Chapter 13

Linear Programming

Solutions (Set-1)

Very Short Answer Type Questions :

1. If the corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0) and the objective function $Z = 4x + 3y$ is given, then find the maximum value of Z.

Sol. At (0, 0); $Z = 4x + 3y = 0$

at (0, 40); $Z = 4x + 3y = 120$

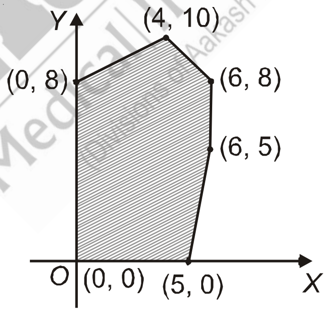
at (20, 40); $Z = 4x + 3y = 200$

at (60, 20); $Z = 4x + 3y = 300$

at (60, 0); $Z = 4x + 3y = 240$

so, Z is maximum at (60, 20) so, its maximum value = 300.

2. The feasible solution for a LPP is shown in figure given below. Let $Z = 3x - 4y$ be the objective function. Find the point at which minimum of Z occurs.

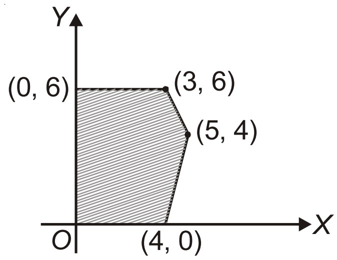


Sol.

Point (x, y)	**$Z = 3x - 4y$**
(0, 0)	$Z = 0$
(5, 0)	$Z = 15$
(6, 5)	$Z = -2$
(6, 8)	$Z = -14$
(4, 10)	$Z = -28$
(0, 8)	$Z = -32$

i.e. Z is minimum at the point (0, 8).

3. The feasible solution for a LPP is shown in the figure given below



Sol. Point(x, y) $Z = 4x + 3y$

O(0, 0) $Z = 0$

(4, 0) $Z = 16$

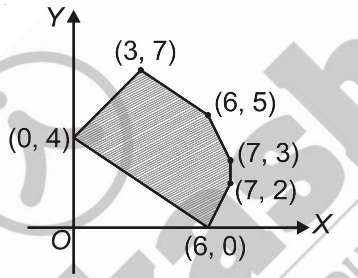
(5, 4) $Z = 32$

(3, 6) $Z = 30$

(0, 6) $Z = 18$

so, Z is the maximum at the point (5, 4).

4. The feasible solution for a LPP is shown in the figure given below



Let $Z = 6x + 4y$ be the objective function. Find the point at which Z is maximum.

Sol. Point(x, y) $Z = 6x + 4y.$

(6, 0) $Z = 36$

(7, 2) $Z = 50$

(7, 3) $Z = 54$

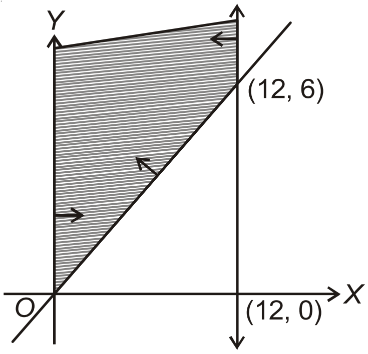
(6, 5) $Z = 56$

(3, 7) $Z = 46$

(0, 4) $Z = 16$

so, Z is maximum at the point (6, 5).

5. The feasible region for an LPP is shown in the figure given below.



Let $P = 3x – 4y$ be the objective function. Then find the maximum value of P.

Sol. Point (x, y) $P = 3x - 4y$

(0, 0) $P = 0$

(12, 6) $P = 12$

i.e. maximum value of P is 12.

6. Refer to question number 5; find the minimum value of P.

Sol. From solution in question number 5

Minimum value of P does not exist.

7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).

If $F = 4x + 6y$ be the objective function, then find the points or region where minimum value of F occurs.

Sol. Point(x, y) $F = 4x + 6y$

(0, 2) $F = 12$

(3, 0) $F = 12$

(6, 0) $F = 24$

(6, 8) $F = 72$

(0, 5) $F = 30$

i.e minimum of F occurs at any point on the line segment joining the points (0, 2) and (3, 0).

8. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3,0). Let $Z = px + qy$, where $p, q > 0$, find the condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1).

Sol. Given that, values of Z are equal at the points (3, 0) and (1, 1).

i.e. $3p + q \times 0 = p + q$

$\Rightarrow$ $2p = q$.

9. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Find the condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20).

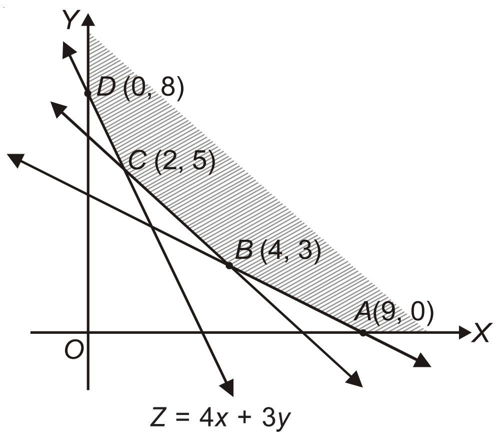
Sol. Given that, values of Z are equal at the points (15, 15) and (0, 20).

So, $15p + 15q = 20q$

$\Rightarrow$ $15p = 5q$

$\Rightarrow$ $3p = q$.

10. Feasible region (shaded) for a LPP is shown in the figure given below. If $Z = 4x + 3y$ be the objective function, then find the point, where minimum of Z occurs.

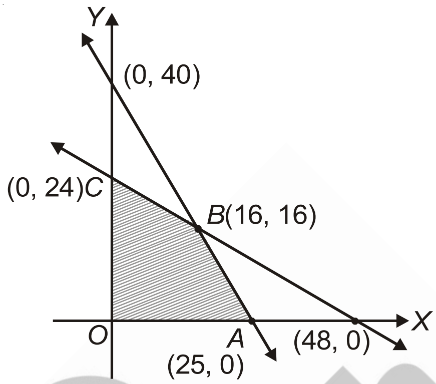


Sol. Point (x, y)

$A(9, 0)$	$Z = 36$
$B(4, 3)$	$Z = 25$
$C(2, 5)$	$Z = 23$
$D(0, 8)$	$Z = 24$

i.e. minimum of Z occurs at the point $C(2, 5)$.

11. Determine the maximum value of $Z = 4x + 3y$, if the feasible region for a LPP is shown in the figure, given below.

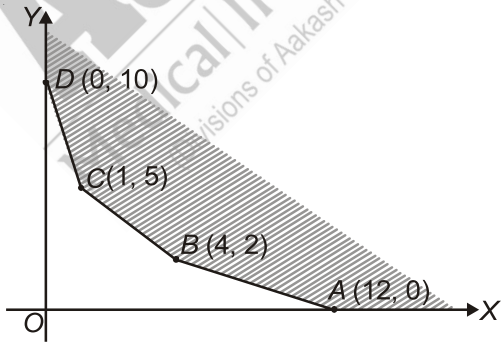


Sol.

Corner Point	Value of $Z = 4x + 3y$	
O (0, 0)	0	
A (25, 0)	100	
B (16, 16)	112	→ maximum
C (0, 24)	72	

Hence the maximum value of Z is 112.

12. Determine the minimum value of $Z = 3x + 2y$ (if any), if the feasible region for a LPP is shown in the figure.

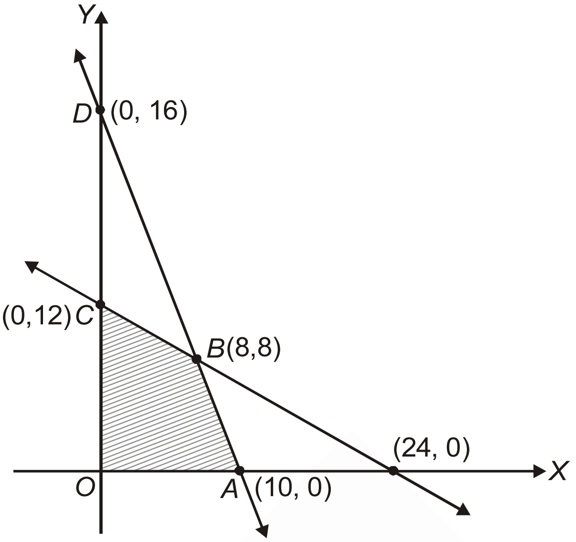


Sol.

Corner Point	Value of $Z = 3x + 2y$	
A (12, 0)	36	
B (4, 2)	16	
C (1, 5)	13	→ smallest
D (0, 10)	20	

i.e. minimum value of Z is 13.

13. Determine the maximum value of $Z = 9x + y$, if the feasible region for a LPP is shown in the figure given below



Sol.

Corner Point	$Z = 9x + y$
$O(0, 0)$	0
A (10, 0)	90
B (8, 8)	80
C (0, 12)	12

i.e. maximum value of Z is 90.

14. What do you mean by multiple optimal points?

Sol. If two corner points of the feasible region are optimal solutions of the same type, *i.e.*, both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

15. What are the theorems, which are used to solve LPPs?

Sol. Following theorems are fundamental in solving LPPs:

Theorem I: Let R be the feasible region for an LPP and let $Z = ax + by$ be the objective functions when Z has an optimal value, where x and y are subject to constraints described by the linear inequalities, this optimal value must occur at a corner point of the feasible region.

Theorem II: If feasible region R is bounded, then the objective function Z has both a maximum and a minimum value on R.

If the feasible region R is unbounded, then a maximum or a minimum value of the objective function may or may not exist.

16. Solve the following linear programming problem graphically:

Maximize $Z = 4x + y$

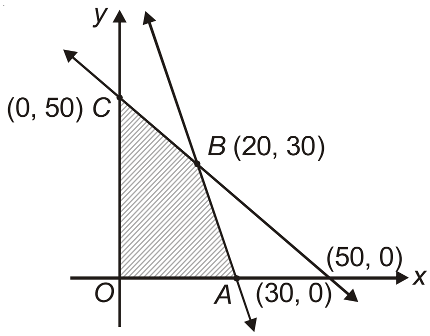
Subject to the constraints:

$x + y \leq 50$

$3x + y \leq 90$

and $x \geq 0,\ y \geq 0$.

Sol. The feasible region determined by the system of constraints is *OABC*, shaded in the figure given below



Corner Point	**$Z = 4x + y$**	
$O(0, 0)$	0	
$B(30, 0)$	120	→ maximum
$C(20, 30)$	110	
$D(0, 50)$	50	

Hence maximum value of Z is 120 at the point (30, 0)

17. Solve the linear programming problem graphically:

Maximize $Z = 200x + 500y$

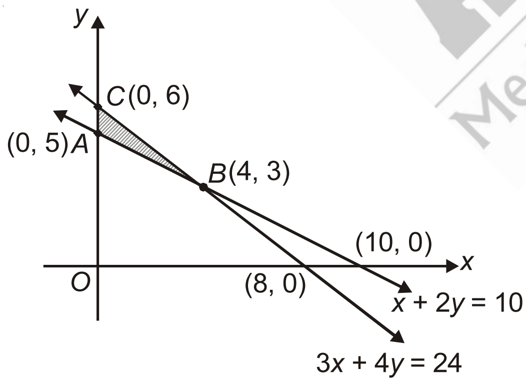
Subject to the constraints:

$$x + 2y \geq 10$$

$$3x + 4y \leq 24$$

and $x \geq 0, \; y \geq 0.$

Sol. The feasible region is *ABC*, shaded in the figure given below



Corner Point	**$Z = 200x + 500y$**
$A(0, 5)$	2500
$B(4, 3)$	2300
$C(0, 6)$	3000

Hence, Z is maximum at the point C.

18. Solve the following problem graphically:

Minimize and Maximize $Z = 3x + 9y$

Subject to the constraints

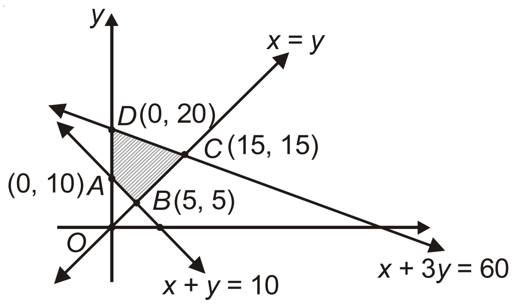
$x + 3y \le 60$

$x + y \ge 10$

$x \le y$

and $x \ge 0$, $y \ge 0$,

Sol. The feasible region is $ABCD$, shaded in the figure given below



Corner Point	$Z = 3x + 9y$
$A(0, 10)$	90
$B(5, 5)$	60
$C(15, 15)$	180
$D(0, 20)$	180

Hence, the minimum value of Z is 60 as well as the maximum value of Z is 180.

19. Determine graphically the minimum value of the objective function

$Z = -50x + 20y$

Subject to the constraints:

$2x - y \ge -5$

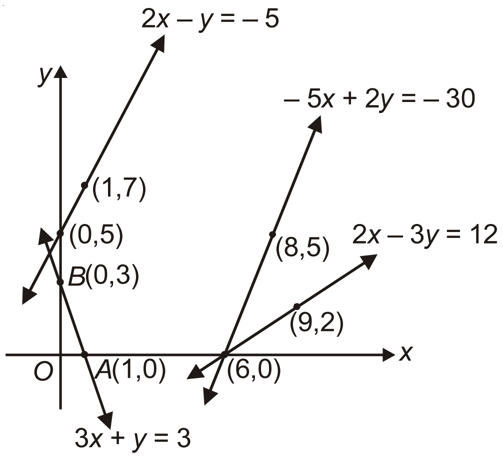
$3x + y \ge 3$

$2x - 3y \le 12$

$-5x + 2y \ge -30$

and $x \ge 0$, $y \ge 0$.

Sol.



Corner Point $Z = -50x + 20y$

Corner Point	$Z = -50x + 20y$	
(0, 5)	100	
(0, 3)	60	
(1, 0)	– 50	
(6, 0)	– 300	→ smallest

Hence, Z has no minimum value subject to the given constraints.

20. Minimize $Z = 3x + 2y$

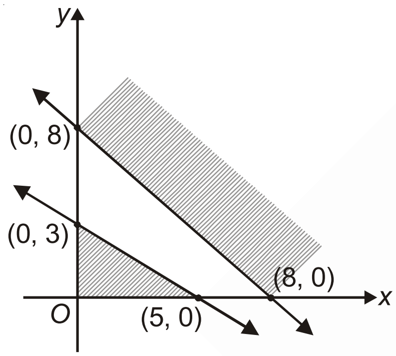
subject to the constraints:

$x + y \geq 8$

$3x + 5y \leq 15$

and $x \geq 0,\ y \geq 0$.

Sol. Let us graph the inequalities:



From figure, it is clear that there is no point satisfying all the constraints simultaneously.

Hence, the problem is having no feasible region and hence no feasible solution.

Long Answer Type Questions :

21. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods x and y are available at a cost of Rs 4 and Rs 3 per units respectively. One unit of the food x contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, whereas one unit of food y contains 100 units of vitamins. 2 units of minerals and 40 units of calories. Find the combination of x and y to be used to have least cost, satisfying the requirement.

Sol. Let x units of food x and y units of food y be taken, then the cost of the diet

$Z = 4x + 3y$.

Now, Linear Programing Problem is

Minimize $Z = 4x + 3y$ subject to the constraints

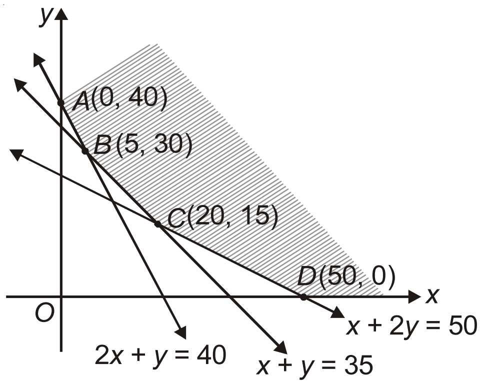
$2x + y \geq 40$

$x + 2y \geq 50$

$x + y \geq 35$

and $x, y \geq 0$.

The feasible region is shaded in the figure given



Corner Point	$Z = 4x + 3y$
A (0, 40)	120
B (5, 30)	110
C (20, 15)	125
D (50, 0)	200

Minimum value of Z is 110 at B(5, 30).

$\therefore$ The cost is minimum, when 5 units of type x and 30 units of type y food are combined.

22. A dietician wishes to mix two types of foods in such way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg. to purchase food 'I' and Rs 70 per kg to purchase food II. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

Sol. Let the mixture contain x kg of food 'I' and y kg of food II.

Hence, the mathematical formulation of the problem is

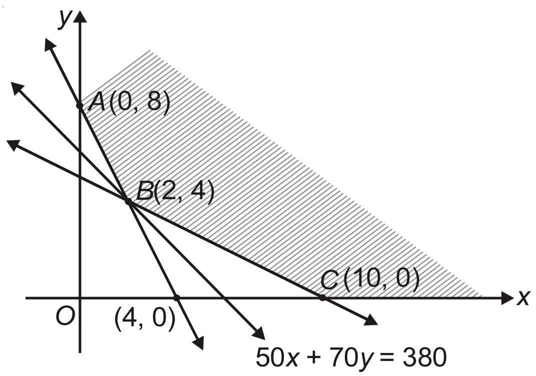
Minimize $Z = 50x + 70y$

subject to the constraints:

$2x + y \geq 8$

$x + 2y \geq 10$

and $x, y \geq 0$.



Corner Point	$Z = 50x + 70y$
(0, 8)	560
(2, 4)	380
(10, 0)	500

As here feasible region is unbounded, therefore we have to draw the graph of the inequality

$50x + 70y < 380$

$\Rightarrow \quad 5x + 7y < 38.$

The minimum cost of the mixture will be Rs 380.

23. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 57,60 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine Rs 240. He expects to sell a fan at a profit of Rs 22 and a sewing machine for a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize his profit. Solve it graphically.

Sol. Let the dealer purchases x fans and y sewing machines.

So, the linear programing problem is as follows:

Maximize $Z = 22x + 18y$

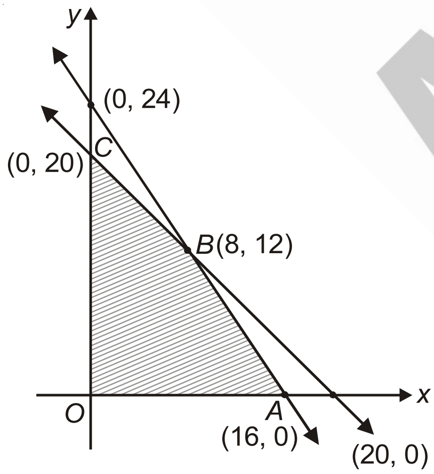
Subject to the constraints:

$3x + 2y \leq 48$

$x + y \leq 20$

and $x, y \leq 0$.

The feasible region is shaded in the figure given below.



Corner Point	$Z = 22x + 18y$
O (0, 0)	0
A (16, 0)	352
B (8, 12)	392
C (0, 20)	360

so, the maximum profit is 392 at the point (8, 12).

24. A cooperative society of farmers has 50 hectares of land to grow two crops x and y. The profit from crops x and y per hectare are estimated at Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops x and y at the rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crops so as to maximize the total profit of the society?

Sol. Let x hectare of land be allocated to crop x and y hectare of crop y.

So, the mathematical formulation of the problem is as follows:

Maximize $Z = 10500x + 9000y$

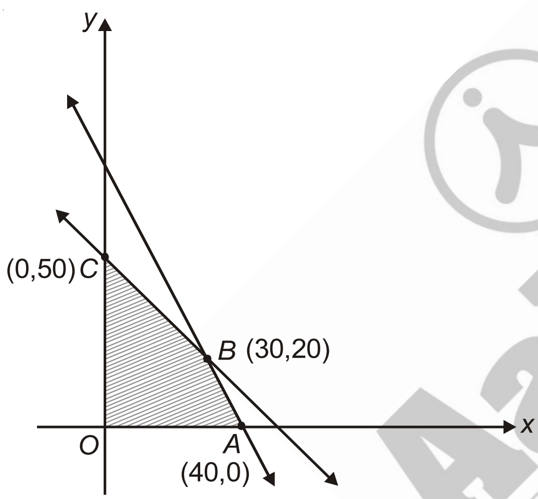
subject to the constraints:

$x + y \le 50$

$2x + y \le 80$

and $x \ge 0$, $y \ge 0$.

The feasible region is O*ABC*, shaded in the figure given below



Corner Point	$Z = 10500x + 9000y$	
O (0, 0)	0	
A (40, 0)	420000	
B (30, 20)	495000	→ maximum
C (0, 50)	450000	

Hence the society will get the maximum profit of Rs 4,95,000 by allocating 30 hectares for crop x and 20 hectares for crop y.

25. A manufacturing company makes two models A and B of a product. Each pieces of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hours for finishing for fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of model B. How many pieces of model A and model B should be manufactured per week to realise a maximum profit?

Sol. Suppose there are x number of pieces of model A and y number of pieces of model B are required.

Now we have the following mathematical problem;

Maximize $Z = 8000x + 12000y$

subject to the constraints:

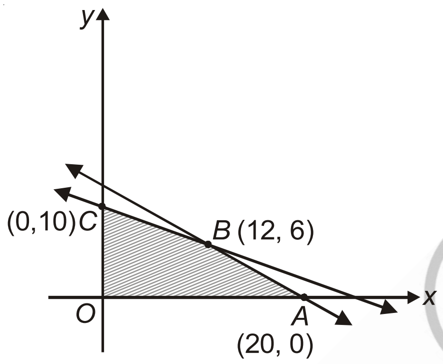
$9x + 12y \leq 180$

i;e. $3x + 4y \leq 60$

$x + 3y \leq 30$

and $x \geq 0,\ y \geq 0.$

The feasible region is $OABC$ shaded in the figure given below.



Corner Point	$Z = 8000x + 12000y$	
O (0, 0)	0	
A (20, 0)	160000	
B (12, 6)	168000	→ maximum
C (0, 10)	120000	

We find the maximum value of Z is at (12, 6).

26. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

Sol. Let x and y be the number of packets of food P and Q respectively.

Mathematical formulation of the given problem is as follows:

Minimize $Z = 6x + 3y$

subject to the constraints

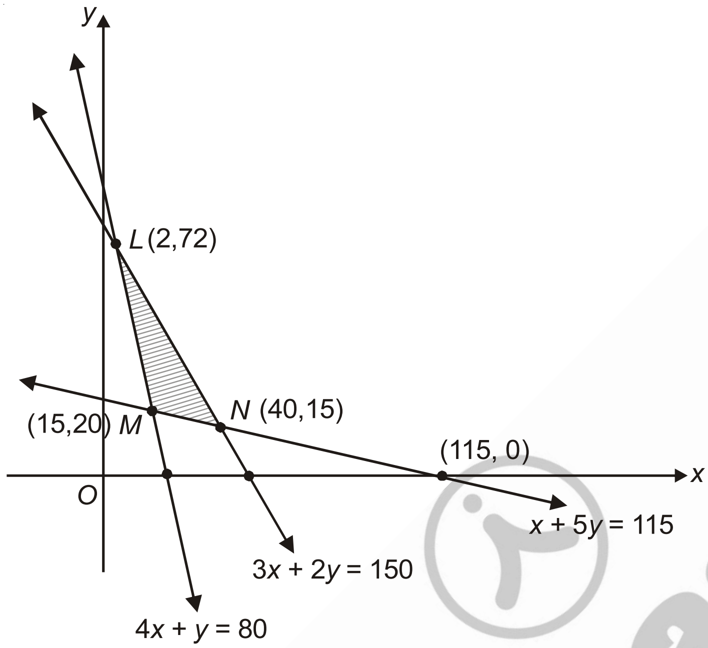
$12x + 3y \geq 240$ or $4x + y \geq 80$

$\Rightarrow$ $4x + 20y \geq 460$ or $x + 5y \geq 115$

$\Rightarrow$ $6x + 4y \leq 300$ or $3x + 2y \leq 150$

and $x \geq 0,\ y \geq 0$

The feasible region is LMN shaded in the figure given below



Corner Point	$Z = 6x + 3y$	
(2, 72)	228	
(15, 20)	150	→ maximum
(40, 15)	285	

So, Z is minimum at the point (15, 20).

27. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. He produces only two items M and N each requiring the use of all the twice machines.

The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should be produce so as to maximize his profit?

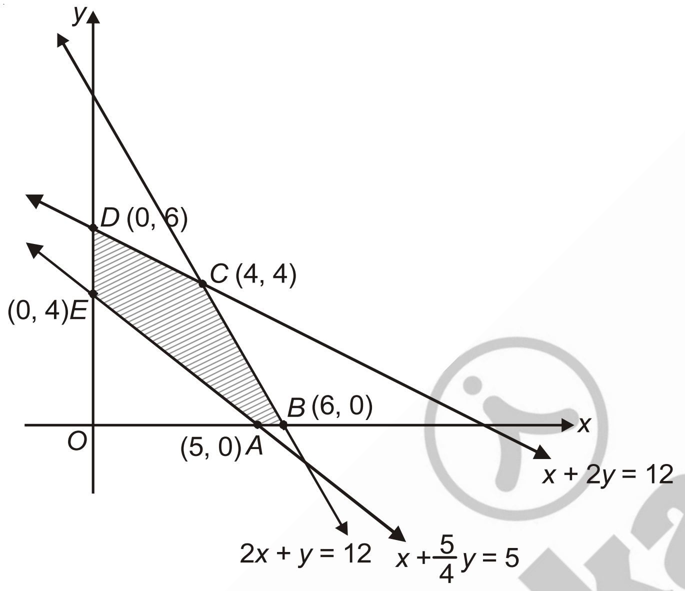
Sol. Let x and y be the number of items M and N respectively

so, mathematical formulation of the given problem is as follows:

Maximize $Z = 600x + 400y$

Subject to the constraints $x + 2y \le 12$, $2x + y \le 12$, $x + \frac{5}{4}y \ge 5$ and $x \ge 0$, $y \ge 0$.

The feasible region is $ABCDE$ shaded in the figure given below



Corner Point	**Z = 600x + 400y**	
(5, 0)	3000	
(6, 0)	3600	
(4, 4)	4000	→ maximum
(0, 6)	2400	
(0, 4)	1600	

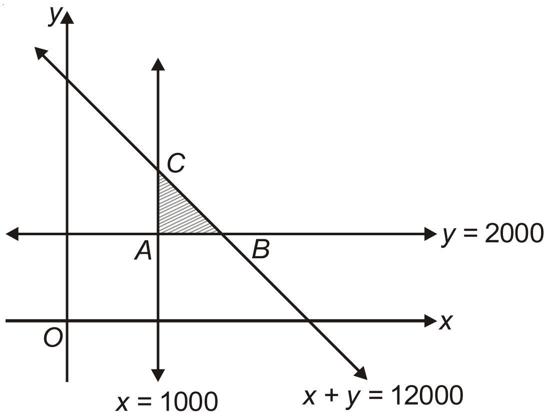
So, the manufacturer has to produce 4 units of each item to get the maximum profit of Rs 4000.

28. Mr. Das wants to invest Rs 12000 in Public Provident Fund (PPF) and in National Bonds. He has to invest at least Rs 1000 in PPF and at least Rs 2000 in bonds. If the rate of interest on PPF is 12% per annum and that on bonds is 15% per annum, how should he invest the money to earn maximum annual income?

Sol. Let Mr. Das has invested x Rs in PPF and y rupees in Bonds. Mathematical formulation of the given problem is as follows Maximize $Z = \frac{12x}{100} + \frac{15y}{100}$

Subject to the constraints; $x \ge 1000$, $y \ge 2000$, $x + y \le 12000$ and $x \ge 0$, $y \ge 0$.

The feasible region is ABC.



Corner Point	$Z = \frac{12x}{100} + \frac{15y}{100}$
A (1000, 2000)	420
B (10000, 2000)	1500
C (1000, 11000)	1770

i.e., Z is maximum at the point (1000, 11000).

29. Two tailors, A and B earn Rs 300 and Rs 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. How many days should each of them work if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost?

Sol. Let A and B work for x days and y days respectively.

Now, the mathematical formulation of the given problem is as follows:

Minimize $Z = 300x + 400y$

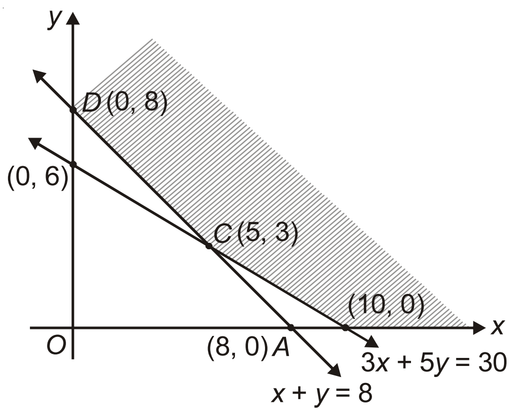
Subject to the constraints:

$6x + 10y \geq 60$

$4x + 4y \geq 32$

and $x \geq 0,\ y \geq 0$

The feasible region is unbounded.



Corner Point	$Z = 300x + 400y$	
B (10, 0)	3000	
C (5, 3)	2700	→ minimum
D (0, 8)	3200	

i.e., Z is minimum at (5, 3).

30. A housewife wishes to mix together two kinds of food x and y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of each food are given below

	Vitamin A	Vitamin B	Vitamin C
Food x	1	2	3
Food y	2	2	1

If 1 kg of food x costs Rs 6 and 1 kg of food y costs Rs 10, find the minimum cost of the mixture which will produce the diet

Sol. Let x kg of the food X and y kg of the food Y be taken.

Now, mathematical formulation of the problem is as

follows

Minimize $Z = 6x + 10y$

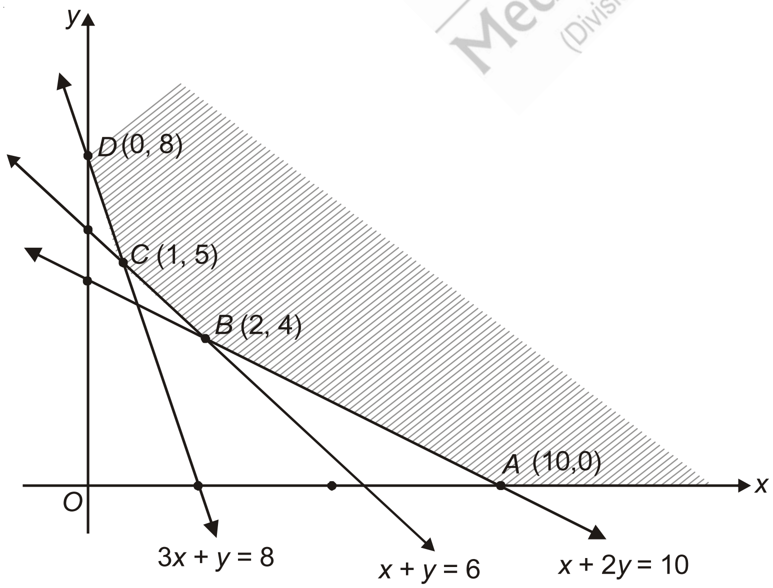
Subject to the constraints

$x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

and $x \geq 0,\ y \geq 0$



The feasible region is unbounded

Corner Point	$Z = 6x + 10y$	
A (10, 0)	60	
B (2, 4)	52	→ minimum
C (1, 5)	56	
D (0, 8)	80	

Hence, Z has minimum value at the point $B(2, 4)$.

31. A small firm manufactures items A and B. The total number of items that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs 300 and that on one unit of item B be Rs 160, how many of each type of item should be produced to maximize the profit? solve the problem graphically.

Sol. Let x items of A and y items of B be produced.

Now, mathematical formulation of the problem is as follows:

Maximize $P = 300x + 160y$

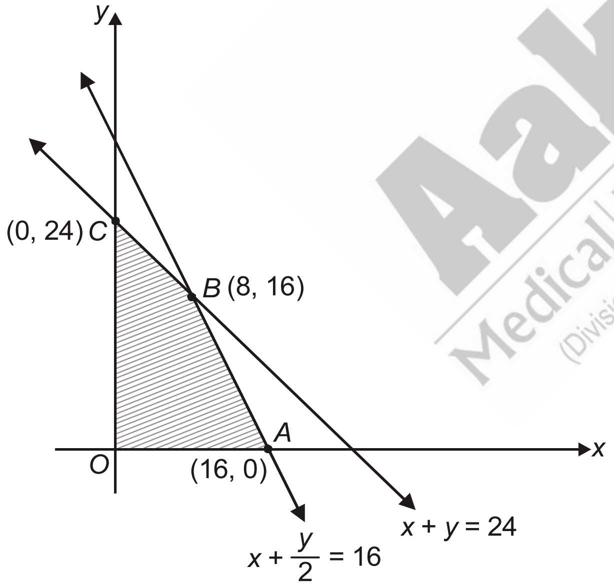
subject to the restrictions:

$x + y \le 24$

$x + \frac{y}{2} \le 16$

and $x \ge 0$, $y \ge 0$

The feasible region is $OABC$



Corner Point	$P = 300x + 160y$
O (0, 0)	0
A (16, 0)	4800
B (8, 16)	4960
C (0, 24)	3840

so, Z is maximum at the point (8, 16).

32. Kellogy is a new cereal formed of a mixture of bean and rice, that contains at least 88 grams of proteins and at least 36 mg of iron. Knowing that bean contains 80 gm of protein and 40 mg of iron per kg and that rice contains 100 gm of protein and 30 mg of iron per kg, find the minimum cost of producing this new cereal if bean costs Rs 5 per kg and rice costs Rs 4 per kg.

Sol. Let the cereal contains x kg of bean and y kg of rice.

Now, mathematical formulation of the problem is as follows:

Minimize

$Z = 5x + 4y$

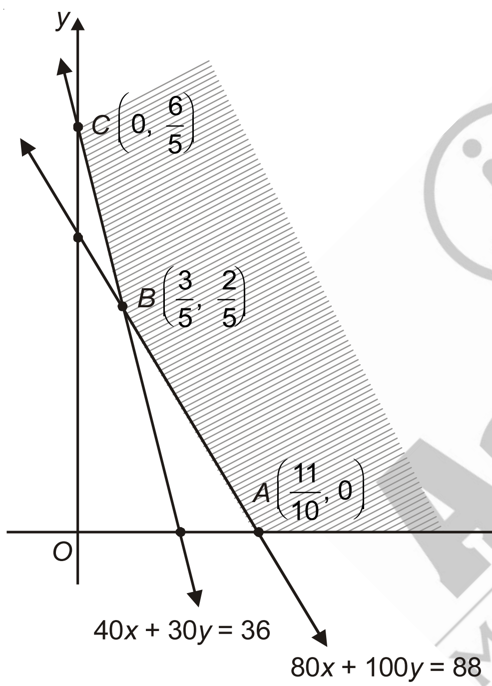
Subject to the constraints

$80x + 100y \geq 88$

$40x + 30y \geq 36$

and $x \geq 0,\ y \geq 0$

Here feasible region is unbounded shaded in the figure given below.



Corner Point	$Z = 5x + 4y$	
$A\left(\frac{11}{10}, 0\right)$	$\frac{11}{2}$	
$B\left(\frac{3}{5}, \frac{2}{5}\right)$	$\frac{23}{5}$	$\rightarrow$ minimum
$C\left(0, \frac{6}{5}\right)$	$\frac{24}{5}$	

Hence Z is minimum

at $B\left(\frac{3}{5}, \frac{2}{5}\right)$.

33. A gardener has a supply of fertilizers of the type-I which consists of 10% nitrogen and 6% phosphoric acids and of type-II which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type-I fertilizer costs 60 paise per kg and the type-II fertilizer costs 40 paise per kg, determine how many kgms of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost?

Sol. Let x kg of type-I and y kg of type-II be used. Then mathematical formulation of the problem is as follows:

Minimize $Z = \frac{3}{5}x + \frac{2}{5}y$

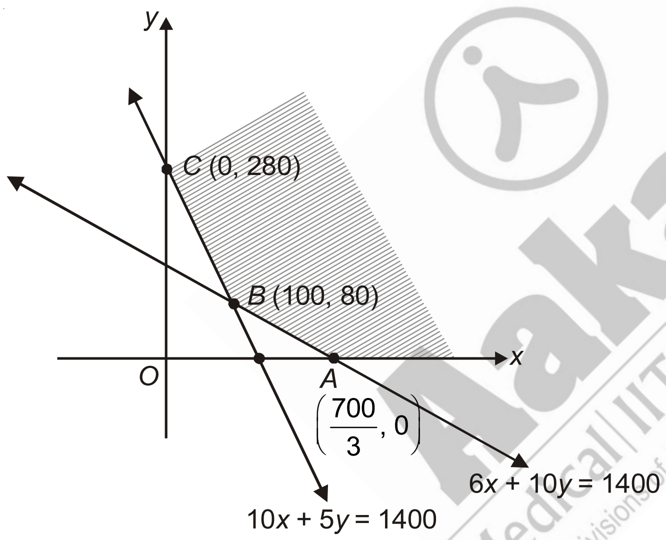
Subject to the constraints:

$10x + 5y \geq 1400$

$6x + 10y \geq 1400$

and $x \geq 0,\ y \geq 0$

The feasible region is unbounded.



Corner Point	$Z = \frac{3x + 2y}{5}$
$A\left(\frac{700}{3}, 0\right)$	140
B (100, 80)	92
C (0, 280)	112

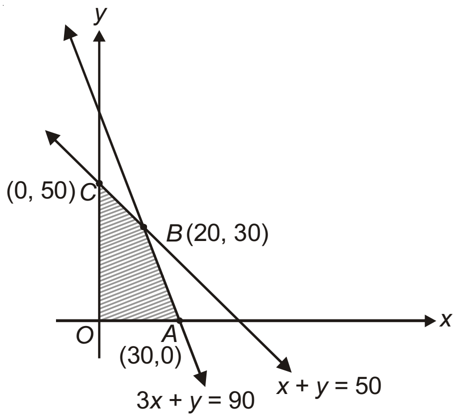
Hence, Z is minimum at the point B(100, 80).

34. Maximize $Z = 60x + 15y$

subject to the constraints:

$x + y \leq 50,\ 3x + y \leq 90,\ x \geq 0,\ y \geq 0$

Sol. The feasible region is *OABC* shaded in the figure given below



Corner Point	**$P = 60x + 15y$**
O (0, 0)	0
A (30, 0)	1800
B (20, 30)	1650
C (0, 50)	750

Hence, Z is maximum at the point (30, 0) and its maximum value is 1800.

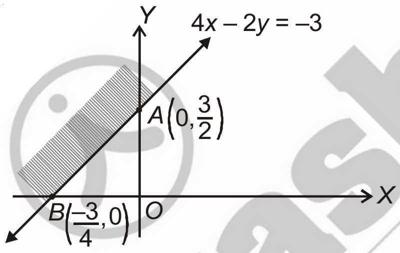
❑ ❑ ❑

Chapter 13

Linear Programming

Solutions (Set-2)

1. Shaded region is represented by



(1) $4x - 2y \le 3$ (2) $4x - 2y \le -3$ (3) $4x - 2y \ge 3$ (4) $4x - 2y \ge -3$

Sol. Answer (2)

Origin is not present in given shaded area. So $4x - 2y \le -3$ satisfying this condition.

2. A LPP means

(1) Only objective function is linear (2) Only constraints are linear

(3) Either objective or constraints are linear (4) All objective function and constraints are linear

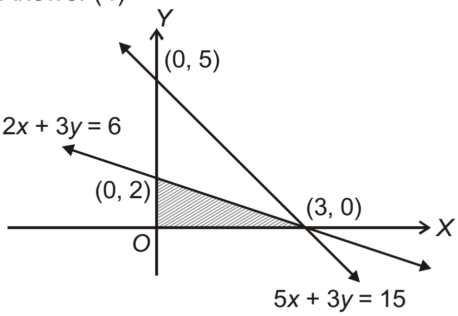
Sol. Answer (4)

A LPP means all objective function and constraints are linear.

3. Which of the following is not a vertex of the positive region bounded by the inequalities $2x + 3y \le 6$, $5x + 3y \le 15$ and $x, y \ge 0$?

(1) (0, 2) (2) (0, 0) (3) (3, 0) (4) (4, 0)

Sol. Answer (4)

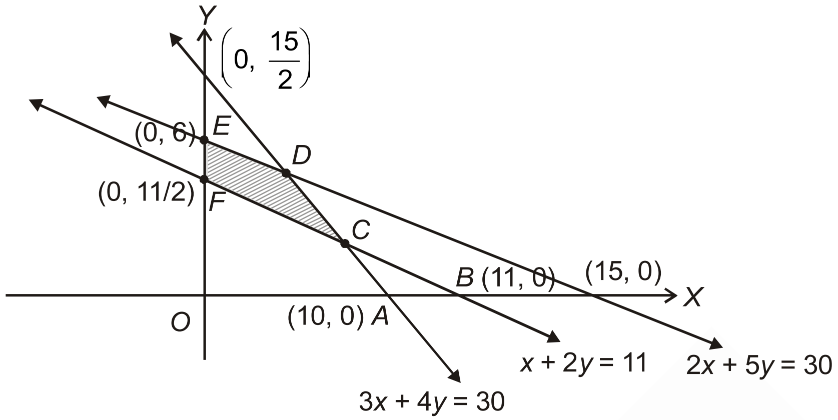


Here, (0, 2); (0, 0) and (3, 0) all are vertices of feasible region. (4, 0) not a vertex.

4. The solution set of constraints $x + 2y \geq 11$, $3x + 4y \leq 30$, $2x + 5y \leq 30$, $x \geq 0$, $y \geq 0$ includes the point

(1) (2, 3) (2) (4, 3) (3) (3, 2) (4) (3, 4)

Sol. Answer (4)



The feasible region is *FCDEF*.

The solution set of constraints includes the point (3, 4).

5. Inequation $y - x \leq 0$ represents

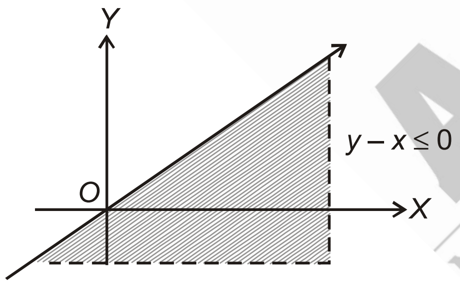
(1) The half plane that contains the positive x-axis

(2) Closed half plane above the line $y = x$ which contains positive y-axis

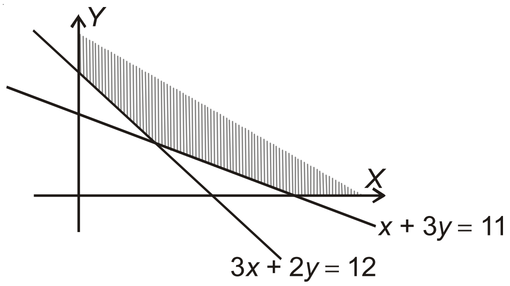
(3) Half plane that contains the negative x-axis

(4) Half plane that contains the positive y-axis

Sol. Answer (1)



6. For the following feasible region, the linear constraints are



(1) $x \geq 0$, $y \geq 0$, $3x + 2y \geq 12$, $x + 3y \geq 11$ (2) $x \geq 0$, $y \geq 0$, $3x + 2y \leq 12$, $x + 3y \geq 11$

(3) $x \geq 0$, $y \geq 0$, $3x + 2y \leq 12$, $x + 3y \leq 11$ (4) $x \geq 0$, $y \geq 0$, $3x + 2y \geq 12$, $x + 3y \leq 11$

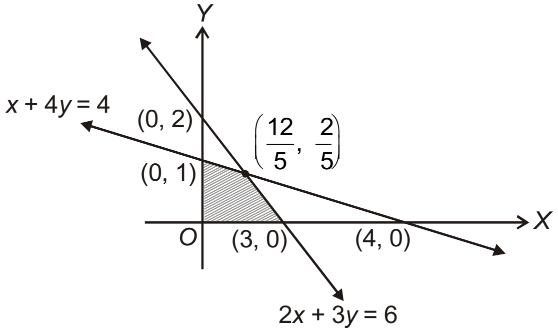
Sol. Answer (1)

We will consider the region above for both the lines in first quadrant.

7. A vertex of the linear inequalities $2x + 3y \le 6$, $x + 4y \le 4$, $x, y \ge 0$, is

(1) (1, 0) (2) (1, 1) (3) $\left(\frac{12}{5}, \frac{2}{5}\right)$ (4) $\left(\frac{2}{5}, \frac{12}{5}\right)$

Sol. Answer (3)



Hence a vertex is $\left(\frac{12}{5}, \frac{2}{5}\right)$.

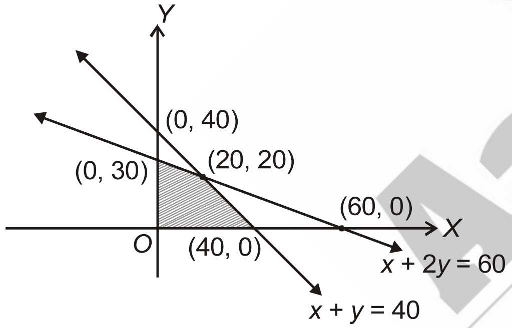
8. The maximum value of $Z = 3x + 4y$, subject to the condition

$x + y \le 40$

$x + 2y \le 60$, $x, y \ge 0$ is

(1) 40 (2) 120 (3) 130 (4) 140

Sol. Answer (4)



Maximum $Z = 3(20) + 4(20) = 60 + 80 = 140$.

9. The linear programming problem : Max $Z = x_1 + x_2$ such that $2x_1 - x_2 \ge -1$, $x_1 \le 2$, $x_1 + x_2 \le 3$ and $x_1, x_2 \ge 0$ has

(1) One solution (2) Three solutions

(3) An infinite number of solution (4) Two solutions

Sol. Answer (3)

Objective function is same as constraint so there are infinite solutions possible.

10. In which quadrant, the bounded region for inequations $x + y \le 1$ and $x - y \le 1$ is situated?

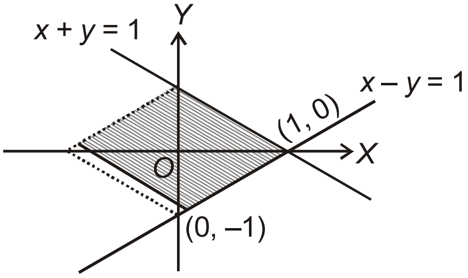
(1) I, II

(2) II, III

(3) I, III

(4) All the four quadrants

Sol. Answer (4)



The bounded region situated in all four quadrants.

11. Which of the following statements is correct?

(1) Every LPP admits an optimal solution

(2) A LPP admits a unique optimal solution

(3) If a LPP admits two optimal solutions it has an infinite number of optimal solutions

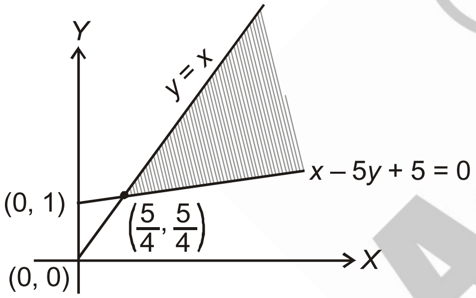
(4) The set of all feasible solutions of a LPP is not a convex set

Sol. Answer (3)

12. The minimum value of the objective function $Z = 2x + 10y$ for linear constraints $x \geq 0$, $y \geq 0$, $x - y \geq 0$, $x - 5y \leq -5$, is

(1) 8 (2) 10 (3) 12 (4) 15

Sol. Answer (4)



Region is unbounded whose vertex is $\left(\frac{5}{4}, \frac{5}{4}\right)$

Hence, the minimum value of $Z = 2x + 10y$ is

$$2\left(\frac{5}{4}\right) + 10\left(\frac{5}{4}\right) = \frac{60}{4} = 15.$$

13. The minimum value of $Z = 2x + 3y$

subject to the constraints

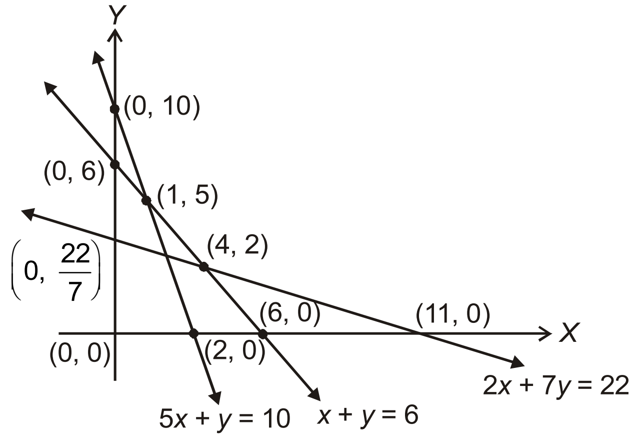
$2x + 7y \geq 22$

$x + y \geq 6$

$5x + y \geq 10$, $x, y \geq 0$ is equal to

(1) 10 (2) 14 (3) 16 (4) 20

Sol. Answer (2)



Obviously, at point (4, 2) the minimum value of Z

$= 2x + 3y$

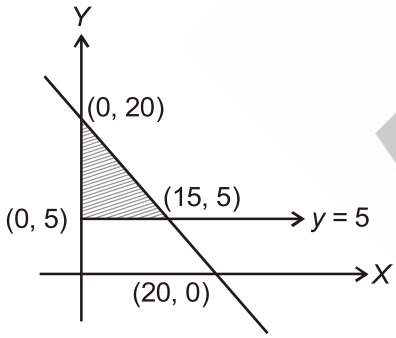
$= 2 \times 4 + 3 \times 2$

$= 14.$

14. The co-ordinate of the point for minimum value of $Z = 7x - 8y$ subject to the conditions $x + y - 20 \leq 0$, $y - 5 \geq 0$, $x \geq 0$, $y \geq 0$ is (α, β). Then $\alpha + \beta$ is equal to

(1) 20 (2) 30 (3) 50 (4) 60

Sol. Answer (1)



Here $Z = 7x - 8y$ will be minimum at (0, 20)

$\therefore$ Min $z = 7 \times 0 - 8 \times 20 = -160$

$\alpha = 0, \beta = 20$

$\alpha + \beta = 20.$

15. A firm makes pants and shirts. A shirt takes 2 hours on machine and 3 hours of man labour while a pant takes 3 hours on machine and 2 hours of man labour. In a week there are 70 hours of machine and 75 hours of man labour available. If the firm determines to make x shirts and y pants per week, then for this the linear constraints are

(1) $2x + 3y \geq 70$, $3x + 2y \geq 75$, $x \geq 0$, $y \geq 0$

(2) $2x + 3y \leq 70$, $3x + 2y \geq 75$, $x \geq 0$, $y \geq 0$

(3) $2x + 3y \geq 70$, $3x + 2y \leq 75$, $x \geq 0$, $y \geq 0$

(4) $2x + 3y \leq 70$, $3x + 2y \leq 75$, $x \geq 0$, $y \geq 0$

Sol. Answer (4)

Types of items	Shirt (x)	Pant (y)	Availability
Working time on machine	2 hours	3 hours	70 hours
Man Labour	3 hours	2 hours	75 hours

Linear constraints are $2x + 3y \le 70$, $3x + 2y \le 75$, $x \ge 0$ and $y \ge 0$.

16. Let X_1 and X_2 are optimal solutions of a LPP for which objective function is maximum. Then

(1) $X = \lambda X_1 + (1 - \lambda)X_2$, $\lambda \in R$ is also an optimal solution

(2) $X = \lambda X_1 + (1 - \lambda)X_2$, $0 \le \lambda \le 1$, gives an optimal solution

(3) $X = \lambda X_1 + (1 + \lambda)X_2$, $\lambda \in R$, gives an optimal solution

(4) $X = \lambda X_1 + (1 + \lambda)X_2$, $0 \le \lambda \le 1$, gives an optimal solution

Sol. Answer (2)

17. We have to purchase two articles A and B of cost ₹ 45 and ₹ 25 respectively. I can purchase total article maximum of ₹ 1000. After selling the articles A and B the profit per unit is ₹ 5 and ₹ 3 respectively. If I purchase x and y number of articles A and B respectively, then the mathematical formulation of problem is

(1) $Z = 5x + 3y$, $45x + 25y \ge 1000$, $x \ge 0$, $y \ge 0$

(2) $Z = 5x + 3y$, $45x + 25y \le 1000$, $x \ge 0$, $y \ge 0$

(3) $Z = 3x + 5y$, $45x + 25y \le 1000$, $x \ge 0$, $y \ge 0$

(4) $Z = 3x + 5y$, $45x + 25y \ge 1000$, $x \ge 0$, $y \ge 0$

Sol. Answer (2)

Here profit is 5 per unit for article A and 3 per unit for article B.

$\therefore \quad Z = 5x + 3y$

Total purchased price $45x + 25y \le 1000$ and $x \ge 0$, $y \ge 0$.

18. A company manufactures two types of products A and B. The storage capacity of its godown is 100 units. Total investment amount is ₹ 30,000. The cost price of A and B are ₹ 400 and ₹ 900 respectively. If all the products have sold and per unit profit is ₹ 100 and ₹ 120 of A and B respectively. If x units of A and y units of B be produced, then the mathematical formulation of problem is

(1) $x + y = 100$, $4x + 9y = 300$, $Z = 100x + 120y$

(2) $x + y \le 100$, $4x + 9y \le 300$, $Z = x + 2y$

(3) $x + y \le 100$, $4x + 9y \le 300$, $Z = 100x + 120y$

(4) $x + y \le 100$, $9x + 4y \le 300$, $Z = x + 2y$

Sol. Answer (3)

Storage capacity is 100 units

$\therefore \quad x + y \le 100$ ...(i)

Total investment is ₹ 30,000

Cost price of one unit of A = ₹ 400

Cost price of x units of A = ₹ $400x$

Cost price of one unit of B = ₹ 900

Cost price of y units of B = ₹ $900y$

$\therefore \quad 400x + 900y \le 30{,}000$

$\Rightarrow \quad 4x + 9y \le 300$ ...(ii)

Total profit $Z = 100x + 120y$ ...(iii)

19. If the number of available constraints is 3 and the number of parameters to be optimized is 4, then

(1) The objective function can be optimized (2) The constraints are short in number

(3) The solution is problem oriented (4) The constraints are more in number

Sol. Answer (2)

20. In a test of mathematics, there are two types of questions to be answered-short answered and long answered. The relevant data is given below

Types of Questions	Time taken solve	Marks	Number of questions
Short answered question	5 minutes	3	10
Long answered question	10 minutes	5	14

The total marks is 100. Students can solve all the questions. To secure maximum marks, a student solves x short answered and y long answered questions in three hours. The linear constraints except $x \ge 0$, $y \ge 0$, are

(1) $5x + 10y \le 180$, $x \le 10$, $y \le 14$ (2) $x + 10y \ge 180$, $x \le 10$, $y \le 14$

(3) $5x + 10y \ge 180$, $x \ge 10$, $y \ge 14$ (4) $5x + 10y \le 180$, $x \ge 10$, $y \ge 14$

Sol. Answer (1)

Obviously, $x \le 10$, $y \le 14$

Max time = 180 minutes

$5x + 10y \le 180$.

21. The objective function for the above question

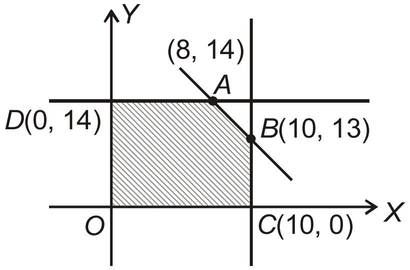
(1) $10x + 14y$ (2) $5x + 10y$ (3) $3x + 5y$ (4) $5x + 3y$

Sol. Answer (3)

22. The number of vertices of a feasible region of the above question

(1) 3 (2) 4 (3) 5 (4) 6

Sol. Answer (3)



Here, required feasible region is given by *ODABCO* and vertices are (0, 0), (0, 14), (8, 14), (10, 13) and (10, 0).

So, number of vertices are 5.

23. The maximum value of the objective function in the above question is

(1) 95 (2) 92 (3) 94 (4) 100

Sol. Answer (1)

The objective function is $Z = 3x + 5y$.

The points are (8, 14) and (10, 13)

$Z = (3 \times 8) + (5 \times 14) = 94$ and $Z = (10 \times 3) + (13 \times 5) = 30 + 65 = 95$

Max $Z = 95$.

24. A firm produces two types of products *A* and *B*. The profit on both is ₹2 per item. Every product requires processing on machine M_1 and M_2. For *A*, machines M_1 and M_2 take 1 minute and 2 minutes respectively and for *B*, machines M_1 and M_2 take 1 minute each. The machines M_1, M_2 are not available more than 8 hours and 10 hours on any of day respectively. If the products made *x* of *A* and *y* of *B* then the linear constraints for the LPP except $x \geq 0$, $y \geq 0$, are

(1) $x + y \leq 480$, $2x + y \leq 600$

(2) $x + y \geq 480$, $2x + y \geq 600$

(3) $x + y \leq 8$, $2x + y \leq 10$

(4) $x + y \leq 8$, $2x + y \geq 10$

Sol. Answer (1)

Obviously $x + y \leq (8 \times 60) = 480$ and $2x + y \leq (10 \times 60) = 600$

	A	*B*	Total availability
M_1	x_{min}	y_{min}	8 hours = 480 minutes
M_2	$2x$	y	10 hours = 600 minutes

25. The objective function for the above question is

(1) $2x + y$ (2) $x + 2y$ (3) $2x + 2y$ (4) $8x + 10y$

Sol. Answer (3)

Profit on both the products *A* and *B* per unit is 2. Total *x* units of *A* and *y* units of *B*.

$Z = 2x + 2y$.

26. By graphical method, the solution of LPP Maximize $Z = 3x + 5y$ subject to

$3x + 2y \leq 18$

$x \leq 4$

$y \leq 6$

$x \geq 0,\ y \geq 0$

is

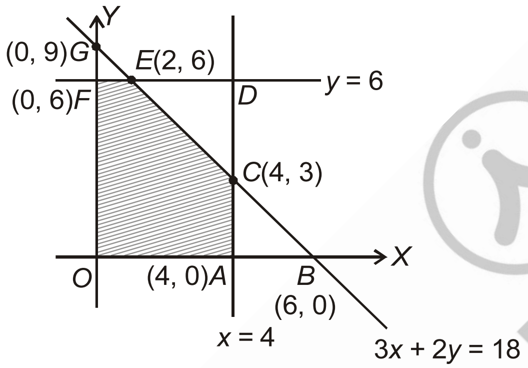
(1) $x = 4,\ y = 6,\ Z = 4.2$

(2) $x = 4,\ y = 3,\ Z = 2.7$

(3) $x = 2,\ y = 6,\ Z = 36$

(4) $x = 2,\ y = 0,\ Z = 6$

Sol. Answer (3)



Required solution lies in the region *OACEF*.

The vertices points are (0, 0), (4, 0), (4, 3), (2, 6), (0, 6).

(2, 6) lies at a greatest distance from the origin.

Thus, the solution of the linear programming problem is $x = 2$, $y = 6$

and $z = 2 \times 3 + 5 \times 6 = 36$.

27. The solution set of a linear programming problem, with an objective to maximize $Z = 3x_1 + 4x_2$ has only four vertices with co-ordinates (0, 6), (4, 4), (6, 0) and (0, 0). The optimal solution of the linear programming problem is

(1) $x_1 = 0,\ x_2 = 6$ (2) $x_1 = 4,\ x_2 = 4$ (3) $x_1 = 6,\ x_2 = 0$ (4) $x_1 = 0,\ x_2 = 0$

Sol. Answer (2)

Let $Z = 3(0) + 4 \times 6 = 24$

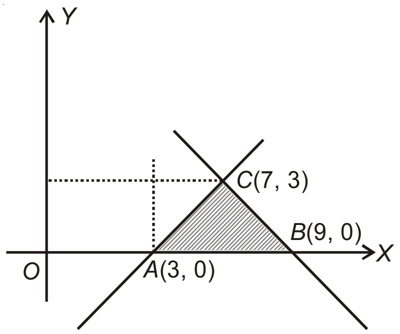
$Z = 3(4) + 4 \times 4 = 28$

$Z = 3 \times 6 + 0 \times 4 = 18$

$Z = 0 + 0 = 0$

$\therefore$ Z is maximum when $x_1 = 4$, $x_2 = 4$.

28. Which of the following system of inequations is represented by the shaded portion of the graph given below ?



(1) $x \ge 3,\ y \ge 3,\ x \le 9$

(2) $3x - 4y \ge 9,\ 3x + 2y \le 27,\ x \ge 0$

(3) $3x - 4y \le 9,\ 3x + 3y \ge 27,\ x \ge 0,\ y \ge 0$

(4) $3x - 4y \ge 9,\ 3x + 2y \le 27,\ y \ge 0$

Sol. Answer (4)

Equation of line AC $y - 0 = \frac{3}{4}(x-3)$

$\Rightarrow\ 4y = 3x - 9$

$\therefore\ 3x - 4y = 9$

Shaded area is below $3x - 4y = 9$

$\therefore\ 3x - 4y \ge 9$

Equation of line BC is $y - 0 = \frac{3}{-2}(x-9)$

$-2y = 3x - 27$

$3x + 2y = 27$

Shaded region is below $3x + 2y = 27$

$\therefore$ *i.e.*, $3x + 2y \le 27$

and $y \ge 0$.

29. A company manufactures two types of fertilizers F_1 and F_2. Each type of fertilizers requires two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the table below

Fertilizers→ / Raw material	F_1	F_2	Availability
A	2	3	40
B	1	4	70

By selling one unit of F_1 and one unit of F_2, the company gets a profit of ₹ 500 and ₹ 750 respectively. Then which of the following option is correct when the profit is maximum and x units of fertilizers F_1 and y units of fertilizers F_2 be produced per day

(1) $2x + 3y \le 40,\ x + 4y \le 70,\ x \ge 0,\ y \ge 0$

(2) $3x + 2y \le 40,\ x + 4y \le 70,\ x \ge 0,\ y \ge 0$

(3) $2x + 3y \le 40,\ x + 4y \ge 70,\ x \ge 0,\ y \ge 0$

(4) $3x + 2y \ge 40,\ x + 4y \ge 70,\ x \ge 0,\ y \ge 0$

Sol. Answer (1)

x units of fertilizer F_1 and y units of fertilizer F_2 be produced per day.

According to the question,

$2x + 3y \le 40$ and $x + 4y \le 70$

Obviously x and y cannot be negative $x \ge 0$, $y \ge 0$.

30. For above question, the objective function for which the profit is maximum is

(1) $500x + 750y$ (2) $700x + 500y$ (3) $2x + 3y$ (4) $500x - 750y$

Sol. Answer (1)

The profit Z obtained by selling x units of F_1 and y units of F_2 is given by

$Z = 500x + 750y$.

31. By graphical method, the solution of linear programming problem

Maximize $Z = 500x + 750y$

subject to $2x + 3y \ge 40$

$x + 4y \le 70$

$x \ge 0,\ y \ge 0$

is

(1) $x = 70,\ y = 0$ (2) $x = 0,\ y = \frac{35}{2}$ (3) $x = 0,\ y = \frac{40}{3}$ (4) $x = 20,\ y = 0$

Sol. Answer (1)

For $x = 70$, $y = 0$

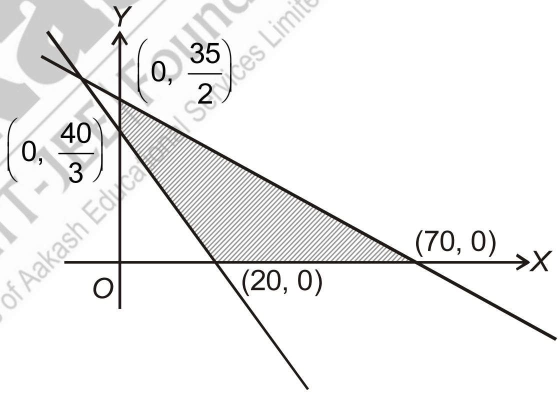
$Z = 500x + 750y = 35{,}000$

$x = 0,\ y = \frac{35}{2},\ Z = 13{,}125$

$x = 0,\ y = \frac{40}{3},\ Z = 10{,}000$

$x = 20,\ y = 0,\ Z = 10{,}000$

$\therefore$ $x = 70$, $y = 0$ is the correct option.



32. An owner of a lodge plans an extension which contains not more than 50 rooms. At least 5 must be executive single room. The number of executive double rooms should be at least 3 times the number of executive single rooms. He charges Rs3000 for executive double room and ₹ 1800 for executive single room per day. The linear programming problem to maximize the profit if x_1 and x_2 denote the number of executive single rooms and executive double rooms respectively, is

(1) Maximize $Z = 1800x_1 + 3000x_2$ subject to $x_1 + x_2 \le 50$, $x_1 \ge 5$, $x_2 \ge 3x_1$, $x_1 \ge 0$, $x_2 \ge 0$

(2) Maximize $Z = 1800x_1 + 3000x_2$ subject to $x_1 + x_2 \ge 50$, $x_1 \le 5$, $x_2 \ge 3x_1$, $x_1 \ge 0$, $x_2 \ge 0$

(3) Maximize $Z = 3000x_1 + 1800x_2$ subject to $x_1 + x_2 \le 50$, $x_1 \ge 5$, $x_2 \ge 3x_1$, $x_1 \ge 0$, $x_2 \ge 0$

(4) Maximize $Z = 3000x_1 + 1800x_2$ subject to $x_1 + x_2 \ge 50$, $x_1 \le 5$, $x_2 \ge 3x_1$, $x_1 \ge 0$, $x_2 \ge 0$

Sol. Answer (1)

Since the owner of the lodge plans an extension of not more than 50 rooms

$\therefore$ $x_1 + x_2 \le 50$ ...(i)

Lodge must have at least 5 executive single rooms

$\therefore \quad x_1 \geq 5$

Since the number of executive double rooms should be at least three times the number of executive single rooms.

$x_2 \geq 3x_1$

The charge for single executive room is ₹ 1800 and that for executive double room is ₹ 3000 per day. Obviously the number of rooms of any type cannot be negative. $x_1 \geq 0$, $x_2 \geq 0$.

The total profit of the lodge owner is $Z = 1800x_1 + 3000x_2$ which is to be maximized.

Hence the LPP is formulated as Maximize $Z = 1800x_1 + 3000x_2$ subject to $x_1 + x_2 \leq 50$, $x_1 \geq 5$, $x_2 \geq 3x_1$, $x_1 \geq 0$, $x_2 \geq 0$.

33. The maximum value of the function $Z = 5x + 2y$ subject to the condition $3x + 5y \leq 15$, $5x + 2y \leq 10$ and $x \geq 0$, $y \geq 0$ is

(1) 10 (2) 20 (3) 30 (4) 40

Sol. Answer (1)

At (0, 3), $Z = 6$

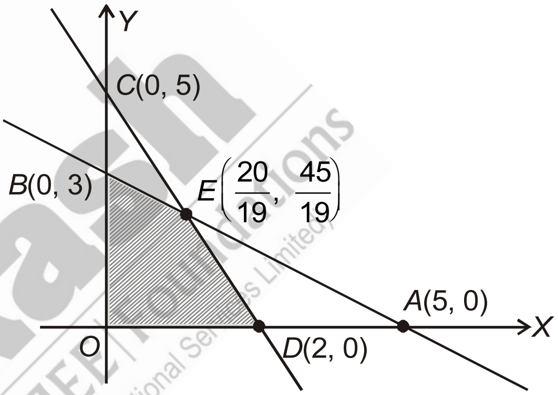
At (0, 0), $Z = 0$

At (2, 0), $Z = 10$

At $\left(\frac{20}{19}, \frac{45}{19}\right)$

$$Z = \frac{100}{19} + \frac{90}{19} = 10$$

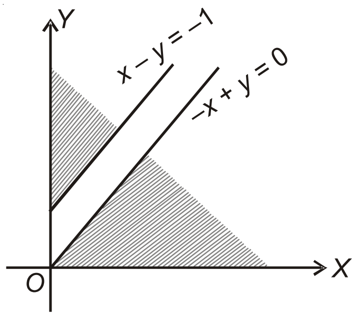
$\therefore \quad Z_{max} = 10.$



34. Maximum value of $Z = 3x + 4y$ subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$ is given by

(1) 1 (2) 4 (3) 6 (4) No feasible solution

Sol. Answer (4)



The shaded portion denotes the region $x - y \leq -1$ and $-x + y \leq 0$.

There is no point in the first quadrant which satisfies the two constraints. Hence, the LPP has no feasible solution.

35. Minimize $Z = \sum_{j=1}^{10}\sum_{i=1}^{30} C_{ij} \cdot X_{ij}$

subject to

$$\sum_{j=1}^{10} x_{ij} \le a_i,\ i = 1, 2, 3, ...30$$

$$\sum_{i=1}^{30} x_{ij} \le b_j,\ j = 1, 2, 3, ...10$$

is a LPP with number of constraints

(1) 40 (2) 20 (3) 300 (4) 3

Sol. Answer (1)

Let i = 1, 2, ..., m, j = 1, 2, 3, ..., n

For $i = 1$, $x_{11} + x_{12} + x_{13} + ... + x_{1n}$

$i = 2$, $x_{21} + x_{22} + x_{23} + ... + x_{2n}$

•
•
•
•

$\therefore$ $i = m, x_{m1} + x_{m2} + x_{m3} + ... + x_{mn} \rightarrow m$ constraints

For $j = 1$, $x_{11} + x_{21} + x_{31} + ... + x_{m1}$

$j = 2$, $x_{12} + x_{22} + x_{32} + x_{42} + ... + x_{m2}$

•
•
•
•

$j = n$, $x_{1n} + x_{2n} + x_{3n} + x_{4n} + ... + x_{mn} \rightarrow n$ constraints

Total constraints = $m + n$

Here $m = 30$ and $n = 10$

$\therefore m + n = 40$

36. If Salim drives a car at a speed of 60 km/h, he has to spend ₹ 5 per km on petrol. If he drives at a faster speed of 90 km/h, the cost of petrol increases to ₹ 8 per km. He has ₹ 600 to spend on petrol and wishes to travel the maximum distance within an hour. If x_1 and x_2 denote the distance (in km) travelled at a speed of 60 km/h and 90 km/h respectively, then the linear programming problem is

(1) Maximize $Z = x_1 + x_2$ subject to $\frac{x_1}{60} + \frac{x_2}{90} \le 1$, $5x_1 + 8x_2 \le 600$, $x_1 \ge 0$, $x_2 \ge 0$

(2) Minimize $Z = x_1 + x_2$ subject to $\frac{x_1}{60} + \frac{x_2}{90} \le 1$, $5x_1 + 8x_2 \le 600$, $x_1 \ge 0$, $x_2 \ge 0$

(3) Maximize $Z = x_1 + x_2$ subject to $\frac{x_1}{90} + \frac{x_2}{60} \le 1$, $5x_1 + 8x_2 \le 600$, $x_1 \ge 0$, $x_2 \ge 0$

(4) Minimize $Z = x_1 + x_2$ subject to $\frac{x_1}{90} + \frac{x_2}{60} \le 1$, $5x_1 + 8x_2 \le 600$, $x_1 \ge 0$, $x_2 \ge 0$

Sol. Answer (1)

Time required to driving distance of x_1 kms is $\frac{x_1}{60}$ hours.

Similarly, time required to drive distance of x_2 kms is $\frac{x_2}{90}$ hours.

Total time required is $\left(\frac{x_1}{60}+\frac{x_2}{90}\right)$ hours.

According to the question,

$$\frac{x_1}{60}+\frac{x_2}{90} \le 1$$

Total cost of travelling is ₹ $5x_1 + 8x_2$

Since Salim has only ₹ 600 to spend on petrol,

$5x_1 + 8x_2 \le 600$

Also total distance covered is $(x_1 + x_2)$ kms which should be maximum. Let us denote it by Z

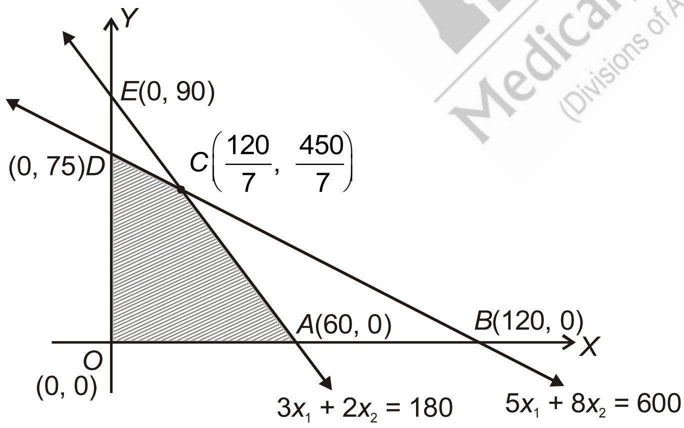
$\therefore$ Maximize $Z = x_1 + x_2$ subject to $\frac{x_1}{60}+\frac{x_2}{90} \le 1$,

$5x_1 + 8x_2 \le 600$, $x_1 \ge 0$, $x_2 \ge 0$.

37. In the above question (i.e. Q.No. 36), the maximum distance covered is

(1) $\frac{120}{7}$ km (2) 60 km (3) 75 km (4) $\frac{570}{7}$ km

Sol. Answer (4)



The shaded region is $OACDO$

For the point $C\left(\frac{120}{7}, \frac{450}{7}\right)$, the function

$Z = x_1 + x_2$ is going to be maximum.

The maximum distance travelled is

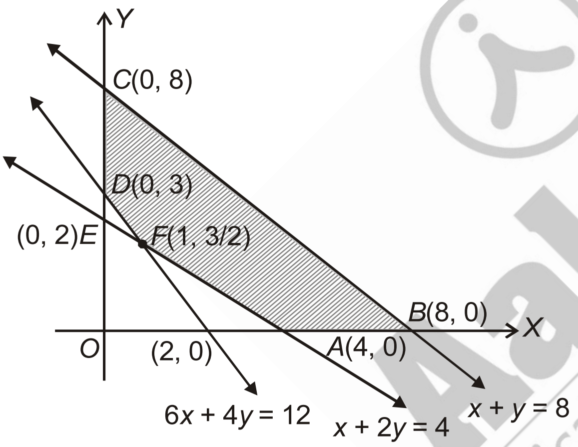
$$Z = \frac{120+450}{7} = \frac{570}{7} \text{ km}$$

Among the points (0, 0), (60, 0), (0, 75), $\left(\frac{120}{7}, \frac{450}{7}\right)$

The point $\left(\frac{120}{7}, \frac{450}{7}\right)$ gives the maximum value of Z.

38. $Z = 30x + 20y$, $x + y \le 8$, $x + 2y \ge 4$, $3x + 2y \ge 6$, $x \ge 0$, $y \ge 0$ has
    (1) Minimum value does not exist
    (2) Minimum value 90
    (3) Minimum at only one point
    (4) Minimum at infinite values

Sol. Answer (4)



Feasible region is $ABCDFA$ and $Z = 30x + 20y$

Now at $A(4, 0)$, $z = 120$

at $B(8, 0)$, $z = 240$

at $C(0, 8)$, $z = 160$

at $D(0, 3)$, $z = 60$

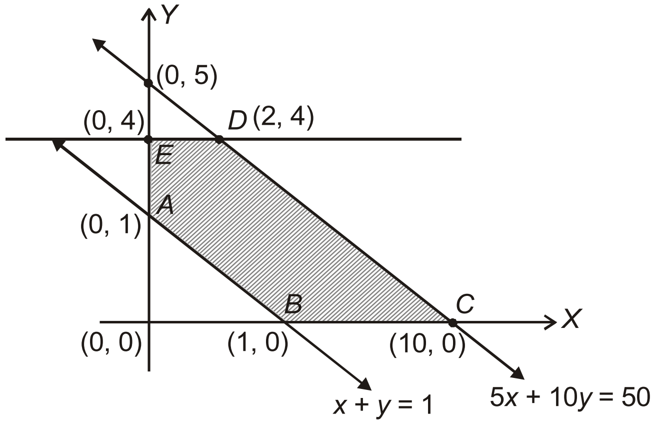
at $F\left(1, \frac{3}{2}\right)$, $z = 30\times 1 + 20\times\frac{3}{2} = 60$.

It is clear that minimum value of Z is 60 at the points $D(0, 3)$ and $F\left(1, \frac{3}{2}\right)$.

39. For the LPP, minimize $Z = 2x + y$ subject to $x + 2y \le 10$, $x + y \ge 1$, $y \le 4$ and $x, y \ge 0$, then Z is

(1) 0	(2) 1	(3) 2	(4) 12

Sol. Answer (2)



Feasible region is *ABCDEA* and vertices of the feasible region are *A*(0, 1), *B*(1, 0), *C*(10, 0), *D*(2, 4) and *E*(0, 4).

Thus the minimum value of objective function is at (0, 1).

$\therefore \quad Z = 0 \times 2 + 1$

$= 1$

40. The maximum value of $Z = 9x + 13y$ subject to constraints $2x + 3y \le 18$, $2x + y \le 10$, $x \ge 0$, $y \ge 0$ is

(1) 79 (2) 81 (3) 49 (4) 130

Sol. Answer (1)

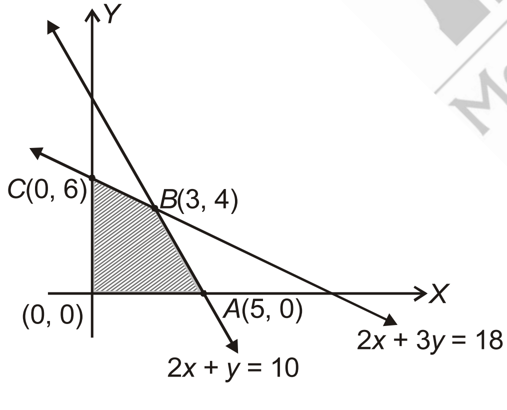
The feasible region is *OABCO*

At *O*(0, 0), $Z = 0$

At *A*(5, 0), $Z = 45$

At *B*(3, 4), $Z = 27 + 52 = 79$

At *C*(0, 6), $Z = 78$



Maximum value of $Z = 79$.

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