Vectors

Summary

1. Vectors & Their Representation:

Vector quantities are specified by definite magnitude and definite directions. A vector is generally represented by a directed line segment, say $\overrightarrow{AB}$. is called the Initial point & B is called the terminal point. The magnitude of vector $\overrightarrow{AB}$ is expressed by $\left|\overrightarrow{AB}\right|$.

Zero Vector : A vector of zero magnitude is a zero vector.

Unit Vector : A vector of unit magnitude in the direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\vec{a}$ symbolically $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$ **.**

Equal Vectors: Two vectors are said to be equal if they have the same magnitude, direction & represent the same physical chemistry.

Collinear Vectors: Two vectors are said to be collinear if their directed line segments are Parallel irrespective of their directions. If $\vec{a}$ *and* $\vec{b}$ are collinear, then $\vec{a}=k\vec{b},$ where $k\in R, k\neq 0$

Vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ *and* $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are collinear if $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

Coplanar Vectors:
A given number of vectors are called coplanar if their line segments are all parallel to the same plane. Note that ''Two Vectors Are Always Coplanar''

2. Angle Between two Vectors

It is the smaller angle formed when the initial points or the terminal points of the two vectors are brought together. It should be noted that $0^0 \leq \theta \leq 180^0$

3. Addition Of Vectors:

- If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$ then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB
- $\vec{a}+\vec{b}=\vec{b}+\vec{a}\,(commutatuve)$
- $\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)(associativity)$
- $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$
- $\vec{a}+\left(-\vec{a}\right)=\vec{0}=\left(-\vec{a}\right)+\vec{a}$
- $\left|\vec{a}+\vec{b}\right|\leq|\vec{a}|+\left|\vec{b}\right|$
- $\left|\vec{a}-\vec{b}\right|\geq\left||\vec{a}|-\left|\vec{b}\right|\right|$
- $\left|\vec{a}\pm\vec{b}\right|=\sqrt{|\vec{a}|^2+\left|\vec{b}\right|^2\pm 2|\vec{a}|\left|\vec{b}\right|\cos\theta}$ where θ is the angle between the vectors

- A vector in the direction of the bisector of the angle between the two vectors $\vec{a}$ & $\vec{b}$ is $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}$. Hence bisector of the angle between the two vectors $\vec{a}$ and $\vec{b}$ $\lambda\left(\hat{a}+\hat{b}\right)$, where $\lambda \in R^{+}$ Bisector of the exterior angle between $\vec{a}$ & $\vec{b}$ is $\lambda\left(\hat{a}-\hat{b}\right), \lambda \in R^{+}$

4. **Multiplication Of A Vector By A Scalar:**

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times that of $\vec{a}$ This multiplication is called **SCALAR MULTIPLICATION**: If $\vec{a}$ and $\vec{b}$ are vectors & m, n are scalars then :

$m\left(\vec{a}\right)=\left(\vec{a}\right)m=m\vec{a}$ $\qquad$ $m\left(n\vec{a}\right)=n\left(m\vec{a}\right)=\left(mn\right)\vec{a}$

$\left(m+n\right)\vec{a}=m\vec{a}+n\vec{a}$ $\qquad$ $m\left(a+\vec{b}\right)=m\vec{a}+m\vec{b}$

5. **Position Vector of A Point:**

Let O be a fixed origin, then the position of a vector $\overrightarrow{OP}$. If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then, $\overrightarrow{AB}=\vec{b}-\vec{a}=pv\ of\ B-pv$ of A.

DISTANCE FORMULA: Distance between the two points $A\left(\vec{a}\right)$ and $B\left(\vec{a}\right)$ is $AB=|\vec{a}-\vec{b}|$

SECTION FORMULA: $\vec{r}=\frac{n\vec{a}+m\vec{b}}{m+n}$. Mid point of AB $AB=\frac{\vec{a}+\vec{b}}{2}$

6. **Scalar Product of Two Vectors:** $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$, where $|\vec{a}|,|\vec{b}|$ are magnitude of $\vec{a}$ and $\vec{b}$ respectively and θ is angle between $\vec{a}$ and $\vec{b}$.

(i) $i.i=j.j=k.k=1;$ $\qquad$ $i.j=j.k=k.i=0$ $\quad$ project of $\vec{a}$ on $\vec{b}=\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

(ii) $\vec{a}=a_1i+a_2j+a_3k$ & $\vec{b}=b_1i+b_2j+b_3k$ then $\vec{a}.\vec{b}=a_1b_1+a_2b_2+a_3b_3$

$|\vec{a}|\sqrt{a_1^2+a_2^2+a_3^2}$ $\quad$, $\quad$ $|\vec{b}|\sqrt{b_1^2+b_2^2+b_3^2}$

(iii) the angle ϕ between $\vec{a}$ and $\vec{b}$ is given by $\cos\phi=\frac{\vec{a}.\vec{b}}{|\vec{a}|\ |\vec{b}|}$, $0\le\phi\le\pi$

(iv) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta(0 \le \theta \le \pi)$, note that if θ is acute, then $\vec{a}.\vec{b} > 0$ & if θ is obtuse, then $\vec{a}.\vec{b} < 0$

(v) $\vec{a}.\vec{a}\ |\vec{a}|^2 = \vec{a}^2$, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ (commutative) $\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ (distributive)

(vi) $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \text{b}$ $(\vec{a} \ne 0\ \vec{b} \ne 0)$

(vii) $(m\ \vec{a}).\ \vec{b} = \vec{a}.(m\ \vec{b}) = (\vec{a}.\ \vec{b})$ (associative) where m is scalar.

Note:

(a) Maximum value $\vec{a}.\vec{b}\ is\ |\vec{a}|\ |\vec{b}|$

(b) Minimum value of $\vec{a}.\vec{b}\ is - |\vec{a}|\ |\vec{b}|$

(c) Any vector $\vec{a}$ can be written as, $\vec{a} = (\vec{a}.\hat{j})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$.

7. Vector Product of Two Vectors:

(i) If $\vec{a}\&\vec{b}$ are two vectors & θ is the angle between them $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\ \vec{n}$, where $\vec{n}$ is the unit vector perpendicular to both $\vec{a}\&\vec{b}$ such that $\vec{a},\vec{b}$ such that $\vec{a},\vec{b}\&\vec{n}$ forms a right handed screw system.

(ii) Geometrically $|\vec{a}\times\vec{b}| =$ area of the parallelogram whose two adjacent sides are represented by $\vec{a}\&\vec{b}$

(iii) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}; \hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$

(iv) $If\ \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\ \ \&\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\ then\ \vec{a}\times\vec{b} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$

(v) $\vec{a}\times\vec{b} \ne \vec{b}\times\vec{a}\,(not\ commutative)$

(vi) $(m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = m(\vec{a}\times\vec{b})(associtative)$ where m is a scalar.

(vii) $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})(distributive)$

(viii) $\vec{a}\times\vec{b}=\vec{0}\Leftrightarrow \vec{a}\, and\, \vec{b}$ are parallel (collinear) $\left(\vec{a}\neq 0,\vec{b}\neq 0\right)$ $i.e. \vec{a}=K\vec{b}$, where K is a scalar

(ix) Unit vector perpendicular to the plane of $\vec{a}\times\vec{b}$ $is\ \hat{n}=\pm\dfrac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

- If θ is the angle between $\vec{a}\times\vec{b}$ then $\sin\theta=\dfrac{\left|\vec{a}\times\vec{b}\right|}{|\vec{a}||\vec{b}|}$

- If $\vec{a},\vec{b}$ & $\vec{c}$ are the pv's of 3 points A, B & C then the vector area of triangle ABC =

$\frac{1}{2}\left[\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right]$.The points A, s & care collinear if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

- Area of any quadrilateral whose diagonal vectors are $\vec{d}_1$ & $\vec{d}_2$ is given by $\frac{1}{2}\left|\vec{d}_1\times\vec{d}_2\right|$
- Lagrange's identity for any two vectors $\vec{a}\,\&\,\vec{b};\left(\vec{a}\times\vec{b}\right)^2=|\vec{a}|^2|\vec{b}|^2-\left(\vec{a}.\vec{b}\right)^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b}\\ \vec{a}.\vec{b} & \vec{b}.\vec{b}\end{vmatrix}$

8. Scalar Triple Product:

- The scalar triple product of three vectors $\vec{a},\vec{b}\ \&\vec{c}$ is defined as : $\vec{a}\times\vec{b}.\vec{c}$ = $|\vec{a}||\vec{b}||\vec{c}|\sin\theta\cos\phi$ where θ is the angle between $\vec{a}\,\&\vec{b}$ $\&\phi$ is the angle between $\vec{a}\times\vec{b}\,\&\vec{c}$. It is also written as $\left[\vec{a}\vec{b}\vec{c}\right]$ and spelled as box product
- Scalar triple product geometrically represents the volume of the parallelopiped whose three coterminous edges are represented by $\vec{a},\vec{b}$ & $\vec{c}$ $i.e.\ V=\left[\vec{a}\vec{b}\vec{c}\right]$
- In a scalar triple product the position of dot & cross can be interchanged i.e . $\vec{a}.\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right).\vec{c}$ OR $\left[\vec{a}\vec{b}\vec{c}\right]=\left[\vec{b}\vec{c}\vec{a}\right]=\left[\vec{c}\vec{a}\vec{b}\right]$
- $\vec{a}..\left(\vec{b}\times\vec{c}\right)=-\vec{a}\left(\vec{c}\times\vec{b}\right)$ $i.e\left[\vec{a}\vec{b}\vec{c}\right]=-\left[\vec{a}\vec{c}\breve{b}\right]$

- $If\ \vec{a}=a_1i+a_2j+a_3k;\vec{b}=b_1i+b_2j+b_3k\ \&\ \vec{c}=c_1i+c_2j+c_3k\ then\left[\vec{a}\vec{b}\vec{c}\right]=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_3 & c_3\end{vmatrix}$

In general, If $\vec{a}=a_1\vec{i}+a_2\vec{m}+a_3\vec{n};\vec{b}=b_1\vec{i}+b_2\vec{m}+b_3\vec{n}\ \&\ \vec{c}=c_1\vec{i}+c_2\vec{m}+c_3\vec{n}$

then $\left[\vec{a}\vec{b}\vec{c}\right]=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_3 & c_3\end{vmatrix}\left[\vec{l}\vec{m}\vec{n}\right]$; where $\vec{l},\vec{m}\ \&\ \vec{n}$ are non coplanar vectors

- If $\vec{a},\vec{b},\vec{c}$ are coplanar $\Leftrightarrow K\left[\vec{a}\vec{b}\vec{c}\right]=0$
- $[i\,j\,k]$
- $\left[K\vec{a}\vec{b}\vec{c}\right]=K\left[\vec{a}\vec{b}\vec{c}\right]$
- $\left[\left(\vec{a}+\vec{b}\right)\vec{c}\vec{d}\right]=\left[\vec{a}\vec{c}\vec{d}\right]+\left[\vec{b}\vec{c}\vec{d}\right]$
- The positon vector of the centroid of a tetrahedron if the pv's of its vertices are $\vec{a},\vec{b},\vec{c}\ \&\ \vec{d}$ are given by $\frac{1}{4}\left[\vec{a}+\vec{b}+\vec{c}+\vec{d}\right]$

9. Vector Triple Product : $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c},\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{b}-\vec{c}\right)\vec{a}$

- $\left(\vec{a}\times\vec{b}\right)\times\vec{c}\neq\vec{a}\times\left(\vec{b}\times\vec{c}\right)$, in general

10. Reciprocal System Of Vectors:

If $\vec{a},\vec{b},\vec{c}\ \&\ \vec{a},\vec{b},\vec{c}$ are two sets of non coplanar vectors such that $\vec{a}.\vec{a}'=\vec{b}.\vec{b}'=\vec{c}.\vec{c}'=1$ then the two systems are called Reciprocal system of vectors, where $\vec{a}'=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]},\vec{b}'=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]},\vec{c}=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$

11. Linear Combinations:

Given a finite set of vectors $\vec{a},\vec{b},\vec{c},$............. then the vector $\vec{r}=x\vec{a}+y\vec{b}+z\vec{c}+$......... is called a linear combination of $\vec{a},\vec{b},\vec{c},$............. for any x, y, z…….. $\in$ R. We have the following results:

(i) If $\vec{a},\vec{b}$ are non zero, non-collinear vectors then

$x\vec{a}+y\vec{b}=x'\vec{a}+y'\vec{b}\ \Rightarrow x=x';y=y'$

(ii) **Fundamental Theorem:** Let $\vec{a},\vec{b}$ be non zero, non collinear vectors. Then any vector $\vec{r}$ coplanar with $\vec{a},\vec{b}$ can be expressed uniquely as a linear combination of $\vec{a},\vec{b}$

i.e. There exist some uniquely $x, y \in R$ such that $x\vec{a} + y\vec{b} = \vec{r}$

(iii) If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non-coplanar vectors then:
$x\vec{a} + y\vec{b} + z\vec{c} = x'\vec{a} + y'\vec{b} + z'\vec{c} \Rightarrow x = x', y = y', z = z'$

(iv) **Fundamental Theorem In Space:** Let $\vec{a}, \vec{b}, \vec{c}$ be non-zero, non-coplanar vector in space. Then any vector $\vec{r}$, can be uniquely expressed as a linear combination of $\vec{a}, \vec{b}, \vec{c}$ *i.e.* There exist some unique x, y ∈ R such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{r}$

(v) If $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ are non zero vectors, & $k_1, k_2 .... k_n$ are n scalars & if the linear combination $k_1\vec{x}_1 + k_2\vec{x}_2 + ...k_n\vec{x}_n = 0 \Rightarrow k_1 = 0, k_2 = 0.........k_n = 0$ then we say that vectors $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ ar Linearly Independent Vectors .

(vi) If $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ are not Linearly Independent then they are said to be Linearly Dependent Vectors.

i.e,. If $k_1\vec{x}_1, k_2\vec{x}_2, ...k\vec{x}_n = 0$ & if there exists at least one $k_r \neq 0$ then $\vec{x}_1, \vec{x}_2, ...\vec{x}_n$ *are* said to Linearly Dependent

Note 1: If $k_1\vec{x}_1 + k_2\vec{x}_2 + k_3\vec{x}_3 + ...k_n\vec{x}_n = \vec{0}$, for some $k_r \neq 0$, *then* $\vec{x}_1\vec{x}_2, \vec{x}_3, ..., \vec{x}_n$ form a linearly dependent set of vectors

- $\vec{i}, \vec{j}, \vec{k}$ are **Linearly Independent** set of vectors. For $K_1\hat{i} + K_2\hat{j} + k_3\hat{k} = 0 \Rightarrow K_1 = K_2 = K_3 = 0$
- Two vectors $\vec{a}$ & $\vec{b}$ are linearly dependent $\Rightarrow$ $\vec{a}$ is parallel to $\vec{b}$ i.e. $\vec{a} \times \vec{b} = 0 \Rightarrow$ linear dependence of $\vec{a}$ & $\vec{b}$ .Conversely if $\vec{a} \times \vec{b} = \vec{0}$ then $\vec{a}$ *and* $\vec{b}$ are linearly dependent.
- If three vectors $\vec{a}, \breve{b}, \vec{c}$ are linearly dependent, then they are coplanar i.e. $\left[\vec{a}, \vec{b}, \vec{c}\right]$ =0, conversely if $\left[\vec{a}, \vec{b}, \vec{c}\right] \neq 0$, then the vectors are linearly independent.

Note: Test of Collinearity :

Three points A,B,C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively are collinear, if & only if there exist scalars $x'y'z$ not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where x + y + z = 0

Note: Test of Coplanarity:

Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if there exist scalars x, y, z, w not all zero simultaneously such ,that $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where x+ y + z + w = 0.

Practice Questions

1. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3j - k$ and $\vec{b} = j + k$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$. than equal to: (2018)

(a) 84
(b) 336
(c) 315
(d) 256

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vector such that $\vec{a} + 2\vec{b} + 2\vec{c} = 0$, then $\left|\vec{a} \times \vec{c}\right|$ is equal to: (2018)

(a) $\frac{\sqrt{15}}{4}$
(b) $\frac{\sqrt{15}}{16}$
(c) $\frac{1}{4}$
(d) $\frac{15}{4}$

3. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = 0$, then $\left|\vec{a} \times \vec{c}\right|$ is equal to: (2018)

(a) $\frac{1}{4}$
(b) $\frac{15}{16}$
(c) $\frac{\sqrt{15}}{4}$
(d) $\frac{\sqrt{15}}{16}$

4. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a}.\vec{b} = 3$. Then $\left|\vec{b}\right|$ equals:
(2018)

(a) $\frac{11}{3}$
(b) $\frac{11}{\sqrt{3}}$
(c) $\sqrt{\frac{11}{3}}$
(d) $\frac{\sqrt{11}}{3}$

5. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

OP.OQ + OR.OS = OR.OP + OQ.OS

= OQ.OR + OP.OS

Then the triangle PQR has S as its (2017)

(a) centroid

(b) orthocenter

(c) incenter

(d) circumcenter

6. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by (2011)

(a) $\hat{i} - 3\hat{j} + 3\hat{k}$

(b) $-3\hat{i} - 3\hat{j} + \hat{k}$

(c) $3\hat{i} - \hat{j} + 3\hat{k}$

(d) $\hat{i} + 3\hat{j} - 3\hat{k}$

7. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by (2010)

(a) $\frac{8}{9}$

(b) $\frac{\sqrt{17}}{9}$

(c) $\frac{1}{9}$

(d) $\frac{4\sqrt{5}}{9}$

8. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$, respectively. The quadrilateral PQRS must be a (2010)

(a) parallelogram, which is neither a rhombus nor a rectangle

(b) square

(c) rectangle, but not a square

(d) rhombus, but not a square

9. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form and acute angle. A point P moves, so that at any time *t* the position vector $\overrightarrow{OP}$ (where *O* is the origin) is given by $\hat{a}$ cos *t* + $\hat{b}$ sin *t*. When *P* is farthest from origin *O*, let *M* be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(2008)

(a) $\hat{u} = \frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|}$ *and* $M = \left(1+\hat{a}.\hat{b}\right)^{1/2}$

(b) $\hat{u} = \frac{\hat{a}-\hat{b}}{\left|\hat{a}-\hat{b}\right|}$ *and* $M = \left(1+\hat{a}.\hat{b}\right)^{1/2}$

(c) $\hat{u} = \frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|}$ *and* $M = \left(1+2\hat{a}.\hat{b}\right)^{1/2}$

(d) $\hat{u} = \frac{\hat{a}-\hat{b}}{\left|\hat{a}-\hat{b}\right|}$ *and* $M = \left(1+2\hat{a}.\hat{b}\right)^{1/2}$

10. Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k}, \vec{b} = \hat{i}-\hat{j}+\hat{k}, \vec{c} = \hat{i}+\hat{j}-\hat{k}$. A vector coplanar to $\vec{a}$ and $\vec{b}$ has a projection along $\vec{c}$ of magnitude $\frac{1}{\sqrt{3}}$, then the vector is (2006)

(a) $4\hat{i}-\hat{j}+4\hat{k}$

(b) $4\hat{i}+\hat{j}-4\hat{k}$

(c) $2\hat{i}+\hat{j}+\hat{k}$

(d) None of these

11. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and

$$\overrightarrow{b_1} = \vec{b} - \frac{\vec{b}.\vec{a}}{\left|\vec{a}\right|^2}\vec{a}, \overrightarrow{b_2} = \vec{b} + \frac{\vec{b}.\vec{a}}{\left|\vec{a}\right|^2}\vec{a}, \overrightarrow{c_1} = \vec{c} - \frac{\vec{c}.\vec{a}}{\left|\vec{a}\right|^2}\vec{a} - \frac{\vec{c}.\vec{b}}{\left|\vec{b}\right|^2}\vec{b}, \overrightarrow{c_2} = \vec{c} - \frac{\vec{c}.\vec{a}}{\left|\vec{a}\right|^2}\vec{a} - \frac{\vec{c}.\overrightarrow{b_1}}{\left|\vec{b}\right|^2}\overrightarrow{b_1},$$

$$\overrightarrow{c_3} = \vec{c} - \frac{\vec{c}.\vec{a}}{\left|\vec{a}\right|^2}\vec{a} - \frac{\vec{c}.\overrightarrow{b_2}}{\left|\overrightarrow{b_2}\right|^2}\overrightarrow{b_2}, \overrightarrow{c_4} = \vec{a} - \frac{\vec{c}.\vec{a}}{\left|\vec{a}\right|^2}\vec{a}$$

Then, which of the following is a set of mutually orthogonal vectors? (2005)

(a) $\left\{\vec{a},\overrightarrow{b_1},\overrightarrow{c_1}\right\}$

(b) $\left\{\vec{a},\overrightarrow{b_1},\overrightarrow{c_2}\right\}$

(c) $\left\{\vec{a},\overrightarrow{b_2},\overrightarrow{a_3}\right\}$

(d) $\left\{\vec{a},\overrightarrow{b_2},\overrightarrow{c_4}\right\}$

12. If $\vec{a}$ and $\vec{b_1}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$, are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is (2002)

(a) 45°
(b) 60°
(c) $\cos^{-1}\left(\frac{1}{3}\right)$
(d) $\cos^{-1}\left(\frac{2}{7}\right)$

13. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors, then $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ does not exceed (2001)

(a) 4
(b) 9
(c) 8
(d) 6

14. Let $\vec{u},\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=0$. If $\left|\vec{u}\right|=3,\left|\vec{v}\right|=4$ and $\left|\vec{w}\right|=5$ then $\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}$ is (1995)

(a) 47
(b) – 25
(c) 0
(d) 25

15. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1,1,0)$ and $\vec{b} = (0,1,1)$ is

(1987)

(a) one
(b) two
(c) three
(d) infinite

16. A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has components p + 1 and 1, then (1986)

(a) $p = 0$

(b) $p = 1 \; or \; p = -\frac{1}{3}$

(c) $p = -1 \; or \; p = -\frac{1}{3}$

(d) $p = -1 \; or \; p = -1$

17. The points with position vectors $60\hat{i} + 3\hat{j}, 40\hat{i} - 8\hat{j}, a\hat{i} - 52\hat{j}$ are collinear, if (1983)

(a) a = - 40
(b) a = 40
(c) a = 20
(d) None of these

18. Let a = $2\hat{i} + \hat{j} - 2\hat{k}$, $b = \hat{i} + \hat{j}$ and c be a vector such that |c – a| = 3, |(a × b × c)| = 3 and the angle between c and a × b is 30. Then, a . c is equal to (2017)

(a) $\frac{25}{8}$

(b) 2
(c) 5

(d) $\frac{1}{8}$

19. If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{29}$ and $\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\vec{b}$ then a possible value of $\left(\vec{a}+\vec{b}\right).\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is (2012)

(a) 0
(b) 3
(c) 4
(d) 8

20. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct?

(2007)

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$
(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$
(c) $\vec{b}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=\vec{0}$
(d) $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ are mutually perpendicular

21. If the vectors $\vec{a},\vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a ABC, then (2000)

(a) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$
(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$
(c) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$
(d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

22. If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are the unit vectors such that $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$, then (2009)

(a) $\vec{a},\vec{b},\vec{c}$ are non-coplanar
(b) $\vec{a},\vec{b},\vec{d}$ are non-coplanar
(c) $\vec{b},\vec{d}$ are non-parallel
(d) $\vec{a},\vec{d}$ are parallel and $\vec{b},\vec{c}$ are parallel

23. The edges of a parallelepiped are of unit length and are parallel to non-coplanar unit vector $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = \frac{1}{2}$. Then, the volume of the parallelepiped is (2008)

(a) $\frac{1}{\sqrt{2}}$ cu unit

(b) $\frac{1}{2\sqrt{2}}$ cu unit

(c) $\frac{\sqrt{3}}{2}$ cu unit

(d) $\frac{1}{\sqrt{3}}$ cu unit

24. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}, \hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is (2007)

(a) 0
(b) 1
(c) 2
(d) 3

25. The value of a, so that the volume of parallelepiped formed by $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ become minimum, is (2003)

(a) 0
(b) 1
(c) 2
(d) 3

26. If $\vec{V} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{W} = \vec{i} + 3\vec{k}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U}\vec{V}\vec{W}\right]$ is (2002)

(a) – 1
(b) $\sqrt{10} + \sqrt{6}$
(c) $\sqrt{59}$
(d) $\sqrt{60}$

27. If $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\vec{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ depends on

(2001)

(a) only x
(b) only y
(c) neither x nor y
(d) both x and y

28. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b}\;\; 2\vec{b} - \vec{c}\;\; 2\vec{c} - \vec{a}\right]$ is

(2000)

(a) 0
(b) 1
(c) $-\sqrt{3}$
(d) $\sqrt{3}$

29. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three? (1998)

(a) $\vec{u}.(\vec{v} \times \vec{w})$
(b) $(\vec{v} \times \vec{w}).\vec{u}$
(c) $\vec{v}.(\vec{u} \times \vec{w})$
(d) $(\vec{u} \times \vec{v}).\vec{w}$

30. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, b = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linear dependent vectors and $\left|\vec{c}\right| = \sqrt{3}$, then (1998)

(a) $\alpha = 1, \beta = -1$
(b) $\alpha = 1, \beta = \pm 1$
(c) $\alpha = -1, \beta = \pm 1$
(d) $\alpha = \pm 1, \beta = 1$

31. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $\left(\vec{a}+\vec{b}+\vec{c}\right).\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)\right]$ equals (1995)

(a) 0
(b) $\left[\vec{c}\,\vec{b}\,\vec{c}\right]$
(c) $2.\left[\vec{c}\,\vec{b}\,\vec{c}\right]$
(d) $-\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

32. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{j}-\hat{k}, \vec{c}=\hat{k}-\hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d}=0=\left[\vec{b}\,\vec{c}\,\vec{d}\right]$, then equals

(1995)

(a) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$
(b) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$
(c) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$
(d) $\pm\hat{k}$

33. Let *a, b, c* be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+ck, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then *c* is (1993)

(a) the arithmetic mean of *a* and *b*
(b) the geometric mean of *a* and *b*
(c) the harmonic mean of *a* and *b*
(d) equal to zero

34. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{a} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{a} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{c}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to (1986)

(a) 0

(b) 1

(c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

(d) $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)\left(c_1^2 + c_2^2 + c_3^2\right)$

35. The volume of the parallelepiped whose sides are given by
$\overrightarrow{OA} = 2\hat{i} - 3\hat{j}, \overrightarrow{OB} = \hat{i} + \hat{j} - \hat{k}, \overrightarrow{OC} = 3\hat{i} - \hat{k}$, is (1983)

(a) $\frac{4}{13}$

(b) 4

(c) $\frac{2}{7}$

(d) None of these

36. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $\left|\left(\vec{a} \times \vec{b}\right).\vec{c}\right| = |\vec{a}||\vec{b}||\vec{c}|$ holds, if and only if (1982)

(a) $\vec{a}.\vec{b} = 0, \vec{b}.\vec{c} = 0$

(b) $\vec{b}.\vec{c} = 0, \vec{c}.\vec{a} = 0$

(c) $\vec{c}.\vec{a} = 0, \vec{a}.\vec{b} = 0$

(d) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

37. The scalar $\vec{A}.\left[\left(\vec{B}+\vec{C}\right)\times\left(\vec{A}+\vec{B}+\vec{C}\right)\right]$ equals (1981)

(a) 0
(b) $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$
(c) $\left[\vec{A}\vec{B}\vec{C}\right]$
(d) none of these

38. Let $\hat{a},\hat{b}$ and $\hat{c}$ be three unit vectors such that $\hat{a}\times\left(\hat{b}\times\hat{c}\right)=\frac{\sqrt{3}}{2}\left(\hat{b}+\hat{c}\right)$. If $\hat{b}$ is not parallel to $\hat{c}$, then the angle between $\hat{a}$ and $\hat{b}$ is (2016)

(a) $\frac{3\pi}{4}$
(b) $\frac{\pi}{2}$
(c) $\frac{2\pi}{3}$
(d) $\frac{5\pi}{6}$

39. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is

(a) $\frac{2\sqrt{2}}{3}$
(b) $\frac{-\sqrt{2}}{3}$
(c) $\frac{2}{3}$
(d) $\frac{-2\sqrt{2}}{3}$

40. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is (2004)

(a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$

(b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

(c) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$

(d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

41. If $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right), \vec{a}.\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is equal to (2003)

(a) $\hat{i}-\hat{j}+\hat{k}$

(b) $2\hat{j}-\hat{k}$

(c) $\hat{i}$

(d) $2\hat{i}$

42. Let the vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\vec{0}$. If P_1 and P_2 are planes determined by the pairs of vectors $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively, then the angle between P_1 and P_2 is (2000)

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

43. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between and is 30°, then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ is equal to (1999)

(a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) 2

(d) 3

44. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\left(\vec{b} + \vec{c}\right)}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is (1995)

(a) $\frac{3\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π

45. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is equal to (1999)

(a) $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$

(b) $\frac{1}{\sqrt{3}}\left(-\hat{i} - \hat{j} - \hat{k}\right)$

(c) $\frac{1}{\sqrt{5}}\left(\hat{i} - 2\hat{j}\right)$

(d) $\frac{1}{\sqrt{5}}\left(\hat{i} - \hat{j} - \hat{k}\right)$

46. Let $\vec{p},\vec{q},\vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{p}\times\left[\left(\vec{x}-\vec{q}\right)\times\vec{p}\right]+\vec{q}\times\left[\left(\vec{x}-\vec{r}\right)\times\vec{q}\right]+\vec{r}\times\left[\left(\vec{x}\times\vec{p}\right)\times\vec{r}\right]=\vec{0}$, then $\vec{x}$ is given by

(1997)

(a) $\frac{1}{2}\left(\vec{p}+\vec{q}-2\vec{r}\right)$

(b) $\frac{1}{2}\left(\vec{p}+\vec{q}+\vec{r}\right)$

(c) $\frac{1}{3}\left(\vec{p}+\vec{q}+\vec{r}\right)$

(d) $\frac{1}{3}\left(2\vec{p}+\vec{q}-\vec{r}\right)$

47. Two vectors $\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=5\hat{i}+2\hat{j}-15\hat{k}$ have the same initial point then their angular bisector having magnitude $\frac{7}{3}$ can be:

(a) $\frac{7}{3\sqrt{6}}\left(2\hat{i}+\hat{j}-\hat{k}\right)$

(b) $\frac{7}{3\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(c) $\frac{7}{3\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

(d) $\frac{7}{3\sqrt{3}}\left(\hat{i}-\hat{j}-\hat{k}\right)$

48. One of the diagonals of a parallelopiped is $4\hat{j}-8\hat{k}$. If two diagonals of one of its base are $6\hat{i}+6\hat{k}$ and $4\hat{j}-2\hat{k}$, then its volume is

(a) 60

(b) 80

(c) 100

(d) 120

49. If $\vec{a}$ and $\vec{b}$ unequal unit vectors such that $(\vec{a}-\vec{b})\times\left[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})\right]=\vec{a}+\vec{b}$, then smaller angle θ between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$

(b) 0

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{4}$

50. The distance between line $\vec{r}=\left(2\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+4\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}+5\hat{j}+\hat{k}\right)=5$ is

(a) $\frac{10}{9}$

(b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{10}{3}$

(d) None of these

51. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero and non coplaner vectors and $\vec{p},\vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p}=\vec{a}+\vec{b}-2\vec{c},\vec{q}=3\vec{a}-2\vec{b}+\vec{c}$ and $\vec{r}=\vec{a}-4\vec{b}+2\vec{c}$. If the volume of parallelopiped determined by $\vec{a},\vec{b}$ and $\vec{c}$ is V_1 and volume of tetrahedron formed by $\vec{p},\vec{q}$ and $\vec{r}$ is V_2 then V_2 : V_1 =

(a) 5 : 2

(b) 2 : 5

(c) 5 : 3

(d) None of these

52. Vectors $4\left(\hat{i}+\hat{j}+\hat{k}\right), 7\hat{i}+6\hat{j}-\hat{k}$ and $3\hat{i}+2\hat{j}-5\hat{k}$ form

(a) Right angled triangle

(b) Equilateral triangle

(c) Isosceles triangle

(d) Scalene triangle

53. Number of integer values x for which vector $\vec{a}=x\hat{i}-3\hat{j}+\hat{k}$ and $\vec{b}=x\hat{i}-3x\hat{j}+2\hat{k}$ contain the obtuse angle between them.

(a) 1

(b) 0

(c) 3

(d) 4

54. $\vec{A}, \vec{B}, \vec{C}$ are unit vectors, suppose $\vec{A}.\vec{B}=\vec{A}.\vec{C}=0$ and angle between $\vec{B}$ and $\vec{C}$ is $\frac{\pi}{6}$ then find k is $\vec{A}=k\left(\vec{B}\times\vec{C}\right)$

(a) ± 2

(b) ± 3

(c) $\pm\frac{1}{3}$

(d) $\pm\frac{1}{2}$

55. Let $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ A vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}.\vec{A} = 0$ would be

(a) $-\hat{i} - 8\hat{j} + 2\hat{k}$

(b) $\hat{i} + 8\hat{j} - 2\hat{k}$

(c) $2\hat{i} + 16\hat{j} - 4\hat{k}$

(d) $-2\hat{i} + 16\hat{j} + 4\hat{k}$

56. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$ then

(a) $|\vec{a}| = |\vec{b}| = |\vec{c}|$

(b) $|\vec{a}| = 2|\vec{b}| = |\vec{c}|$

(c) $|\vec{a}| \neq |\vec{b}| \neq |\vec{c}|$

(d) $|\vec{a}| = |\vec{b}| \neq |\vec{c}|$

57. Vector $\vec{a} = -4\hat{i} + 3\hat{k}$, $\vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$. The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$ and is having magnitude is $\sqrt{6}$

(a) $\hat{i} + \hat{j} + 2\hat{k}$

(b) $\hat{i} - \hat{j} + 2\hat{k}$

(c) $\hat{i} + \hat{j} - 2\hat{k}$

(d) none

58. If P is any point within a Δ ABC, then $\overrightarrow{PA}+\overrightarrow{CP}$ =

(a) $\overrightarrow{AC}+\overrightarrow{CB}$

(b) $\overrightarrow{BC}+\overrightarrow{BA}$

(c) $\overrightarrow{CB}+\overrightarrow{AB}$

(d) $\overrightarrow{CB}+\overrightarrow{BA}$

59. If ABCDEF is a regular hexagon and $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\lambda\overrightarrow{AD}$ then $\lambda =$

(a) 2

(b) 3

(c) 4

(d) 6

60. If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\vec{b}=\hat{i}+\hat{j},\vec{c}=\hat{i}$ and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$ then $\lambda+\mu$

(a) 0

(b) 1

(c) 2

(d) 3

61. If three unit vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}}{2}$ then vectors $\vec{a}$ makes angles with $\vec{b}\,\&\,\vec{c}$ respectively

(a) 60°, 90°

(b) 45°, 45°

(c) 30°, 60°

(d) 90°, 60°

62. If $\vec{p}$ & $\vec{s}$ are not perpendicular to each other and $\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$ & $\vec{r}.\vec{s} = 0$ then $\vec{r}$ =

(a) $\vec{p}.\vec{s}$

(b) $\vec{q} - \left(\frac{\vec{q}.\vec{s}}{\vec{p}.\vec{s}}\right)\vec{p}$

(c) $\vec{q} + \left(\frac{\vec{q}.\vec{p}}{\vec{p}.\vec{s}}\right)\vec{p}$

(d) $\vec{q} + \mu\vec{p}$ for all scalars μ

63. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar & $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors, then:
$\left(\overrightarrow{\ell a} + \overrightarrow{mb} + \overrightarrow{nc}\right).\left(\overrightarrow{\ell p} + \overrightarrow{mq} + \overrightarrow{nr}\right)$ is equal to:

(a) $\ell^2 + m^2 + n^2$

(b) $\ell m + mn + n\ell$

(c) 0

(d) none of these

64. If $\vec{a}$ is vector whose initial point divides the join of $5\hat{i}$ and $5\hat{j}$ in the ratio : 1 and terminal point is origin and $|\vec{a}| \le \sqrt{17}$, then the set of exhaustive values of λ, is

(a) $\left[-6, -\frac{1}{6}\right]$

(b) $\left(-\infty, \frac{1}{4}\right) \cup [4,8]$

(c) $\left[\frac{1}{4}, 4\right]$

(d) $\left[-\frac{1}{6}, \infty\right]$

65. The vector equation of the plane containing the lines $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $\vec{r}=\left(\hat{i}+\hat{j}\right)+\mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$

(a) $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=0$

(b) $\vec{r}.\left(\hat{i}-\hat{j}-\hat{k}\right)=0$

(c) $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=3$

(d) none

66. Angle between line $\vec{r}=\left(\hat{i}+2\hat{j}-\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the normal of plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=4$

(a) $\sin^{-1}\frac{2\sqrt{2}}{3}$

(b) $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(c) $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(d) $\cot^{-1}\frac{2\sqrt{2}}{3}$

67. The line of intersection of the planes $\vec{r}.\left(\hat{i}-3\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+5\hat{j}-3\hat{k}\right)=2$ is parallel to vector

(a) $-4\hat{i}+5\hat{j}+11\hat{k}$

(b) $4\hat{i}+5\hat{j}+11\hat{k}$

(c) $4\hat{i}-5\hat{j}+11\hat{k}$

(d) $4\hat{i}-5\hat{j}-11\hat{k}$

68. The value of $\dfrac{\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2}{2\vec{a}^2\vec{b}^2}=$

(a) $\frac{1}{2}$

(b) $\frac{3}{2}$

(c) $\frac{5}{2}$

(d) $\frac{4}{3}$

69. If $\alpha\left(\vec{a}\times\vec{b}\right)+\beta\left(\vec{b}\times\vec{c}\right)+\gamma\left(\vec{c}\times\vec{a}\right)=\vec{0}$ and at least one of the number α,β and γ is non-zero, then vectors $\vec{a},\vec{b},\vec{c}$ are

(a) perpendicular

(b) parallel

(c) coplanar

(d) none

70. If a, b, c are three non-coplanar vector then $\dfrac{\vec{a}.\vec{b}\times\vec{c}}{\vec{c}\times\vec{a}.\vec{b}}+\dfrac{\vec{b}.\vec{a}\times\vec{c}}{\vec{c}.\vec{a}\times\vec{b}}=$

(a) 0

(b) 2

(c) – 2

(d) 4

71. If the non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then solution of the equation $\vec{r}\times\vec{a}=\vec{b}$ is

(a) $\vec{r}=x\vec{a}+\frac{1}{\vec{a}.\vec{a}}\left(\vec{a}\times\vec{b}\right)$

(b) $\vec{r}=x\vec{b}-\frac{1}{\vec{b}.\vec{b}}\left(\vec{a}\times\vec{b}\right)$

(c) $\vec{r}=x\vec{a}\times\vec{b}$

(d) $\vec{r}=x\vec{b}\times\vec{a}$

72. Image of the point 'P' with position vector $7\hat{i}+\hat{j}+2\hat{k}$ in the line whose vector equation is $\vec{r}=-3\hat{i}-10\hat{k}+\lambda\left(4\hat{i}+3j+5\hat{k}\right)$ has the position vector

(a) $-9\hat{i}+5j+2\hat{k}$

(b) $9\hat{i}+5j-2\hat{k}$

(c) $9\hat{i}-5j-2\hat{k}$

(d) $9\hat{i}+5j+2\hat{k}$

73. Let $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vector such that

$\vec{r_1}=\vec{a}-\vec{b}+\vec{c},\ \vec{r_2}=\vec{b}+\vec{c}-\vec{a},\ \vec{r_3}=\vec{c}+\vec{a}+\vec{b}$

$\vec{r}=2\vec{a}-3\vec{b}+4\vec{c},$ if $\vec{r}=\lambda_1\vec{r_1}+\lambda_2\vec{r_2}+\lambda_3\vec{r_3}$ then $\lambda_1+\lambda_2+\lambda_3=$

(a) 4

(b) 5

(c)3

(d) 2

74. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{c}$ be a unit vector $\perp$ to and coplanar with $\vec{a}$ and $\vec{b}$ then $\vec{c}$ is

(a) $-\frac{1}{\sqrt{6}}\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

(b) $\frac{1}{\sqrt{6}}\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

(c) $\frac{1}{\sqrt{6}}\left(2\hat{i} - \hat{j} + \hat{k}\right)$

(d) $-\frac{\left(2\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{6}}$

75. A new tetrahedron is formed by joining the centroids of the faces of a given tetrahedron. The ratio of the volume of given tetrahedron to that of new tetrahedron is

(a) 3

(b) 9

(c)27

(d) 81

76. A, B and C are three non collinear points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively and plane ABC is not passing through origin, then vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are

(a) parallel vectors

(b) non coplanar vector

(c) coplanar vector

(d) linearly dependent vectors

77. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 0$, then $\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)\right)$ is equal to

(a) $48\vec{b}$

(b) – 48 $\vec{b}$

(c) 48 $\vec{a}$

(d) – 48 $\vec{a}$

78. A rigid body is spinning about a fixed point (3, -2, -1) with angular velocity of 4 rad/s, the axis of rotation being in the direction of (1, 2, -2), then the velocity of the particle at the point (4, 1, 1) is

(a) $\frac{4}{3}(1,-4,10)$

(b) $\frac{4}{3}(4,-10,1)$

(c) $\frac{4}{3}(10,-4,1)$

(d) $\frac{4}{3}(10,4,1)$

79. Let $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $3\vec{a}+4\vec{b}+5\vec{c}=0$. Then which of the following statements is true?

(a) $\vec{a}$ is parallel to $\vec{b}$

(b) $\vec{a}$ is perpendicular to $\vec{b}$

(c) $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$

(d) none of the above

80. The values of x for which the angle between the vectors $2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $7\hat{i}-2\hat{j}+x\hat{k}$ are obtuse and the angle between the z-axis and $7\hat{i}-2\hat{j}+x\hat{k}$ is acute and less than $\frac{\pi}{6}$ is given by

(a) $0<x<\frac{1}{2}$

(b) $x>\frac{1}{2} or\ x<0$

(c) $\frac{1}{2}<x<15$

(d) there is no such value for x

81. Let $\vec{a},\vec{b}$ and $\vec{c}$ be mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation $\vec{a}\times\left[\left(\vec{x}-\vec{b}\right)\times\vec{a}\right]+\vec{b}\times\left[\left(\vec{x}-\vec{c}\right)\times\vec{b}\right]+\vec{c}\times\left[\left(\vec{x}-\vec{a}\right)\times\vec{c}\right]=0$, then $\vec{x}$ is given by

(a) $\frac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$

(b) $\frac{1}{3}\left(2\vec{a}+\vec{b}+\vec{c}\right)$

(c) $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{c}\right)$

(d) $\frac{1}{2}\left(\vec{a}+\vec{b}-2\vec{c}\right)$

82. A unit tangent vector at $t=2$ on the curve $x=t^2+2, y=4t-5, z=2t^2-6t$ is

(a) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(b) $\frac{1}{3}\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

(c) $\frac{1}{\sqrt{6}}\left(2\hat{i}+\hat{j}+\hat{k}\right)$

(d) none of these

83. When a right handed rectangular Cartesian system *oxyz* is rotated about the *z*-axis through an angle $\frac{\pi}{4}$ in the counter-clockwise direction it is found that a vector $\vec{a}$ has the components $2\sqrt{3}, 3\sqrt{2}$ and 4. The components of $\vec{\alpha}$ in the *oxyz* coordinate system are

(a) 5, - 1, 4

(b) 5, - 1, 4 $\sqrt{2}$

(c) – 1, - 5, 4 $\sqrt{2}$

(d) none of these

84. Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}, \vec{b} = 2\hat{i} - 6\hat{j}$ and $\vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$. Then the resolution of the vector $\vec{\alpha}$ into components with respect to $\vec{a}$ and $\vec{b}$ is given by

(a) $3\vec{a} - 2\vec{b}$

(b) $2\vec{a} - 3\vec{b}$

(c) $3\vec{b} - 2\vec{a}$

(d) none of these

85. For any four points P, Q, R, S, $\left|\overrightarrow{PQ} \times \overrightarrow{RS} - \overrightarrow{QR} \times \overrightarrow{PS} \times \overrightarrow{RP} \times \overrightarrow{QS}\right|$ is equal to 4 times the area of the triangle

(a) PQR

(b) QRS

(c) PRS

(d) PQS

86. If $\sum_{i=1}^{n} \vec{a_i} = 0$ where $\left|\vec{a_i}\right| = 1 \forall i$, then the value of $\sum_{1 \le i < j \le n} \Sigma \vec{a_i} . \vec{a_j}$ is

(a) – n/2

(b) – n

(c) n/2

(d) n

87. If $\vec{a},\vec{b},\vec{c}$ are unit vectors then $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ does not exceed

(a) 4

(b) 9

(c) 8

(d) 6

88. Let $\vec{v}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{w}=\hat{i}+3\hat{k}$. If $\vec{u}$ is a unit vector, then maximum value of the scalar triple product $\left[\vec{u}\,\vec{v}\,\vec{w}\right]$ is

(a) – 1

(b) $\sqrt{10}+\sqrt{6}$

(c) $\sqrt{59}$

(d) $\sqrt{60}$

89. If $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$, then

(a) $\left|\vec{a}\right|=1,\left|\vec{b}\right|=\left|\vec{c}\right|$

(b) $\left|\vec{c}\right|=1,\left|\vec{a}\right|=1$

(c) $\left|\vec{b}\right|=2,\left|\vec{b}\right|=2\left|\vec{a}\right|$

(d) $\left|\vec{b}\right|=1,\left|\vec{c}\right|=\left|\vec{a}\right|$

90. Let $\vec{u}$ and $\vec{v}$ are unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$, then the value of $\left[\vec{u}\,\vec{v}\,\vec{w}\right]$ is

(a) 1

(b) – 1

(c) 0

(d) none of these

Answers

1. (b)	2. (a)	3. (c)	4. (c)	5. (b)	6. (c)	7. (b)	8. (a)	9. (a)	10. (a)	11. (b)	12. (b)
13. (b)	14. (b)	15. (b)	16. (b)	17. (a)	18. (b)	19. (c)	20. (b)	21. (b)	22. (c)	23. (a)	24. (c)
25. (c)	26. (c)	27. (c)	28. (a)	29. (c)	30. (d)	31. (d)	32. (a)	33. (b)	34. (c)	35. (b)	36. (d)
37. (a)	38. (d)	39. (a)	40. (c)	41. (c)	42. (a)	43. (b)	44. (a)	45. (a)	46. (b)	47. (b)	48. (d)
49. (a)	50. (b)	51. (a)	52. (a)	53. (b)	54. (a)	55. (a)	56. (a)	57. (a)	58. (d)	59. (b)	60. (a)
61. (d)	62. (b)	63. (a)	64. (c)	65. (b)	66. (b)	67. (b)	68. (a)	69. (c)	70. (a)	71. (a)	72. (b)
73. (a)	74. (c)	75. (c)	76. (b)	77. (a)	78. (c)	79. (d)	80. (d)	81. (a)	82. (b)	83. (d)	84. (b)
85. (b)	86. (a)	87. (b)	88. (c)	89. (d)	90. (a)						

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