Properties and Solution of Triangles

Type – 1

Choose the most appropriate option (a, b, c or d).

Q 1. In a ΔABC, $\frac{c+b}{c-b}.\tan\frac{A}{2}$ is equal to

(a) $\tan\left(\frac{A}{2}+B\right)$ (b) $\cot\left(\frac{A}{2}+B\right)$ (c) $\tan\left(A+\frac{B}{2}\right)$ (d) none of these

Q 2. If the area of a ΔABC be L then $a^{2+}\sin 2B + b^2 \sin 2A$ is equal to

(a) 2λ (b) λ (c) 4λ (d) none of these

Q 3. If k be the perimeter of the ΔABC then $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$ is equal to

(a) k (b) 2k (c) $\frac{k}{2}$ (d) none of these

Q 4. If R denotes circumradius then in ΔABC, $\frac{b^2-c^2}{2aR}$ is equal to

(a) $\cos(B - C)$ (b) $\sin(B - C)$ (c) $\cos B - \cos C$ (d) none of these

Q 5. In a ΔABC, $\cot\frac{A-B}{2}.\tan\frac{A+B}{2}$ is equal to

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$ (c) $\frac{a(a-b)}{b(a+b)}$ (d) none of these

Q 6. In a ΔABC, $(c + a + b)(a + b - c) = ab$. The measure of $\angle C$ is

(a) 2b (b) 2c (c) 3b (d) 3a

Q 7. In a ΔABC, A : B : C = 3 : 5 : 4. Then $a + b\ c\sqrt{2}$ is equal to

(a) 2b (b) 2c (c) 3b (d) 3a

Q 8. The equation $ax^2 + bx + c = 0$, where a,b,c are the sides of aΔABC, and the equation $x^2 + \sqrt{2}x + 1 = 0$ have a common root. The measure of $\angle C$ is

(a) $90°$ (b) $45°$ (c) $60°$ (d) none of these

Q 9. If in a ΔABC, $a^2\cos^2 A = b^2 + c^2$ then

(a) $A < \frac{\pi}{4}$ (b) $\frac{\pi}{4} < A < \frac{\pi}{2}$ (c) $A > \frac{\pi}{2}$ (d) $A = \frac{\pi}{2}$

Q 10. If in a ΔABC, $\tan\frac{A}{2}$ and $\tan\frac{B}{2}$ satisfy $6x^2 - 5x + 1 = 0$. Then

(a) $a^2 + b^2 > c^2$ (b) $a^2 - b^2 = c^2$ (c) $a^2 + b^2 = c^2$ (d) none of these

Q 11. Two sides of a triangle are given by the roots of the equation $x^2 - 2\sqrt{3}x + 2 = 0$. The angle between the side is $\pi/3$. The perimeter of the triangle is

(a) $6+\sqrt{3}$ (b) $2\sqrt{3}+\sqrt{6}$ (c) $2\sqrt{3}+\sqrt{10}$ (d) none of these

Q 12. The side of a ΔABC are $AB = \sqrt{13}cm, BC4\sqrt{3}cm$ and CA = 7 cm. Then sin θ, where θ is the smallest angle of the triangle, is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (d) none of these

Q 13. Two sides of a triangle are $2\sqrt{2}$ cm and $2\sqrt{3}$ cm and the angle opposite to the shorter side of the two is $\pi/4$. The largest possible length of the third side is

(a) $\sqrt{2}(\sqrt{3}+1)$cm (b) $(6+\sqrt{2})$cm (c) $(\sqrt{6}-\sqrt{2})$cm (d) none of these

Q 14. In a αABC, a = 8, b = 10 and c = 12. The C is equal to

(a) $\frac{A}{2}$ (b) 2A (c) 3A (d) none of these

Q 15. In ΔABC, if tan $\frac{A}{2} = \frac{5}{6}$ and tan $\frac{B}{2} = \frac{20}{37}$ then

(a) 2a = b + c (b) a > b > c (c) 2c = a + b (d) none of these

Q 16. In a ΔABC, a = 2b and |A – B| = $\frac{\pi}{3}$. The measure of $\angle C$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) none of these

Q 17. In a ΔABC, the tangent of half the difference of two angles is one–third the tangent of half the sum of the two angles. The ratio of sides opposite the angles is

(a) 2 : 3 (b) 1 : 3 (c) 1 : 2 (d) 3 : 4

Q 18. In a $\Delta ABC, A = \frac{2\pi}{3}, b - c = 3\sqrt{3}$ cm and ar(ΔABC)= $\frac{9\sqrt{3}}{2}$ cm^2. Then a is

(a) $6\sqrt{3}$cm (b) 9cm (c) 18cm (d) none of these

Q 19. The are of a ΔABC is $a^2 - (b - c)^2$. Then tan A is equal to

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{8}{15}$ (d) none of these

Q 20. In a ΔABC, B = 90°, AC = h and the length of the perpendicular from B to AC is p such that h = 4p. If AB < BC then $\angle C$ has the measure

(a) $\frac{5\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) none of these

Q 21. In a ΔABC, a = 5, b = 4 and tan $\frac{C}{2} = \sqrt{\frac{7}{9}}$. The side c is

(a) 6 (b) 3 (c) 2 (d) none of these

Q 22. If in a ΔABC, AC = 12, BC = 13 and AB = 5, then the distance of A from BC is

(a) $\frac{25}{13}$ (b) $\frac{60}{13}$ (c) $\frac{65}{12}$ (d) none of these

Q 23. In a ΔABC, $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$. The value of cos C can be

(a) $\frac{7}{13}$ (b) $\frac{12}{13}$ (c) $\frac{33}{65}$ (d) none of these

Q 24. In a ΔABC, $B = \frac{\pi}{8}$ and $C = \frac{5\pi}{8}$. The altitude from A to the side BC is

(a) $\frac{a}{2}$ (b) 2a (c) $\frac{1}{2}(b+c)$ (d) none of these

Q 25. Two angles of a triangle are $\frac{\pi}{6}$ and $\frac{\pi}{4}$, and the length of the included side is $(\sqrt{3}+10$ cm. The are of the triangle is

(a) $\frac{\sqrt{3}-1}{2}cm^2$ (b) $\frac{\sqrt{3}}{2}cm^2$ (c) $\frac{\sqrt{3}+1}{2}cm^2$ (d) none of these

Q 26. If in a ΔABC, $\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A . \sin C$, then the value of the value of determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) 0 (b) $(a + b + c)^3$ (c) $(a + b + c)(ab + bc + ca)$ (d) none of these

Q 27. In a ΔABC, $A = 90°$. Then $\tan^{-1}\frac{b}{a+c} + \tan^{-1}\frac{c}{a+b}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\tan^{-1}\frac{a}{b+c}$ (d) none of these

Q 28. If in a ΔABC, the values of cot A, cot B, cot C are in AB, then
(a) a, b, c are in AP (b) a^2, b^2, c^2 are in AP (c) cos A, cos B, cos C are in AP (d) none of these

Q 29. If in a ΔABC, $\frac{a}{\cos A} = \frac{b}{\cos B}$, then

(a) $2\sin A \sin B \sin C = 1$ (b) $\sin^2 A + \sin^2 B = \sin^2 C$ (c) $2\sin A \cos B = \sin C$ (d) none of these

Q 30. If the sides of a triangle are in GP and the largest angle is twice the smallest angle then the common ratio, which is greater than 1, lies in the interval

(a) $(1, \sqrt{3})$ (b) $(1, \sqrt{3})$ (c) $\left(1, \frac{\sqrt{5}+1}{2}\right)$ (d) none of these

Q 31. In a ΔABC, the sides a,b and c are such that they are the roots of $x^3 - 11x^2 + 38x - 40 = 0$. Then $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is equal to

(a) $\frac{3}{4}$ (b) 1 (c) $\frac{9}{16}$ (d) none of these

Q 32. If cos A + cos B + 2 cos C = 2 then the sides of the ΔABC are in

(a) AP (b) GP (c) HP (d) none of these

Q 33. If in the ΔABC, the incentre is the middle point of the median AD then cos A has the value

(a) $\frac{7}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{2}}$

Q 34. If in a ΔABC, 3a = b + c then $\tan\frac{B}{2}$. Tan $\frac{C}{2}$ is equal to

(a) $\tan\frac{A}{2}$ (b) 1 (c) 2 (d) none of these

Q 35. If in a ΔABC, 2cos A sin C = sin B then the triangle is

(a) equilateral (b) isosceles (b) right angled (d) none of these

Q 36. In a ΔABC, a = 1 and the perimeter is six times the AM of the sines of the angle. The measure of $\angle$A is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Q 37. In a ΔABC, $\angle$A> <B. If sin A and sin B satisfy the equation $3\sin x - 4\sin^3 x - k =- 0$, $0 < k < 1$, then $\angle$C is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

Q 38. In a ΔABC the side a, b and c are in AP. Then $\left(\tan\frac{A}{2}+\tan\frac{C}{2}\right):\cot\frac{B}{2}$ is

(a) 3 : 2 (b) 1 : 2 (c) 3 : 4 (d) none of these

Q 39. If the sides of a triangle are proportional to the cosines of the opposite angles then the triangle is

(a) right angled (b) equilateral (c) obtuse angled (d) none of these

Q 40. If in a ΔABC, $\cos^2\frac{A}{2}$ + a $\cos^2\frac{C}{2}=\frac{3b}{2}$, then a, b, c are in

(a) GP (b) HP (c) AP (d) none of these

Q 41. The side of a triangle are in AP and its are is $\frac{3}{5}$ x(area of an equilateral triangle of the same perimeter). Then the ratio of the sides of

(a) 1 : 2 : 3 (b) 3 : 5 : 7 (c) 3 : 4 (d) none of these

Q 42. If BD, BE and CF are the medians of a ΔABC then

(a) 1 : 2 : 3 (b) 3 : 5 : 7 (c) 1 : 3 : 5 (d) none of these

Q 43. sin A, sin B sin C are in AP for the ΔABC Then

(a) the altitudes are in AP (b) the altitudes are in HP

(c) The medians are in GP (d) the medians are in AP

Q 44. AD is median of the ΔABC. If AE and AF are medians of the triangles ABD and ADC respectively, and AD = m_1, AE = m_2, AF = m_3, then $\frac{a^2}{8}$ is

(a) $m_2^2 + m_3^2 - 2m_1^2$ (b) $m_1^2 + m_2^2 - 2m_3^2$ (c) $m_2^2 + m_3^2 - m_1^2$ (d) none of these

Q 45. The ratio of the distance of the orthocenter of an acute–angled ΔABC from the sides BC, AC and AB is

(a) cos A : cos B : cos C (b) sin A : sin B : sin C (c) sec A : sec B : sec C (d) none of these

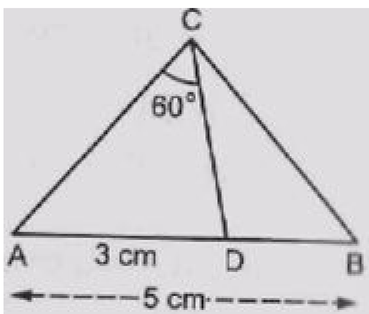
Q 46. In a ΔABC, I is the incentre. The ratio IA : IB : IC is equal to

(a) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$ (b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

(c) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$ (d) none of these

Q 47. In the figure, ABC is a triangle in which C = 90° and AB = 5 cm.

D is a point on AB such that AD = 3 cm and $\angle$ACD = =60°. Then the length of AC is



(a) $5\sqrt{\frac{3}{7}}$cm (b) $\sqrt{\frac{7}{3}}$cm (c) $\frac{3}{\sqrt{7}}$cm (d) none of these

Q 48. If α, β, γ are the altitudes of ΔABC and 2s denotes its perimeter then $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is equal to

(a) $\frac{\Delta}{s}$ (b) $\frac{s}{\Delta}$ (c) s.Δ (d) none of these

Q 49. In a ΔABC, the sides are in the ratio 4 : 5 : 6. The ratio of the circumradius and the inradius is

(a) 8 : 7 (b) 3 : 2 (c) 7 : 3 (d) 16 : 7

Q 50. In a ΔABC, R = circumradius and r = inradius. The value of is equal to $\frac{a\cos A + b\cos B + \cos C}{a + b + c}$

(a) $\frac{R}{r}$ (b) $\frac{R}{2r}$ (c) $\frac{r}{R}$ (d) $\frac{2r}{R}$

Q 51. In a ΔABC, 2s = perimeter and R = circumradius. Then s/R is equal to

(a) sin A + sin B + sin C (b) cos A + cos B + cos C

(c) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$ (d) none of these

Q 52. The ratio of the circumradius and inradius of an equilateral triangle is

(a) 3 : 1 (b) 1 : 1 (c) 2 : $\sqrt{3}$ (d) 2 : 1

Q 53. If in a ΔABC, $a^2 + b^2 + c^2 = 8R^2$, where R = circumradius, then the triangle is

(a) equilateral (b) isosceles (c) right angled (d) none of these

Q 54. In an equilateral triangle, (circumradius): (inradius): (exradius) is equal to

(a) 1 : 1 : 1 (b) 1 : 2 : 3 (c) 2 :1 : 3 (d) 3 : 2 : 4

Q 55. The diameter of the circumcircle of a triangle with sides 5 cm, 6 cm and 7 cm is

(a) $\frac{3\sqrt{6}}{2}$ cm (b) $2\sqrt{6}$ cm (c) $\frac{35}{48}$ cm (d) none of these

Q 56. If in a triangle, R and r are the circumradius and inradius respectively then the HM of the exradii of the triangle is

(a) 3r (b) 2R (c) R + r (d) none of these

Q 57. The angles of a right-triangle are in AP. The ratio of the inradius and the perimeter is

(a) $(2-\sqrt{3}):2\sqrt{3}$ (b) $1:8\sqrt{3}(2+\sqrt{3})$ (c) $(2+\sqrt{3}):4\sqrt{3}$ (d) none of these

Q 58. A ΔABC is right angled at B. Then the diameter of the incircle of the triangle is

(a) 2(c + a – b) (b) c + a – 2b (c) c + a – b (d) none of these

Q 59. If the exradii of a triangle are in HP then corresponding sides are in

(a) AP (b) GP (c) HP (d) none of these

Q 60. In a ΔABC, the inradius and three exradii are r, r_1, r_2 and r_3 respectively In usual notations the value of r : r_1. r_2. r_3 is equal to

(a) 2Δ (b) Δ^2 (c) $\frac{abc}{4R}$ (d) none of these

Q 61. In a ΔABC, the perimeter = 2s and the exradii are r_1, r_2 and r_3. Then $r_1r_2 + r_2r_3 + r_3r_1$ is equal to

(a) s^2 (b) $2s^2$ (c) $3s^2$ (d) $4s^2$

Q 62. In a ΔABC, tan A.tan B. tan C = 9. For such triangles, if then $\tan^2$ A + $\tan^2$ B + tan+2 C = k

(a) $9.\sqrt[3]{3} < k < 27$ (b) $k \not\leq 27$ (c) $k < 9.\sqrt[3]{3}$ (d) $k \not< 27$

Q 63. If the ΔABC is acute angled at C then

(a) cos 2A + cos 2B $-$ cos 2C < 1 (b) cos 2A + cos 2B + cos 2C < 1

(c) $\cos^2$ A + $\cos^2$B + $\cos^2$ C < 1 (d) none of these

Q 64. If for a ΔABC, cot A . cot B. cot C > 0 then the triangle is

(a) right angled (b) acute angled (c) obtuse angled (d) all these potions are possible

Q 65. In a ΔABC, cos B. cos C + sin B. sin B. sin C. $\sin^2$ A = 1. Then the triangle is

(a) right–angled isosceles (b) isosceles whose equal angle are greater than $\pi/4$

(c) equilateral (d) none of these

Q 66. In a ΔABC, the angles A and B are two values of θ satisfying $\sqrt{3}\cos\theta + \sin\theta = k$, k|k| < 2. The triangle

(a) is acute angled (b) is right angled (c) is obtuse angled (d) has one angle = $\pi/3$

Q 67. If in an obtuse–angled triangle the obtuse angle is $3\pi/4$ and the other two angles are equal to two values of θ satisfying atan θ + bsec θ = c, where $|b| \leq \sqrt{a^2+c^2}$,then $a^2 - c^2$ is equal to

(a) ac (b) 2ac (c) a/c (d) none of these

Q 68. Let A_0, A_1, A_2, A_3, A_4 and A_5 be the consecutive vertices of a regular hexagon inscribed in a unit circle. The product of the lengths of A_0A_1, A_0A_2 and A_0A_4 is

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$ (c) 3 (d) $\frac{3\sqrt{3}}{2}$

Q 69. The area of a circle is A_1 and the area of a regular pentagon inscribed in the circle is A_2. Then $A_1 : A_2$ is

(a) $\frac{\pi}{5}\cos\frac{\pi}{10}$ (b) $\frac{2\pi}{5}\sec\frac{\pi}{10}$ (c) $\frac{2\pi}{5}\operatorname{cosec}\frac{\pi}{10}$ (d) none of these

Q 70. The are of a cyclic quadrilateral ABCD is $(3\sqrt{3})/4$. The radius of the circle circumscribing it is 1. If AB = 1, BD = $\sqrt{3}$ then BC. CD is equal to

(a) 2 (b) $3-\frac{1}{\sqrt{3}}$ (c) $3\sqrt{3}+1$ (d) none of these

Type 2

Choose the correct options. One or more options may be correct.

Q 71. If in aAABC, a = 6, b = 3 and cos(A - B)= $\frac{4}{5}$ then

(a) $C = \frac{\pi}{4}$ (b) $A = \sin^{-1}\frac{2}{\sqrt{5}}$ (c) ar(ΔABC) = 9 (d) none of these

Q 72. The number of possible triangles ABC in which BC = $\sqrt{11}$ cm, CA = $\sqrt{13}$ cm and A = 60° is

(a) 0 (b) 1 (c) 2 (d) none of these

Q 73. In a ΔABC, A = $\frac{\pi}{3}$ and b : c = 2 :3. If $\tan\alpha = \frac{\sqrt{3}}{5}, 0 < \alpha = \frac{\pi}{2}$, then

(a) B = 60° + α (b) C-60° + α (c) B = 60°- α (d) C = 60°- α

Q 74. In a triangle the cosines of two angles are inversely proportional to the sides opposite the angles. The triangle is

(a) isosceles (b) equilateral (c) right angled (d) none of these

Q 75. In a ΔABC, the line segments AD, BE and CF are three altitudes. If R is the circumradius of the ΔABC, a side of the ΔDEF will be

(a) Rsin 2A (b) ccos B (c) asin A (d) bcos B

Q 76. In a ΔABC, tan A and tan B are the roots of the equation $ab(x^2 +1) = c^2x$, where a, b and c are the sides of the triangle. Then

(a) $\tan(A - B) = \frac{a^2-b^2}{2ab}$ (b) cot C = 0 (c) $\sin^2 A + \sin.^2 B = 1$ (d) none of these

Q 77. The distances of the circumcentre of the acute-angled ΔABC from the sides BC, CA and AB are in the ratio

(a) asin A : bsinB: Ccsin,C (b) cosA : cos B: cos C

(c) acot A: bcot B:ccotC (d) none of these

Q 78. In any ΔABC, $\Sigma a^3 \sin(B - C)$ is equal to

(a) $\Sigma a(\sin B - \sin C)$ (b) $\Sigma a^2(\cos^2 B - \cos^2 C)$ (c) 0 (d) none of these

Q 79. In aΔABC, $2\cos\frac{A-C}{2} = \frac{a+c}{\sqrt{a^2+c^2-ac}}$. Then

(a) $B = \frac{\pi}{3}$ (b) B = C (c) A,B,C are in AP (d) B + C = A

Q 80. Let Lsin 0 = 10 + log sin 0. The number of triangles ABC such that log b + 10 = log c + Lsin B is

(a) one (b) two (c) infinite (d) none of these

Q 81. In any triangle ABC, $\sin\frac{A}{2}$ is

(a) less than $\frac{a}{b+c}$ (b) less than or equal to $\frac{b+c}{a}$

(c) greater than $\frac{2a}{a+b+c}$ (d) none of these

Q 82. In a ΔABC, tan C < 0. Then

(a) tan A . tan B < 1 (b) tan A tan B > 1

(c) tan A + tan B + tan C < 0 (d) tan A + tan B + tan C > 0

Q 83. In a ΔABC, tan A and tan B satisfy the inequation $3x^2 - 4x + 3 < 0$ Then

(a) $a^2 + b^2 + ab > c^2$ (b) $a^2 + b^2 - ab < c^2$ (c) $a^2 + b^2 > c^2$ (d) none of these

Q 84. In a ΔABC, cosA + cos B + cosC > 1 only if the triangle is

(a) acute angled

(b) obtuse angled

(c) right angled

(d) the nature of the triangle caanot be detemined

Answers

1a	2c	3c	4b	5a	6c	7c	8b	9c	10c
11b	12b	13a	14b	15b	16b	17c	18b	19c	20c
21a	22b	23c	24a	25c	26a	27a	28b	29c	30b
31c	32a	33b	34d	35b	36c	37c	38d	39b	50c
41b	42c	43b	44a	45c	46a	47a	48b	49d	50c
51a	52d	53c	54c	55d	56a	57a	58c	59a	60b
61a	62b	63a	64b	65a	66c	67b	68c	69b	70a
71b,c	72c	73b,c	74a,c	75a,d	76a,b,c	77b,c	78a,b,c	79a,c	80a
81b	82a,c	83a,b	84d						