is $x^2 + y^2 = 9$

$\therefore$ Equations of tangents

$$y = -\frac{3}{4}x \pm 3\sqrt{\left(1 + \left(-\frac{3}{4}\right)^2\right)}$$

$\Rightarrow$ $4y = -3x \pm 15$

or $3x + 4y \pm 15 = 0$

(ii) slope of $2x + 3y + 7 = 0$ is $-\frac{2}{3}$

$\therefore$ Slope of perpendicular to $2x + 3y + 7 = 0$ is $\frac{3}{2} = m$ (say)

and given circle is $x^2 + y^2 = 9$

$\therefore$ Equations of tangents perpendicular to $2x + 3y + 7 = 0$ is

$$y = \frac{3}{2}x \pm 3\sqrt{1 + \left(\frac{3}{2}\right)^2}$$

$\Rightarrow$ $2y = 3x \pm 3\sqrt{13}$

or $3x - 2y \pm 3\sqrt{13} = 0$

(iii) Since tangent make an angle 60° with the x-axis

$\therefore$ $m = \tan 60° = \sqrt{3}$

and given circle $x^2 + y^2 = 9$

$\therefore$ Equation of tangents $y = \sqrt{3}x \pm 3\sqrt{1 + (\sqrt{3})^2}$

or $\sqrt{3}x - y \pm 6 = 0$

Example – 50

Show that the line $3x - 4y = 1$ touches the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Find the co-ordinates of the point of contact.

Sol. The centre and radius of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$ are $(1, -2)$ and $\sqrt{(-1) + (2)^2 - 1} = 2$ respectively.

Since length of perpendicular from centre $(1, -2)$ on $3x - 4y = 1$ is

$$\frac{|3 \times 1 - 4 \times (-2) - 1|}{\sqrt{(3)^2 + (-4)^2}} = \frac{10}{5}$$

$= 2 =$ radius of the circle

Hence $3x - 4y = 1$ touches the circle

$x^2 + y^2 - 2x + 4y + 1 = 0$

Second part : Let point of contact is (x_1, y_1) then tangent at (x_1, y_1) on $x^2 + y^2 - 2x + 4y + 1 = 0$ is

$xx_1 + yy_1 - (x + x_1) + 2(y + y_1) + 1 = 0$

$\Rightarrow$ $x(x_1 - 1) + y(y_1 + 2) - x_1 + 2y_1 + 1 = 0$ ... (1)

and given line $3x - 4y - 1 = 0$ ... (2)

Since (1) and (2) are identical, then comaring (1) and (2), we get

$$\frac{x_1 - 1}{3} = \frac{y_1 + 2}{-4} = \frac{-x_1 + 2y_1 + 1}{-1}$$

or $x_1 = -\frac{1}{5}$ and $y_1 = -\frac{2}{5}$

$\therefore$ Point of contact is $\left(-\frac{1}{5}, -\frac{2}{5}\right)$.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Basic

1. Centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$ is –

(a) (–1, 2) (b) (1, –2)
(c) (1, 2) (d) (–1, –2)

2. If (6, –3) is the one extremity of diameter to the circle $x^2 + y^2 - 3x + 8y - 3 = 0$ then its other extremity is–

(a) $\left(\frac{3}{2}, -4\right)$ (b) (–3, –5)
(c) (3, –5) (d) (3, 5)

3. If $y = 2x + K$ is a diameter to the circle $2(x^2 + y^2) + 3x + 4y - 1 = 0$, then K equals –

(a) 0 (b) 1
(c) 2 (d) 1/2

4. The line joining (5, 0) to (10 cosθ, 10 sinθ) is divided internally in the ratio 2 : 3 at P. If θ varies then the locus of P is :

(a) a pair of straight lines (b) a circle
(c) a straight line (d) none of these

5. The number of circles of a given radius which touch both the axes is

(a) 1 (b) 2
(c) 3 (d) 4

Condition for General Equation being a Circle

6. If the equation $\frac{\lambda(x+1)^2}{3} + \frac{(y+2)^2}{4} = 1$ represents a circle then λ =

(a) 1 (b) $\frac{3}{4}$
(c) 0 (d) $-\frac{3}{4}$

7. The value of k, such that the equation $2x^2 + 2y^2 - 6x + 8y + k = 0$ represents a point circle, is

(a) 0 (b) 25
(c) $\frac{25}{2}$ (d) $-\frac{25}{2}$

8. The equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle only if

(a) $a = b, h = 0$
(b) $a = b \neq 0, h = 0$
(c) $a = b \neq 0, h = 0, g^2 + f^2 - c > 0$
(d) $a = b \neq 0, h = 0, g^2 + f^2 - ac > 0$

9. The equation $2x^2 + 3y^2 - 8x - 18y + 35 = \lambda$ represents

(a) a circle for all λ (b) an ellipse if $\lambda < 0$
(c) the empty set if $\lambda > 0$ (d) a-point if $\lambda = 0$

Finding equation of Circle under given condition

10. Find the equation of a circle whose centre is (2, –1) and radius is 3

(a) $x^2 + y^2 + 4x - 2y + 4 = 0$
(b) $x^2 + y^2 - 4x + 2y - 4 = 0$
(c) $x^2 + y^2 + 4x + 2y - 4 = 0$
(d) $x^2 + y^2 + 2x - 4y - 4 = 0$

11. The equation of the circle touches y axis and having centre is (–2, –3)–

(a) $x^2 + y^2 - 4x - 9y - 4 = 0$
(b) $x^2 + y^2 + 4x + 9y + 4 = 0$
(c) $x^2 + y^2 + 4x + 6y + 9 = 0$
(d) $x^2 + y^2 - 4x - 6y - 9 = 0$

12. A circle touches x axis at +3 distance and cuts an intercept of 8 in +ve direction of y axis. Its equation is–

(a) $x^2 + y^2 + 6x + 10y - 9 = 0$
(b) $x^2 + y^2 - 6x - 10y - 9 = 0$
(c) $x^2 + y^2 - 6x - 10y + 9 = 0$
(d) $x^2 + y^2 + 6x + 10y + 9 = 0$

13. The equation of circle with centre (1, 2) and tangent $x + y - 5 = 0$ is

(a) $x^2 + y^2 + 2x - 4y + 6 = 0$

(b) $x^2 + y^2 - 2x - 4y + 3 = 0$

(c) $x^2 + y^2 - 2x + 4y + 8 = 0$

(d) $x^2 + y^2 - 2x - 4y + 8 = 0$

14. Equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$ is

(a) $x^2 + y^2 - 4x - 8y - 97 = 0$

(b) $x^2 + y^2 - 4x - 6y - 87 = 0$

(c) $x^2 + y^2 - 2x - 8y - 95 = 0$

(d) None of these

15. Equation of a circle concentric with the circle $2x^2 + 2y^2 - 6x + 8y + 1 = 0$ and of double its area is

(a) $2x^2 + 2y^2 - 12x + 16y + 1 = 0$

(b) $4x^2 + 4y^2 - 3x + 16y + 2 = 0$

(c) $4x^2 + 4y^2 - 12x + 16y - 21 = 0$

(d) None of these

16. Equation of the circle of radius 5 whose centre lies on y-axis in first quadrant and passes through (3, 2) is

(a) $x^2 + y^2 - 12y + 11 = 0$ (b) $x^2 + y^2 - 6y - 1 = 0$

(c) $x^2 + y^2 - 8y + 3 = 0$ (d) None of these

17. The equation of the circle which touches the axis of y at the origin and passes through (3, 4) is-

(a) $4(x^2 + y^2) - 25x = 0$

(b) $3(x^2 + y^2) - 25x = 0$

(c) $2(x^2 + y^2) - 3x = 0$

(d) $4(x^2 + y^2) - 25x + 10 = 0$

18. If (4, –2) is a point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, which is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, then value of c is-

(a) –4 (b) 0

(c) 4 (d) 1

19. The equation of the image of the circle $(x - 3)^2 + (y - 2)^2 = 1$ by the line mirror $x + y = 19$ is

(a) $(x - 14)^2 + (y - 13)^2 = 1$

(b) $(x - 15)^2 + (y - 14)^2 = 1$

(c) $(x - 16)^2 + (y - 15)^2 = 1$

(d) $(x - 17)^2 + (y - 16)^2 = 1$

20. The equation of a circle which passes through the point (1, –2) and (4, –3) and whose centre lies on the line $3x + 4y = 7$ is

(a) $15(x^2 + y^2) - 94x + 18y - 55 = 0$

(b) $15(x^2 + y^2) - 94x + 18y + 55 = 0$

(c) $15(x^2 + y^2) + 94x - 18y + 55 = 0$

(d) none of these

21. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. Then the equation of the circle is.

(a) $x^2 + y^2 + 2x - 2y = 62$ (b) $x^2 + y^2 + 2x - 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 47$ (d) $x^2 + y^2 - 2x + 2y = 62$

22. Two vertices of an equilateral triangle are (–1, 0) and (1, 0), and its third vertex lies above the x-axis. Then the equation of the circumcircle of the triangle is :

(a) $x^2 + y^2 = 1$

(b) $\sqrt{3}(x^2 + y^2) + 2y - \sqrt{3} = 0$

(c) $\sqrt{3}(x^2 + y^2) - 2y - \sqrt{3} = 0$

(d) none of these

Equation of circle passing through 3 point

23. The equation of a circle which passes through the three points (3, 0), (1, –6), (4, –1) is-

(a) $2x^2 + 2y^2 + 5x - 11y + 3 = 0$

(b) $x^2 + y^2 - 5x + 11y - 3 = 0$

(c) $x^2 + y^2 + 5x - 11y + 3 = 0$

(d) $2x^2 + 2y^2 - 5x + 11y - 3 = 0$

24. The radius of the circle passing through the points (1, 2), (5, 2) & (5, –2) is :

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$

(c) $3\sqrt{2}$ (d) $2\sqrt{2}$

Diametric forms

25. The equation of the incircle of the triangle formed by the axes and the line $4x + 3y = 6$ is

(a) $x^2 + y^2 - 6x - 6y + 9 = 0$

(b) $4(x^2 + y^2 - x - y) + 1 = 0$

(c) $4(x^2 + y^2 + x + y) + 1 = 0$

(d) none of these

26. The abscissae and ordinates of the points A and B are the roots of the equations $x^2 + 2ax + b = 0$ and $x^2 + 2cx + d = 0$ respectively, then the equation of circle with AB as diameter is

(a) $x^2 + y^2 + 2ax + 2cy + b + d = 0$

(b) $x^2 + y^2 - 2ax - 2cy - b - d = 0$

(c) $x^2 + y^2 - 2ax - 2cy + b + d = 0$

(d) $x^2 + y^2 + 2ax + 2cy - b - d = 0$

27. The equation to a circle passing through the origin and cutting of intercepts each equal to + 5 of the axes is-

(a) $x^2 + y^2 + 5x - 5y = 0$ (b) $x^2 + y^2 - 5x + 5y = 0$

(c) $x^2 + y^2 - 5x - 5y = 0$ (d) $x^2 + y^2 + 5x + 5y = 0$

Intercept made by Circle

28. The length of intercept on y-axis, by a circle whose diameter is the line joining the points (–4, 3) & (12, –1) is-

(a) $2\sqrt{13}$ (b) $\sqrt{13}$

(c) $4\sqrt{13}$ (d) None of these

29. For the circle $x^2 + y^2 + 4x - 7y + 12 = 0$ the following statement is true-

(a) the length of tangent from (1, 2) is 7

(b) Intercept on y-axis is 2

(c) intercept on x-axis is $2-\sqrt{2}$

(d) None of these

30. The circle $x^2 + y^2 + 4x - 7y + 12 = 0$ cuts an intercept on y-axis is equal to

(a) 7 (b) 4

(c) 3 (d) 1

31. The circle $x^2 + y^2 - 3x - 4y + 2 = 0$ cuts x-axis at

(a) (2, 0), (–3, 0) (b) (3, 0), (4, 0)

(c) (1, 0), (–1, 0) (d) (1, 0), (2, 0)

32. The length of the chord of the circle $(x - 3)^2 + (y - 5)^2 = 80$ cut off by the line $3x - 4y - 9 = 0$ is

(a) 16 (b) 8

(c) $\sqrt{96}$ (d) $2\sqrt{96}$

Parametric Forms

33. The Cartesian equation of the curve $x = 7 + 4\cos\alpha$, $y = -3 + 4\sin\alpha$ is

(a) $x^2 + y^2 - 14x + 6y + 42 = 0$

(b) $x^2 + y^2 - 6x + 14y + 21 = 0$

(c) $x^2 + y^2 - 10x + 12y + 28 = 0$

(d) None of these

34. α, β, γ are the parametric angles of three point P, Q and R respectively on the circle $x^2 + y^2 = 1$ and A is the point (–1, 0). If $\alpha, \beta, \gamma \in (0, \pi)$ and the lengths of the chords AP, AQ and AR are in G.P., then $\cos\frac{\alpha}{2}$, $\cos\frac{\beta}{2}$ and $\cos\frac{\gamma}{2}$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

Position of point w.r.t circle

35. The greatest distance of the point P (10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is-

(a) 5 (b) 15

(c) 10 (d) None of these

36. The co-ordinate of the point on the circle $x^2 + y^2 - 12x - 4y + 30 = 0$, which is farthest from the origin are:

(a) (9, 3) (b) (8, 5)

(c) (12, 4) (d) none of these

37. The point $(\sin\theta, \cos\theta)$, θ being any real number, lie inside the circle $x^2 + y^2 - 2x - 2y + \lambda = 0$, if

(a) $\lambda < 1 + 2\sqrt{2}$ (b) $\lambda > 2\sqrt{2} - 1$

(c) $\lambda < -1 - 2\sqrt{2}$ (d) $\lambda > 1 + 2\sqrt{2}$

38. The range of values of $\theta \in [0, 2\pi]$ for which $(1 + \cos\theta, \sin\theta)$ is on interior points of the circle $x^2 + y^2 = 1$, is

(a) $(\pi/6, 5\pi/6)$ (b) $(2\pi/3, 5\pi/3)$

(c) $(\pi/6, 7\pi/6)$ (d) $(2\pi/3, 4\pi/3)$

39. If the point $\left(-5+\frac{\lambda}{\sqrt{2}}, -3+\frac{\lambda}{\sqrt{2}}\right)$ is an interior point of the larger segement of the circles $x^2 + y^2 = 16$ cut off by the line $x + y = 2$, then

(a) $\lambda \in \left(-\infty, 5\sqrt{2}\right)$

(b) $\lambda \in \left(4\sqrt{2}-\sqrt{14}, 5\sqrt{2}\right)$

(c) $\lambda \in \left(4\sqrt{2}-\sqrt{14}, 4\sqrt{2}+\sqrt{14}\right)$

(d) none of these

Positon of line w.r.t circle and equation of tangents

40. The line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ at only one point, if k^2 is

(a) $20r^2$ (b) $52r^2$

(c) $\frac{52}{9}r^2$ (d) $\frac{20}{9}r^2$

41. If the straight line $y = mx$ is outside the circle $x^2 + y^2 - 20y + 90 = 0$, then

(a) $m < 3$ (b) $|m| < 3$

(c) $m > 3$ (d) $|m| > 3$

42. The value of c, for which the line $y = 2x + c$ is a tangent to the circle $x^2 + y^2 = 16$, is

(a) $-16\sqrt{5}$ (b) $4\sqrt{5}$

(c) $16\sqrt{5}$ (d) 20

43. The equation of the tangents to the circle $x^2 + y^2 = 4$, which are parallel to $x + 2y + 3 = 0$, are

(a) $x - 2y = 2$ (b) $x+2y=\pm 2\sqrt{3}$

(c) $x+2y=\pm 2\sqrt{5}$ (d) $x-2y=\pm 2\sqrt{5}$

44. The equations of the tangents to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ which are parallel to the line $4x + 3y + 5 = 0$, are

(a) $4x + 3y + 11 = 0$ and $4x + 3y + 8 = 0$

(b) $4x + 3y - 9 = 0$ and $4x + 3y + 7 = 0$

(c) $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$

(d) $4x + 3y - 10 = 0$ and $4x + 3y + 12 = 0$

45. If the equation of the tangent to the circle $x^2 + y^2 - 2x + 6y - 6 = 0$ parallel to $3x - 4y + 7 = 0$ is $3x - 4y + k = 0$, then the values of k are

(a) 5, – 35 (b) –5, 35

(c) 7, –32 (d) –7, 32

46. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through (–2, 11) is

(a) $4x + 3y = 25$ (b) $7x - 24y = 320$

(c) $3x + 4y = 38$ (d) $24x + 7y + 125 = 0$

47. If the straight line $ax + by = 2$, $b \neq 0$, touches the circle $x^2 + y^2 - 2x = 3$ and is normal to the circle $x^2 + y^2 - 4y = 6$, then the values of a and b can be

(a) a = 1, b = 2 (b) a = 1, b = –1

(c) a = –4/3, b = 1 (d) None of these

Family of Circles

48. The equation of tangent drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ is-

(a) $y = 0$ (b) $x - y = 0$

(c) $(h^2 - r^2)x - 2rhy = 0$ (d) None of these

49. The equation of the circle passing through (1, –3) & the points common to the two circle, $x^2 + y^2 - 6x + 8y - 16 = 0$, $x^2 + y^2 + 4x - 2y - 8 = 0$ is.

(a) $x^2 + y^2 - 4x + 6y + 24 = 0$

(b) $2x^2 + 2y^2 + 3x + y - 20 = 0$

(c) $3x^2 + 3y^2 - 5x + 7y - 19 = 20$

(d) none of these

50. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points :

(a) $\left(1+\sqrt{3},0\right)$ (b) $\left(-1+\sqrt{3},0\right)$

(c) $\left(-\sqrt{3}-1,0\right)$ (d) $\left(1-\sqrt{3},0\right)$

51. The radius of the circle passing through the points (1, 2), (5, 2) & (5, –2) is :

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$

(c) $3\sqrt{2}$ (d) $2\sqrt{2}$

52. If $y = 2x$ is a chord of the circle $x^2 + y^2 = 10x$, then the equation of the circle whose diameter is this chord is

(a) $x^2 + y^2 + 2x + 4y = 0$ (b) $x^2 + y^2 + 2x - 4y = 0$

(c) $x^2 + y^2 - 2x - 4y = 0$ (d) None of these

Position of two circles and number of Common tangents

53. The circles $x^2 + y^2 + 6x + 6y = 0$ and $x^2 + y^2 - 12x - 12y = 0$

(a) touch each other internally

(b) touch each other externally

(c) intersect in two points

(d) cut orthogonally

54. The number of common tangents that can be drawn to two circles $x^2 + y^2 = 6x$ and $x^2 + y^2 + 6x + 2y + 1 = 0$ is

(a) 4 (b) 5

(c) 2 (d) 1

55. The number of common tangents to the circles

$x^2 + y^2 - 2x - 4y + 1 = 0$

and $x^2 + y^2 - 12x - 16y + 91 = 0$ is

(a) 1 (b) 2

(c) 3 (d) 4

56. The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and

$x^2 + y^2 - 5x + 6y + 15 = 0$

(a) intersect (b) are concentric

(c) touch internally (d) touch externally

57. Which of the following statements is true regarding the following two circles ?

$x^2 + y^2 - 4x - 8y = 0$ and $x^2 + y^2 - 8x - 6y + 20 = 0$

(a) These circles do not touch each other

(b) These circles touch each other internally

(c) These circles touch each other externally

(d) None of these

58. The circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and

$x^2 + y^2 - 2x - 2y + 1 = 0$ touch each other

(a) externally at (0,1) (b) internally at (0,1)

(c) externally at (1,0) (d) internally at (1,0)

Orthogonal Circles & Geometry related to Circles

59. The value of λ, for which the circle

$x^2 + y^2 + 2\lambda x + 6y + 1 = 0$ inersects the circle

$x^2 + y^2 + 4x + 2y = 0$ orthogonally, is

(a) 11/8 (b) –1

(c) –5/4 (d) 5/2

60. The value of k for which the circles $x^2 + y^2 + 5x + 3y + 7 = 0$ and $x^2 + y^2 - 8x + 6y + k = 0$ intersects orthogonally.

(a) – 18 (b) 18

(c) 9 (d) – 9

61. The locus of the centre of the circle passing through (1,1) and intersecting the circle $x^2 + y^2 = 9$ orthogonally is

(a) $x + y - 10 = 0$ (b) $x + y - 5 = 0$

(c) $2x + 2y - 5 = 0$ (d) None of these

62. The square of the length of the tangent from (3, –4) to the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ is

(a) 20 (b) 30

(c) 40 (d) 50

63. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their point of contact is :

(a) $\frac{192}{25}$ (b) 192

(c) 25 (d) 250

64. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$, and B (1,7) and D (4, –2) are points on the circle then, if tangents be drawn at B and D, which meet at C, then area of quadrilateral ABCD is-

(a) 150 units (b) 75 units

(c) 75/2 units (d) None

65. The angle between the tangents from (α, β) to the circle $x^2 + y^2 = a^2$ is –

(a) $\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ (b) $2\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$

(c) $2\tan^{-1}\left(\frac{\sqrt{S_1}}{a}\right)$ (d) None of these

Where $S_1 = \alpha^2 + \beta^2 - a^2$

66. If the line $y = x + 3$ meets the circle $x^2 + y^2 = a^2$ at A and B, then the equation of the circle having AB as a diameter will be-

(a) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(b) $x^2 + y^2 + 3x + 3y - a^2 + 9 = 0$

(c) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$

(d) None of these

67. A point (2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is :

(a) $(x + g)(x - 2) + (y + f)(y - 1) = 0$

(b) $(x + g)(x - 2) - (y + f)(y - 1) = 0$

(c) $(x - g)(x + 2) + (y - f)(y + 1) = 0$

(d) none of these

68. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, ($\alpha$ is acute) then the angle between the tangents is :

(a) α (b) 2α

(c) $\frac{\alpha}{2}$ (d) none of these

69. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) none of these

70. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their point of contact is :

(a) $\frac{192}{25}$ (b) 192

(c) 25 (d) 250

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are **(2002)**
(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$

2. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is **(2002)**
(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) none of these

3. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is **(2002)**
(a) $4 \le x^2 + y^2 \le 64$ (b) $x^2 + y^2 \le 25$
(c) $x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$

4. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is **(2002)**
(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, -\sqrt{2}\right)$
(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{3}{2}\right)$

5. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length 3a is **(2002)**
(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

6. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then **(2003)**
(a) $r < 2$ (b) $r = 2$
(c) $r > 2$ (d) $2 < r < 8$

7. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is **(2003)**
(a) $x^2 + y^2 + 2x - 2y = 47$
(b) $x^2 + y^2 - 2x + 2y = 47$
(c) $x^2 + y^2 - 2x + 2y = 62$
(d) $x^2 + y^2 + 2x - 2y = 62$

8. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is **(2004)**
(a) $2ax - 2by + (a^2 + b^2 + 4) = 0$
(b) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(c) $2ax + 2by + (a^2 + b^2 + 4) = 0$
(d) $2ax - 2by - (a^2 + b^2 + 4) = 0$

9. A variable circle passes through the fixed point A(p, q) and touches x–axis. The locus of the other end of the diameter through A is **(2004)**
(a) $(y - p)^2 = 4qx$ (b) $(x - q)^2 = 4py$
(c) $(x - p)^2 = 4qy$ (d) $(y - q)^2 = 4px$

10. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is **(2004)**
(a) $x^2 + y^2 + 2x + 2y - 23 = 0$
(b) $x^2 + y^2 - 2x - 2y - 23 = 0$
(c) $x^2 + y^2 - 2x + 2y - 23 = 0$
(d) $x^2 + y^2 + 2x - 2y - 23 = 0$

11. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is **(2004)**
(a) $x^2 + y^2 + x + y = 0$ (b) $x^2 + y^2 - x + y = 0$
(c) $x^2 + y^2 - x - y = 0$ (d) $x^2 + y^2 + x - y = 0$

12. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for **(2005)**
(a) no value of a (b) exactly one value of a
(c) exactly two value of a (d) infinitely many values of a

13. A circle touches the x–axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is **(2005)**
(a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola

14. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is **(2005)**

(a) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(b) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

(c) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$

15. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then **(2005)**

(a) $3a^2 - 2ab + 3b^2 = 0$

(b) $3a^2 - 10ab + 3b^2 = 0$

(c) $3a^2 + 2ab + 3b^2 = 0$

(d) $3a^2 + 10ab + 3b^2 = 0$

16. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, then the equation of the circle is **(2006)**

(a) $x^2 + y^2 + 2x - 2y - 47 = 0$

(b) $x^2 + y^2 + 2x - 2y - 62 = 0$

(c) $x^2 + y^2 - 2x + 2y - 62 = 0$

(d) $x^2 + y^2 - 2x + 2y - 47 = 0$

17. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of chord of the circle C that subtend an angle of $2\pi/3$ at its centre is **(2006)**

(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$

18. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x–axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval **(2007)**

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$

(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

19. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2ax + 4y - 3 = 0$ is **(2008)**

(a) (3, 4) (b) (3, –4)

(c) (–3, 4) (d) (–3, –4)

20. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) and **(2009)**

(a) all values of p (b) all except one value of p

(c) all except two values of p (d) exactly one value of p

21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points, if **(2010)**

(a) $-85 < m < -35$ (b) $-35 < m < 15$

(c) $15 < m < 65$ (d) $35 < m < 85$

22. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is **(2011)**

(a) $x^2 + y^2 + x + y - 2 = 0$ (b) $x^2 + y^2 - 2x - 2y + 1 = 0$

(c) $x^2 + y^2 - x - y = 0$ (d) $x^2 + y^2 + 2x + 2y - 7 = 0$

23. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if **(2011)**

(a) $|a| = c$ (b) $a = 2c$

(c) $|a| = 2c$ (d) $2|a| = c$

24. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is **(2012)**

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$

(c) $\frac{6}{5}$ (d) $\frac{5}{3}$

25. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **(2013)**

(a) (–5, 2) (b) (2, –5)

(c) (5, –2) (d) (–2, 5)

26. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to : **(2014)**

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

27. If the point (1, 4) lies inside the circle $x^2 + y^2 - 6x - 10y + p = 0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of p is the interval: **(2014/Online Set–1)**

(a) (0, 25) (b) (25, 39)
(c) (9, 25) (d) (25, 29)

28. The set of all real values of λ for which exactly two common tangents can be drawn to the circles $x^2 + y^2 - 4x - 4y + 6 = 0$ and $x^2 + y^2 - 10x - 10y + \lambda = 0$ is the interval: **(2014/Online Set–2)**

(a) (12, 32) (b) (18, 42)
(c) (12, 24) (d) (18, 48)

29. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is/are: **(2014/Online Set–3)**

(a) one pair of common tangents
(b) two pairs of common tangents
(c) three common tangents
(d) no nommon tangent

30. The equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 + = 16$ as diameter is **(2014/Online Set–4)**

(a) $x^2 + y^2 + 3x + y - 11 = 0$
(b) $x^2 + y^2 + 3x + y + 1 = 0$
(c) $x^2 + y^2 + 3x + y - 2 = 0$
(d) $x^2 + y^2 + 3x + y - 22 = 0$

31. The number of common tangents to the circles $x^2 + y^2 – 4x – 6y – 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is: **(2015)**

(a) 3 (b) 4
(c) 1 (d) 2

32. Let the tangents drawn to the circle, $x^2 + y^2 = 16$ from the point P(0, h) meet the x-axis at points A and B. If the area of ΔAPB is minimum, then h is equal to : **(2015/Online Set–1)**

(a) $4\sqrt{2}$ (b) $4\sqrt{3}$
(c) $3\sqrt{2}$ (d) $3\sqrt{3}$

33. If a circle passing through the point (-1, 0) touches y-axis at (0, 2) then the length of the chord of the circle along the x-axis is : **(2015/Online Set–2)**

(a) $\frac{3}{2}$ (b) 3
(c) $\frac{5}{2}$ (d) 5

34. If one of the diameters of the circle, given by the equation, $x^2 + y^2 – 4x + 6y – 12 = 0$ is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is : **(2016)**

(a) $5\sqrt{3}$ (b) 5
(c) 10 (d) $5\sqrt{2}$

35. A circle passes through (–2, 4) and touches the y-axis at (0, 2). Which one of the following equations can represent a diameter of this circle ? **(2016/Online Set–1)**

(a) $4x + 5y – 6 = 0$ (b) 2x “ 3y + 10 = 0
(c) $3x + 4y – 3 = 0$ (d) $5x + 2y + 4 = 0$

36. Equation of the tangent to the circle, at the point (1, –1), whose centre is the point of intersection of the straight lines $x – y = 1$ and $2x + y = 3$ is : **(2016/Online Set–2)**

(a) $4x + y – 3 = 0$ (b) $x + 4y + 3 = 0$
(c) $3x – y – 4 = 0$ (d) $x – 3y – 4 = 0$

37. The radius of a circle, having minimum area, which touches the curve $y = 4 – x^2$ and the lines, $y = |x|$ is: **(2017)**

(a) $2\left(\sqrt{2}+1\right)$ (b) $2\left(\sqrt{2}-1\right)$
(c) $4\left(\sqrt{2}-1\right)$ (d) $4\left(\sqrt{2}+1\right)$

38. A line drawn through the point P (4, 7) cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then PA.PB is equal to : **(2017/Online Set–1)**

(a) 53 (b) 56
(c) 74 (d) 65

39. If a point P has co-ordinates (0, –2) and Q is any point on the circle, $x^2+y^2-5x-y+5=0$, then the maximum value of $(PQ)^2$ is : **(2017/Online Set–1)**

(a) $\frac{25+\sqrt{6}}{2}$ (b) $14+5\sqrt{3}$

(c) $\frac{47+10\sqrt{6}}{2}$ (d) $8+5\sqrt{3}$

40. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is : **(2017/Online Set–1)**

(a) $\frac{4}{\sqrt{7}}$ (b) $\frac{8}{\sqrt{7}}$

(c) $\frac{8}{7}$ (d) $\frac{16}{7}$

41. Let the orthocentre and centroid of a triangle be A (–3,5) and B (3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is : **(2018)**

(a) $\frac{3\sqrt{5}}{2}$ (b) $\sqrt{10}$

(c) $2\sqrt{10}$ (d) $3\sqrt{\frac{5}{2}}$

42. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to : **(2018/Online Set–1)**

(a) 2 (b) $\sqrt{5}$

(c) $\sqrt{2}$ (d) 1

43. The tangent to the circle C_1 : $x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2, whose centre is (3, -2). The radius of C_2 is : **(2018/Online Set–2)**

(a) 2 (b) $\sqrt{2}$

(c) 3 (d) $\sqrt{6}$

44. If a circle C, whose radius is 3, touches externally the circle, $x^2+y^2+2x-4y-4=0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to : **(2018/Online Set–3)**

(a) $2\sqrt{5}$ (b) $3\sqrt{2}$

(c) $\sqrt{5}$ (d) $2\sqrt{3}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. A variable circle having fixed radius 'a' passes through origin and meets the coordinate axes in point A and B. Locus of centroid of triangle OAB, 'O' being the origin, is

(a) $9(x^2+y^2)=4a^2$ (b) $9(x^2+y^2)=a^2$

(c) $9(x^2+y^2)=2a^2$ (d) $9(x^2+y^2)=8a^2$

2. A circle and a square have the same perimeter. Then

(a) their areas are equal

(b) the area of the circle is large

(c) the area of the square is larger

(d) the area of the circle is π times the area of the square

3. The equation of a circle which touches x-axis and the line $4x-3y+4=0$, its centre lying in the third quadrant and lies on the line $x-y-1=0$, is-

(a) $9(x^2+y^2)+6x+24y+1=0$

(b) $9(x^2+y^2)-6x-24y+1=0$

(c) $9(x^2+y^2)-6x+2y+1=0$

(d) None of these

4. The equation of the circle whose radius is 3 and which touches the circle $x^2+y^2-4x-6y-12=0$ internally at the point $(-1,-1)$ is-

(a) $\left(x-\frac{4}{5}\right)^2+\left(y+\frac{7}{5}\right)^2=3^2$

(b) $\left(x-\frac{4}{5}\right)^2+\left(y-\frac{7}{5}\right)^2=3^2$

(c) $(x-8)^2+(y-1)^2=3^2$

(d) None of these

5. The equation of the circle which is touched by $y=x$, has its centre on the positive x-axis & cuts off a chord of length 2 units along the line $\sqrt{3}y-x=0$ is-

(a) $x^2+y^2-4x+2=0$ (b) $x^2+y^2-8x+8=0$

(c) $x^2+y^2-4x+1=0$ (d) $x^2+y^2-4y+2=0$

6. From the point A(0, 3) on the circle $x^2+4x+(y-3)^2=0$ a chord AB is drawn & extended to a point M such that AM = 2AB. The equation of the locus of M is :

(a) $x^2+8x+y^2=0$ (b) $x^2+8x+(y-3)^2=0$

(c) $(x-3)^2+8x+y^2=0$ (d) $x^2+8x+8y^2=0$

7. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x-axis in points whose abscissae are roots of the equation :

(a) $x^2+ax+b=0$ (b) $x^2-ax+b=0$

(c) $x^2+ax-b=0$ (d) $x^2-ax-b=0$

8. The abscissa of two points A and B are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are the roots of the equation $y^2+2py-q^2=0$. The radius of the circle with AB as a diameter will be-

(a) $\sqrt{a^2+b^2+p^2+q^2}$ (b) $\sqrt{b^2+q^2}$

(c) $\sqrt{a^2+b^2-p^2-q^2}$ (d) $\sqrt{a^2+p^2}$

9. If the squares of the lengths of the tangents from a point P to the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$ and $x^2+y^2=c^2$ are in A.P., then

(a) a, b, c are in GP (b) a, b, c are in AP

(c) a^2, b^2, c^2 are in AP (d) a^2, b^2, c^2 are in GP

10. If a chord of the circle $x^2+y^2=8$ makes equal intercepts of length a on the coordinate axes, then-

(a) $|a|<8$ (b) $|a|<4\sqrt{2}$

(c) $|a|<4$ (d) $|a|>4$

11. The angle of intersection of the two circles $x^2+y^2-2x-2y=0$ and $x^2+y^2=4$, is–

(a) 30° (b) 60°

(c) 90° (d) 45°

12. The value of 'c' for which the set, $\{(x, y) \mid x^2+y^2+2x \le 1\} \cap \{(x, y) \mid x-y+c \ge 0\}$ contains only one point in common is :

(a) $(-\infty,-1)\cup[3,\infty)$ (b) $\{-1, 3\}$

(c) $\{-3\}$ (d) $\{-1\}$

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13. The circle $x^2 + y^2 - 6x - 10y + c = 0$ does not intersect or touch either axis & the point (1, 4) is inside the circle. Then the range of possible values of c is given by :

(a) $c > 9$ (b) $c > 25$

(c) $c > 29$ (d) $25 < c < 29$

14. Let x and y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M & m respectively, then the numerical value of M – m is :

(a) 2 (b) 8

(c) 15 (d) none of these

15. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is :

(a) $\sqrt{q-p}$ (b) $\sqrt{p-q}$

(c) $\sqrt{q+p}$ (d) none of these

16. The centre of the smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is :

(a) (3, 2) (b) (4, 4)

(c) (2, 7) (d) (2, 5)

17. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles. The area of the rhombus is :

(a) $8\sqrt{3}$ sq. units (b) $4\sqrt{3}$ sq. units

(c) $16\sqrt{3}$ sq. units (d) none of these

18. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co-ordinates of the centre of C_2 are :

(a) $\left(\pm\frac{9}{5}, \pm\frac{12}{5}\right)$ (b) $\left(\pm\frac{9}{5}, \mp\frac{12}{5}\right)$

(c) $\left(\pm\frac{12}{5}, \pm\frac{9}{5}\right)$ (d) $\left(\pm\frac{12}{5}, \mp\frac{9}{5}\right)$

19. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then p + q is equal to :

(a) 25 (b) 100

(c) 10 (d) 48

20. The distance from the centre of the circle $x^2 + y^2 = 2x$ to the straight line passing through the points of intersection of the two circle ;

$x^2 + y^2 + 5x - 8y + 1 = 0$, $x^2 + y^2 - 3x + 7y - 25 = 0$ is

(a) 1 (b) 2

(c) 3 (d) none of these

21. The value of 'a' for which the common chord of the circles $c_1 : x^2 + y^2 = 8$ and $c_2 : (x - a)^2 + y^2 = 8$ subtends a right angle at the origin are :

(a) $\pm\sqrt{2}$ (b) ± 2

(c) ± 4 (d) none of these

22. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other then

(a) $f_1g_1 = f_2g_2$ (b) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$

(c) $f_1f_2 = g_1g_2$ (d) none of these

23. If r_1 be the radius of smallest circle which passes through (5, 6) and touches the circle $(x - 2)^2 + y^2 = 4$, then r_1 is

(a) $\frac{3\sqrt{5}+2}{2}$ (b) $\frac{3\sqrt{5}-2}{2}$

(c) $\frac{3\sqrt{5}+4}{2}$ (d) $\frac{41}{6}$

24. A line meets the co-ordinate axes in A & B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively, the diameter of the circle is :

(a) $\frac{2d_1+d_2}{2}$ (b) $\frac{d_1+2d_2}{2}$

(c) $d_1 + d_2$ (d) $\frac{d_1d_2}{d_1+d_2}$

25. If a circle passes through the point (1, 2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is-

(a) $x^2 + y^2 - 2x - 6y - 7 = 0$

(b) $x^2 + y^2 - 3x - 8y + 1 = 0$

(c) $2x + 4y - 9 = 0$

(d) $2x + 4y - 1 = 0$

26. The length of the common chord of the circles $(x - a)^2 + y^2 = c^2$ and $x^2 + (y - b)^2 = c^2$ is-

(a) $\sqrt{c^2 + a^2 + b^2}$ (b) $\sqrt{4c^2 + a^2 + b^2}$

(c) $\sqrt{4c^2 - a^2 - b^2}$ (d) $\sqrt{c^2 - a^2 - b^2}$

27. Two circles whose radii are equal to 4 and 8 intersect at right angles. The length of their common chord is :

(a) $\frac{16}{\sqrt{5}}$ (b) 8

(c) $4\sqrt{6}$ (d) $\frac{8\sqrt{5}}{5}$

28. A circle touches a straight line $lx + my + n = 0$ & cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circle is :

(a) $(lx + my + n)^2 = (l^2 + m^2)(x^2 + y^2 - 9)$

(b) $(lx + my - n)^2 = (l^2 + m^2)(x^2 + y^2 - 9)$

(c) $(lx + my + n)^2 = (l^2 + m^2)(x^2 + y^2 + 9)$

(d) none of these

29. If a circle passes through the point (a, b) & cuts the circle $x^2 + y^2 = K^2$ orthogonally, then the equation of the locus of its centre is :

(a) $2ax + 2by - (a^2 + b^2 + K^2) = 0$

(b) $2ax + 2by - (a^2 - b^2 + K^2) = 0$

(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - K^2) = 0$

(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - K^2) = 0$

30. Minimum radius of circle which is orthogonal with both the circle $x^2 + y^2 - 12x + 35 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ is

(a) 4 (b) 3

(c) $\sqrt{15}$ (d) 1

31. Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is :

(a) $x^2 + y^2 = a^2 + b^2$ (b) $x^2 - y^2 = a^2 - b^2$

(c) $x^2 + y^2 = a^2 - b^2$ (d) $x^2 - y^2 = a^2 + b^2$

32. If tangent at (1, 2) to the circle $c_1 : x^2 + y^2 = 5$ intersects the circle $c_2 : x^2 + y^2 = 9$ at A and B and tangents at A and B to the second circle meet at point C, then the co-ordinates of C are :

(a) (4, 5) (b) $\left(\frac{9}{15}, \frac{18}{5}\right)$

(c) (4, – 5) (d) $\left(\frac{9}{5}, \frac{18}{5}\right)$

33. A circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches the line pair $x^2 - y^2 - 2x + 1 = 0$. The co-ordinates of the centre of the circle are :

(a) (4, 0) (b) (5, 0)

(c) (6, 0) (d) (0, 4)

34. If $C_1 : x^2 + y^2 = (3 + 2\sqrt{2})^2$ be a circle and PA and PB are pair of tangents on C_1 where P is any point on the director circle of C_1, then the radius of smallest circle which touch C_1 externally and also the two tangents PA and PB is

(a) $2\sqrt{2} - 3$ (b) $2\sqrt{2} - 1$

(c) $2\sqrt{2} + 1$ (d) 1

35. From a point R(5, 8) two tangents RP and RQ are drawn to a given circle S = 0 whose radius is 5. If circumcentre of the triangle PQR is (2, 3), then the equation of circle S = 0 is

(a) $x^2 + y^2 + 2x + 4y - 20 = 0$

(b) $x^2 + y^2 + x + 2y - 10 = 0$

(c) $x^2 + y^2 - x - 2y - 20 = 0$

(d) $x^2 + y^2 - 4x - 6y - 12 = 0$

36. On the line segment joining (1, 0) and (3, 0) an equilateral triangle is drawn having its vertex in the fourth quadrant, then radical centre of the circle described on its sides as diameter is

(a) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (b) $\left(3,-\sqrt{3}\right)$

(c) $\left(2,-\frac{1}{\sqrt{3}}\right)$ (d) $\left(2,-\sqrt{3}\right)$

37. If the tangents are drawn from any point on the line x + y = 3 to the circle $x^2 + y^2 = 9$, then the chord of contact passes through the point

(a) (3, 5) (b) (3, 3)

(c) (5, 3) (d) none of these

38. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$ such that P lies on the major arc QR. If Q and R have coordinates (3, 4) and (–4, 3) respectively, then ∠QPR is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

39. Equation of chord of the circle $x^2 + y^2 - 3x - 4y - 4 = 0$ which passes through the origin such that origin divides it in the ratio 4 : 1, is

(a) x = 0 (b) 24x + 7y = 0

(c) 7x + 24y = 0 (d) 7x – 24y = 0

40. If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 6 units, then minimum distance of T from the director circle of the given circle is :

(a) 6 (b) 12

(c) $6\sqrt{2}$ (d) $12-4\sqrt{2}$

41. P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the co–ordinate axes cut at right angles, then

(a) $a^2 - 6ab + b^2 = 0$ (b) $a^2 + 2ab - b^2 = 0$

(c) $a^2 - 4ab + b^2 = 0$ (d) $a^2 - 8ab + b^2 = 0$

Multiple Type Questions

42. Consider the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. Let O be the centre of the circle and tangent at A(7, 3) and B(5, 1) meet at C. Let S = 0 represents family of circles passing through A and B, then

(a) area of quadrilateral OACB = 4

(b) the radical axis for the family of circles S=0 is x+y=10

(c) the smallest possible circle of the family S = 0 is $x^2 + y^2 - 12x - 4y + 38 = 0$

(d) the coordinates of point C are (7, 1)

43. Let x, y be real variable satisfying the $x^2 + y^2 + 8x - 10y - 40 = 0$. Let a = max $\left\{\sqrt{(x+2^2)+(y-3)^2}\right\}$ and

b = min $\left\{\sqrt{(x+2^2)+(y-3)^2}\right\}$, then

(a) a + b = 18 (b) a + b = $4\sqrt{2}$

(c) a – b = $4\sqrt{2}$ (d) a . b = 73

44. Coordinates of the centre of a circle, whose radius is 2 unit and which touches the line pair $x^2 - y^2 - 2x + 1 = 0$, are

(a) (4, 0) (b) $\left(1+2\sqrt{2},0\right)$

(c) (4, 1) (d) $\left(1, 2\sqrt{2}\right)$

45. Point M moved on the circle $(x-4)^2 + (y-8)^2 = 20$. Then it broke away from it and moving along a tangent to the circle, cuts the x–axis at the point (–2, 0). The co–ordinates of a point on the circle at which the moving point broke away is

(a) $\left(-\frac{3}{5},\frac{46}{5}\right)$ (b) $\left(-\frac{2}{5},\frac{44}{5}\right)$

(c) (6, 4) (d) (3, 5)

46. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the radii corresponding to the points of contact is 15, then a value of c is

(a) 9 (b) 4

(c) 5 (d) 25

47. Two circles passing through A(1, 2) and B(2, 1) touch the lines 4x + 8y – 7 = 0 at C and D such that ACED is parallelogram, then

(a) mid point of AE must lie on CD.

(b) mid point of AE must lie on AC.

(c) E must be the point $\left(\frac{13}{2}, -4\right)$

(d) E must be the point $\left(\frac{15}{2}, -\frac{9}{2}\right)$

48. A (3, 0), B (0, 4), and C(0, 0) be the vertices of a triangle. If r is the radius of incircle and r_1, r_2, r_3 be the exradii then

(a) $r = 1$ (b) $r_1 < r_2 < r_3$

(c) $r_2 < r_1 < r_3$ (d) $r_1 < r_3 < r_2$

49. If the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is inscribed in a triangle whose two sides are axes and one side has negative slope cutting intercepts a and b on x and y axis, then

(a) $\frac{1}{a} + \frac{1}{b} - 1 = -\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

(b) $\frac{1}{a} + \frac{1}{b} - 1$ is negative

(c) $\frac{1}{a} + \frac{1}{b} > 1$

(d) $\frac{1}{a} + \frac{1}{b} - 1 = +\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

50. A straight line through the vertex P of a triangle PQR, intersects the side QR at the point S and the circum-circle of the triangle PQR at the point T. If S is not the circumcentre of the triangle, then

(a) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$

(b) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(c) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$

(d) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

51. The equation of the chords of length 5 and passing through the point (3, 4) on the circle $4x^2 + 4y^2 - 24x - 7y = 0$ are

(a) $4x + 3y = 0$ (b) $4x - 3y = 0$

(c) $4x + 3y - 24 = 0$ (d) $4x + 3y - 12 = 0$

Assertion Reason

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false

(D) If ASSERTION is false, REASON is true

52. **Assertion :** Number of common tangents of $x^2 + y^2 - 2x - 4y - 95 = 0$ and $x^2 + y^2 - 6x - 8y + 16 = 0$ is zero.

Reason : If $C_1C_2 < |r_1 - r_2|$, then there will be no common tangent. (where C_1, C_2 are the centre and r_1, r_2 are radii of circles)

(a) A (b) B
(c) C (d) D

53. **Assertion :** Let $S1 : x^2 + y^2 - 10x - 12y - 39 = 0$
$S2 : x^2 + y^2 - 2x - 4y + 1 = 0$
$S3 : 2x^2 + 2y^2 - 20x - 24y + 78 = 0$

The radical centre of these circles taken pairwise is (–2, –3)

Reason : Point of intersection of three radical axis of three circles taken in pairs is known as radical centre

(a) A (b) B
(c) C (d) D

54. **Assertion :** The equations of the straight lines joining origin to the points of intersection of $x^2 + y^2 - 4x - 2y = 4$ and $x^2 + y^2 - 2x - 4y - 4 = 0$ is $(y - x)^2 = 0$

Reason : $y + x = 0$ is a common chord of $x^2 + y^2 - 4x - 2y = 4$ and $x^2 + y^2 - 2x - 4y - 4 = 0$

(a) A (b) B
(c) C (d) D

55. **Assertion :** Two orthogonal circles intersect to generate a common chord which subtends complimentary angles at their circumferences.

Reason : Two orthogonal circles intersect to generate a common chord which subtends supplementary angles at their centres

(a) A (b) B
(c) C (d) D

56. **Assertion :** for two non–intersecting circles, direct common tangents subtends a right angle at either of point of intersection of circles with line segment joining the centres of circles.

Reason : If distance between the centres is more than sum of radii, then circles are non–intersecting

(a) A (b) B
(c) C (d) D

Match the column

57.

	Column–I	Column–II
(A)	If $ax + by - 5 = 0$ is the equation of the chord of the circle $(x - 3)^2 + (y - 4)^2 = 4$, which passes through (2, 3) and at the greatest distance from the centre of the circle, then $\lvert a + b\rvert$ is equal to	(p) 6
(B)	Let O be the origin and P be a variable point on the circle $x^2 + y^2 + 2x + 2y = 0$. If the locus of mid–point of OP is $x^2 + y^2 + 2gx + 2fy = 0$, then the value of $(g + f)$ is equal to	(q) 3
(C)	The x–coordinates of the centre of the smallest circle which cuts the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 - 10x + 12y + 52 = 0$ orthogonally, is	(r) 2
(D)	If θ be the angle between two tangents which are drawn to the circle $x^2 + y^2 - 6\sqrt{3}\,x - 6y + 27 = 0$ from the origin, then $2\sqrt{3}\tan\theta$ equals to	(s) 1

58.	**Column–I**	**Column–II**
(A)	The length of the common chord of two circles of radii 3 and 4 units which intersect orthogonally is k/5, then k equals to	(p) 1
(B)	The circumference of the circle $x^2 + y^2 + 4x + 12y + p = 0$ is bisected by the circle $x^2 + y^2 - 2x + 8y - q = 0$, then p + q is equal to :	(q) 24
(C)	Number of distinct chords of the circle $2x(x-\sqrt{2}) + y(2y-1) = 0$; chords are passing through the point $\left(\sqrt{2}, \frac{1}{2}\right)$ and are bisected on x–axis is	(r) 32
(D)	one of the diameters of the circle circumscribing the rectangle AB CD is 4y = x + 7. If A and B are the points (–3, 4) and (5, 4) respectively, then the area of the rectangle is.	(s) 36

59. Match the items of Column I with those of Column II.

	Column I	**Column II**
(A)	Equations of the circle circumscribing the rectangle whose sides are $x - 3y - 4 = 0$, $3x + y - 22 = 0$, $x - 3y - 14 = 0$ and $3x + y - 62 = 0$ is	(P) $x^2 + y^2 - 9x - 9y + 36 = 0$
(B)	Two vertices of an equilateral triangle are (–1, 0) and (1, 0) and its third vertex lies above the x-axis. The equation of its circumcircle is	(Q) $\sqrt{3}(x^2 + y^2) - 2y - \sqrt{3} = 0$
(C)	The equation of a circle with origin at centre and passing through the vertices of an equilateral triangle whose median is of length 6 units is	(R) $x^2 + y^2 - 9 = 0$
(D)	The vertices of a triangle are (6, 0), (0, 6) and (7, 7). The equation of the incircle of the triangle is	(S) $x^2 + y^2 - 27x - 3y + 142 = 0$
		(T) $x^2 + y^2 - 16 = 0$

Use the following passage, solve Q. 60 to Q. 62

PASSAGE –1

A system of circles is said to be coaxial when every pair of the circles has the same radical axis. It follows from this definition that :

1. The centres of all circles of a coaxial system system lie on one straight line, which is perpendicular to the common radical axis.

2. Circles passing through two fixed points form a coaxial system for which the line joining the fixed points is the common radical axis.

3. The equation to a coaxial system, of which two members are $S_1 = 0$ and $S_2 = 0$, is $S_1 + \lambda S_2 = 0$, λ is parameter. If we choose the line of centres as x–axis and the common radical axis as y–axis, then the simplest form of equation of coaxial circles is $x^2 + y^2 + 2gx + c = 0$ ...(1)

   where c is fixed and g is arbitrary.

If $g = \pm\sqrt{c}$, then the radius $\sqrt{g^2 - c}$ vanishes and the circles become point circles. The points $(\pm\sqrt{c}, 0)$ are called the limiting points of the system of coaxial circles given by (1).

60. The equation of the circle which belongs to the coaxial system of circles for which the limiting points are (1, –1), (2, 0) and which passes through the origin is

(a) $x^2 + y^2 - 4x = 0$ (b) $x^2 + y^2 + 4x = 0$

(c) $x^2 + y^2 - 4y = 0$ (d) $x^2 + y^2 + 4y = 0$

61. If origin be a limiting point of a coaxial system one of whose member is $x^2 + y^2 - 2\alpha x - 2\beta y + c = 0$, then the other limiting point is

(a) $\left(\frac{c\alpha}{\alpha^2+\beta^2}, -\frac{c\beta}{\alpha^2+\beta^2}\right)$ (b) $\left(\frac{c\alpha}{\alpha^2+\beta^2}, \frac{c\beta}{\alpha^2+\beta^2}\right)$

(c) $\left(\frac{\alpha\beta}{\alpha^2+\beta^2}, \frac{c\alpha}{\alpha^2+\beta^2}\right)$ (d) $\left(-\frac{c\beta}{\alpha^2+\beta^2}, \frac{c\alpha}{\alpha^2+\beta^2}\right)$

62. The equation of the radical axis of the system of coaxial circles $x^2 + y^2 + 2ax + 2by + c + 2\lambda(ax - by + 1) = 0$ is

(a) $ax - by + 1 = 0$ (b) $bx + ay - 1 = 0$

(c) $2(ax + by) + 1 = 0$ (d) $2(bx - ay) + 1 = 0$

Use the following passage, solve Q.63 to Q.65

PASSAGE –2

P is a variable point on the line L = 0. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQSR is completed

63. If $L \equiv 2x + y - 6 = 0$, then the locus of circumcentre of ΔPQR is

(a) $2x - y = 4$ (b) $2x + y = 3$

(c) $x - 2y = 4$ (d) $x + 2y = 3$

64. If $P \equiv (6, 8)$, then the area of ΔQRS is

(a) $\frac{(6)^{3/2}}{25}$ sq. units (b) $\frac{(24)^{3/2}}{25}$ sq. units

(c) $\frac{48\sqrt{6}}{25}$ sq. units (d) $\frac{192\sqrt{6}}{25}$ sq. units

65. If $P \equiv (3, 4)$, then coordinate of S is

(a) $\left(-\frac{46}{25}, -\frac{63}{25}\right)$ (b) $\left(-\frac{51}{25}, -\frac{68}{25}\right)$

(c) $\left(-\frac{46}{25}, -\frac{68}{25}\right)$ (d) $\left(-\frac{68}{25}, -\frac{51}{25}\right)$

66. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact. If $OA = \lambda + \sqrt{\mu}$, then the value of $\lambda^2 + \mu$ must be

67. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is

$x + y - xy + k\,(x^2 + y^2)^{1/2} = 0$, then the value of k must be

68. If the minimum radius of the circle which contains the three circles

$x^2 + y^2 - 4y - 5 = 0$ $x^2 + y^2 + 12x + 4y + 31 = 0$

and $x^2 + y^2 + 6x + 12y + 36 = 0$ is $\left(3 + \frac{5}{36}\sqrt{\lambda}\right)$, then the value of λ must be

69. Circle with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangent to these circles at their point of contact. And the distance of P from point of contact is d. Then [D] is.

70. The line $Ax + By + C = 0$ cuts the circle $x^2 + y^2 + ax + by + c = 0$ in P and Q. The lines $A'x + B'y + C' = 0$ cuts the circle $x^2 + y^2 + a'x + b'y + c' = 0$ in R and S. If P, Q, R, S are concyclic, then the value of

$$\begin{vmatrix} a-a' & b-b' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix}$$ is

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Answer Questions

1. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is : **(1980)**

(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$

(c) $x^2 + y^2 - 4y + 2 = 0$ (d) none of these

2. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is : **(1983)**

(a) $4x^2 + 4y^2 - 30x - 10y = 25$

(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$

(d) none of these

3. The centre of the circle passing through the point (0,1) and touching the curve $y = x^2$ at (2, 4) is : **(1983)**

(a) $\left(-\frac{16}{5}, \frac{27}{10}\right)$ (b) $\left(-\frac{16}{7}, \frac{53}{10}\right)$

(c) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ (d) none of these

4. *AB* is a diameter of a circle and *C* is any point on the circumference of the circle. Then : **(1983)**

(a) the area of ΔABC is maximum when it is isosceles

(b) the area of ΔABC is minimum when it is isosceles

(c) the perimeter of ΔABC is minimum when it is isosceles

(d) none of these

5. The locus of the mid-point of a chord of the circle $x^2+y^2 = 4$ which subtends a right angle at the origin, is : **(1984)**

(a) $x + y = 2$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = 2$ (d) $x + y = 1$

6. The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is : **(1992)**

(a) (3/2, 1/2) (b) (1/2, 3/2)

(c) (1/2, 1/2) (d) $\left(\frac{1}{2} \pm \sqrt{2}\right)$

7. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis is given by the equation : **(1993)**

(a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$

8. The angle between a pair of tangents drawn from a point *P* to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point *P* is : **(1996)**

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$

(c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$

9. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is : **(1998)**

(a) 0 (b) 1

(c) 3 (d) 4

10. If two distinct chords, drawn from the point (*p, q*) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then : **(1999)**

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$

(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

11. The triangle *PQR* is inscribed in the circle $x^2 + y^2 = 25$. If *Q* and *R* have coordinates (3, 4) and (–4, 3) respectively, then $\angle QPR$ is equal to : **(2000)**

(a) $\pi/2$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/6$

12. If the circle $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then *k* is : **(2000)**

(a) 2 or –3/2 (b) –2 or –3/2

(c) 2 or 3/2 (d) –2 or 3/2

13. Let *AB* be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle *PAB* as *P* moves on the circle is : **(2001)**

(a) a parabola (b) a circle

(c) an ellipse (d) a pair of straight lines.

14. Let *PQ* and *RS* be tangents at the extremities of the diameter *PR* of a circle of radius *r*. If *PS* and *RQ* intersect at a point *X* on the circumference of the circle, then 2*r* equals. **(2001)**

(a) $\sqrt{PQ.RS}$ (b) $\frac{PQ + RS}{2}$

(c) $\frac{2PQ \cdot RS}{PQ + RS}$ (d) $\sqrt{\frac{PQ^2 + RS^2}{2}}$

15. If the tangent at the point P on the circle $x^2+y^2+6x+6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is : **(2002)**

(a) 4 (b) $2\sqrt{5}$

(c) 5 (d) $3\sqrt{5}$

16. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is : **(2003)**

(a) (4, 7) (b) (7, 4)

(c) (9, 4) (d) (4, 9)

17. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is : **(2004)**

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) 3 (d) 2

18. The locus of the centre of circle which touches $(y-1)^2 + x^2 = 1$ externally and also touches x axis is : **(2005)**

(a) $x^2 = 4y \cup (0, y), y < 0$

(b) $x^2 = y$

(c) $y = 4x^2$

(d) $y^2 = 4x \cup (0, y), y \in R$

19. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is **(2012)**

(a) $20(x^2 + y^2) - 36x + 45y = 0$

(b) $20(x^2 + y^2) + 36x - 45y = 0$

(c) $36(x^2 + y^2) - 20x + 45y = 0$

(d) $36(x^2 + y^2) + 20x - 45y = 0$

Use the following passage, solve Q. 20 and 21

PASSAGE

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3}, 1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

20. A common tangent of the two circles is **(2012)**

(a) $x = 4$ (b) $y = 2$

(c) $x + \sqrt{3}y = 4$ (d) $x + 2\sqrt{2}y = 6$

21. A possible equation of L is **(2012)**

(a) $x - \sqrt{3}y = 1$ (b) $x + \sqrt{3}y = 1$

(c) $x - \sqrt{3}y = -1$ (d) $x + \sqrt{3}y = 5$

Use the following passage, solve Q. 22 to 24

PASSAGE

Let ABCD be a square of side length 2 unit. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of square ABCD. L is the line through A.

22. If P is a point of C_1 and Q is a point on C_2, then $\dfrac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to **(2006)**

(a) 0.75 (b) 1.25

(c) 1 (d) 0.5

23. A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is **(2006)**

(a) ellipse (b) hyperbola

(c) parabola (d) parts of straight line

24. A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1 T_2 T_3$ is **(2006)**

(a) $\dfrac{1}{2}$ sq unit (b) $\dfrac{2}{3}$ sq unit

(c) 1 sq unit (d) 2 sq unit

25. Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is : **(2007)**

(a) 3 (b) 2

(c) $\dfrac{3}{2}$ (d) 1

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Use the following passage, solve Q. 26 to Q. 28

PASSAGE

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

26. The equation of circle C is **(2008)**

(a) $(x-2\sqrt{3})^2 + (y-1)^2 = 1$

(b) $(x-2\sqrt{3})^2 + \left(y+\frac{1}{2}\right)^2 = 1$

(c) $(x-\sqrt{3})^2 + (y+1)^2 = 1$

(d) $(x-\sqrt{3})^2 + (y-1)^2 = 1$

27. Points E and F are given by **(2008)**

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

28. Equations of the sides QR, RP are **(2008)**

(a) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$

(b) $y = \frac{1}{\sqrt{3}}x, y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$

(d) $y = \sqrt{3}x, y = 0$

29. Tangents drawn from the point P (1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is **(2009)**

(a) $x^2 + y^2 + 4x - 6y + 19 = 0$

(b) $x^2 + y^2 - 4x - 10y + 19 = 0$

(c) $x^2 + y^2 - 2x + 6y - 29 = 0$

(d) $x^2 + y^2 - 6x - 4y + 19 = 0$

PASSAGE 30 and 31

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant. **(2016)**

30. The orthocentre of the triangle F_1MN is

(a) $\left(-\frac{9}{10}, 0\right)$ (b) $\left(\frac{2}{3}, 0\right)$

(c) $\left(\frac{9}{10}, 0\right)$ (d) $\left(\frac{2}{3}, \sqrt{6}\right)$

31. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(a) 3 : 4 (b) 4 : 5

(c) 5 : 8 (d) 2 : 3

PASSAGE 32 and 33

Let S be the circle in the xy-plane defined by the equation $x_2 + y_2 = 4$. **(2018)**

32. Let $E_1 E_2$ and F_1F_2 be the chords of S passing through the point P0 (1, 1) and parallel to the x-axis and the y-axis, respectively. Let $G_1 G_2$ be the chord of S passing through P 0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve

(a) $x + y = 4$ (b) $(x-4)^2 + (y-4)^2 = 16$

(c) $(x-4)(y-4) = 4$ (d) $xy = 4$

33. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(a) $(x+y)^2 = 3xy$ (b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2^{\frac{4}{3}}$

(c) $x^2 + y^2 = 2xy$ (d) $x^2 + y^2 = x^2y^2$

Multiple Question

34. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **(2013)**

(a) $x^2 + y^2 - 6x + 8y + 9 = 0$ (b) $x^2 + y^2 - 6x + 7y + 9 = 0$

(c) $x^2 + y^2 - 6x - 8y + 9 = 0$ (d) $x^2 + y^2 - 6x - 7y + 9 = 0$

35. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **(2014)**

(a) radius of S is 8 (b) radius of S is 7

(c) centre of S is (–7, 1) (d) centre of S is (–8, 1)

36. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **(2016)**

(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

37. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then. **(2016)**

(a) $Q_2Q_3 = 12$

(b) $R_2R_3 = 4\sqrt{6}$

(c) area of the triangle OR_2R_3 is $6\sqrt{2}$

(d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Match the Column

38. Match the conditions/expressions in Column I with statement in Column II. **(2007)**

Column I	Column II
(A) Two intersecting circles	(p) have a common tangent
(B) Two mutually external circles.	(q) have a common normal
(C) Two circles, one strictly inside the other	(r) do not have a common tangent
(D) Two branches of a hyperbola	(s) do not have a common normal

Assertion Reason

39. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Assertion : The tangents are mutually perpendicular.

Reason : The locus the points from which a mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$. **(2007)**

(a) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(b) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(c) If ASSERTION is true, REASON is false

(d) If ASSERTION is false, REASON is true

40. Consider

$$L_1 : 2x + 3y + p - 3 = 0$$
$$L_2 : 2x + 3y + p + 3 = 0$$

where p is a real number and

$$C : x^2 + y^2 - 6x + 10y + 30 = 0$$

Assertion : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C. and

Reason : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C. **(2008)**

(a) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(b) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(c) If ASSERTION is true, REASON is false

(d) If ASSERTION is false, REASON is true

41. Consider the two curves

$C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 - 6x + 1 = 0$, then **(2008)**

(a) C_1 and C_2 touch each other only at one point

(b) C_1 and C_2 touch each other exactly at two points

(c) C_1 and C_2 intersect (but do not touch) at exactly two points

(d) C_1 and C_2 neither intersect nor touch each other

Subjective

42. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 unit from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangents to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is ...... **(2009)**

43. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d, prove that the area of the circle is

$$\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$ **(1996)**

44. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line drawn from the point P intersect the curve at points Q and R. If the product PQ . PR is independent of the slope of the line, then show that the curve is a circle. **(1997)**

45. Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on C.

(A rational point is a point both of whose coordinates are rational numbers.) **(1997)**

46. C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. **(1998)**

47. Let T_1, T_2 and be two tangents drawn from (–2, 0) onto the circle $C : x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles when taken two at a time. **(1999)**

48. Consider the family of circles $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the coordinate axes at A and B, then find the equation of the locus of the mid points of AB. **(1999)**

49. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **(2001)**

50. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. **(2001)**

51. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at the point $(1, -1)$ and is orthogonal to the circle which has the line segment having end points $(0, -1)$ and $(-2, 3)$ as the diameter. **(2004)**

52. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ? **(2017)**

True/False

53. No tangent can be drawn from the point (5/2, 1) to the circumcircle of the triangle with vertices $(1, \sqrt{3}), (1, -\sqrt{3}), (3, \sqrt{3})$ **(1985)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (b)	2. (b)	3. (d)	4. (b)	5. (d)	6. (b)	7. (c)	8. (d)	9. (d)	10. (b)
11. (c)	12. (c)	13. (b)	14. (b)	15. (c)	16. (a)	17. (b)	18. (a)	19. (d)	20. (b)
21. (c)	22. (c)	23. (d)	24. (d)	25. (b)	26. (a)	27. (c)	28. (c)	29. (d)	30. (d)
31. (d)	32. (a)	33. (a)	34. (b)	35. (b)	36. (a)	37. (c)	38. (d)	39. (b)	40. (b)
41. (b)	42. (b)	43. (c)	44. (c)	45. (a)	46. (a)	47. (c)	48. (c)	49. (b)	50. (a,d)
51. (d)	52. (c)	53. (b)	54. (a)	55. (d)	56. (c)	57. (b)	58. (a)	59. (c)	60. (a)
61. (d)	62. (c)	63. (a)	64. (b)	65. (b)	66. (a)	67. (a)	68. (b)	69. (c)	70. (a)

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (b)	2. (d)	3. (a)	4. (b)	5. (c)	6. (d)	7. (b)	8. (b)	9. (c)	10. (c)
11. (c)	12. (a)	13. (c)	14. (c)	15. (c)	16. (d)	17. (d)	18. (d)	19. (d)	20. (c)
21. (b)	22. (c)	23. (a)	24. (a)	25. (c)	26. (a)	27. (d)	28. (b)	29. (d)	30. (b)
31. (a)	32. (a)	33. (b)	34. (a)	35. (b)	36. (b)	37. (c)	38. (b)	39. (b)	40. (b)
41. (d)	42. (a)	43. (d)	44. (a)						

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (a)	2. (b)	3. (a)	4. (b)	5. (a)	6. (b)	7. (b)	8. (a)	9. (c)	10. (c)
11. (d)	12. (d)	13. (d)	14. (b)	15. (a)	16. (d)	17. (a)	18. (b)	19. (c)	20. (b)
21. (c)	22. (b)	23. (b)	24. (c)	25. (c)	26. (c)	27. (a)	28. (a)	29. (a)	30. (c)
31. (b)	32. (d)	33. (a,c)	34. (d)	35. (a)	36. (c)	37. (b)	38. (c)	39. (b)	40. (d)
41. (c)	42. (a,c,d)	43. (a,c,d)	44. (b,d)	45. (b,c)	46. (a,d)	47. (a, d)	48. (a, c)	49. (a, b)	50. (b,d)
51. (b,c)	52. (a)	53. (d)	54. (c)	55. (a)	56. (d)	57. (A–r; B–s; C–q; D–p)			
58. (A–q; B–s; C–p; D–r)			59. A–S; B–Q; C–T; D–P						
60. (d)	61. (b)	62. (a)	63. (b)	64. (d)	65. (b)	66. (0171)	67. (0001)	68. (0949)	
69. 2	70. 0								

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b)	2. (b)	3. (c)	4. (a)	5. (c)	6. (d)	7. (d)	8. (d)	9. (b)	10. (d)
11. (c)	12. (a)	13. (b)	14. (a)	15. (c)	16. (a)	17. (c)	18. (a)	19. (a)	20. (d)
21. (a)	22. (a)	23. (c)	24. (c)	25. (b)	26. (d)	27. (a)	28. (d)	29. (b)	30. (a)
31. (c)	32. (a)	33. (d)	34. (a,c)	35. (b,c)	36. (a,c)	37. (a,b,c)	38. (A–p,q; B–p,q; C–q,r; D–q,r)		
39. (a)	40. (c)	41. (b)	42. (8)	47. $\left(x+\frac{4}{3}\right)^2+y^2=\left(\frac{1}{3}\right)^2$ and $(x-4)^2+y^2=(3)^2$				48. $4x^2+25y^2=4x^2y^2$	
49. $3\left(3+\sqrt{10}\right)$		50. Ellipse		51. $2x^2+2y^2-10x-5y+1=0$			52. (2)	53. True	

Dream on !!