DAY THIRTY ONE

# Dual Nature of Matter

Learning & Revision for the Day

- Photon
- Particle Nature of Light
- Photoelectric Effect
- Laws of Photoelectric Effect
- de-Broglie Waves
- Davisson-Germer Experiment

## Photon

A particle of light called a **photon** has energy $E$ that is related to the frequency $f$ and wavelength $\lambda$ of light wave.

By the Einstein equation, $E = hf = \dfrac{hc}{\lambda}$ ...(i)

where, $c$ is the speed of light (in vacuum) and $h$ is **Planck's constant**.

$$h = 6.626 \times 10^{-34} \text{ J-s} = 4.136 \times 10^{-15} \text{ eV-s}$$

Since, energies are often given in electron volt $(1\text{eV} = 1.6 \times 10^{-19} \text{ J})$ and wavelengths are in Å, it is convenient to the combination $hc$ in eV-Å. We have,

$$hc = 12375 \text{ eV-Å}$$

Hence, Eq. (i), in simpler form can be written as,

$$E \text{ (in eV)} = \frac{12375}{\lambda \text{ (in Å)}} \quad \text{...(ii)}$$

The propagation of light is governed by its wave porperties whereas the exchange of energy between, light with matter is governed by its particle properties. The wave-particle duality is a general property in nature. For example, electrons (and other so called particles) also propagate as waves and exchange energy as particles.

## Particle Nature of Light

Photoelectric effect gave evidence to the strange fact that, light in interaction with matter behaved as if it was made of quanta or packets of energy, each of energy $h\nu$.

Einstein stated that the light quantum can also be associated with momentum $\left(\dfrac{h\nu}{c}\right)$.

This particle like behaviour of light was further confirmed, in 1924, by the Compton experiment of scattering of X-rays from electrons.

# Photoelectric Effect

- Photoelectric effect is the phenomenon of emission of electrons (known as photoelectrons) from the surface of metals when light radiation of suitable frequency are incident on them.
- The minimum energy of incident radiation needed to eject the electrons from metal surface is known as **work function** ($\phi_0$) of that surface.
- The frequency or wavelength corresponding to the work function is called **threshold frequency** or **threshold wavelength**. Work function is related to threshold frequency as,

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

where, $\lambda_0 =$ threshold wavelength.

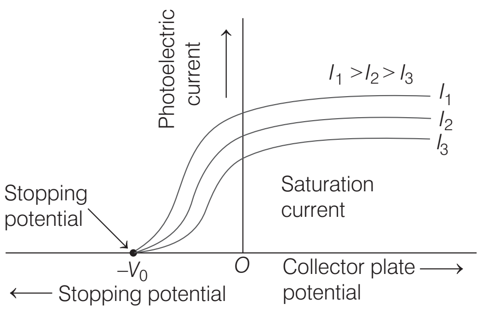
- In electron volt units, $\phi\,(\text{eV}) = \dfrac{hc}{e\lambda_0} = \dfrac{12400}{\lambda\,(\text{Å})}$
- For photoemission to take place energy of incident light ($E$) is related as, $E \geq p_0$
- According to Einstein's photoelectric equation,

$$h\nu = \phi_0 + K_{\max}$$

where, $K_{\max} = \dfrac{1}{2} mv_{\max}^2 =$ maximum kinetic energy of ejected photoelectron.

## Effect of Intensity on Photoelectric Emission

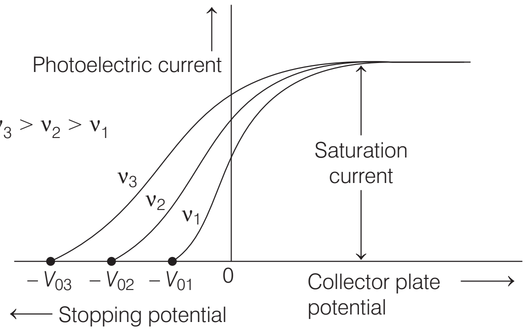
For a light of given frequency $\nu > \nu_0$ (or given wavelength $\lambda < \lambda_0$), if the intensity of light incident on photosensitive metal surface is increased, the number of photoelectrons and consequently the photoelectric current $I$ increases. However, the stopping potential $V_0$ remain constant.




Photoelectric current *versus* stopping potential curve

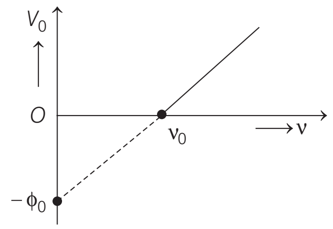
## Effect of Frequency on Photoelectric Emission

If keeping the intensity of incident light constant, the frequency of incident light is increased, then the stopping potential $V_0$ (and hence, $K_{\max}$) increases, but the photoelectric current $I$ remains unchanged.




$I$-$V_0$, when frequency changed

A photon may collide with a material particle. The total energy and the total momentum remain conserved in such a collision. Photoelectric emission is an instantaneous phenomenon.




Cut-off voltage *versus* frequency of incident light

Variation of stopping potential $V_0$ with frequency $\nu$ of incident radiation is as shown in above figure.

As, $eV_0 = h(\nu - \nu_0) = h\nu - \phi_0 \Rightarrow V_0 = \dfrac{h}{e}\nu - \dfrac{\phi_0}{e}$

Thus, $V_0$- $\nu$ graph is a straight line whose slope is $\dfrac{h}{e}$ and intercept is $-\phi_0$ eV. The graph meets the $\nu$-axis at $\nu_0$.

Photocurrent $\propto \dfrac{1}{\nu} \propto \lambda$

## Energy and Momentum of Photon

- From Einstein's mass-energy relation $E = h\nu = mc^2$

  Kinetic mass of photon is $m = \dfrac{h\nu}{c^2}$

  But $\nu = \dfrac{c}{\lambda}$, where $\lambda$ is wavelength of the photon.

  $\therefore$Kinetic mass of photon, $m = \dfrac{h}{c^2}\left(\dfrac{c}{\lambda}\right) = \dfrac{h}{c\lambda}$

  Kinetic mass of photon, $m = \dfrac{h\nu}{c^2} = \dfrac{h}{c\lambda}$

- Momentum of photon,

$$p = \text{kinetic mass of photon} \times \text{velocity of photon}$$

$$= \frac{h\nu}{c^2} \times c = \frac{h\nu}{c}$$

  Also, $\nu = \dfrac{c}{\lambda}$

  $\therefore$Momentum of photon, $p = \dfrac{h}{c}\left(\dfrac{c}{\lambda}\right) = \dfrac{h}{\lambda}$

## Laws of Photoelectric Effect

Lenard and Millikan gave the following laws on the basis of experiments on photoelectric effect.

- The rate of emission of photoelectrons from the surface of a metal varies directly as the intensity of the incident light falling on the surface.
- The maximum kinetic energy of the emitted photoelectrons is independent of the intensity of the incident light.
- The maximum kinetic energy of the photoelectrons increases linearly with increase in the frequency of the incident light.
- As soon as, the light is incident on the surface of the metal, the photoelectrons are emitted instantly, i.e. there is no time lag between incidence of light and emission of electrons ($\approx 10^{-9}$ s).

## de-Broglie Waves

Light is said to have dual character, i.e. it behaves like matter (particle) and wave both. Some properties like interference, diffraction can be explained on the basis of wave nature of light, while the phenomena like photoelectric effect, black body radiation, etc. can be explained on the basis of particle nature of light.

In 1942, Louis de-Broglie explained that like light, matter also show dual behaviour, there is a wave associated with moving particle, known as **matter waves or de-Broglie waves**.

### de-Broglie Relation

According to quantum theory, energy of photon

$$E = h\nu \quad \text{...(i)}$$

If mass of the photon is taken as $m$, then as per Einstein's equation

$$E = mc^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get, $h\nu = mc^2$

$$h\frac{c}{\lambda} = mc^2,$$

where, $\lambda$ = wavelength of photon

$$\lambda = \frac{h}{mc}$$

de-Broglie asserted that the above equation is completely a general function and applies to photon as well as all other moving particles.

So, $$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

where, $m$ is mass of particle and $v$ is its velocity.

- **de-Broglie wavelength associated with charged particle**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

- **de-Broglie wavelength of a gas molecule**

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

where, $T$ = absolute temperature

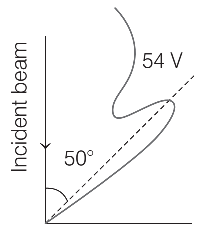
and $k$ = Boltzmann's constant $= 1.38 \times 10^{-23}$ J / K

- **Ratio of wavelength of photon and electron** The wavelength of photon of energy $E$ is given by $\lambda_p = \frac{hc}{E}$ while the wavelength of an electron of kinetic energy $K$ is given by $\lambda_c = \frac{h}{\sqrt{2mK}}$. Therefore for same energy, the ratio

$$\frac{\lambda_p}{\lambda_e} = \frac{c}{E}\sqrt{2mK} = \sqrt{\frac{2mc^2K}{E^2}}$$

## Davisson-Germer Experiment

- The de-Broglie hypothesis was confirmed by Davisson-Germer experiment. It is used to study the scattering of electron from a solid or to verify the wave nature of electron.
- A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. Ni crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.
- The diffracted beam of electrons is received by the detector which can be positioned at any angle by rotating it about the point of incidence.
- The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.
- According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer found that the intensity of scattered beam of electrons was not same but different at different angles of scattering.
- It is maximum for diffracting angle 50° at 54 V potential difference.
- If the de-Broglie waves exist for electrons, then these should be diffracted as X-rays.
Using the Bragg's formula $2d \sin\theta = n\lambda$, we can determine the wavelength of these waves,



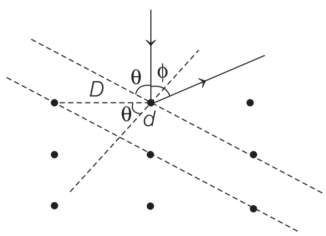
Diffraction angle *versus* intensity curve

where $d$ = distance between diffracting planes,

$$\theta = \frac{180 - \phi}{2}$$

= glancing angle for incident beam = Bragg's angle.

Clearly from figure, we have $\theta + \phi + \theta = 180°$



Reflection of electron (beam) by atoms

DAY PRACTICE SESSION 1

# FOUNDATION QUESTIONS EXERCISE

**1** If a source of power 4kW produces $10^{20}$ photons/second, the radiation belong to a part of the spectrum called

(a) X-rays (b) ultraviolet rays
(c) microwaves (d) $\gamma$-rays

**2** What will be the number of photons emitted per second by a 10 W sodium vapour lamp assuming that 90% of the consumed energy is converted into light? (Wavelength of sodium light is 590 nm and $h = 6.63 \times 10^{-34}$ J-s)

(a) $0.267 \times 10^{18}$ (b) $0.267 \times 10^{19}$
(c) $0.267 \times 10^{20}$ (d) $0.267 \times 10^{17}$

**3** Two monochromatic beams *A* and *B* of equal intensity *I*, hit a screen. The number of photons hitting the screen by beam *A* is twice that by beam *B*. Then, what inference can you make about their frequencies?

(a) $\nu_B = 2\nu_A$ (b) $\nu_B = \nu_A$
(c) $\nu_A = 2\nu_B$ (d) $\nu_B > \nu_A$

**4** The eye can detect $5 \times 10^4$ photons $m^{-2}s^{-1}$ of light of wavelength 500 nm. The ear can hear intensity upto $10^{-13}$ $Wm^{-2}$. As a power detector, which is more sensitive?

(a) Sensitivity of eye is one-fifth of the ear
(b) Sensitivity of eye is five times that of the ear
(c) Both are equally sensitive
(d) Eye cannot be used as a power detector

**5** The threshold wavelength for photoelectric emission from a material is 5200Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a

(a) 50 W infrared lamp (b) 1 W infrared lamp
(c) 50 W ultraviolet lamp (d) 1 W ultraviolet lamp

**6** The wavelength of the photoelectric threshold for silver is $\lambda_0$. The energy of the electron ejected from the surface of silver by an incident light of wavelength $\lambda$ ($\lambda < \lambda_0$) will be

(a) $hc(\lambda_0 - \lambda)$ (b) $\frac{hc}{\lambda_0 - \lambda}$
(c) $\frac{h}{c} + \left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ (d) $hc\left(\frac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$

**7** In photoelectric effect match the following column I with column II.

| | Column I | | Column II |
|---|---|---|---|
| A. | If frequency of incident light is increased | 1. | Stopping potential may increase |
| B. | If intensity of incident light is increased | 2. | Stopping potential must increase |
| C. | If work function of metal is increased | 3. | Photo effect may stop |

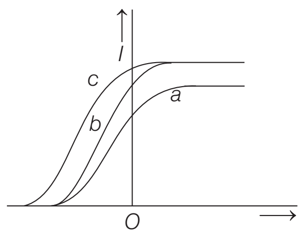
**Codes**

| | A | B | C | | A | B | C |
|---|---|---|---|---|---|---|---|
| (a) | 1 | 2 | 3 | (b) | 3 | 2 | 1 |
| (c) | 2 | 1 | 3 | (d) | 2 | 3 | 1 |

**8** When a point source of monochromatic light is at a distance of 0.2 m form a photoelectric cell the cut-off voltage and saturation current are 0.6 V and 18 mA respectively. If the same source is placed 0.6 m away from the photoelectric cell, then

(a) the stopping potential will be 0.2 V
(b) the stopping potential will be 0.6 V
(c) the saturation current will be 6 mA
(d) the saturation current will be 18 mA

**9** The figure shows the variation of photocurrent (*I*) with anode potential ($V_a$) of a photosensitive surface for three different radiations. Let $I_a, I_b$ and $I_c$ be the intensities and $f_a$, $f_b$ and $f_c$ the frequencies for the waves *a,b* and *c* respectively

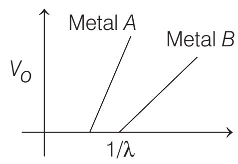



(a) $f_a = f_b$ and $I_a \neq I_b$ (b) $f_a = f_c$ and $I_a = I_c$
(c) $f_a = f_b$ and $I_a = I_c$ (d) $f_b = f_c$ and $I_b = I_c$

**10** In a photoelectric effect measurement, the stopping potential for a given metal is found to be $V_0$ volt when radiation of wavelength $\lambda_0$ is used. If radiation of wavelength $2\lambda_0$ is used with the same metal, then the stopping potential (in volt) will be

(a) $\frac{V_0}{2}$ (b) $2V_0$
(c) $V_0 + \frac{hc}{2e\lambda_0}$ (d) $V_0 - \frac{hc}{2e\lambda_0}$

**11** In an experiment on photoelectric effect, a student plots stopping potential $V_0$ against reciprocal of the wavelength $\lambda$ of the incident light for two different metals *A* and *B*. These are as shown in the figure.



Looking at the graphs, you can most appropriately say that

(a) Work function of metal $B$ is greater than that of metal $A$
(b) Work function of metal $A$ is greater than that of metal $B$
(c) Students data is not correct
(d) None of the above

**12** A copper ball of radius 1cm and work function 4.47 eV is irradiated with ultraviolet radiation of wavelength 2500 Å. The effect of irradiation results in the emission of electrons from the ball. Further the ball will acquire charge and due to this there will be a finite value of the potential on the ball. The charge acquired by the ball is

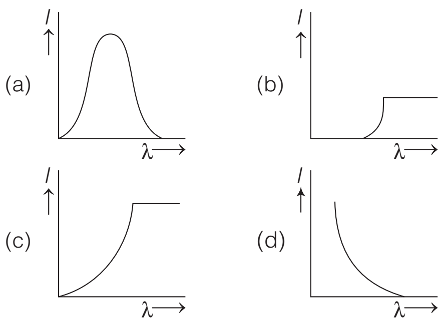
(a) $5.5\times 10^{-13}$ C  (b) $7.5\times 10^{-13}$C
(c) $4.5\times 10^{-12}$ C  (d) $2.5\times 10^{-11}$ C

**13** Match List I (fundamental experiment) with List II (its conclusion) and select the correct option from the choices given below the list.

| List I | List II |
|---|---|
| A. Franck-Hertz experiment | 1. Particle nature of light |
| B. Photo-electric experiment | 2. Discrete energy levels of atom |
| C. Davisson-Germer experiment | 3. Wave nature of electron |
| | 4. Structure of atom |

| | A | B | C | | A | B | C |
|---|---|---|---|---|---|---|---|
| (a) | 1 | 4 | 3 | (b) | 2 | 4 | 3 |
| (c) | 2 | 1 | 3 | (d) | 4 | 3 | 2 |

**14** The anode voltage of a photocells kept fixed. The wavelength $\lambda$ of the light falling on the cathode is gradually changed. The plate current $I$ of photocell varies as follows




**15** de-Broglie wavelength of an electron accelerated by a voltage of 50V is close to ($e = 1.6\times10^{-19}$ C, $m_e = 9.1\times10^{-31}$kg, $h = 6.6\times10^{-34}$ J-s).

(a) 0.5 Å  (b) 1.7Å
(c) 2.4 Å  (d) 1.2 Å

**16** Photons of an electromagnetic radiation has an energy 11keV each. To which region of electromagnetic spectrum does it belong?

(a) X-ray region  (b) Ultraviolet region
(c) Infrared region  (d) Visible region

**17** The voltage applied to an electron microscope to produce electrons of wavelength 0.50 Å is

(a) 602 V  (b) 50 V  (c) 138 V  (d) 812 V

**18** The energy of photon is equal to the kinetic energy of a proton.The energy of photon is $E$. Let $\lambda_1$ be the de-Broglie wavelength, of the proton and $\lambda_2$ be the wavelength of photon. The ratio $\lambda_1 / \lambda_2$ is proportional to

(a) $E^0$  (b) $E^{\frac{1}{2}}$
(c) $E^{-1}$  (d) $E^{-2}$

**19** An electron is moving with an initial velocity $v = v_0\mathbf{i}$ and is in a magnetic field $B = B_0\mathbf{j}$. Then its de-Broglie wavelength

(a) remains constant
(b) increase with time
(c) decrease with time
(d) increases and decreases periodically

**20** Orbits of a particle moving in a circle are such that the perimeter of the orbit equals an integer number of de-Broglie wavelengths of the particle. For a charged particle moving in a plane perpendicular to a magnetic field, the radius of the $n$th orbital will therefore be proportional to

(a) $n^2$  (b) $n$
(c) $n^{1/2}$  (d) $n^{1/4}$

**Direction** (Q. Nos. 21-27) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

**21** **Statement I** As intensity of incident light (in photoelectric effect) increases, the number of photoelectrons emitted per unit time increases.

**Statement II** More intensity of light means more energy per unit area per unit time.

**22** **Statement I** The relative velocity of two photons travelling in opposite directions is the velocity of light.

**Statement II** The rest mass of photon is zero.

**23** **Statement I** Work function of copper is greater than the work function of sodium but both have same value of threshold frequency and threshold wavelength.

**Statement II** The frequency is inversely proportional to wavelength.

**24** **Statement I** The de-Broglie wavelength of a molecules varies inversely as the square root of temperature.

**Statement II** The root mean square velocity of molecule depends on the temperature.

**25** **Statement I** Davisson-Germer experiment established the wave nature of electrons.

**Statement II** If electrons have wave nature, they can interfere and show diffraction.

**26** **Statement I** A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are $K_{max}$ and $V_0$, respectively. If the frequency incident on the surface is doubled, both the $K_{max}$ and $V_0$ are also doubled.

**Statement II** The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

**27** **Statement I** When ultraviolet light is incident on a photocell, its stopping potential is $V_0$ and the maximum kinetic energy of the photoelectrons is $K_{max}$. When the ultraviolet light is replaced by X-rays, both $V_0$ and $K_{max}$ increase.

**Statement II** Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

DAY PRACTICE SESSION 2

# PROGRESSIVE QUESTIONS EXERCISE

**1** An electron of mass $m$ and charge $e$ are initially at rest. It gets accelerated by a constant electric field $E$. The rate of change of de-Broglie wavelength of this electron at time $t$ is

(a) $\frac{-h}{eEt^2}$ (b) $\frac{-nh}{eEt^2}$ (c) $\frac{-h}{eE}$ (d) $\frac{-eht}{E}$

**2** When a surface 1cm thick is illuminated with light of wavelength $\lambda$, the stopping potential is $V_0$, but when the same surface is illuminated by light of wavelength $3\lambda$, the stopping potential is $V_0/6$, the threshold wavelength for metallic surface is

(a) $4\lambda$ (b) $5\lambda$ (c) $3\lambda$ (d) $2\lambda$

**3** A photocell with a constant potential difference of $V$ volt across it is illuminated by a point source from a distance of 25 cm. When the source is moved to a distance of 1m, the electrons emitted by the photocell

(a) carry 1/4th their previous energy
(b) are 1/16th as numerous as before
(c) are 1/4th as numerous as before
(d) carry 1/4th their previous momentum

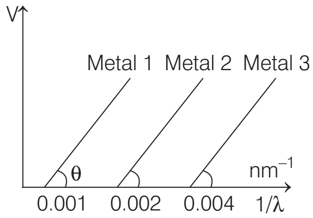
**4** Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

(a) 2.83 eV (b) 2 eV (c) 1.02 eV (d) 3.42 eV

**5** A metallic surface is illuminated with monochromatic light of wavelength $\lambda$, the stopping potential for photoelectric current is $3V_0$ and when the same surface is illuminated with light of wavelength $2\lambda$, the stopping potential is $V_0$. The threshold wavelength of this surface for photoelectric effect is

(a) $4\lambda/3$ (b) $6\lambda$ (c) $8\lambda$ (d) $4\lambda$

**6** The graph between $1/\lambda$ and stopping potential ($V$) of three metals having work functions $\phi_1, \phi_2$ and $\phi_3$ in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statement (s) is/are correct? (Here, $\lambda$ is the wavelength of the incident ray)




(a) Ratio of work function $\phi_1:\phi_2:\phi_3 = 1:2:4$
(b) Ratio of work function $\phi_1:\phi_2:\phi_3 = 4:2:1$
(c) $\tan\theta$ is directly proportional to $hc/e$ where, $h$ is Planck's constant and $c$ is speed of light
(d) The violet colour light can eject photoelectrons from metal 2 and 3

**7** Electrons are accelerated through a potential difference $V_0$ and protons are accelerated through a potential difference 4V. The de-Broglie wavelength are $\lambda_e$ and $\lambda_p$ for electrons and protons respectively.

The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by (Given, $m_e$ is mass of electrons and $m_p$ is mass of proton).

(a) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$ (b) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$

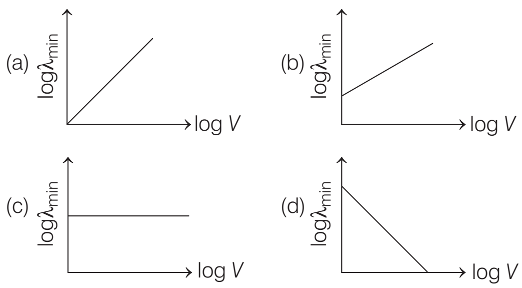
(c) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2}\sqrt{\frac{m_e}{m_p}}$ (d) $\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$

**8** An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let $\lambda_n, \lambda_g$ be the de-Broglie wavelength of the electron in the $n$th state and the ground state, respectively. Let $\Lambda_n$ be the wavelength of the emitted photon in the transition from the $n$th state to the ground state. For large $n$, ($A, B$ are constants)

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n^2$

(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$

**9** An electron beam is accelerated by a potential difference $V$ to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If $\lambda_{min}$ is the smallest possible wavelength of X-rays in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in




**10** A particle $A$ of mass $m$ and initial velocity $v$ collides with a particle $B$ of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths $\lambda_A$ to $\lambda_B$ after the collision is

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

**11** Radiation of wavelength $\lambda$ is incident on a photocell. The fastest emitted electron has speed $v$. If the wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be

(a) $> v\left(\frac{4}{3}\right)^{1/2}$ (b) $< v\left(\frac{4}{3}\right)^{1/2}$ (c) $= v\left(\frac{4}{3}\right)^{1/2}$ (d) $= v\left(\frac{3}{4}\right)^{1/2}$

**12** The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of $3 \times 10^{-4}$ T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(a) 1.8 eV (b) 1.1 eV (c) 0.8 eV (d) 1.6 eV

**13** The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is ($hc = 1240$ eV-nm)

(a) 3.09 eV (b) 1.42 eV (c) 151 eV (d) 1.68 eV

## ANSWERS

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| SESSION 1 | **1** (a) | **2** (c) | **3** (a) | **4** (b) | **5** (c,d) | **6** (d) | **7** (c) | **8** (b) | **9** (a) | **10** (d) |
| | **11** (c) | **12** (a) | **13** (c) | **14** (d) | **15** (b) | **16** (b) | **17** (a) | **18** (b) | **19** (a) | **20** (c) |
| | **21** (a) | **22** (b) | **23** (d) | **24** (b) | **25** (a) | **26** (d) | **27** (d) | | | |
| SESSION 2 | **1** (a) | **2** (b) | **3** (b) | **4** (c) | **5** (d) | **6** (a,c) | **7** (d) | **8** (a) | **9** (d) | **10** (a) |
| | **11** (a) | **12** (b) | **13** (b) | | | | | | | |

# Hints and Explanations

## SESSION 1

**1** $4 \times 10^3 = 10^{20} \times hf$

$$f = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$$

$$f = 6.64 \times 10^{16} \text{ Hz}$$

The obtained frequency lies in the band of X-rays.

**2** Energy of photon,

$$E_1 = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{590 \times 10^{-9}}$$

$$= \frac{6.63 \times 3}{59} \times 10^{-18}$$

Light energy produced per second

$$E = \frac{90}{100} \times 10$$

$$= 9 \text{ W}$$

Number of photons emitted per sec $= \frac{E}{E_1}$

$$= \frac{9 \times 59}{6.63 \times 3 \times 10^{-18}}$$

$$= 2.67 \times 10^{19} = 0.267 \times 10^{20}$$

**3** Intensity $A$ = Intensity $B$

The number of photons of beam $A = n_A$

The number of photons of beam $B = n_B$

According to question, $n_A = 2n_B$

Let $\nu_A$ be the frequency of beam $A$ and $\nu_B$ be the frequency of beam $B$.

$\therefore$ Intensity $\propto$ Energy of photons

$I \propto (h\nu) \times$ Number of photons

$$\therefore \quad \frac{I_A}{I_B} = \frac{n_A \nu_A}{n_B \nu_B}$$

According to question, $I_A = I_B$

$\therefore \quad n_A \nu_A = n_B \nu_B \Rightarrow \frac{\nu_A}{\nu_B} = \frac{n_B}{n_A} = \frac{1}{2}$

So, $\quad \nu_B = 2\nu_A$

**4** Sensitivity of eye = energy detected per square meter

$$= \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}}$$

$$= 0.2 \times 10^{-13}\ \text{Wm}^{-2}$$

The sensitivity of ear $= 1 \times 10^{-13}\ \text{Wm}^{-2}$

Thus, the sensitivity of eye is five times more than that of the ear.

**5** For photo emission to take place, wavelength of incident light should be less than the threshold wavelength of ultraviolet light $< 5200$ Å while that of infrared radiations $> 5200$Å.

**6** $E_k = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left[\frac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right]$

**7** As we know,

$eV_0 = h(\nu - \nu_0)$ and $h\nu = \text{KE} + W_0$

So, A $\to$ 2, B $\to$ 1, C $\to$ 3

**8** By changing distance the intensity changes but frequency remains same, so stopping potential remains same.

**9** Threshold voltage for $a$ and $b$ is same and it depends on frequency. So, $f_a = f_b$.

Photo current of $a$ and $b$ are unequal and photo current depends on intensity.

So, $I_a \neq I_b$

**10** $eV_0 = \frac{hc}{\lambda_0} - W_0$ and $eV' = \frac{hc}{2\lambda_0} - W_0$

Subtracting them, we have

$$e(V_0 - V') = \frac{hc}{\lambda_0}\left[1 - \frac{1}{2}\right] = \frac{hc}{2\lambda_0}$$

or $\quad V' = V_0 - \frac{hc}{2e\lambda_0}$

**11** We have, $eV_0 = \frac{hc}{\lambda} - \phi \Rightarrow V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$

$$V_0 = mx + c$$

$\because$ Data is not sufficient.

**12** As, $\frac{1}{4\pi\varepsilon_0} \frac{Q}{1 \times 10^{-2}} = \frac{\frac{hc}{\lambda} - \phi}{Q}$

$\Rightarrow \quad Q = 5.5 \times 10^{-13}\ \text{C}$

**13** (A) Franck-Hertz experiments is associated with discrete energy levels of atom.

(B) Photo-electric experiment is associated with particle nature of light.

(C) Davisson-Germer experiment is associated with wave nature of electron.

**14** As $\lambda$ is increased, there will be a value of $\lambda$ above which photoelectron will be cease to come out, so photocurrent will becomes zero.

**15** de-Broglie wavelength is,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}} = 1.7\ \text{Å}$$

**16** As, $\quad E = \frac{hc}{\lambda}$

and $\quad \lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{11 \times 1.6 \times 10^{-19}}$

$$= 1.125 \times 10^{-7}\ \text{m}$$

Hence, UV region.

**17** de-Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

But $\quad E = eV$

$$\lambda = \frac{h}{\sqrt{2meV}} \Rightarrow V = \frac{h^2}{2me\lambda^2}$$

$$V = \frac{(6.62 \times 10^{-34})^2}{(0.5 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}$$

$\Rightarrow \quad V = 601.98\ \text{V} \approx 602\ \text{V}$

**18** As, $\lambda_1 = \frac{h}{\sqrt{2m_pE}}$, $\lambda_2 = \frac{hc}{E}$

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{h/\sqrt{2m_pE}}{hc/E} = \frac{\sqrt{E}}{\sqrt{2m_pc^2}}$

$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} \propto \sqrt{E} \Rightarrow \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$

**19** Here, $v = v_o\mathbf{i}$ $B = B_o\mathbf{j}$

Force on moving electron due to magnetic field is

$$\text{F} = -e(v \times \text{B}) = -e[v_o\mathbf{i} \times B_o\mathbf{j}]$$
$$= -e\, v_o B_o \text{k}$$

As this force is perpendicular to $v$ and **B**, so the magnitude of $v$ will not change i.e. momentum ($mv$) will remain constant is magnitude. Hence, de-Broglie wavelength $\lambda = h/mv$ remains constant.

**20** As, $2\pi r = n\lambda \quad \Rightarrow \quad r = \frac{n\lambda}{2\pi}$

Now, de-Broglie equation $\lambda = \frac{h}{p}$

$\Rightarrow \quad mv_n = \frac{h}{\lambda} = \frac{h}{\frac{2\pi r_n}{n}} = \frac{nh}{2\pi r_n}$

Also, for charged particle moving in a magnetic field

$$r_n = \frac{mv_n}{qB} = \frac{nh}{(2\pi r_n)qB}$$

$\Rightarrow \quad r_n^2 = \frac{nh}{2\pi qB}$

$\therefore \quad r_n \propto n^{1/2}$

**21** From quantum theory of light, as intensity of light increases means number of photons/area/time increases and hence more photons take part in ejecting the photoelectron, thus increasing the number of photoelectrons.

**22** Velocity of first photon $= u = c$

Velocity of second photon $= v = -c$

Now relative velocity of first photon with respect to second photon

$$= \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{c - (-c)}{1 - \frac{c \times (-c)}{c^2}}$$

$$= \frac{2c}{1 + \frac{c^2}{c^2}}$$

$$= \frac{2c}{1 + 1} = \frac{2c}{2} = c$$

Also, the rest of mass of photon is zero.

**23** When work function of copper is greater than the work function of sodium,then

$$\phi_{\text{Cu}} > \phi_{\text{Na}}$$

$$(h\nu_o)_{\text{Cu}} > (h\nu_o)_{\text{Na}} \quad \ldots\text{(i)}$$

But we know that, $\nu_o = \frac{c}{\lambda_c}$

Hence, Eq. (i), becomes

$\left(\frac{hc}{\lambda_o}\right)_{\text{Cu}} > \left(\frac{hc}{\lambda_o}\right)_{\text{Na}}$ $(\lambda_o)_{\text{Na}} > (\lambda_o)_{\text{Cu}}$

**24** de-Broglie wavelength associated with gas molecules varies

$$\lambda \propto \frac{1}{\sqrt{T}}$$

Also, root mean square velocity of gas molecules is $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$.

**25** Davisson and Germer experimentally established wave nature of electron by observing diffraction pattern while bombarding electrons on Ni crystal.

**26** Maximum kinetic energy $(\text{KE})_{\text{max}}$ is given by $(\text{KE})_{\text{max}} = h\nu - h\nu_0$.

When frequency is increased $(\text{KE})_{\text{max}}$ will increase stopping potential is that negative voltage given to the anode at which photocurrent stops, hence doubling frequency will not effect it, also

If $\quad \nu' = 2\nu$

$\therefore \quad K'_{\text{max}} = e\nu'_0 = h(2\nu_1 - \nu_0)$

$\quad K'_{\text{max}} = 2K_{\text{max}} + h\nu_0$

$\therefore \quad K'_{\text{max}} > 2K_{\text{max}} \Rightarrow \nu'_0 > 2\nu_0$

Hence, $(\text{KE})_{\text{max}}$ and stopping potential are linearly dependent on the frequency of incident light.

**27** Since, the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays

$$KE_{photoelectron} = h\nu - \phi$$

Stopping potential is to stop the fastest photoelectron

$$V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

So, $KE_{max}$ and $V_0$ both increases.

But KE ranges from zero to $KE_{max}$ because of loss of energy due to subsequent collisions before getting ejected and not due to range of frequencies in the incident light.

## SESSION 2

**1** Here, the initial velocity is $u = 0$

Since, $a = \frac{eE}{m}$, $v = ?$ at $t = t$

So we get, using $v = u + at = 0 + \frac{eE}{m}t$

This gives $\lambda = \frac{h}{mv} = \frac{h}{m(eEt/m)} = \frac{h}{eEt}$

The rate of change of de-Broglie wavelength is

$$\frac{d\lambda}{dt} = \frac{h}{eE} \times \left(\frac{-1}{t^2}\right) = \frac{-h}{eEt^2}$$

**2** From Einstein's photoelectric equation, we have

$$eV_0 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \quad \ldots(i)$$

$$\frac{eV_0}{6} = hc\left[\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right] \quad \ldots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$6 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right)}$$

$$\frac{6}{3\lambda} - \frac{6}{\lambda_0} = \frac{1}{\lambda} - \frac{1}{\lambda_0} \Rightarrow \frac{1}{\lambda} = \frac{5}{\lambda_0}$$

$$\Rightarrow \quad \frac{\lambda_0}{\lambda} = 5 \Rightarrow \lambda_0 = 5\lambda$$

**3** Photoelectric current $I$ is directly proportional to intensity of light and intensity $\propto \frac{1}{(\text{distance})^2}$

$$I \propto \frac{1}{r^2}$$

$$I_{25} \propto \frac{1}{(25)^2} \quad \ldots(i)$$

$$I_{100} \propto \frac{1}{(100)^2} \quad [1 \text{ m} = 100 \text{ cm}]\ldots(ii)$$

$$\therefore \frac{I_{25}}{I_{100}} = \frac{(100)^2}{(25)^2} = 16 \Rightarrow I_{100} = \frac{I_{25}}{16}$$

**4** Given, wavelength

$\lambda_1 = 600\,\text{nm} = 600 \times 10^{-9}\text{m}$

Energy correspond to $\lambda_1 = E_1$

Again, wavelength

$\lambda_2 = 400\,\text{nm} = 400 \times 10^{-9}\text{m}$

Energy correspond to $\lambda_2 = E_2$

Let the work function of metal is $\phi$.

According to question $2E_1 = E_2$

i.e. $$2\left(\frac{hc}{\lambda_1} - \phi_0\right) = \frac{hc}{\lambda_2} - \phi_0$$

or $$\frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} = 2\phi_0 - \phi_0$$

or $$hc\left[\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right] = \phi_0$$

$$\phi_0 = 6.63 \times 10^{-34} \times 3 \times 10^8 \left[\frac{2}{600 \times 10^{-9}} - \frac{1}{400 \times 10^{-9}}\right]$$

or $$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-7}}\left[\frac{1}{3} - \frac{1}{4}\right]$$

or $$= 6.63 \times 3 \times 10^{-19}\left[\frac{4-3}{12}\right]\text{J}$$

$$= \frac{6.63 \times 3 \times 10^{-19}}{1.6 \times 10^{-19} \times 12} = 1.02 \text{ eV}$$

**5** According to the Einstein's photoelectric effect

$$h\nu - h\nu_0 = \frac{1}{2}mv^2 = eV$$

$$\Rightarrow \left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = eV$$

where, $\lambda_0$ = threshold wavelength.

Now for the first case,

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = e(3V_0) \quad \ldots(i)$$

For the second case,

$$\frac{hc}{2\lambda} - \frac{hc}{\lambda_0} = e(V_0) \quad \ldots(ii)$$

On dividing Eq. (i) by Eq.(ii), we get

$$\frac{hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)} = \frac{3eV_0}{eV_0}$$

$$\Rightarrow \quad \frac{\frac{\lambda_0 - \lambda}{\lambda\lambda_0}}{\frac{\lambda_0 - 2\lambda}{2\lambda \cdot \lambda_0}} = \frac{3eV_0}{eV_0}$$

$$\lambda_0 = 4\lambda$$

**6** From the relation,

$$eV = \frac{hc}{\lambda} - \phi \text{ or } V = \left(\frac{hc}{e}\right)\frac{1}{\lambda} - \frac{\phi}{e}$$

This is the equation of straight line slope is $\tan\theta = hc/e$

$$\phi_1 : \phi_2 : \phi_3 = \frac{hc}{\lambda_{01}} : \frac{hc}{\lambda_{02}} : \frac{hc}{\lambda_{03}}$$

$$\frac{1}{\lambda_{01}} : \frac{1}{\lambda_{02}} : \frac{1}{\lambda_{03}} = 1:2:4$$

$$\frac{1}{\lambda_0} = 0.001\,\text{nm}^{-1}$$

$$\lambda_0 = 10{,}000\,\text{Å}$$

$$\frac{1}{\lambda_{02}} = 0.002\,\text{nm}^{-1}$$

or $$\lambda_{02} = 5000\,\text{Å}$$

$$\frac{1}{\lambda_{03}} = 0.004\,\text{nm}^{-1}$$

$$\lambda_{03} = 2500\,\text{Å}$$

Violet colour has wavelength 4000Å.

So, violet colour can eject photoelectrons from metal 1 and metal 2.

**7** We have, $E = qV$, we know that

$$E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$$

as, $$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2E}{m}}}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mqV}} \quad \ldots(i)$$

For electron, $$\lambda_e = \frac{h}{\sqrt{2m_eqV}} \quad \ldots(ii)$$

For protron $$\lambda_p = \frac{h}{\sqrt{2m_pqV}}$$

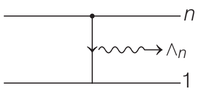
$$\therefore \quad \lambda_p = \frac{h}{\sqrt{2m_pq \cdot 4V}}$$

$$(\because V = 4\text{V}) \ldots(iii)$$

On dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$$

**8** If wavelength of emitted photon in de-excitation is $\Lambda_n$;




Then, $$\frac{hc}{\Lambda_n} = E_n - E_g$$

$$\frac{hc}{\Lambda_n} = \frac{p_n^2}{2m} - \frac{p_g^2}{2m} \quad \left[\because E = \frac{p^2}{2m}\right]$$

As energies are negative, we get

$$\frac{hc}{\Lambda_n} = \frac{p_g^2}{2m} - \frac{p_n^2}{2m} = \frac{p_g^2}{2m}\left(1 - \left(\frac{p_n}{p_g}\right)^2\right)$$

$$= \frac{h^2}{2m\lambda_g^2}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)\left[\because p \propto \lambda^{-1}, p = \frac{h}{\lambda}\right]$$

$$\Rightarrow \quad \Lambda_n = \frac{2m\lambda_g^2 c}{h}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)^{-1}$$

$$\Rightarrow \quad \Lambda_n = \frac{2m\lambda_g^2 c}{h}\left(1 + \frac{\lambda_g^2}{\lambda_n^2}\right)$$

$[\because (1 - x)^{-n} = 1 + nx]$

$$\Rightarrow \quad \Lambda_n \simeq A + \frac{B}{\lambda_n^2}$$

where, $A = \left[\frac{2mc\lambda_g^2}{h}\right]$ and $B = \left[\frac{2mc\lambda_g^4}{h}\right]$ are constants.

**9** $\lambda_{\min} = \frac{hc}{eV}$

$$\log(\lambda_{\min}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - mx$$

So, the required graph is given in option (d).

**10** For elastic collision,

$$p_{\text{before collision}} = p_{\text{after collision}}.$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \quad \ldots \text{(i)}$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collision $e = 1$

$$\therefore 1 = \frac{v_B - v_A}{v} \Rightarrow v = v_B - v_A \quad \ldots \text{(ii)}$$

From Eq. (i) and Eq. (ii),

$$v_A = \frac{v}{3} \quad \text{and} \quad v_B = \frac{4v}{3}$$

Hence, $\frac{\lambda_A}{\lambda_B} = \frac{\left(\frac{h}{mV_A}\right)}{\left(\frac{h}{\frac{m}{2}.V_B}\right)}$

$$= \frac{V_B}{2V_A} = \frac{4/3}{2/3} = 2$$

**11** According to Einstein's photoelectric emission of light, $E = (\text{KE})_{\max} + \phi$

As, $\frac{hc}{\lambda} = (\text{KE})_{\max} + \phi$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$$\Rightarrow \frac{4}{3}\frac{hc}{\lambda} = \left(\frac{4}{3}(\text{KE})_{\max} + \frac{\phi}{3}\right) + \phi$$

For fastest emitted electron,

$$(\text{KE})_{\max.} = \frac{1}{2}mv'^2 + \phi$$

$$\Rightarrow \quad \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

i.e. $\quad v' > v\left(\frac{4}{3}\right)^{1/2}$

**12** When an electron moves in a circular path, then

Radius, $r = \frac{mv}{eB} \Rightarrow \frac{r^2e^2B^2}{2} = \frac{m^2v^2}{2}$

$$\text{KE}_{\max} = \frac{(mv)^2}{2m} \Rightarrow \frac{r^2e^2B^2}{2m} = (\text{KE})_{\max}$$

Work function of the metal ($W$),

i.e. $\quad W = h\nu - \text{KE}_{\max}$

$$1.89 - \phi = \frac{r^2e^2B^2}{2m}\frac{1}{2}\text{eV}$$

$$= \frac{r^2eB^2}{2m}\text{eV}$$

[$h\nu \rightarrow 1.89$ eV, for the transition on from third to second orbit of H-atom]

$$= \frac{100 \times 10^{-6} \times 1.6 \times 10^{-19} \times 9 \times 10^{-8}}{2 \times 9.1 \times 10^{-31}}$$

$$\phi = 1.89 - \frac{1.6 \times 9}{2 \times 9.1}$$

$$= 1.89 - 0.79 = 1.1 \text{ eV}$$

**13** $\because \text{KE}_{\max} = eV_0$

$$\Rightarrow \frac{1}{2}mv^2 = eV_0 = 1.68 \text{ eV}$$

$$\Rightarrow h\nu = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{400 \text{ nm}} = 3.1 \text{ eV}$$

$$\Rightarrow \quad 3.1 \text{ eV} = W_0 + 1.68 \text{ eV}$$

[From Einstein equation, $E = W_0 + K_{\max}$]

$$W_0 = 1.42 \text{ eV}$$