Derivatives of Trigonometric Functions

Q.1. If $y = \sqrt{[(1 - \cos x) / (1 + \cos x)]}$, find dy / dx.

Solution : 1

We are given : $y = \sqrt{[(1 - \cos x) / (1 + \cos x)]}$

Or, $y = \sqrt{[(1 - \cos x)(1 + \cos x) / (1 - \cos^2 x)]}$

Or, $y = (1 - \cos x) / \sin x$

Or, $y = \text{cosec } x - \cos x$

Or, $dy / dx = - \text{cosec } x \cot x + \text{cosec}^2 x$

$= - \text{cosec } x [\cot x - \text{cosec } x]$.

Q.2. If $\sin y = x \sin (a + y)$, show that $dy / dx = \sin^2 (a + y) / \sin a$.

Solution : 2

We are given : $\sin y = x \sin (a + y)$

Or, $x = \sin y / \sin (a + y)$

Differentiating both sides with respect to y we get

$dx / dy = [\sin (a + y). \cos y - \sin y \cos (a + y)] / \sin^2 (a + y)$

$= \sin (a + y - y) / \sin^2 (a + y)$

$= \sin a / \sin^2 (a + y)$

Hence, $dy / dx = \sin^2 (a + y) / \sin a$. **[Proved.]**

Q.3. $\sin x^2$ If $y = e$, find dy / dx.

Solution : 3

We are given, sin x^2.

y = e sin x^2

dy / dx = e d / dx (sin x^2)

sin x^2 = e × cos x^2 × d / dx (x^2) sin x^2

= 2x cos x^2 e

Q.4. If sin (xy) + cos (xy) = 1 and tan (xy) ≠ 1, then show that dy / dx = – y / x.

Solution : 4

We are given,

sin (xy) + cos (xy) = 1,

Differentiating both sides with respect to x we get,

cos (xy). d/dx (xy) – sin (xy). d/dx (xy) = 0

Or, d/dx (xy) (cos xy – sin xy) = 0

Or, (x dy/dx + y) (sin xy – cos xy) = 0

Either sin xy – cos xy = 0 Or, x dy/dx + y = 0

We get sin xy – cos xy = 0 => sinxy = cosxy => tan xy = 1 but tan xy ≠ 1.

Hence, x dy/dx + y = 0 => x dy/dx = – y => dy/dx = – y/x. **[Proved.]**

Q.5. If x = a $\sin^3$ t and y = a $\cos^3$ t, find dy/dx.

Solution : 5

We have, x = a $\sin^3$ t => dx / dt = 3a $\sin^2$ t. cos t --- --- (i)

Also y = a $\cos^3$ t => dy / dt = 3a $\cos^2$ t. (– sin t) --- --- (ii)

Dividing (ii) by (i) we get, (dy/dt)/(dx/dt) = – 3a $\cos^2$ t. sin t / 3a $\sin^2$ t. cos t

Or, dy / dx = – cot t.

Q.6. If y = sin √ (sin x + cos x), find dy / dx.

Solution : 6

Using chain rule,

dy / dx = cos √ (sin x + cos x) . d/dx [√ (sin x + cos x)]

= cos √(sin x + cos x). 1/{2√ (sin x + cos x)}. d/dx (sin x + cos x)

= [{cos √ (sin x + cos x)} / {2 √(sin x + cos x)}]. (cos x – sin x)

= [cos {√ (sin x + cos x)}. (cos x – sin x)]/[2 √(sin x + cos x)]

Q.7. If y = √{(1 – tan x)/(1 + tan x)}, find dy/dx.

Solution : 7

Using chain rule,

dy / dx = 1/[2 √{(1 – tan x)/(1 + tan x)}]. d/dx {(1 – tan x)/(1 + tan x)}

= 1/[2 √{(1 – tan x)/(1 + tan x)}]. [(1 + tan x). d/dx (1 – tan x) – (1 – tan x) ×d/dx (1 + tan x)]

= 1/[2 √ {(1 – tan x)/(1 + tan x)}]. [{(1 + tan x) (– sec^2 x) – (1 – tan x) (sec^2 x)}] / [$(1 + tan x)^2$]

= – sec^2 x / {$(1 + tan x)^2$}. √(1 + tan x)/√(1 – tan x)

= 0 – sec^2 x / [(1 + tan x)3/2(1 – tan x)1/2]

Q.8. If y = sin [√ {sin √x }], find dy / dx.

Solution : 8

Using chain rule,

dy / dx = cos [√ {sin √x }] . d/dx [√ { sin √x }]

= cos [√ { sin √x }] . 1/[2 sin √x] . d/dx [sin √x]

= cos [√ { sin √x }] . 1/[2 sin √x] . cos √x . d/dx (√x)

= cos [√ { sin √x }] . 1/[2 sin √x] . cos √x . 1/2√x

= (cos [√ { sin √x }] . cot √x) / 4 √x .

Q.9. If y = √[(1 − sin 2x)/(1 + sin 2x)], show that dy/dx + sec 2 (π/4 − x) = 0 .

Solution : 9

We have , y = √[(1 − sin 2x)/(1 + sin 2x)]

= √[(cos x − sin x)2/(cos x + sin x)^{2}]

= (cos x − sin x)/(cos x + sin x)

= (1 − tan x)/(1 + tan x) = tan (π/4 − x)

Differentiating w. r. t. x , we get

dy/dx = sec 2(π/4 − x) .(−1)

Or, dy/dx + sec 2(π/4 − x) = 0 . **[Proved.]**

Q.10. Find dy/dx , when : x = a (1 − cos θ) ; y = a (θ + sin θ) .

Solution : 10

We have , x = a (1 − cos θ) ; y = a (θ + sin θ)

Then dx/dθ = a sin θ ; dy/dθ = a (1 + cos θ)

Therefore , dy/dx = (dy/dθ)/(dx/dθ) = a (1 + cos θ)/a sin θ

= {2 cos 2(θ/2)}/[2 sin (θ/2) cos (θ/2)]

= cot (θ/2) .

Q.11. Find dy/dx , when y = a (θ + sin θ) ; x = a (1 + cos θ) .

Solution : 11

We have , $y = a(\theta + \sin\theta)$; $x = a(1 + \cos\theta)$

Then $dy/d\theta = a(1 + \cos\theta) = 2a\cos 2(\theta/2)$;

$dx/d\theta = -a\sin\theta = -2a\sin(\theta/2)\cos(\theta/2)$.

Therefore , $dy/dx = (dy/d\theta)/(dx/d\theta) = -\cot\theta/2$.