Optical Instruments

OPTICAL INSTRUMENTS

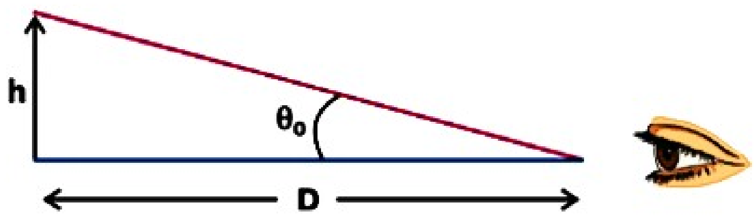
How big an object will appear to us depends on the angle subtended by the object at our eye.

When an object is too small or far away, then the angle subtended by an object is very small, and it appears very small, the purpose of optical instruments is to magnify this angle.

The magnifying power (MP) or angular magnification of an optical instrument is defined as the ratio of visual angle with the instrument to the maximum visual angle for clear vision when the eye is unaided (i.e., when the object is at least distance of distinct vision).

i.e., magnifying power M.P., $M = \frac{\text{Visual angle with instrument}}{\text{Max. visual angle for unadded eye}} = \frac{\theta}{\theta_0}$

Maximum visual angle for unaided eye will be then object is kept at a distance D (least distance for clear vision) from the eye.



unaided-eye

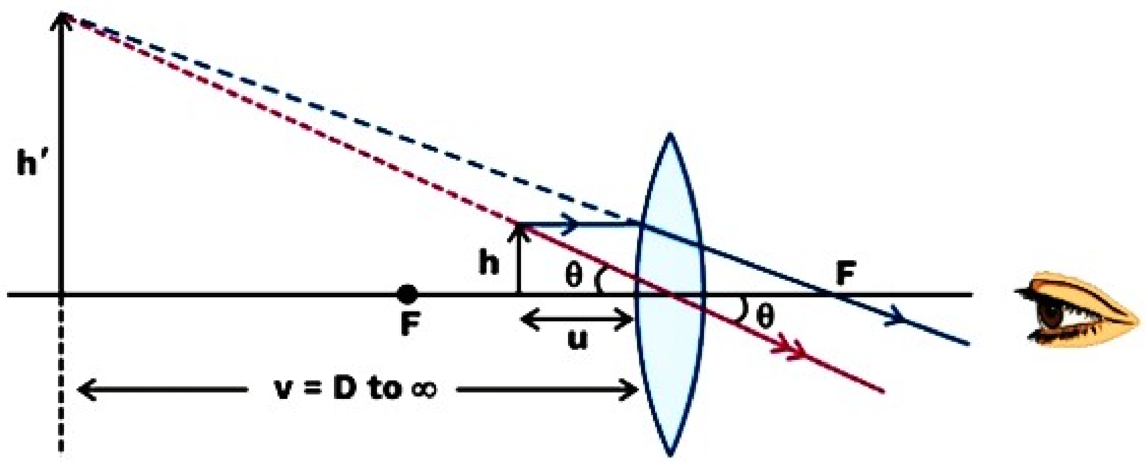
$$\theta_0 = \frac{h}{D}$$

Microscope

It is an optical instrument used to increase the visual angle of near objects which are too small to be seen by naked eye.

Simple Microscope

It is also known as a magnifying glass or simply magnifier and consists of a convergent lens with the object between its focus and optical center and eye close to it. The image formed by the lens is erect, virtual, enlarged, and is located between object and infinity.



eye with instrument

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max. visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

If an object of size h is placed at a distance u (< D) from the lens and its image size h' is formed at a distance v (≥ D) from the eye

$$\theta = \frac{h'}{v} = \frac{h}{u} \text{ with } \theta_0 = \frac{h}{D}$$

So, magnifying power $MP = \frac{\theta}{\theta_0} = \frac{h}{u} \times \frac{D}{h} = \frac{D}{u}$ ...(1)

Applying Lens Formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \text{Putting with sign } u = -u,\ v = -v$$

$$\Rightarrow \frac{1}{u} = \frac{1}{v} + \frac{1}{f}$$

$MP = \frac{D}{u} = D\left(\frac{1}{v} + \frac{1}{f}\right)$ (put values of D, v, and f without sign)

It can be used in two ways

(i) The normal adjustment When image formed by the lens is at infinity.

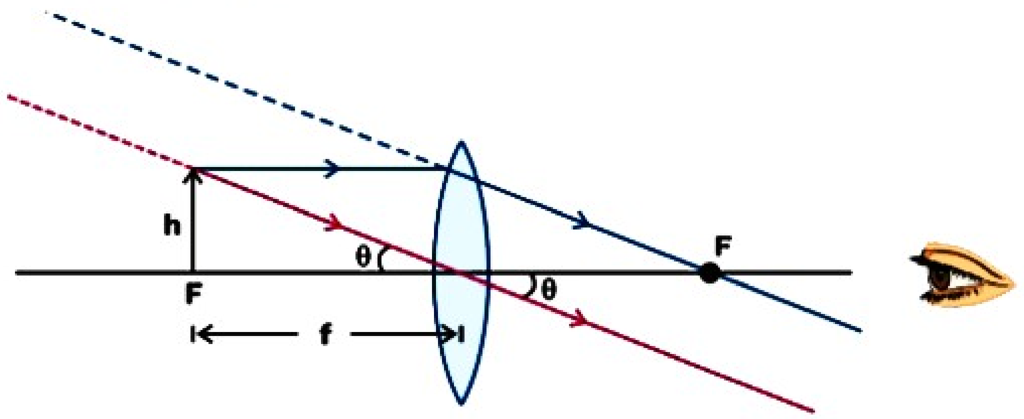
(ii) Near point adjustment When image formed by the lens is at D (Least distance of distinct vision).

Normal adjustment

$$v \to \infty \Rightarrow MP = D\left(\frac{1}{\infty} + \frac{1}{f}\right) = \frac{D}{f} \quad \boxed{MP = \frac{D}{f}}$$

$$\frac{1}{u} = \frac{1}{\infty} + \frac{1}{f} \Rightarrow u = f$$

As here, u is maximum (as the object is to be within focus), MP is minimum, and as in this situation, a parallel beam of light enters the eye, the eye is least strained and is said to be normal, relaxed, or unstrained.



Near Point adjustment

$$v = D,\ MP = D\left(\frac{1}{D} + \frac{1}{f}\right) = 1 + \frac{D}{f}$$

$$\Rightarrow \boxed{MP = 1 + \frac{D}{f}}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{D} + \frac{1}{f}$$

$$\Rightarrow u = \frac{Df}{D+f}$$

As the minimum value of v for clear vision is D, in this situation, u is minimum, and hence this is the maximum possible MP of a simple microscope, and as in this situation final image is closest to the eye, the eye is under maximum strain.

In general, $\frac{D}{f} \le MP \le 1 + \frac{D}{f}$

Special Points

(a) Simple magnifier is an essential part of most optical instruments (such as microscope or telescope) in the form of eye piece or ocular.

(b) The magnifying power (MP) has no unit. It is different from the power of a lens which is expressed in dioptre (D).

(c) With the increase in wavelength of light used, the focal length of the magnifier will increase, and hence its MP will decrease.

Q **A man with normal near point 25 cm reads a book with a small print using a magnifying glass, a thin convex lens of focal length 5 cm.**
(a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass?
(b) What is the maximum and minimum MP possible using the above simple microscope?

Sol: **(a)** $u_{min} = \frac{Df}{D+f} = \frac{25 \times 5}{25+5} = \frac{25}{6} cm$

$u_{max} = f = 5 cm$

(b) $(MP)_{min} = \frac{D}{f} = \frac{25}{5} = 5$

$(MP)_{max} = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$

COMPOUND-MICROSCOPE

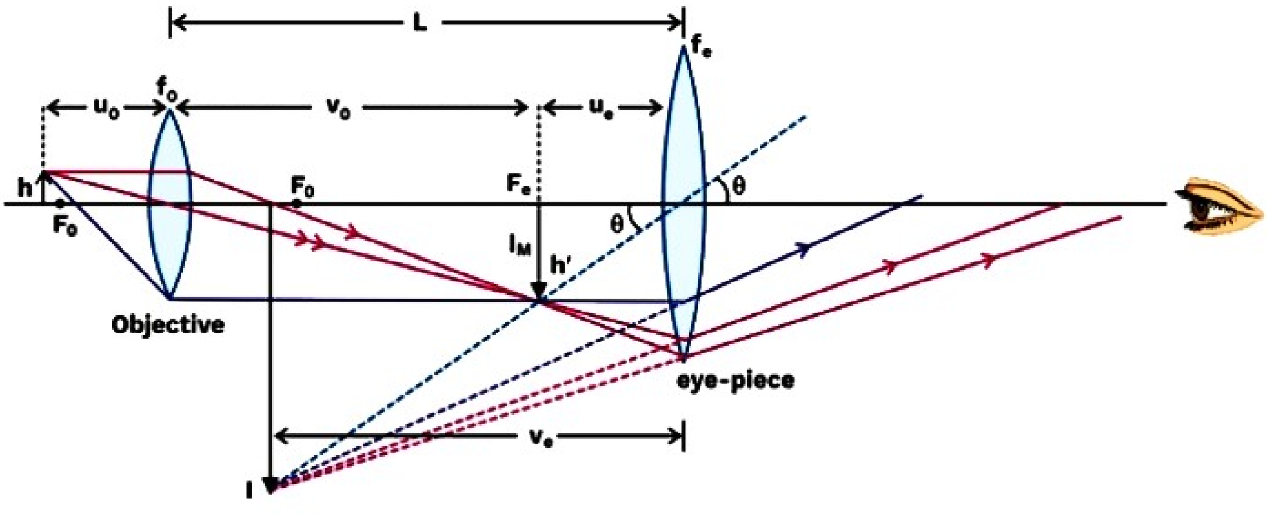
A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope.

Construction

It consists of two convergent lenses of short focal lengths and apertures arranged co-axially lens (of focal length f_0) facing the object is called objective or field lens while the lens (of focal length f_e) facing the eye is called eye-piece or ocular. The objective has a smaller aperture and smaller focal length than the eye piece. The separation between objective and eye-piece can be varied.

Image Formation

The object is placed between F and 2F of the objective, so the image I_M formed by objective (called intermediate image) is inverted, real, enlarged, and at a distance greater than f_0 on the other side of the objective lens. This image I_M acts as an object for eye-piece and is within its focus. So eye-piece forms final image I which is erect, virtual and enlarged with respect to intermediate image I_M. So the final image I with respect to the object is inverted, virtual, enlarged, and at a distance D to infinity from an eye on the same side of the eye-piece as I_M. This is all shown in the figure.



Magnifying power (MP) of the compound microscope,

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max. visual angle for unadded eye}} = \frac{\theta}{\theta_0} = \frac{h'/u_e}{h/D}$$

$$MP = \left[\frac{h'}{u_e}\right] \times \left[\frac{D}{h}\right] = \left[\frac{h'}{h}\right]\left[\frac{D}{u_e}\right]$$

But for objective, $u = -u_0$ and $v = +v_0$

$$m = \frac{I}{O} = \frac{v}{u} \text{ i.e., } \frac{h'}{h} = \frac{v_o}{-u_o} = -\frac{v_o}{u_o}$$

So, $MP = -\frac{v_o}{u_o}\left[\frac{D}{u_e}\right]$

with a length of the tube

$$L = v_o + u_e \qquad ...(1)$$

Put values of v_o, u_o, D and u_e without sign

Now there are two possibilities.

(a) If the final image is at infinity (far point)

This situation is called normal adjustment as in this situation eye is least strained or relaxed. In this situation, as for eye-piece $v_e = \infty$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \text{ i.e., } u_e = f_e = \text{maximum}$$

Substitution this value of in equation (1), we have

$$MP = -\frac{v_o}{u_o}\left[\frac{D}{f_e}\right] \text{ with } L = v_o + f_e \qquad ...(2)$$

A microscope is usually considered to operate in this mode unless otherwise stated. In this mode, as u_e is maximum MP is minimum for a given microscope.

(b) If the final image is at D (near point)

In this situation as for eye-piece $v_e = D$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \text{ i.e., } \frac{1}{u_e} = \frac{1}{D}\left[1 + \frac{D}{f_e}\right]$$

Substituting this value of u_e in equation (1), we have

$$MP = -\frac{v_o}{u_o}\left[1 + \frac{D}{f_e}\right] \text{ with } L = v_o + \frac{f_e D}{f_e + D}$$

In this situation, as u_e is minimum, MP is maximum, and the eye is most strained.

Q **The focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm, respectively, and the distance between them is 20 cm. Find the distance of an object from the objective when the final image seen by the eye is 25 cm from the eyepiece. Also, find the magnifying power.**

Sol: Given $f_o = 2$ cm, $f_e = 5$ cm

$|v_o| + |u_e| = L = 20\,cm$

$\therefore \quad v_e = -25$ cm

From lens formula $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$

$$\therefore \quad u_e = -\frac{25}{6}\,cm$$

Distance of real image from objective, $|v_o| = L - |u_e|$

$$v_o = 20 - \frac{25}{6} = \frac{120 - 25}{6} = \frac{95}{6}\,cm$$

Now $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{(95/6)} - \frac{1}{2}$

i.e., $\frac{1}{u_o} = \frac{6}{95} - \frac{1}{2} = \frac{12 - 95}{190} = -\frac{83}{190}$

$$\therefore \quad u_o = -\frac{190}{83} = -2.3\,cm$$

Magnifying power $M = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) = -\frac{95/6}{(190/83)}\left(1 + \frac{25}{5}\right) = -41.5$

TELESCOPE

Astronomical Telescope

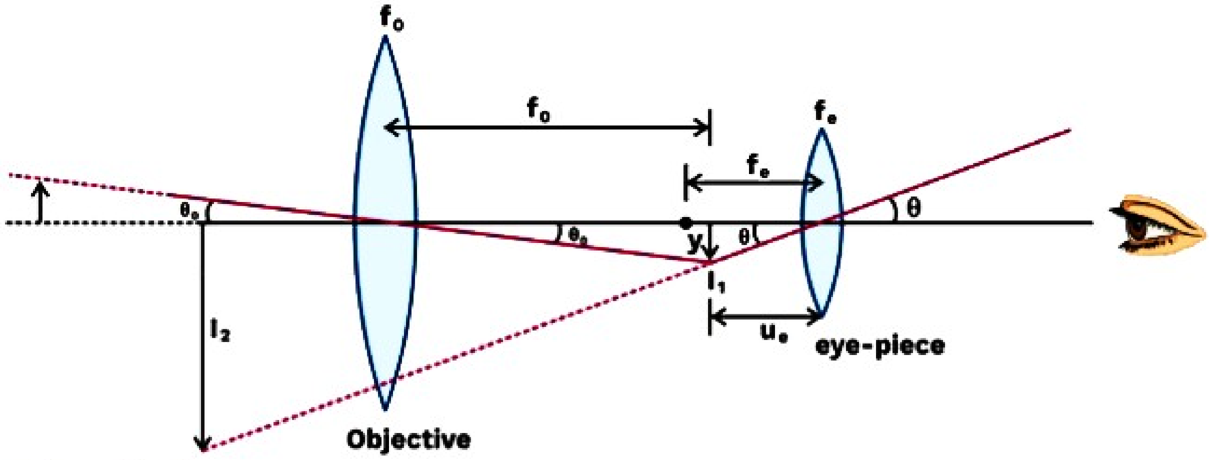
It is an optical instrument used to increase the visual angle of distant large objects such as a star, a planet or a cliff, etc. An astronomical telescope consists of two converging lenses. The one facing the object is called an objective or field lens and has a large focal length and aperture. The distance between the two lenses is adjustable.

In the telescope, the focal length of the objective is very large in comparison to the focal length of the eye piece.

Aperture of objective is also very large in comparison to an aperture of the eyepiece. A telescope is used to see distant objects; in it object is between ∞ and 2F of objective and hence image I_1 formed by objective is real, inverted, and diminished and is between F and 2F on the other side of it. This image (is called intermediate image) acts as an object for eye-piece, and shifting the position of the eyepiece can bring this object within its focus. So final image I_2, with respect to the intermediate image (I_1) is erect, virtual, enlarged, and at a distance D to ∞ from the eye. This, in turn, implies that the final image with respect to the object is inverted, enlarged, and at a distance D to from the eye.

Magnifying Power (MP) astronomical telescope

$$MP = \frac{\text{Visual angle with instrument}}{\text{Visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$



But from the figure,

$$\theta_0 = \left(\frac{y}{f_0}\right) \text{ and } \theta = \left(\frac{y}{-u_e}\right)$$

So, $MP = \frac{\theta}{\theta_0} = -\left[\frac{f_0}{u_e}\right]$ with length of tube

$$L = (f_0 + u_e) \qquad ...(1)$$

Now there are two possibilities

(a) If the final image is at infinity (far point)

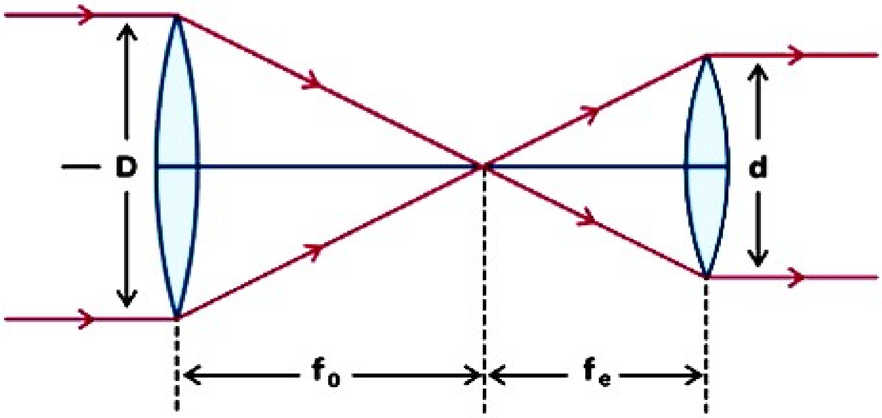
This situation is called a normal adjustment, and as in this situation, the eye is least strained or relaxed. In this situation, as for eye-piece $v_e = \infty$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e.,} \quad u_e = f_e$$

So, substituting this value of u_e in equation (1), we have

$$MP = -\left(\frac{f_0}{f_e}\right) \text{ and } L = (f_0 + f_e)$$

Usually, the telescope operates in this mode unless stated otherwise. In this mode, as u_e is maximum for a given telescope MP is minimum while the length of tube maximum.



$$\frac{f_0}{f_e} = \frac{\text{Aperture of objective}}{\text{Aperture of eye piece}} \text{ i.e., } MP = \frac{-f_0}{f_e} = \frac{-D}{d}$$

(b) If the final image is at D (near point)

In this situation, as for eye-piece $v_e = D$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \text{ i.e., } \frac{1}{u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

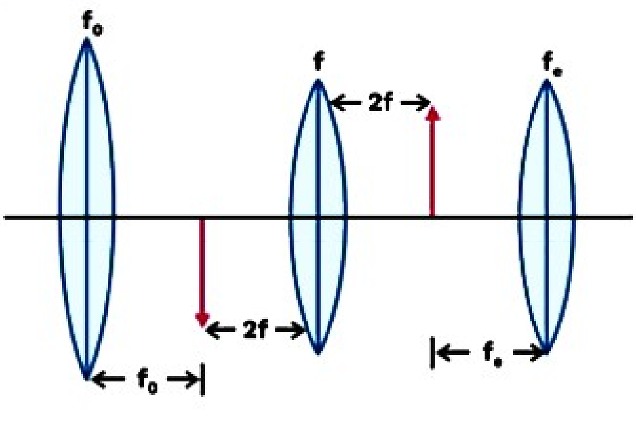
So, substituting this value of u_e in Equation (1), we have

$$MP = \frac{-f_0}{f_e}\left[1 + \frac{f_e}{D}\right] \text{ with } L = f_0 + \frac{f_e D}{f_e + D}$$

In this situation, u_e is minimum, so for a given telescope MP is maximum while the length of tube minimum and the eye is most strained.

Terrestrial Telescope

Uses a third lens in between objective and eyepieces so as to form final image erect. This lens simply inverts the image formed by the objective without affecting the magnification. Length of tube $L = f_0 + f_e + 4f$

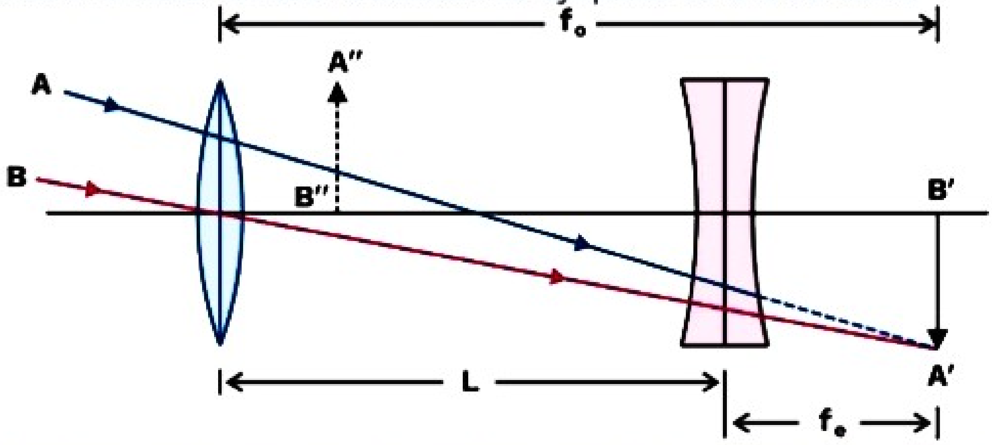


Galileo's Telescope

The convex lens as the objective

The concave lens as the eyepiece

The field of view is much smaller because the eyepiece lens is concave.



Results for Galileo's telescope can be written simply by replacing f_e with $-f_e$ in the above-derived results.

$$M = \frac{f_o}{f_e}\left[1 - \frac{f_e}{v_e}\right]$$

(a) If the final image is at infinity (far point)

$$M = \frac{f_o}{f_e} \text{ and } L = f_o - f_e$$

(b) If the final image is at D (near point)

$$M = \frac{f_o}{f_e}\left[1 - \frac{f_e}{D}\right] \text{ and } L = f_0 - u_e$$

Q **A telescope consists of two convex lenses of focal length 16 cm and 2 cm. What is the angular magnification of the telescope for a relaxed eye? What is the separation between the lenses? If an object subtends an angle of 0.5° on the eye, what will be the angle subtended by its image?**

Sol: If nothing is mentioned, considered normal adjustment.

Angular magnification $M = -\frac{f_o}{f_e} = -\frac{16}{2} = -8$

$\Rightarrow |M| = 8$

Separation between lenses = $f_o + f_e = 16 + 2 = 18$ cm

Here $\theta_0 = 0.5°$

Angular subtended by image $\theta = M\theta_0 = 8 \times 0.5° = 4°$

Q **The magnifying power of the telescope is found to be 9, and the separation between the lenses is 20 cm for a relaxed eye. What are the focal lengths of components lenses?**

Sol: Magnification $M = -\frac{f_o}{f_e} = -9 \Rightarrow f_o = 9f_e$ ...(1)

Separation between lenses, $L = f_o + f_e \Rightarrow f_o + f_e = 20$ ...(2)

Putting value of f_o from (1) in (2), we get

$9f_e + f_e = 20$

$\Rightarrow 10f_e = 20 \Rightarrow f_e = 2$ cm

$\therefore f_o = 9f_e = 9 \times 2 = 18$ cm

$\therefore f_o = 18$ cm, $f_e = 2$ cm

Q **The focal lengths of the objective and the eyepiece of an astronomical telescope are 60 cm and 5 cm, respectively. Calculate the magnifying power and the length of the telescope when the final image is formed at (i) infinity, (ii) the least distance of distinct vision (25 cm).**

Sol: (i) When the final image is at infinity, then

$MP = -\frac{f_o}{f_e} = -\frac{60}{5} = -12$ and length of the telescope is $L = f_o + f_e = 60 + 5 =$ 65 cm

(ii) For least distance of distinct vision, the magnifying power is

$$MP = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) = -\frac{60}{5}\left(1+\frac{5}{25}\right) = -\frac{12\times 6}{5} = -14.4$$

Now $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

$\Rightarrow \frac{1}{5} = -\frac{1}{25} - \frac{1}{u_e}$

$\Rightarrow \frac{-1}{u_e} = \frac{1}{25} + \frac{1}{5}$

$\Rightarrow u_e = -4.17$ cm

$\Rightarrow |u_e| = 4.17$ cm

The length of the telescope in this position is $L = f_0 + |u_e| = 60 + 4.17 = 64.17$ cm

S. No.	Compound-Microscope	Astronomical-Telescope
1.	It is used to increase the visual angle of the near tiny object	It is used to increase the visual angle of distant large objects.
2.	In it field and eye lens both are convergent, of short focal length and aperture.	In it, the objective lens is of large focal length and aperture, while the eye lens is of short focal length and aperture, and both are convergent.
3.	The final image is inverted, virtual, and enlarged, and at a distance D to ∝ from the eye.	Final image is inverted, virtual, and enlarged at a distance D to ∝ from the eye.
4.	MP does not change appreciably if objective and eye lens are interchanged as [MP ~ (LD / f_o f_e)].	MP becomes ($1/m^2$) times of its initial value if objective and eye-lenses are interchanged as MP ~ [f_o / f_e].
5.	MP is increased by decreasing the focal length of both the lenses.	MP is increased by increasing the focal length of an objective lens and by decreasing the focal length of the eyepiece.
6.	RP is increased by decreasing the wavelength of light used. $\left(\because RP = \frac{2\mu \sin\theta}{1.22\lambda}\right)$	RP is increased by increasing the aperture of the objective. $\left(\because RP = \frac{D}{1.22\lambda}\right)$