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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)
Questions & Solutions

Date: 17 March, 2021 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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1. There are two teams A and B. Team A consists of 7 boys and n girls whereas Team B consists of 4 boys and 6 girls. Matches occur between one male player of each team and between one female player of each team. The total matches played are 52. The value of n is :

(1) 6 (2) 4 (3) 8 (4) 2

Ans. (2)

Sol. Total matches between boys of both teams = ${}^7C_1.{}^4C_1 = 28$

Total matches between girls of both teams = ${}^nC_1.{}^6C_1 = 6n$

$\because 28 + 6n = 52$

$\Rightarrow n = 4$

2. If matrix $A = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$ follows an equation $A^2 - \frac{1}{2}I = 0$, where $\alpha \in \left[0, \frac{\pi}{2}\right]$, then value of α is :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (3)

Sol. $A = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}\begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$

$A^2 - \frac{I}{2} = \begin{bmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \sin^2\alpha - \frac{1}{2} = 0$

$\sin\alpha = -\frac{1}{\sqrt{2}}$ (rejected)

$\Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}$

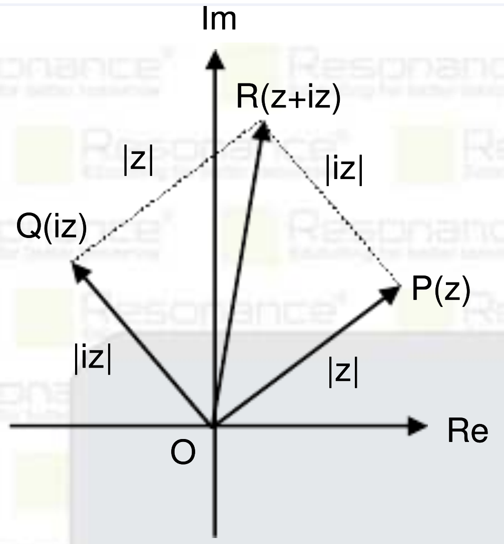
$\alpha = \frac{\pi}{4}$

3. If three vertices of a triangle are z, iz and z + iz . Find its area.

(1) $|z|^2$ (2) $\frac{|z^2|}{2}$ (3) $\frac{z^2}{4}$ (4) 1

Ans. (2)

Sol.



Required area = area of ΔOPR

$= \frac{1}{2}(OP)\ (PR) = \frac{1}{2}|z||iz|$

$= \frac{|z^2|}{2}$

4. The minimum distance between the circles having equations $x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 24x - 10y + 160 = 0$ is :

(1) 4 (2) 3 (3) 1 (4) 6

Ans. **(3)**

Sol. Let $C_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

$\therefore$ Centre $(C_1) = (5,5)$ and Radius $(r_1) = 3$

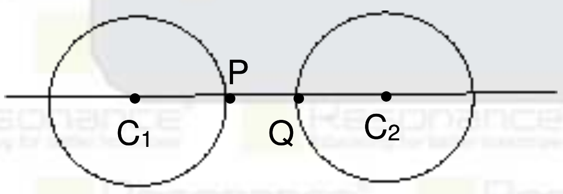
Let $C_2 \equiv x^2 + y^2 - 24x - 10y + 160 = 0$

$\therefore$ Centre $(C_2) = (12,5)$ and Radius $(r_2) = 3$

$C_1C_2 = \sqrt{49+10} = 7$

$\sqrt{49+0} = 7$

$r_1 + r_2 = 6 \Rightarrow C_1C_2 > r_1 + r_2$ means circle are completely apart from each other



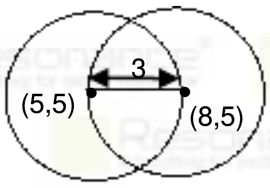
Minimum distance between circle $= C_1C_2 - (r_1 + r_2) = 7 - 6 = 1$

5. If two circles C_1 and C_2 having equations $x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 16x - 10y + 80 = 0$ are given. The incorrect statement among the following is :

(1) Distance between centres is average of their radii
(2) Both circle passes through centre of each other
(3) Centre of each circle is contained in other circle
(4) Both circle intersect at 2 point

Ans. **(3)**

Sol.



$x^2 + y^2 - 10x - 10y + 41 = 0$
$C_1 = (5,5)$ and $r_1 = 3$
$x^2 + y^2 - 16x - 10y + 80 = 0$
$C_2 = (8,5)$ and $r_2 = 3$
$C_1C_2 = 3 = r_1 = r_2$

6. If $\cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots\ldots + 100$ terms $= \cot^{-1}\alpha$, then find α.
(1) 1.00 (2) 1.01 (3) 1.02 (4) 1.03

Ans. **(2)**

Sol. $T_n = \cot^{-1}(2n^2) = \tan^{-1}\left(\frac{1}{2n^2}\right) = \tan^{-1}\left(\frac{2}{4n^2}\right)$

$T_n = \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$
$T_1 = \tan^{-1}(3) - \tan^{-1}(1)$
$T_2 = \tan^{-1}(5) - \tan^{-1}(3)$
$T_3 = \tan^{-1}(7) - \tan^{-1}(5)$
$\vdots$
$T_{100} = \tan^{-1}(201) - \tan^{-1}(199)$

$T_1 + T_2 + \ldots + T_{100} = \tan^{-1}(201) - \tan^{-1}(1) = \tan^{-1}\left(\frac{200}{202}\right) = \cot^{-1}\left(\frac{202}{200}\right) = \cot^{-1}\alpha$

$\alpha = 1.01$

7. If system of linear equation has no solution

$kx + y + z = 1$
$x + ky + z = k$
$x + y + kz = k^2$

then find k.
(1) –2 (2) –1 (3) 1 (4) 0

Ans. **(1)**

Sol. $\Delta = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}$

After expansion
$\Delta = (k-1)^2(k+2) = 0$
$\Rightarrow k = 1, -2$
But, k = 1, three planes are identical
So, $k = -2$

8. If $f(x) = x^{\log_{10} 5}$. Then the inverse of f(x) is
(1) $y = x^{\log_5 10}$ (2) $y = 10^{\log_x 5}$ (3) $y = 5^{\log_x 5}$ (4) $y = 5^{\log_x \sqrt{5}}$

Ans. **(1)**

Sol. $y = x^{\log_{10} 5}$

$\Rightarrow f^{-1}(y) = y^{\log_5 10}$

$\Rightarrow f^{-1}(x) = x^{\log_5 10}$

9. If $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ sum of all values of x is :

(1) $-\frac{31}{4}$ (2) $\frac{33}{4}$ (3) $\frac{31}{4}$ (4) $-\frac{29}{4}$

Ans. **(1)**

Sol. $\tan^{-1}(x+1) + \frac{\pi}{2} - \tan^{-1}\frac{1}{x-1} = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}\frac{1}{x-1} - \tan^{-1}(x+1) = \frac{\pi}{2} - \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}\left(\dfrac{\frac{1}{x-1}-(x+1)}{1+\frac{1}{(x-1)}(x+1)}\right) = \cot^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1-(x^2-1)}{2x}\right) = \tan^{-1}\left(\frac{31}{8}\right)$

$\Rightarrow \frac{2-x^2}{2x} = \frac{31}{8} \Rightarrow 4x^2 + 31x - 8 = 0$

$\therefore$ sum of roots $= -\frac{31}{4}$

10. A curve follows the differential equation $\frac{dy}{dx} = xy+x-y-1$ and it passes through origin. Then the value of y(1) is :

(1*) $e^{-\frac{1}{2}} - 1$ (2) $e^{-\frac{1}{2}} + 1$ (3) $\frac{1}{2}$ (4) $1 - e^{\frac{1}{2}}$

Ans. **(1)**

Sol. $\Rightarrow \frac{dy}{dx} = (x-1)(y+1)$

$\Rightarrow \frac{dy}{y+1} = (x-1)dx$

Integrating both side

$\Rightarrow \ln(y+1) = \frac{x^2}{2} - x + c$

passes through origin (0,0)

$\Rightarrow c = 0$

$\Rightarrow \ln(y+1) = \frac{x^2}{2} - x$

so, y(1) is

$\Rightarrow \ln(y+1) = -\frac{1}{2} \Rightarrow y = e^{-\frac{1}{2}} - 1$

equals to x)

(1) $\frac{3\pi}{2}$ (2) $\frac{\pi}{2}$ (3) π (4) $-\frac{\pi}{2}$

Ans. (2)

Sol. $\lim\limits_{x\to0^+}\frac{\sin^{-1}(x-[x]^2)\cos^{-1}(x-[x]^2)}{x-x^3}$

$= \lim\limits_{x\to0^+}\frac{\sin^{-1}(x).\cos^{-1}(x)}{x(1-x^2)}$

$= \lim\limits_{x\to0^+}\left(\frac{\sin^{-1}x}{x}\right)\frac{\cos^{-1}x}{(1-x^2)} = 1.\frac{(\cos^{-1}0)}{1-0} = \frac{\pi}{2}$

12. If $(2021)^{3762}$ is divided by 17, then find the remainder

Ans. (04.00)

Sol. $2021 = 17m - 2,\ m \in I$

$(2021)^{3762} = 17\lambda + 2^{3762}$

$= 17\lambda + 2^{4\times 940+2}$

$= 17\lambda + (16)^{940}.4$

$= 17\lambda + 4(17-1)^{940}$

$= 17\lambda + 17\mu + 4$

Remainder = 4

13. In the expansion of $\left(x + x^{\log_2 x}\right)^7$ fourth term is 4480 then find value of 'x'.

(1) 1 (2) 2 (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Ans. (2)

Sol. $T_{r+1} = {}^7C_r.\ x^{7-r}.\ \left(x^{\log_2 x}\right)^r$

$T_4 = {}^7C_3.\ x^4.\ \left(x^{\log_2 x}\right)^3 = 4480$

$\Rightarrow 35\ x^4.\ x^{3\log_2 x} = 4480$

$\Rightarrow x^4.\ x^{3\log_2 x} = 128$

$\Rightarrow x^4.\ x^{3\log_2 x} = 2^4.2^3$

$\Rightarrow x = 2$

14. A dice has 1,2,3,5,7,11 digit on its face. If two such dice are rolled then probability of getting sum of digits on both dice is less than or equal to 8.

(1) $\frac{13}{36}$ (2) $\frac{11}{36}$ (3) $\frac{17}{36}$ (4) $\frac{19}{36}$

Ans. (3)

Sol. n(S) = 36

A = sum $\le$ 8

= {(1, 1), (1, 2), (1, 3), (1, 5), (1, 7), (2, 1), (2, 2), (2, 3), (2, 5), (3, 1), (3, 2), (3, 3), (3, 5), (5, 1), (5, 2), (5, 3), (7, 1)}

$\therefore$ n(A) = 17

$p(A) = \frac{17}{36}$

x^4 (k)

Ans. 6

Sol. $f(0) = \lim\limits_{x\to 0}\dfrac{\cos(\sin x) - \cos x}{x^4}$

$$= \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$= \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x + x}{2}\right)}{\left(\frac{\sin x + x}{2}\right)}\cdot\left(\frac{\frac{\sin x + x}{2}}{x}\right)\cdot\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\cdot\frac{x-\sin x}{2x^3}$$

$$= 2.1.\frac{(1+1)}{2}.1.\lim_{x\to 0}\frac{x - \left(x - \frac{x^3}{3!} + ...\right)}{2x^3}$$

$$= 2.\frac{1}{12} = \frac{1}{6} = \frac{1}{k} \Rightarrow k = 6$$

16. If $\dfrac{d}{dx}\left(\sin\left(\cos^{-1}\left(\dfrac{1-2^x}{1+2^x}\right)\right)\right)_{(x=1)} = \dfrac{p}{q}\log 2$. then $|q^2 - p^2| =$

Ans. 79.00

Sol. $\dfrac{d}{dx}\left(\sqrt{1-\left(\dfrac{1-2^x}{1+2^x}\right)^2}\right)$

$$= \frac{d}{dx}\left(\sqrt{\frac{1+(2^x)^2 + 2.2^x - 1 - (2^x)^2 + 2.2^x}{(1+2^x)^2}}\right)$$

$$= \frac{d}{dx}\left(\frac{2.2^{x/2}}{1+2^x}\right)$$

$$= \frac{d}{dx}\left(\frac{2}{2^{x/2} + 2^{-x/2}}\right)$$

$$= \frac{-2.(2^{x/2}\frac{1}{2}\ell n2 - \frac{1}{2}.2^{-x/2}\ell n2)}{(2^{x/2} + 2^{-x/2})^2}$$

$$\left.\frac{d}{dx}\right|_{x=1} = \frac{-2}{\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)^2}\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\frac{1}{2}\ell n2$$

$$= \frac{-2(2-1)}{\sqrt{2}\left(\frac{9}{2}\right)}\frac{1}{2}\ell n2 = \frac{-\sqrt{2}}{9}\ell n2$$

$|q^2 - p^2| = |81 - 2| = 79$

17. If $(q \to p) \leftrightarrow ((\sim q) * p)$ is a tautology then $(\sim p) * q$ is

(1) $q \to p$ (2) $p \to q$ (3) $\sim q \to p$ (4) $\sim p \to q$

Ans. **(2)**

Sol. If * is taken OR ($\vee$)

p	q	$q \to p$	$((\sim q) \vee p)$	$(q \to p) \leftrightarrow ((\sim q) \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

OR is justified so * is used for OR operator

$\therefore \ (\sim p) * q \equiv (\sim p) \vee q$

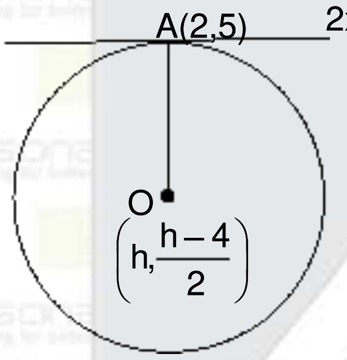
$\equiv p \to q$

18. If 2x – y + 1 = 0 is tangent to the circle at (2, 5) and centre of circle lies on the line x – 2y = 4, then the radius of circle is

(1) $2\sqrt{5}$ (2) $3\sqrt{5}$ (3) $4\sqrt{5}$ (4) $6\sqrt{5}$

Ans. **(2)**

Sol. Centre lies in the line **x** –2y–4=0, Let centre be $\left(h, \frac{h-4}{2}\right)$



Slope of OA (m_{OA}) = $\frac{5-\left(\frac{h-4}{2}\right)}{2-h} = -\frac{1}{2}$

$\Rightarrow \frac{10-h+4}{4-2h} = -\frac{1}{2}$

$\Rightarrow \frac{14-h}{4-2h} = -\frac{1}{2}$

$\Rightarrow h = 8$

$\therefore$ Centre O = (8, 2)

$\therefore$ Radius = $\sqrt{(8-2)^2+(2-5)^2} = 3\sqrt{5}$

19. The value of $4 + \cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\ldots\infty}}}$ is

(1) $2+\frac{2\sqrt{30}}{5}$ (2) $2-\frac{2\sqrt{30}}{5}$ (3) $4+\frac{2\sqrt{30}}{5}$ (4) $4-\frac{2\sqrt{30}}{5}$

Ans. (1)

$$5+\cfrac{1}{4+\cfrac{1}{5+\ldots\infty}}$$

$$y = 4+\cfrac{1}{5+\cfrac{1}{y}} \Rightarrow y = 4+\frac{y}{1+5y} \quad \Rightarrow y = \frac{4+21y}{1+5y}$$

$$\Rightarrow y + 5y^2 = 4 + 21y \Rightarrow 5y^2 - 20y - 4 = 0$$

$$\Rightarrow y = \frac{20 \pm \sqrt{400+80}}{10}$$

$$\Rightarrow y = \frac{20+4\sqrt{30}}{10},\ y = \frac{20-4\sqrt{30}}{10}$$ (rejected because y cannot be negative)

$$\therefore y = \frac{20+4\sqrt{30}}{10} = 2 + \frac{2\sqrt{30}}{5}$$

20. If $f(\alpha)$ is defined as :

$$f(\alpha) = \int_{\pi/6}^{\pi/3} \frac{(\sin x)^{\alpha}}{(\sin x)^{\alpha} + (\cos x)^{\alpha}}\,dx$$, then which of the following statement is correct about $f(\alpha)$.

(1) $f(\alpha)$ is strictly increasing

(2) $f(\alpha)$ is strictly decreasing

(3) $f(\alpha)$ has point of inflection at $\alpha = -\frac{1}{2}$

(4) $f(\alpha)$ is even

Ans. **(4)**

Sol. $$f(\alpha) = \int_{\pi/6}^{\pi/3} \frac{(\sin x)^{\alpha}}{(\sin x)^{\alpha} + (\cos x)^{\alpha}}\,dx \quad ....(1)$$

Applying $f(x) = f(a + b - x)$

$$f(\alpha) = \int_{\pi/6}^{\pi/3} \frac{(\cos x)^{\alpha}}{(\cos x)^{\alpha} + (\sin x)^{\alpha}}\,dx \quad .....(2)$$

Adding (1) & (2)

$$2f(\alpha) = \int_{\pi/6}^{\pi/3} 1\,dx$$

$$f(\alpha) = \frac{\pi}{12}$$

21. If k is defined as, $k = \int_0^{} \left[[x^2] + \cos x\right] dx$ where [x] represent greatest integer less than or equal to x, then value of k is

(1) $\sqrt{\frac{3\pi}{2}} - 1$ (2) $\sqrt{\frac{\pi}{2}} - 1$ (3) $1 - \sqrt{\frac{\pi}{2}}$ (4) $\sqrt{\pi} - 1$

Ans. (2)

Sol. $k = \int_0^1 [\cos x]\, dx + \int_1^{\sqrt{\frac{\pi}{2}}} [1 + \cos x]\, dx$

$k = \int_0^1 0\, dx + \int_1^{\sqrt{\frac{\pi}{2}}} 1\, dx + \int_1^{\sqrt{\frac{\pi}{2}}} [\cos x]\, dx$

$= 0 + \sqrt{\frac{\pi}{2}} - 1 + 0$

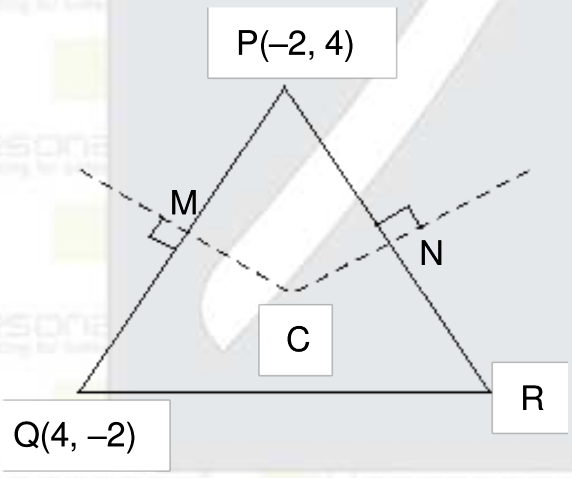
$k = \sqrt{\frac{\pi}{2}} - 1$

22. If P(–2, 4) and Q(4, –2) are the two vertices of ΔPQR and 2x – y + 2 = 0 is perpendicular bisector of of side PR, then circumcentre of ΔPQR is

(1) (–2, –1) (2) (–2, –2) (3) (–1, –2) (4) (–1, –1)

Ans. (2)

Sol.



Equation of line CN, 2x – y + 2 = 0 .....(1)

Mid point of PQ = M(1, 1)

Slope of PQ = $\frac{4+2}{-2-4} = -1$

Slope of CM = 1 (Perpendicular to PQ)

Equation of CM, y – 1 = 1(x – 1)

y = x .....(2)

Intersection point of line CM and CN is circumcentre C.

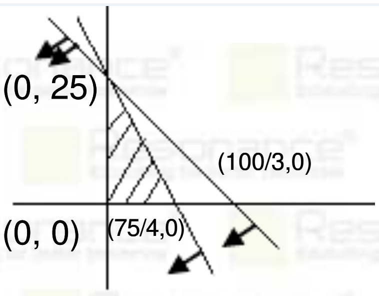
by solving equation (1) & (2)

C(–2, –2)

23. If $4x+3y \le 75$, $3x+4y \le 100$, $x \ge 0$, $y \ge 0$ and $z = 6xy+y^2$, then maximum value of z is :

(1) 635 (2) 625 (3) 525 (4) 665

Ans. (2)



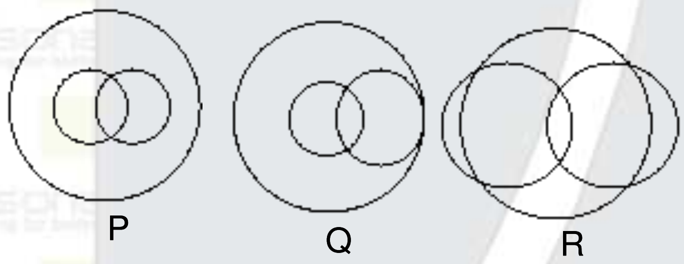
z (0,25) =0+625 =625

$z\left(\frac{75}{4},0\right)=0$

z (0,0) =0

$\therefore z_{max} = 625$

24. 3 games are played in school. If some students play exactly 2 games and no student play all 3 games, then which of the following venn diagram represent the above situation :



(1) P & Q (2) P& R (3) Q & R (4) none of the above

Ans. **(4)**

Sol. None of P,Q,R represent the except condition given in the problem. So option (4) is right answer

25. If a matrix A is $\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of det (A^4) – det$(A^{10}$– adj$(2A)^{10})$

Ans. **16**

Sol. $A^n = \begin{bmatrix} 2^n & 2^n - 1 \\ 0 & 1 \end{bmatrix}$ $n \in 2N$

and $A^n = \begin{bmatrix} 2^n & 2^n + 1 \\ 0 & -1 \end{bmatrix}$ $n \in 2N+1$

det (A^4) = $(\det A)^4 = (-2)^4 = 16$

$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix}$

$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$

adj(2A) = $\begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$

(adj (2A))^{10} = $2^{10}\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$

$$\begin{bmatrix} 0 & 2^{10} \end{bmatrix}$$

$$= 2^{10}\begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$$A^{10} - (\text{adj }(2A))^{10} = \begin{bmatrix} 0 & 2^{11} \times 1023 \\ 0 & 1-(1024)^2 \end{bmatrix}$$

$|A^{10} - (\text{adj}(2A)| = 0$

So, $\det(A^4) - \det(A^{10} - (\text{adj}(2A)^{10}) = 16 - 0 = 16$

26. If a plane $ax + by + cz - 7 = 0$ passes through the point $(-2, 1, 3)$ and point of intersection of two planes $P_1 : 2x - 7y + 4z - 11 = 0$ and $P_2 : 3x + 5y - 4z + 3 = 0$ then the value of $2a + b + c + 5$ is :

Ans. **(2)**

Sol. Required plane is $(2x - 7y + 4z - 11) + \lambda(3x + 5y - 4z + 3) = 0$

As it passes through $(-2,1,3)$ so it will satisfy the required plane

$\therefore (-4 - 7 + 12 - 11) + \lambda(-6 + 5 - 12 + 3) = 0$

$-10 + \lambda(-10) = 0$

So $\lambda = -1$

So equation of plane is $(2x - 7y + 4z - 11) - (3x + 5y - 4z + 3) = 0$

$= -x - 12y + 8z - 14 = 0$

$= x + 12y - 8z + 14 = 0$

So $\dfrac{a}{1} = \dfrac{b}{12} = \dfrac{c}{-8} = \dfrac{-7}{14}$

$a = -\dfrac{1}{2}$, $b = -6$, $c = 4$

So $2a + b + c + 5 = -1 - 6 + 4 + 5 = 2$