Chapter 2

Units and Measurements

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. Name two derived quantities.

Sol. Energy, Potential difference

2. Express 1 N in CGS system.

Sol. 1 kg m s^{-2} = 1 × (1000 g) × (100 cm) × (1 s^{-2})

= 10^5 g cm s^{-2}

3. Express 1 J in CGS system.

Sol. 1 kg m^2 s^{-2}

= 1 × (10^3 g) (10^2) cm^2 s^{-2}

= 10^7 g cm^2 s^{-2}

4. What unit does a relative error in a measurement have?

Sol. Relative error is the ratio of two quantities having similar units, hence has no units.

5. Is it possible that a dimensionless quantity has unit?

Sol. Yes, a solid angle and a plane angle have the units steradian and radian, but are dimensionless.

6. How many zeroes appearing in the number 0.0040500, are significant?

Sol. Three

7. What error does a vernier callipers have when zero mark of vernier scale does not coincide with zero mark of main scale?

Sol. Zero error

8. Write the dimensional formula for power.

Sol. $[ML^2T^{-3}]$

9. Round off the number 9.8953 to three significant figures.

Sol. 9.90

10. Write 623.5 cm in scientific notation, and its order of magnitude.

Sol. 6.235×10^2 cm, Order of magnitude = 2

Short Answer Type Questions :

11. What is the relation between 1 AU and 1 parsec?

Sol. 1 AU = 1.496×10^{11} m

1 parsec = 3.08×10^{16} m

$$\therefore \quad \frac{1\,\text{parsec}}{1\,\text{AU}} = \frac{3.08 \times 10^{16}}{1.496 \times 10^{11}} = 2.06 \times 10^5$$

Thus $\boxed{1\,\text{parsec} = 2.06 \times 10^5\,\text{AU}}$

12. Name various instruments and methods used to estimate very small distances.

Sol. (i) Optical microscope : Limit of resolution 10^{-7} m.

(ii) Electron microscope : Limit of resolution 0.6×10^{-7} m.

(iii) Tunnelling microscope

(iv) Volumetric method

13. What principle is used in a mass spectrograph to estimate the mass of a charged particle?

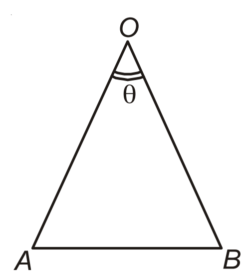
Sol. It is based on the principle that the radius of the trajectory of a charged particle moving in magnetic field is proportional to its mass.

14. What do you mean by parallax angle?

Sol. When a far off object *O*, is observed from two different positions *A* and *B* such that *AB* << *AO* or *BO*, Angle AOB is called the parallax angle θ.

15. Write the SI units of following quantities.

Gravitational constant, Refractive index, Solar constant, Electric current



Sol. Gravitational constant $\rightarrow$ N m^2 kg^{-2}

Refractive index $\rightarrow$ No units

Solar constant $\rightarrow$ J m^{-2} s^{-1}

Electric current $\rightarrow$ A

16. In a new system of units, the unit of mass is a kg, the units of length and time are respectively b m and c s. What is the magnitude of 6 W of power, in this system?

Sol. 6 W = 6 kg m^2 s^{-3}

Let it be equal to *x* in the new system

Thus $x\,(a\text{ kg})\,(b\text{m})^2\,(cs)^{-3} = 6\text{ kg m}^2\text{ s}^{-3}$

$$x = \frac{6\text{ kg m}^2\text{s}^{-3}}{ab^2c^{-3}\text{ kg m}^2\text{s}^{-3}}$$

$\Rightarrow$ $x = 6\,a^{-1}\,b^{-2}\,c^3$

17. A quantity X is given as $X = \frac{ab^2}{c^3}$. The percentage error in the measurement of a, b and c are ± 1%, ± 3% and ± 2% respectively. Find the percentage error in X.

Sol. $X = \frac{ab^2}{c^3}$

Percentage error in X, $\frac{\Delta X}{X} \times 100 = \frac{\Delta a}{a} \times 100 + \frac{2\Delta b}{b} \times 100 + \frac{3\Delta c}{c} \times 100$

= 1 + 2 × 3 + 3 × 2

= 13%

18. A distance of 100 cm is measured using a metre scale of least count 0.1 cm. Find the percentage error in the measurement.

Sol. Percentage error = $\frac{0.1}{100} \times 100$ = 0.1%

19. The mass of a beaker is found to be (35.6 ± 0.2) gram when empty, and (46.4 ± 0.2) gram when filled partially with a liquid. Find the mass of the liquid with proper uncertainty.

Sol. Mass of the liquid = Mass of filled beaker – Mass of empty beaker

= (46.4 ± 0.2) – (35.6 ± 0.2)

= (10.8 ± 0.4) g

20. Define least count of an instrument. What does the least count error indicate?

Sol. The smallest value that can be measured by a measuring instrument is called its least count. For example, a metre scale can accurately measure a minimum distance of 1 mm. Hence, the metre scale has a least count 1 mm.

The least count error indicates the inability of an instrument to measure a value lesser than its least count.

21. Write the number of significant figures in the following values.

(i) 2.340

(ii) 8.049

(iii) 36000

(iv) 0.0042

Sol. (i) 2.340 → Four significant figures

(ii) 8.049 → Four significant figures

(iii) 36000 → Two significant figures

(iv) 0.0042 → Two significant figures

22. Solve with due regard to significant figures.

$6.30 \times 10^{-3} + 4.2 \times 10^{-2}$

Sol. $6.30 \times 10^{-3} = 0.630 \times 10^{-2}$

$(4.2 + 0.630) \times 10^{-2}$

$= 4.830 \times 10^{-2}$

$= 4.8 \times 10^{-2}$

23. Round off the following numbers to three significant figures.

(i) 12.132 (ii) 2.1603

(iii) 120.82 (iv) 0.4265

Sol. (i) $12.132 \rightarrow 12.1$ (ii) $2.1603 \rightarrow 2.16$

(iii) $120.82 \rightarrow 121$ (iv) $0.4265 \rightarrow 0.426$

24. What is the importance of making rough estimates of physical quantities using common observations?

Sol. When it is not possible to measure a quantity precisely, we describe its magnitude using rough estimates or indirect methods.

For example, the distance between two cities cannot be measured precisely using a metre scale. It calls for a lot of human effort and hence, seems impractical. However we can estimate it roughly by measuring the speed of the vehicle and the time taken by it to cover the distance.

The knowledge of this distance can help us decide the ticket fare for trains and buses, or it can help us estimate the expenditure for laying a road between the two cities. This example shows the importance of making rough estimates of physical quantities.

25. A boy drops a ball from rest which reaches the ground in 2.1 s. Find the maximum height from where the ball is dropped to appropriate significant figures.

[given g = 9.8 ms^{-2}]

Sol. Ball reaches the ground in time t = 2.1 s

The height from where the ball is dropped,

$$H = \frac{1}{2}g(t)^2$$

$$= \frac{1}{2}\times 9.8\times(2.1)^2$$

$$= 21.609 \text{ m}$$

Rounding off to two significant figures, H = 22 m

26. Does dimensional consistency ensure the physical correctness of a mathematical equation?

Sol. No, dimensional consistency does not ensure the physical correctness of a mathematical relation.

For example; consider the two mathematical relations giving the absolute refractive index μ of a medium.

$$\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of ligh in the medium}};\ \mu = \frac{\text{Speed of light in medium}}{\text{Speed of light in vacuum}}$$

Checking dimensional consistency for both.

$$[\mu] = \frac{[LT^{-1}]}{[LT^{-1}]} = [M^0L^0T^0]\ ;\ [\mu] = [M^0L^0T^0]$$

Both the expressions are dimensionally consistent. But we know that the correct expression of μ is the first one.

27. What are the dimensional formulae for the quantities having following units?

(i) electron volt (ii) kW h

Sol. (i) Electron volt = [Charge] × $\frac{[\text{Work}]}{[\text{Charge}]}$ [Electron represents the charge on an electron]

$\Rightarrow$ [eV] = [Work]

= $[ML^2T^{-2}]$

(ii) kW h = $[ML^2T^{-3}]$ [T]

= $[ML^2T^{-2}]$

28. Can we use dimensional method to find an expression for gravitational force acting between two objects of mass m_1 and m_2?

Sol. A dimensional method can be used to find out the relation between quantities having different dimensions. The given example involves to find the dependence of force on two masses m_1 and m_2 which have similar dimension *i.e.* [M] Hence, this method cannot be used to find the empirical expression for gravitational force.

29. The viscous force acting between liquid layers of area A is given as

$$F = -\ \eta A\frac{dv}{dx}$$

If dv has dimensions of speed and dx has dimensions of distance, find the dimensions of coefficient of viscosity η.

Sol. $F = -\eta A\dfrac{d\nu}{dx}$.

Applying the principle of dimensional homogeneity, we get

$$[F] = \left[\eta A\frac{d\nu}{dx}\right]$$

$$\Rightarrow\ [F] = \frac{[\eta][A][d\nu]}{[dx]}$$

$$\Rightarrow\ [M\,L\,T^{-2}] = [\eta]\frac{[L^2][LT^{-1}]}{[L]}$$

$$\Rightarrow\ [MLT^{-2}] = [\eta][L^2T^{-1}]$$

$$[\eta] = [ML^{-1}\,T^{-1}]$$

30. Check the dimensional consistency of the following relation.

$v = \dfrac{mgr}{\eta}$; where , v = Terminal velocity of a ball

r = Radius of the ball, g = Acceleration due to gravity

η = Coefficient of viscosity, m = Mass of ball

Sol. $v = \dfrac{mgr}{\eta}$

Here v = terminal velocity of a ball

r = radius of the ball

η = coefficient of viscosity

g = acceleration due to gravity

L.H.S. $\rightarrow [v] = [LT^{-1}]$

R.H.S. $\rightarrow \left[\dfrac{mgr}{\eta}\right] = \dfrac{[ML^2T^{-2}]}{[ML^{-1}T^{-1}]} = [L^3T^{-1}]$

Thus $[v] \neq \left[\dfrac{mgr}{\eta}\right]$

Hence, dimensionally inconsistent.

Long Answer Type Questions :

31. What do you mean by measurement? Explain why is it necessary.

Sol. Measurement is a process of determining how large or small a physical quantity is as compared to a basic arbitrarily chosen reference standard called unit.

To express the measurement of a physical quantity, we need two things –

(i) The unit in which the quantity is measured.

(ii) The magnitude of the quantity i.e., the number of times that unit is contained in the given physical quantity. So measure of a quantity = Numerical value × size of the unit.

Importance of Measurement : Physics is a quantitative science based on measurement of physical quantities. A scientific observation becomes more relevant when it is provided with precise and quantitative details.

32. What is a system of units? Discuss various popular system of units.

Sol. A complete set of both the base units and derived units, is called the system of units. Historically many systems of units were in use in different parts of the world. A few of them which were quite popular till recently are given below.

1. **CGS System :** In this system centimetre, gram and second are used as the base units for length, mass and time respectively.
2. **FPS System :** It uses foot, pound and second as base units for length, mass and time respectively.
3. **MKS System :** It uses metre, kilogram and second as the respective units for length, mass and time.
4. **S.I. System :** It is used internationally at present.

- The SI system is a decimal system, also known as metric system, a modernised and extended form of metric systems like CGS and MKS.
- There are seven base units and two supplementary units in SI. These units with their names and symbols are given below.

Base Quantity	Base SI Unit	Unit Symbol	Unit Definition
Length	metre	m	
Mass	kilogram	kg	
Time	second	s	
Electric Current	ampere	A	
Thermodynamic Temperature	kelvin	K	
Amount of substance	Mole	Mol	
Luminous intensity	Candela	cd	

SUPPLEMENTARY UNITS

Plane angle	Radian	rad
Solid Angle	Steradian	sr

33. Discuss in brief the various techniques employed to measure very small and very large distances. Will a distant star show greater parallax than a near star for the same basis?

Sol. Measurement of very small distances :

(i) Optical Microscope : It uses visible light. It can resolve particles upto the size of 10^{-7} m.

(ii) Electron Microscope : It uses electron beams focused by electric and magnetic fields. Its resolution is better than that of optical microscope. It can resolve particles as small as 0.6×10^{-10} m.

(iii) Tunnelling Microscopy : Its limit of resolution is even better than that of an electron microscope.

` **(iv) Volumetric Method :** This method can give an estimate of the sizes of molecules.

Measurement of Large Distances : The size or the distance of a far off objects can be estimated using indirect methods like parallax method and triangulation method. A nearer star shows greater parallax than that by a farther star for the same basis.

34. What are the base units in SI? Define the supplementary units and the quantities they measure.

Sol. Base Units : The units of the base quantities, i.e., the quantities which are independent and are not usually defined in terms of other physical quantities, are called base units. The seven base quantities along with the respective base SI units are as follows:

Length (metre), Mass (kilogram), Time (second), Electric current (ampere), Thermodynamic temperature (kelvin), Luminous intensity (candela) and Amount of substance (mole)

The Two more Dimensionless Quantities Defined in SI.

1. **Plane angle :** Its SI unit is radian

   One radian is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

2. **Solid angle :** Its SI unit is steradian.

   One steradian is defined as the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.

35. (i) Does a more accurate of the two values have greater precision too? Explain with example.

(ii) What is the order of magnitude of a quantity?

Sol. (i) It is not necessary that a more accurate value among the given values is more precise too. For example, let the actual thickness of a block be 2.10 cm. If measured with a metre scale having precision 0.1 cm it comes out to be 2.0 m. Let the value of this length when measured with a vernier callipers be 1.98 cm. This is a more precise value as compared to 2.0 cm. The absolute error in the two cases is

Metre scale $\rightarrow$ 2.0 – 2.10 = – 0.1 cm

Vernier callipers $\rightarrow$ 1.98 – 2.10 $\rightarrow$ 0.12 cm $\leftarrow$ greater error

Since the measurement with vernier callipers has greater error, we say that metre scale is more accurate.

(ii) If the magnitude of a physical quantity is expressed as $a \times 10^b$, where a is a number lying between 1 and 10 and b is an exponent of 10. Then the exponent b is called the order of magnitude of the physical quantity.

For example, the speed of light is given as 3.00×10^8 m s^{-1}. So the order of magnitude of the speed of light is 8. The order of magnitude, gives an estimate about the magnitude of the quantity. If the charge on an electron is 1.6×10^{-19} C. Therefore, we can say that the change possessed by an electron is of the order 10^{-19} on its order of magnitude is – 19.

The size of an atom is of the order 10^{-10} m and the size of a nucleus $\approx 10^{-14}$ m.

36. Discuss the various errors normally associated with a measurement, and also the ways to reduce them.

Sol. Systematic errors : The errors which occur in one direction only, i.e. either positive or negative are called systematic errors. If the measured value is greater than the true value, the error is said to be positive. And if the measured value is less than the true value, the error is said to be negative.

Some of the sources of systematic errors are as follows:

(1) **Instrumental Errors :** These errors arise when the measuring instrument itself has some defect in it, such as

(i) Improper Designing or Calibration : It means the instrument is not graduated properly. For example, if an ammeter reads a current of 1.5 A, when a 2 A current is actually flowing through the circuit, it has an imperfect calibration.

(ii) Zero Error : If the zero mark of vernier scale does not coincide with the zero mark of the main scale, the instrument is said to have zero error. A metre scale having worn off zero mark also has zero error.

(2) **Imperfection in Experimental Technique or procedure :** The measurement may be systematically affected by external conditions such as changes in temperature humidity, wind velocity etc. For example, the temperature of a human body measured by a thermometer placed under the armpit will always be less than the actual temperature.

(3) **Personal errors.**

Minimising Systematic Errors : Systematic errors can be minimised by using more accurate instruments, and improved experimental techniques. One should take proper precautions and remove personal bias as far as possible while doing the experiments. Necessary corrections can be done for the instruments having zero errors, after taking the readings.

Random errors : The errors which are random in sign as well as in size i.e. it may be positive or negative or both. These errors can be minimised by taking large number of observations and then arithmetic mean of that the instrument used should have high precision.

37. Define absolute, relative and percentage errors with examples.

Sol. Absolute Errors : Let the values for the measurement of a physical quantity are taken carefully *x* number of times. If the values are a_1, a_2, ...a_n then their arithmetic mean is taken as the most accurate value

$$a_{\text{mean}} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

The magnitude of the difference between the individual measured value and the true value of the quantity is called the absolute error of the measurement.

$$\Delta a_{\text{absolute}_1} = |a_1 - a_{\text{mean}}|$$

$$\Delta a_{\text{absolute}_2} = |a_2 - a_{\text{mean}}|$$

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$$\Delta a_{\text{absolute(n)}} = |a_n - a_{\text{mean}}|$$

Relative Error : The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the measured quantity. It is also called fractional error.

Relative error = $\dfrac{\Delta a_{mean}}{a_{mean}}$

Percentage Error : The relative error expressed in percent gives the percentage error. It is denoted by δa.

Thus $\delta a = \dfrac{\Delta a_{mean}}{a_{mean}} \times 100$

38. What are significant figures? Write the rules to round off insignificant digits in measured values.

Sol. All the certain digits and the one uncertain digit in a measured value are called the **significant figures** present in it. Significant figures indicate the precision of the measurement which depends on the least count of the measuring instrument.

Rounding Off the Uncertain Digits

1. **If the insignificant digit to be dropped is more than 5, the preceding digit is raised by 1.** Let the insignificant digit in the number 3.78 be 8 (circled). Since 8 > 5, we raise the preceding digit 7 by 1. Hence, the number becomes 3.8.

2. **If the insignificant digit to be dropped is less than 5, the preceding digit is left unchanged.** Let the insignificant digit in the number 3.74 be 4 (circled). Since 4 < 5, we keep the preceding digit 7 unchanged. Hence the number becomes 3.7.

3. **If the insignificant digit to be dropped is 5, the preceding digit is raised by 1 if it is odd, and is left unchanged if it is even.** Let 5 (circled) be the insignificant digit in the numbers 3.745 and 3.775. In the first number, since the preceding digit 4 is even, it remains as such and the number becomes 3.74.

   In the second number, the preceding digit 7 is odd, hence it is raised by 1 and the number is written as 3.78.

4. When a complex multi-step calculation is involved all the numbers occurring in the intermediate steps should retain a digit more than the significant digits present in them. The final answer at the end of the calculation, can then be rounded off to the appropriate significant figures.

5. The exact numbers like π, 2, 3, 4 etc. that appear in formulae and are known to have infinite significant figures, can be rounded off to a limited number of significant figures as per the requirement.

39. A boy measures the diameter of a small metal ball using a vernier callipers. He takes the following five readings. 2.48 cm, 2.49 cm, 2.51 cm, 2.50 cm, 2.51 cm

Find its radius with the appropriate % error.

Sol. True value of the diameter of the ball = Arithmetic mean of all readings.

$$\therefore \quad D_{mean} = \frac{2.48 + 2.49 + 2.51 + 2.50 + 2.51}{5}$$

$$= 2.498 \text{ cm} \approx 2.50 \text{ cm}$$

Radius $R = \dfrac{D_{mean}}{2} = \dfrac{2.498}{2} = 1.249$ cm

Rounding off to the two decimal places R = 1.25 cm.

To find absolute errors

$|\Delta D_1| = |2.48 - 2.50| = 0.02$ cm

$|\Delta D_2| = |2.49 - 2.50| = 0.01$ cm

$|\Delta D_3| = |2.51 - 2.50| = 0.01$ cm

$|\Delta D_4| = |2.50 - 2.50| = 0.00$ cm

$|\Delta D_5| = |2.51 - 2.50| = 0.01$ cm

Hence, mean absolute error = $\dfrac{|\Delta D_1| + |\Delta D_2| + ... + |\Delta D_5|}{5}$

$$= \frac{0.02 + 0.01 + 0.01 + 0.00 + 0.01}{5}$$

$\Delta D_{mean} = 0.01$ cm

Now $R_{mean} = \dfrac{D_{mean}}{2}$ ...(i)

$\Rightarrow \Delta R_{mean} = \dfrac{\Delta D_{mean}}{2}$ ...(ii)

From equations (i) and (ii), we have

Relative error in $R = \dfrac{\Delta R_{mean}}{R_{mean}} = \dfrac{\Delta D_{mean}}{D_{mean}} = \dfrac{0.01}{2.50} = .004$

$\therefore$ Percentage error in $R = \dfrac{\Delta R_{mean}}{R_{mean}} \times 100 = 0.4\%$

Hence, the radius of the ball is 1.25 cm with an error of 0.4%.

40. A physical quantity is given by the relation $z = \dfrac{ab^2}{c}$ where $a = (5.34 \pm 0.01)$ g, $b = (3.2 \pm 0.1)$ cm and $c = (0.42 \pm 0.01)$ s. Find the relative error in z. Which of the three quantities should be measured most precisely and why?

Sol. Given that $z = \dfrac{ab^2}{c}$

and $a = 5.34 \pm 0.01$ g

$b = 3.2 \pm 0.1$ cm

$c = 0.42 \pm 0.01$ s

Relative error in z, $\dfrac{\Delta z}{z} = \dfrac{\Delta a}{a} + 2\dfrac{\Delta b}{b} + \dfrac{\Delta c}{c}$

$$= \frac{0.01}{5.34} + 2 \times \frac{0.1}{3.2} + \frac{0.01}{0.42}$$

$= .00187 + .062 + .0238$

$0.0876 \approx 0.09$

The largest relative error is contributed in *z* by the quantity b (relative error 0.062). Hence, this quantity should be measured with greatest precision.

41. (i) What are the applications of dimensional analysis? The position (x) of a particle depends on its velocity (v) and time(t) as given by the relation. $x = Pv + \frac{Q}{P+t}$. Find the dimensions of PQ.

(ii) If kinetic energy K depends on velocity v, acceleration a and density (ρ) of an object, find the expression for K.

Sol. (i) Application of Dimensional Analysis

- Checking the dimensional consistency of physical formulae and equation.

  Each of the terms appearing in a mathematical relation should have the same dimensions. If it is the not so the relation is proved wrong. This is called the principle of dimensional homogeneity.

- Deducing relation among various physical quantities.

  Given equation $x = Pv + \frac{Q}{P+t}$

  Since P is added to t, therefore

  $[P] = [t]$

  $\Rightarrow [P] = [T^1]$ ...(i)

  Also all the terms in an equation should have the same dimensions

  $\therefore \left[\frac{Q}{P+t}\right] = [x]$

  $\Rightarrow \frac{[Q]}{[T]} = [L]$

  $[Q] = [LT]$ ...(ii)

  From (i) and (ii)

  $\therefore [PQ] = [T^1][LT] = [LT^2]$

(ii) Let the required expression be

$K \propto v^x \rho^y a^z$

$[K] = [LT^{-1}]^x [ML^{-3}]^y [LT^{-2}]^z$

$[ML^2 T^{-2}] = [M^y L^{x-3y+z} T^{-x-2z}]$

Equating the dimensions of like quantities on both sides.

$\Rightarrow \boxed{y = 1}$

$\Rightarrow x - 3y + z = 2$

$\Rightarrow x + z = 5,$

$-x - 2z = -2$

$\Rightarrow x + 2z = 2$

or $\boxed{z = -3}$

and $\boxed{x = 8}$

$\therefore K \propto v^8 \rho a^{-3}$

$K \propto \frac{v^8 \rho}{a^3}$

42. What points do we conclude from dimensional analysis? Explain the principle of homogeneity of dimensions.

Sol. Dimensional Analysis : Dimensional formulae of various physical quantities help us to understand their physical behaviour. When a calculation involves the multiplication or division of the magnitude of two physical quantities, the dimensions of the resultant quantity can be simply obtained by algebraically multiplying or dividing the respective dimensions of the quantities. The dimensions of the physical quantities can be multiplied or divided simply even if their units are expressed in different systems of units. We need not bother about the the conversion of units and the magnitudes of the quantities)

Also only the quantities having similar dimensions and similar nature can be added or subtracted from each other.

Principle of Homogeneity of Dimensions : If a mathematical expression is physically correct, all the terms appearing in it should have same dimensions. It can be used to check the dimensional consistency of various equations. If an equation fails this consistency test, it is proved wrong, but if it passes, it is not proved right.

43. (i) How many significant figures are there in the following quantities?

(a) 10.163,

(b) 1.67×10^{-17},

(c) 0.270,

(d) 1.496,

(e) 15000,

(f) 2.4300,

(g) 0.001040

(ii) Round off the following numbers to three significant figures.

(a) 3.264,

(b) 0.9462,

(c) 1.667,

(d) 1.285,

(e) 45.875

Sol. (i) (a) 10.163

All non-zero digits and the zeroes lying between two non-zero digits are significant. Hence five significant figures.

(b) 1.67×10^{-17}

All digits lying in a base number of a scientific notation are significant.

Hence, three significant figures.

(c) 0.270

In a number having decimal point, trailing zeroes are significant, but zeroes in the beginning are not significant. Hence, three significant figures.

(d) 1.496

Four significant figures.

(e) 15000

In a number without a decimal point, trailing zeroes are not significant.

Hence, two significant figures.

(f) 2.4300

Five significant figures.

(g) 0.001040

Four significant figures.

(ii) (a) 3.264 $\rightarrow$ 3.26

(b) 0.9462 $\rightarrow$ 0.946

(c) 1.667 $\rightarrow$ 1.67

(d) 1.285 $\rightarrow$ 1.28

(e) 45.875 $\rightarrow$ 45.9

44. Deduce the dimensional formulae of the following physical quantities.

(i) Specific heat,

(ii) Mechanical equivalent of heat,

(iii) Coefficient of viscosity.

(iv) Time period,

(v) Relative density,

(vi) Angular speed,

(vii) π (a pure number)

Sol. (i) Specific heat $= \dfrac{\text{Heat}}{\text{Mass} \times \text{Temperature}} = \dfrac{[ML^2T^{-2}]}{[M][K]} = [M^0L^2T^{-2}\ K^{-1}]$

(ii) Mechanical equivalent of heat $= J = \dfrac{W}{H} = \dfrac{ML^2T^{-2}}{ML^2T^{-2}} = [M^0L^0T^0]$

(iii) Coefficient of viscosity $= \dfrac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Velocity}} = \dfrac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$

(iv) Time period = Time = $[M^0L^0T^1]$

(v) Relative density $= \dfrac{\text{Density of a substance}}{\text{Density of water at } 4°\text{C}} = \dfrac{ML^{-3}}{ML^{-3}} = [M^0L^0T^0]$

(vi) Angular speed $= \dfrac{\text{Angle}}{\text{Time}} = \dfrac{1}{T} = = [M^0L^0T^{-1}]$

(vii) π (pure number) = $[M^0L^0T^0]$

45. (i) Find the dimensions of $\left(\dfrac{a}{b}\right)$ in the equation

$P = \dfrac{a - t^2}{bx}$ where P is pressure, x is distance and t is time.

(ii) Discuss the advantages and two limitations of using dimensional method.

Sol. (i) $P = \dfrac{a - t^2}{bx}$

Since t^2 is subtracted from a,

$\Rightarrow [a] = [t^2]$ ...(i)

$\Rightarrow [a] = [M^0\,L^0\,T^2]$

Applying dimensional homogeneity

$[P] = \dfrac{[a - t^2]}{[bx]}$

$[ML^{-1}T^{-2}] = \dfrac{[T^2]}{[b][L]}$

$[b] = \dfrac{[L^{-1}T^2]}{[ML^{-1}T^{-2}]}$

$[b] = [M^{-1}L^0T^4]$ ...(ii)

Using equation (i) and (ii),

$\dfrac{[a]}{[b]} = \dfrac{[T^2]}{[M^{-1}L^0T^4]} = [MT^{-2}]$

(ii) **Advantages of dimensional Analysis :**

(a) It is extremely useful in checking the correctness of an equation.

(b) We need not worry about conversions among multiples and submultiples of the units while analysing the dimensions.

(c) We can easily deduce relation among various physical quantities.

Limitations of Dimensional Analysis

(1) We cannot obtain relations if it has trigonometric ratios, logarithmic functions or exponential function.

(2) We cannot use this method to find expression for the quantities which depend on two or more quantities having similar dimensions *e.g.*, we cannot deduce expression for gravitational force between two masses m_1 and m_2.

SECTION - B

Model Test Paper

1. Give the SI units of solid angle and plane angle.

Sol. Steradian, radian

2. The magnification produced by a convex lens is 2. Write its value in both SI and CGS units.

Sol. Magnification does not have units, hence its value is 2 in both unit systems.

3. Name the errors which can have both positive and negative signs.

Sol. Random errors.

4. What is the value of 1J in CGS units?

Sol. 10^7 erg.

5. How many significant figures are there in the value 163.602?

Sol. Six

6. What is the order of magnitude of 1 unified a.m.u. in SI units?

Sol. – 27

7. Round of 66.7200 to three significant figures.

Sol. 66.7

8. Name two dimensional constants.

Sol. Gravitational constant, solar constant.

9. Express light year in metres.

Sol. 1ly = Distance travelled by light in vacuum during one year

$= 3.0 \times 10^8 \text{ (m/s)} \times 365.25 \times 24 \times 3600$

$= 9.46 \times 10^{15}$ m

10. How much distance does a body moving with a speed 27 km h^{-1} cover in one second?

Sol. Speed = 27 km h^{-1}

$= 27 \times (1000 \text{ m}) (3600 \text{ s})^{-1}$

$= \frac{27000}{3600} \text{m s}^{-1}$

$= 7.5 \text{ m s}^{-1}$

Distance covered in one second = 7.5 × 1 = 7.5 m

11. Find the total length when two rods of lengths (6.42 ± 0.01) m and (10.32 ± 0.01) m are joined end to end.

Sol. (6.42 ± 0.01) m + (10.32 ± 0.01) m

$\Rightarrow$ 16.74 ± 0.02 m

12. What do you mean by the order of magnitude of a physical quantity?

Sol. If the magnitude of a physical quantity is expressed as $a \times 10^b$, where a is a number lying between 1 and 10 and b is an exponent of 10, then the exponent b is called the order of magnitude of the physical quantity. If a body as certain mass 3.46×10^5 kg, the order of magnitude of its mass is 5.

13. Find the momentum of a 2.53 g mass moving with speed 16.2 m s^{-1} upto appropriate significant figures.

Sol. Momentum = $m \times v$ = (2.53) g × (16.2 m s^{-1}) = 40.986 g m s^{-1}

Rounding off to three significant figures, = 41.0 g m s^{-1}

14. Check the dimensional consistency of the equation, Kinetic Energy = $\dfrac{(\text{momentum})^2}{2\times(\text{mass})}$

Sol. L.H.S. → [Kinetic energy]

= [ML^{2}T^{-2}]

R.H.S. → $\dfrac{[\text{Momentum}^2]}{[2\times\text{mass}]}$

$= \dfrac{[MLT^{-1}]^2}{[M]} = \dfrac{[M^2L^2T^{-2}]}{[M]}$ = [M L^2 T^{-2}]

Comparing dimensions on both sides, equation is dimensionally consistent.

15. What do you mean by random errors?

Sol. The errors which are random in sign and size (i.e., they can be positive as well as negative) are called random errors.

16. Name any three physical quantities having the same dimensions and also give their dimensions.

Sol. Work, Energy, Torque.

All have the dimensions [ML^{2}T^{-2}]

17. Which techniques are used for measuring small distances?

Sol. We can measure small distances using

(i) Optical microscopes

(ii) Electron microscope

(iii) Tunnelling microscope

(iv) Volumetric method (Rough estimate)

18. Define 'unit' of a physical quantity. Give SI units for work, power and angular speed.

Sol. Unit : An arbitrarily chosen magnitude of a physical quantity which is used as a basic reference standard to measure the quantity of similar nature is called a unit.

e.g. SI unit for work is joule or N m, power it is W and for angular speed it is rad s^{-1}.

19. A length is expressed in two different units as 23 m and 2300 cm. Find the number of significant figures for each value.

Sol. A mere change of units does not change the number of significant figures present in a measured value. Hence, if the length is measured precisely upto two significant figures, it has two significant figures for any unit used to express it. Thus each of the given values has two significant figures.

20. A quantity z is given by the relation, $z = \dfrac{2\sqrt{ab}}{c^3}$. Find the percentage error in z if the percentage error in a, b and c are respectively 2%, 1% and 4%.

Sol. $z = \dfrac{2\sqrt{ab}}{c^3}$

$$\frac{\Delta z}{z} = \frac{1}{2}\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) + \frac{3\Delta c}{c}$$

$\therefore$ Percentage error $\frac{\Delta z}{z} \times 100 = \frac{1}{2}\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100 + \frac{3\Delta c}{c} \times 100$

$$= \frac{1}{2}(2+1) + 3 \times 4 = 12 + \frac{3}{2} = 13.5\ \%$$

21. If centripetal force is given by the relation $F = \frac{mv^2}{r}$, where m mass = 20.04 g, v velocity = 10.4 m/s and r radius of the circular path = 15.5 m. Find F to appropriate significant figures.

Sol. $F = \frac{mv^2}{r} = \frac{(20.04\text{ g})(10.4\text{ m/s})^2}{(15.5)\text{ m}} = 139.84\text{ g m s}^{-2}$

Rounding off to three significant figures = 140 g m s^{-2}

22. (i) Explain how reporting a measurement in scientific notation removes ambiguities regarding significant figures.

(ii) The period of oscillation of a simple pendulum is given by the relation $T = 2\pi\sqrt{\frac{L}{g}}$. The measured values of length L and the period of oscillations for 50 oscillations are (20.0 ± 0.1) cm and (45 ± 1)s respectively. Find the absolute error in determination of g.

Sol. (i) The scientific notation is used to avoid the confusion arising due to the change in the units of the measured quantity. For example, a thickness measured as 3.560 m can be written in different units as 356.0 cm = 3560 mm = 3560000 μm.

Thus using this notation, the value 3.560 m can be written in different units as

3.560 m = 3.560 × 10^{-3} km

= 3.560 × 10^{2} cm

= 3.560 × 10^{3} mm

= 3.560 × 10^{6} μm

Each number in this case has four significant figures. All the digits appearing in the base number, including the trailing zeroes are significant.

(ii) $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$ ...(i)

Here time period of one oscillation,

$$T = \frac{\text{Period of 50 Oscillations }(t)}{50} = \frac{45}{50} = 0.9\text{ s}$$

$$\Rightarrow g = \frac{4\pi^2 \times 20}{(0.9)^2}$$

$\because$ 973.78 cm s^{-2} = 974 cm s^{-2}

$\because \frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{45}$ [from the values given in the question]

Therefore relative error in g

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = \frac{0.1}{20} + \frac{2}{45} = 0.0494 \simeq 0.05$$

$\therefore$ absolute error

$\Delta g = g \times 0.05 = 973.78 \times 0.05 = 48.68$ ms^{-2}

23. (i) Explain how errors combine after arithmetic operations of two measured values?

(ii) If the heat dissipated in a resistor depends on its resistance R, current I through it and the time t for which the current flows, find an expression for heat H dissipated.

Sol. (i) (a) For addition and subtraction

If $Z = A + B$

Maximum permissible error $\Delta Z = \Delta A + \Delta B$,

If $Z = A - B$

Maximum permissible error in Z :

$\Delta Z = \Delta A + \Delta B$

(b) For multiplication and division

If $Z = AB$

Maximum relative error in Z, $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

If $Z = \dfrac{A}{B}$

Maximum relative error $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

(c) For quantities raised to powers.

If $Z = A^P$

$$\frac{\Delta Z}{Z} = P\frac{\Delta A}{A}$$

(ii) Let the required expression be

$H \propto I^a R^b t^c$

$\Rightarrow [H] = [I]^a [R]^b [T]^c$

$[ML^2T^{-2}] = [A]^a [ML^2T^{-3}A^{-2}]^b [T]^c$

$\Rightarrow [ML^2T^{-2}] = [M^b L^{2b} T^{-3b+c} A^{a-2b}]$

Equating the dimensions on both sides

$\Rightarrow \boxed{b = 1}$

$-3b + c = -2$

$\Rightarrow \boxed{c = 1}$,

$a - 2b = 0$

$\Rightarrow \boxed{a = 2}$

$\therefore H \propto I^2Rt$

Here constant of proportionality is 1.

$\therefore H = I^2Rt$.

❑ ❑ ❑

Solutions (Set-2)

Objective Type Questions

(System of Units)

1. The base quantity among the following is

(1) Speed (2) Weight (3) Length (4) Area

Sol. Answer (3)

There are seven base quantities,

(i) Mass (ii) Length

(iii) Time (iv) Current

(v) Amount of substance (vi) Luminous intensity

(vii) Temperature

2. Which of the following is not a unit of time?

(1) Second (2) Minute (3) Hour (4) Light year

Sol. Answer (4)

Light year is the unit of distance

1 light year = 9.46×10^{15} m

3. One astronomical unit is a distance equal to

(1) 9.46×10^{15} m (2) 1.496×10^{11} m (3) 3×10^{8} m (4) 3.08×10^{16} m

Sol. Answer (2)

One astronomical unit is the average distance between earth and sun

$\boxed{1 \text{ astronomical unit (AU)} = 1.496 \times 10^{11} \text{ m}}$

4. Ampere second is a unit of

(1) Current (2) Charge (3) Energy (4) Power

Sol. Answer (2)

Current $I = \frac{q}{t} \Rightarrow q = It$

$\boxed{q = \text{Ampere second}}$

So, ampere second is the unit of charge.

5. Which of the following is a unit that of force?

(1) N m (2) mN (3) nm (4) N s

Sol. Answer (2)

Nm $\rightarrow$ Unit of torque

mN $\rightarrow$ Milli newton $\Rightarrow 10^{-3}$ N

nm $\rightarrow$ Nano metre

Ns $\rightarrow$ Unit of momentum

6. The value of 60° in radian is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{5}$

Sol. Answer (2)

180º = π radian

$1^\circ = \frac{\pi}{180}$ rad

$60^\circ = \frac{\pi}{180} \times 60$ rad

$\boxed{60^\circ = \frac{\pi}{3} \text{ rad}}$

7. The total plane angle subtended by a circle at its centre is

(1) π rad (2) 2π rad

(3) $\frac{2\pi}{3}$ rad (4) $\frac{\pi}{2}$ rad

Sol. Answer (2)

The total plane angle is 360º or 2π rad.

8. One unified atomic mass unit represents a mass of magnitude

(1) 10^{-30} kg

(2) 1.66×10^{27} kg

(3) 1.66×10^{-27} kg

(4) 10^{30} kg

Sol. Answer (3)

$\boxed{1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}}$

(Measurement of Length, Mass and Time, Accuracy, Precision)

9. A far off planet is estimated to be at a distance *D* from the earth. If its diametrically opposite extremes subtend an angle θ at an observatory situated on the earth, the approximate diameter of the planet is

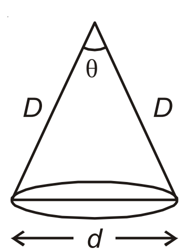
(1) $\frac{\theta}{D}$ (2) $\frac{D}{\theta}$ (3) $D\theta$ (4) $\frac{1}{D\theta}$

Sol. Answer (3)

$\theta = \frac{\text{Arc length}}{\text{Radius}}$

$\theta = \frac{d}{D}$

$\Rightarrow \boxed{d = D\theta}$



10. If the average life of a person is taken as 100 s, the age of the universe on this scale is of the order

(1) 10^{10} s (2) 10^{8} s (3) 10^{17} s (4) 10^{9} s

Sol. Answer (1)

Time span of human life = 10^9 s

Age of universe = 10^{17} s

So, $\frac{\text{Age of universe}}{\text{Time of human}} = \frac{10^{17}}{10^9} = 10^8$

If, $\frac{\text{Age of universe}}{100} = 10^8$

$\Rightarrow$ $\boxed{\text{Age of universe} = 10^{10}\ \text{s}}$

11. The order of the magnitude of speed of light in SI unit is

(1) 16 (2) 8 (3) 4 (4) 7

Sol. Answer (2)

Speed of light $\Rightarrow 3 \times 10^8$ ms^{-1}

Order of magnitude = 8

12. The most precise reading of the mass of an object, among the following is

(1) 20 g (2) 20.0 g (3) 20.01 g (4) 2.0 kg

Sol. Answer (3)

A measurement having more number of decimal places in same unit is the one with the most precision.

So, 20.01 g is most precise.

13. The most accurate reading of the length of a 6.28 cm long fibre is

(1) 6 cm (2) 6.5 cm (3) 5.99 cm (4) 6.0 cm

Sol. Answer (2)

Most accurate reading is the one having minimum error.

So, |6 – 6.28| = 0.28 cm

|6.5 – 6.28| = 0.22 cm

|5.99 – 6.28| = 0.29 cm

|6.0 – 6.28| = 0.28 cm

So, second reading is most accurate.

14. Which of the following is the most precise measurement?

(1) 0.003 m (2) 0.0030 m (3) 0.03 m (4) 0.00300 m

Sol. Answer (4)

In 0.003 m, precision is 0.001 m

0.0030 m, precision is 0.0001 m

0.00300 m, precision is 0.00001 m

So, fourth measurement is most precise.

15. The values of a number of quantities are used in a mathematical formula and are having same power. The quantity that should be most precise and accurate in measurement is the one

(1) Having smallest magnitude (2) Having largest magnitude

(3) Used in the numerator (4) Used in the denominator

Sol. Answer (1)

The quantity having smallest magnitude should be measured very precisely as it is likely to contribute the maximum relative error.

(Errors)

16. Thickness of a pencil measured by using a screw gauge (least count .001 cm) comes out to be 0.802 cm. The percentage error in the measurement is

(1) 0.125% (2) 2.43% (3) 4.12% (4) 2.14%

Sol. Answer (1)

The percentage error is $\frac{\Delta L}{L}\times 100\% = \frac{0.001}{0.802}\times 100\% = 0.1246\% \simeq 0.125\%$

17. The percentage error in the measurement of the voltage V is 3% and in the measurement of the current is 2%. The percentage error in the measurement of the resistance is

(1) 3% (2) 2% (3) 1% (4) 5%

Sol. Answer (4)

$V = IR \Rightarrow R = \frac{V}{I}$

$\Rightarrow \left(\frac{\Delta R}{R}\right)\times 100\% = \left(\frac{\Delta V}{V}+\frac{\Delta I}{I}\right)\times 100\%$

$\Rightarrow \frac{\Delta R}{R}\times 100\% = 3\% + 2\% = 5\%$

18. The relative error in the measurement of the side of a cube is 0.027. The relative error in the measurement of its volume is

(1) 0.027 (2) 0.054 (3) 0.081 (4) 0.046

Sol. Answer (3)

Volume of cube, V = side^3

$\frac{\Delta V}{V} = \frac{3\,\Delta \text{side}}{\text{side}}$

$\frac{\Delta V}{V} = 3\times 0.027 \Rightarrow \boxed{\frac{\Delta V}{V} = 0.081}$

19. Zero error in an instrument introduces

(1) Systematic error (2) Random error (3) Least count error (4) Personal error

Sol. Answer (1)

Zero error is a part of systematic error.

20. A packet contains silver powder of mass 20.23 g ± 0.01 g. Some of the powder of mass 5.75 g ± 0.01 g is taken out from it. The mass of the powder left back is

(1) 14.48 g ± 0.00 g (2) 14.48 ± 0.02 g (3) 14.5 g ± 0.1 g (4) 14.5 g ± 0.2 g

Sol. Answer (2)

m_1 = 20.23 g ± 0.01 g

m_2 = (5.75 ± 0.01) g

$m_1 - m_2$ = [(20.23 – 5.75) ± 0.02] g

$$\boxed{\Delta m = (14.48 \pm 0.02)\text{ g}}$$

21. The radius of a sphere is (2.6 ± 0.1) cm. The percentage error in its volume is

(1) $\frac{0.1}{2.6}\times 100\,\%$ (2) $3 \times \frac{0.1}{2.6}\times 100\,\%$ (3) $\frac{0.1}{3\times 2.6}\times 100\,\%$ (4) $\frac{0.1}{2.6}\,\%$

Sol. Answer (2)

r = (2.6 ± 0.1) cm

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{V}\times 100\% = \frac{3\Delta r}{r}\times 100\%$$

$$\boxed{\frac{\Delta V}{V}\times 100\% = \frac{3\times 0.1}{2.6}\times 100\%}$$

22. We can reduce random errors by

(1) Taking large number of observations (2) Corrected zero error

(3) By following proper technique of experiment (4) Both (1) & (3)

Sol. Answer (1)

The only method of reducing random errors is by taking more and more number of observations.

(Significant figures)

23. The volume of a cube having sides 1.2 m is appropriately expressed as

(1) 1.728×10^6 cm^3 (2) 1.7×10^6 cm^3 (3) 1.8×10^6 cm^3 (4) 1.73×10^6 cm^3

Sol. Answer (2)

The volume of cube is l^3

$$v = (1.2\text{ cm})^3 = 1.728\times 10^6\ \text{ cm}^3$$

$$\boxed{v \simeq 1.7\times 10^6\ \text{cm}^3}$$

Answer should be reported in minimum number of significant figures.

24. The number of significant figures in a pure number 410 is

(1) Two (2) Three (3) One (4) Infinite

Sol. Answer (4)

A pure number has infinite number of significant figures.

25. The addition of three masses 1.6 g, 7.32 g and 4.238 g, addressed upto proper decimal places is

(1) 13.158 g (2) 13.2 g (3) 13.16 g (4) 13.15 g

Sol. Answer (2)

m_1 = 1.6 g

m_2 = 7.32 g

m_3 = 4.238 g

$m_1 + m_2 + m_3$ = 13.158 g

but answer should be reported in one decimal place only.

$\therefore \quad \boxed{m = 13.2\ \text{g}}$

26. The area of a sheet of length 10.2 cm and width 6.8 cm addressed upto proper number of significant figures is

(1) 69.36 cm^2 (2) 69.4 cm^2 (3) 69 cm^2 (4) 70 cm^2

Sol. Answer (3)

l = 10.2 cm

w = 6.8 cm

Area = lw = 10.2 × 6.8 = 69.36

$\Rightarrow$ Area = 69 cm^2

27. The uncertain digit in the measurement of a length reported as 41.68 cm is

(1) 4 (2) 1 (3) 6 (4) 8

Sol. Answer (4)

41.68 cm

The rightmost digit is most insignificant and leftmost is most significant.

So, 8 $\rightarrow$ most insignificant

4 $\rightarrow$ most significant

28. The number of significant figures in the measured value 0.0204 is

(1) Five (2) Three (3) Four (4) Two

Sol. Answer (2)

The non-zero digits after the decimal places are significant.

29. The number of significant figures in the measured value 26000 is

(1) Five (2) Two (3) Three (4) Infinite

Sol. Answer (2)

The trailing zeros are not significant.

So, only two digits are significant.

30. The number of significant zeroes present in the measured value 0.020040, is

(1) Five (2) Two (3) One (4) Three

Sol. Answer (4)

Zeores appearing between and after non-zero numbers are significant.

$0.02\underline{00}4\underline{0}$

31. The number of significant figures in the measured value 4.700 m is the same as that in the value

(1) 4700 m (2) 0.047 m (3) 4070 m (4) 470.0 m

Sol. Answer (4)

4.700 $\Rightarrow$ Four significant figures.

Also, 470.0 m $\Rightarrow$ Four significant figures.

32. If a calculated value 2.7465 g contains only three significant figures, the two insignificant digits in it are

(1) 2 and 7 (2) 7 and 4 (3) 6 and 5 (4) 4 and 6

Sol. Answer (3)

2.7465 g $\Rightarrow$ Last two digits are most insignificant.

33. An object of mass 4.237 g occupies a volume 1.72 cm^3. The density of the object to appropriate significant figures is

(1) 2.46 g cm^{-3} (2) 2.463 g cm^{-3} (3) 2.5 g cm^{-3} (4) 2.50 g cm^{-3}

Sol. Answer (1)

m = 4.237 g

V = 1.72 cm^3

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{4.237\text{ g}}{1.72\text{ cm}^3}$$

$$\Rightarrow \boxed{d = 2.46\text{ gcm}^{-3}}$$

34. Round off the value 2.845 to three significant figures.

(1) 2.85 (2) 2.84 (3) 2.80 (4) 2.83

Sol. Answer (2)

2.845 $\Rightarrow$ 2.84

35. A length 5.997 m rounded off to three significant figures is written as

(1) 6.00 m (2) 5.99 m (3) 5.95 m (4) 5.90 m

Sol. Answer (1)

5.997 $\Rightarrow$ 6.00 m

(Dimensions of Physical Quantities, Formulae and Equations)

36. What are the dimensions of the change in velocity?

(1) $[M^0L^0T^0]$ (2) $[LT^{-1}]$ (3) $[MLT^{-1}]$ (4) $[LT^{-2}]$

Sol. Answer (2)

The dimensions of change in velocity is same as that of velocity $[M^0LT^{-1}]$.

37. The dimensional formula for energy is

(1) $[MLT^{-2}]$ (2) $[ML^2T^{-2}]$ (3) $[M^{-1}L^2T]$ (4) $[M L^2 T]$

Sol. Answer (2)

The dimensional formula is $[ML^2T^{-2}]$

38. The pair of the quantities having same dimensions is

(1) Displacement, velocity (2) Time, frequency

(3) Wavelength, focal length (4) Force, acceleration

Sol. Answer (3)

Wavelength and focal length both are have units of length.

39. The dimensional formula for relative refractive index is

(1) $[M^1L^1 T^1]$ (2) $[M^0L^0T^0]$ (3) $[M^1L^0T^0]$ (4) $[MLT^{-1}]$

Sol. Answer (2)

Refractive index is a pure number, hence dimensionless.

40. The dimensional formula $[ML^{-1}T^{-2}]$ is for the quantity

(1) Force (2) Acceleration (3) Pressure (4) Work

Sol. Answer (3)

The dimensional formula for pressure

$$P = \frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} \Rightarrow [ML^{-1}T^{-2}]$$

41. The dimensionally correct expression for the resistance R among the following is
[P = electric power, I = electric current, t = time, V = voltage and E = electric energy]

(1) $R = \sqrt{PI}$ (2) $R = \frac{E}{I^2t}$ (3) $R = V^2P$ (4) $R = VI$

Sol. Answer (2)

Dimensional formula of power = $\frac{W}{t} = \frac{ML^2T^{-2}}{T}$ = $[ML^2T^{-3}]$

Current $\rightarrow$ [A]

$$V = \frac{W}{q} = \frac{ML^2T^{-2}}{AT} = [ML^2T^{-3}A^{-1}]$$

$$E = [ML^2T^{-2}]$$

So, $R = \frac{E}{I^2t} = \frac{ML^2T^{-2}}{A^2T} \Rightarrow [ML^2T^{-3}A^{-2}]$

and $V = IR \Rightarrow R = \frac{ML^2T^{-3}A^{-1}}{A} \Rightarrow [ML^2T^{-3}A^{-2}]$

So, (2) is the correct formula.

42. Which of the following does not have dimensions of force?

(1) Weight (2) Rate of change of momentum

(3) Work per unit length (4) Work done per unit charge

Sol. Answer (4)

Dimension of $\frac{W}{q} = [ML^2A^{-1}T^{-3}]$

which is different from dimension of force $[MLT^{-2}]$

43. The focal power of a lens has the dimensions

(1) [L] (2) $[ML^2T^{-3}]$ (3) $[L^{-1}]$ (4) $[MLT^{-3}]$

Sol. Answer (3)

Focal length $\Rightarrow f = [L]$

44. Which of the following is a dimensional constant?

(1) Magnification (2) Relative density (3) Gravitational constant (4) Relative error

Sol. Answer (3)

Gravitational constant is a dimensional constant.

$[G] = [M^{-1}L^3T^{-2}]$

45. The dimensions of solar constant (energy falling on earth per second per unit area) are

(1) $[M^0L^0T^0]$ (2) $[MLT^{-2}]$ (3) $[ML^2T^{-2}]$ (4) $[M\,T^{-3}]$

Sol. Answer (4)

Solar constant $[S] = \boxed{\dfrac{\text{Energy}}{\text{Area} \times \text{Time}}} = \dfrac{ML^2T^{-2}}{L^2T} \Rightarrow [MT^{-3}]$

(Application of Dimensions)

46. If the buoyant force F acting on an object depends on its volume V immersed in a liquid, the density ρ of the liquid and the acceleration due to gravity g. The correct expression for F can be

(1) $V\rho\, g$ (2) $\dfrac{\rho g}{V}$ (3) $\rho\, g V^2$ (4) $\sqrt{\rho g V}$

Sol. Answer (1)

$F \propto V^a \rho^b g^c$

$F = [L^3]^a [ML^{-3}]^b [LT^{-2}]^c$

$[MLT^{-2}] = F = [M^b L^{3a-3b+c} T^{-2c}]$

On comparing,

$\boxed{b=1}$, $-2c = -2$

$\Rightarrow \boxed{c=1}$

$3a - 3b + c = 1$

$\Rightarrow 3a - 3 + 1 = 1$

$\Rightarrow 3a - 2 = 1$

$\Rightarrow 3a = 3 \Rightarrow \boxed{a=1}$

So, on putting all these values,

$\boxed{F = V\rho g}$

47. The potential energy u of a particle varies with distance x from a fixed origin as $u = \dfrac{A\sqrt{x}}{x+B}$, where A and B are constants. The dimensions of A and B are respectively

(1) $[ML^{5/2}T^{-2}]$, [L] (2) $[MLT^{-2}]$, $[L^2]$ (3) [L], $[ML^{3/2}\,T^{-2}]$ (4) $[L^2]$, $[MLT^{-2}]$

Sol. Answer (1)

$$u = \frac{A\sqrt{x}}{x+B}$$

By the principle of homogeneity, $x = B$ (dimensionally)

$\Rightarrow$ $\boxed{B = [L]}$

and $[ML^2T^{-2}] = \dfrac{AL^{1/2}}{L}$

$[ML^2T^{-2}] = AL^{-1/2}$

$\boxed{A = [ML^{5/2}T^{-2}]}$

48. A physical quantity P is given by the relation. $P = P_0 e^{(-\alpha t^2)}$ If t denotes the time, the dimensions of constant α are

(1) [T] (2) $[T^2]$ (3) $[T^{-1}]$ (4) $[T^{-2}]$

Sol. Answer (4)

$P = P_0 e^{-\alpha t^2}$

The power of exponent is dimensionless,

$\alpha t^2 = [M^0L^0T^0]$

$\boxed{\alpha = [T^{-2}]}$

49. The dimensions of potential energy of an object in mass, length and time are respectively

(1) 2, 2, 1 (2) 1, 2, – 2 (3) –2, 1, 2 (4) 1, – 1, 2

Sol. Answer (2)

The dimensional formula of energy

$E = [ML^2T^{-2}]$

So, dimensions of i) Mass $\rightarrow$ 1 ii) Length $\rightarrow$ 2 iii) Time $\rightarrow$ –2

50. The amount of heat energy Q, used to heat up a substance depends on its mass m, its specific heat capacity (s) and the change in temperature ΔT of the substance. Using dimensional method, find the expression for s is (Given that $[s] = [L^2T^{-2}K^{-1}]$) is

(1) $Qm\Delta T$ (2) $\dfrac{Q}{m\Delta T}$ (3) $\dfrac{Qm}{\Delta T}$ (4) $\dfrac{m}{Q\Delta T}$

Sol. Answer (2)

$Q = m^a s^b (\Delta T)^c$

$[ML^2T^{-2}] = [M^a][L^{2b}T^{-2b}K^{-b}][K^c]$

$\Rightarrow$ $\boxed{a = 1}$, $2b = 2 \Rightarrow \boxed{b = 1}$

$-b + c = 0$

$\Rightarrow b = c \Rightarrow \boxed{c = 1}$

$Q = ms\Delta T$

$\Rightarrow$ $\boxed{s = \dfrac{Q}{m\Delta T}}$

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