Chapter – 8
Introduction to Trigonometry

Exercise 8.4

Q. 1 Express the trigonometric ratios sin A, sec A and tan A in terms of cot A

Answer: sin A can be expressed in terms of cot A as:

And we know that: $cosec^2 A - cot^2 A = 1$, $cosec^2A=1+cot^2A$,

so cosec $A=\sqrt{(1+cot^2A)}$

$\sin A = \frac{1}{cosec\ A}$

Therefore, $\sin A = \frac{1}{\sqrt{cos^2 A+1}}$

Now, And we know that: $cosec^2 A - cot^2 A = 1$, $cosec^2A=1+cot^2A$,

so cosec $A=\sqrt{(1+cot^2A)}$

$\sin A = \frac{1}{cosec\ A}$

Therefore,

$\sin A = \frac{1}{\sqrt{cos^2 A+1}}$

Now, Sec A can be expressed in terms of cot A as:
We know that: $sec^2 A - tan^2 A = 1$, sec A = $\sqrt{(1 + tan^2 A)}$

$\sec A = \sqrt{1 + tan^2 A}$

$\sec A = \sqrt{1 + \frac{1}{cot^2 A}}$

And also, tan A = 1/ cot A

Therefore,

$\sec A = \sqrt{\frac{1+cot^2 A}{cot^2 A}}$

$\sec A = \frac{1}{\cot A}\sqrt{1 + cot^2 A}$

tanA can be expressed in terms of cotA as:

$\tan A = \frac{1}{\cot A}$

Q. 2 Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

Sin A can be expressed in terms of sec A as:

Sin A = $\sqrt{\sin^2 A}$

sin A = $\sqrt{(1 - \cos^2 A)}$

$Sin\ A = \sqrt{1 - \frac{1}{sec^2 A}}$

$\sin A = \sqrt{\frac{sec^2 A - 1}{sec^2 A}}$

$\sin A = \frac{1}{\sec A}\sqrt{sec^2 A - 1}$

Now,

cos A can be expressed in terms of secA as:

$\cos A = \frac{1}{secA}\ \frac{1}{\sec A}$

tanA can be expressed in the form of sec A as:
As, $1 + \tan^2 A = \sec^2 A$

$\Rightarrow \tan A = \pm\sqrt{(\sec^2 A - 1)}$
As A is acute angle, And tan A is positive when A is acute,So,tan A = $\sqrt{(\sec^2 A - 1)}$
cosec A can be expressed in the form of sec A as:

$cosec\ A = \frac{1}{\sin A}$

$\frac{1}{\frac{\sec A}{\sqrt{1-sec^2 A}}}$

$= \frac{\sqrt{1-sec^2 A}}{\sec A}$

cot A can be expressed in terms of sec A as:

cot A= $\frac{1}{\tan A}$

as, 1 + $\tan^2$A = $\sec^2$A

$\frac{1}{\sqrt{sec^2 A-1}}$

Q. 3 Evaluate:

(i) $\frac{sin^2 63°+sin^2 27°}{cos^2 17°+cos^2 73°}$

(ii) Sin 25° Cos 65° + Cos 25° Sin 65°

Answer:

(i) To Prove: $\frac{sin^2 63°+sin^2 27°}{cos^2 17°+cos^2 73°} = 1$

Proof:

L.H.S. $= \frac{sin^2 63°+sin^2 27°}{cos^2 17°+cos^2 73°}$

$= \frac{sin^2 63°+sin^2 (90°-63°)}{cos^2 17°+cos^2 (90°-17°)}$

$(By\ complementary\ angle\ formula\ that, \sin(90-A = \cos A\ and \cos(90-A)))$

$= \frac{sin^2 63°+cos^2 63°}{cos^2 17°+sin^2 17°}$

$(By\ sin^2 A + cos^2 A = 1)$

$= \frac{1}{1}$

$= 1 = R.H.S$

L.H.S = R.H.S.

Hence, Proved

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$= \sin 25° \cos (90°-25°) + \cos 25° \sin 65°$
$= \sin 25° \sin 25° + \cos 25° \sin 65° = \sin^2 25° + \cos 25° \sin (90°-25°)$

$= \sin^2 25° + \cos^2 25°$

$=1$

Q. 4 A Choose the correct option. Justify your choice.

$9sec^2 A - 9tan^2 A$

A. 1

B. 9

C. 8

D. 0

Answer: Following is the explanation:

$9 \sec^2 A – 9 \tan^2 A$

$= 9 (\sec^2 A – \tan^2 A)$

$= 9 \times 1 = 9$

Q. 4 (B) Choose the correct option. Justify your choice.

$(1 + tan\ \theta + sec\ \theta)(1 + cot\ \theta - cosec\ \theta) =$

A. 0

B. 1

C. 2

D. –1

Answer: Consider $(1+\tan\theta+\sec\theta)(1+\cot\theta-cosec\ \theta)$

By applying formulae

$$\tan\theta=\frac{\sin\theta}{\cos\theta},\cot\theta=\frac{\cos\theta}{\sin\theta},\sec\theta=\frac{1}{\cos\theta},cosec\ \theta=\frac{1}{\sin\theta}$$

$$\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{1+\cos\theta+\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

Multiplying both terms, we get

$$=\frac{\sin\theta+\sin\theta\cos\theta+sin^2\theta+\cos\theta+cos^2\theta+\sin\theta\cos\theta-1-\cos\theta-\sin\theta}{\cos\theta\sin\theta}$$

$=\frac{sin^2\theta+cos^2\theta+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$

$=\frac{1+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$

$=\frac{2\sin\theta\cos\theta}{\cos\theta\sin\theta}$

= 2

Therefore, $(1+tan\theta+sec\theta)(1+cot\theta-cosec\theta)=2$

Q. 4 C Choose the correct option. Justify your choice.

(Sec A + tan A) (1 – sin A) =

A. sec A

B. sin A

C. cosec A

D. Cos A

Answer: (Sec A + tan A) (1 – sin A)

$=\left(\frac{1}{cos\ A}+\frac{sin\ A}{cos\ A}\right)(1-sin\ A)$

$= \frac{(1+\sin A)(1-\sin A)}{\cos\theta}$

$= (1 - \sin^2 A)/\cos A$

$= \cos^2 A/\cos A$

$= \cos A$

Q. 4 D Choose the correct option. Justify your choice.

$\frac{1+tan^2 A}{1+cot^2 A} =$

A. $\sec^2 A$
B. -1
C. $\cot^2 A$
D. $\tan^2 A$

Answer: We know,

$1 + \tan^2\theta = \sec^2\theta$

and
$1 + \cot^2\theta = \mathrm{cosec}^2\theta$

Therefore,

$$\frac{1+tan^2 A}{1+cot^2 A} = \frac{sec^2 A}{cosec^2 A}$$

$$\frac{sec^2 A}{cosec^2 A} = \frac{\frac{1}{cos^2 A}}{\frac{1}{sin^2 A}} = \frac{sin^2 A}{cos^2 A}$$

$$\frac{sin^2 A}{cos^2 A} = tan^2 A$$

$$\Rightarrow \frac{1+tan^2 A}{1+cot^2 A} = tan^2 A$$

Therefore, option (D) is correct

Q. 5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(cosec\ \theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta cosec\ \theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = cosec\ A + \cot A$ using the identity $cosec^2 A = 1 + cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + cosec\ A)^2(\cos A + \sec A)^2 = 7 + tan^2 A + cot^2 A$

(ix) $(cosec\ A - \ sin\ A)(sec\ A - \ cos\ A) = \frac{1}{tan\ A + cot\ A}$

[Hint: simplify LHS and RHS separately]

(x) $\left(\frac{1+tan^2 A}{1+cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = tan^2 A$

Answer: To Prove

$(cosec\ \theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Proof: LHS $= (cosec\ \theta - \cot\theta)^2$

Apply formulas: $cosec\ \theta = \frac{1}{\sin\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\left(\frac{1}{sin\ \theta} - \frac{cos\ \theta}{sin\ \theta}\right)^2$

$\left[\frac{(1-\cos\theta)}{\sin\theta}\right]^2$

Since, $sin^2\theta = 1 - \ cos^2\theta$

$= \frac{(1-\cos\theta)^2}{1-cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

Hence, proved

(ii) To Prove: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Proof:

LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

Proof

LHS: $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

Use the identity $sin^2\theta + cos^2\theta = 1$

$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)}$

$= \frac{2+2\sin A}{(1+\sin A)(\cos A)}$

$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)}$

$= \frac{2}{\cos A}$

$= 2\sec A$

= RHS

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta cosec\ \theta$

[Hint: Write the expression in terms of sin θ and Cos θ]

$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{sin^3\theta-cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

Use the formula $a^3 - b^3 = (a^2 + b^2 + ab)(a - b)$

$$\frac{\sin\theta-\cos\theta(sin^2\theta+cos^2\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

Cancelling (sin θ - cos θ) from numerator and denominator

$$= \frac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

As cosθ = 1/secθ and sinθ = 1/cosecθ

= 1 + sec θ cosec θ

= RHS

(iv) $\frac{1+\sec A}{\sec A} = \frac{sin^2 A}{1-\cos A}$

[Hint: Simplify LHS and EHS separately]

LHS

$$\frac{1+\sec A}{\sec A}$$

Use the formula $\sec A = 1/\cos A = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

= cos A + 1

RHS

$$\frac{sin^2 A}{1-\cos A}$$

Use the identity $sin^2\theta + cos^2\theta = 1$

$$= \frac{1-cos^2 A}{1-\cos A}$$

Use the formula $a^2 - b^2 = (a - b)(a + b)$

$$= \frac{(1-\cos A)(1+\cos A)}{1-\cos A}$$

= cos A +1

LHS= RHS

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing numerator and Denominator by sin A

$$= \frac{\frac{\cos A-\sin A+1}{\sin A}}{\frac{\cos A+\sin A-1}{\sin A}}$$

$$= \frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$$

Use the formula cotθ = cosθ / sinθ $= \frac{\cot A-1+cosec\ A}{\cot A+1-cosec\ A}$

Using the identity $cosec^2A = 1 + cot^2A$

$$= \frac{\cot A-\left(cosec^2A-cot^2A\right)+cosec\ A}{\cot A+1-cosec\ A}$$

$$= \frac{(\cot A+cosec\ A)-(cosec^2A-cot^2A)}{\cot A+1-cosec\ A}$$

Use the formula $a^2 - b^2 = (a - b)(a + b)$

$$= \frac{(\cot A + cosec\ A) - [(cosec\ A - \cot A)(cosec\ A + \cot A)]}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A)[1 - (cosec\ A - \cot A)]}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A)(1 - cosec\ A + \cot A)}{\cot A + 1 - cosec\ A}$$

= cot A + cosec A

= RHS

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Dividing numerator and denominator of LHS by cos A

$$\sqrt{\frac{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}}$$

$As\ cos\theta = 1/sec\theta\ and\ tan\theta = sin\theta\ /cos\theta = \sqrt{\frac{\sec A + \tan A}{\sec A - \tan A}}$

Rationalize the square root to get, $= \sqrt{\frac{(\sec + \tan A)\times(\sec A + \tan A)}{(\sec A - \tan A)\times(\sec A + \tan A)}}$

Use the formula $a^2 - b^2 = (a - b)(a + b)$ to get,

$$= \sqrt{\frac{(sec\ A + tan\ A)^2}{(sec^2 A - tan^2 A)}}$$

$$= \frac{\sqrt{(sec\ A + tan\ A)^2}}{\sqrt{sec^2 A - tan^2 A}}$$

Use the identity $\sec^2\theta = 1 + \tan^2\theta$ to get,

$$= \frac{sec\ A + tan\ A}{1}$$

= sec A + tan A

= RHS

(vii) To prove $\frac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta} = \tan\theta$

Proof: LHS

$\frac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1 - 2sin^2\theta)}{\cos\theta(2cos^2\theta - \cos\theta)}$

Since $sin^2\theta = 1 - cos^2\theta$

$= \frac{\sin\theta(1 - 2(1 - cos^2\theta))}{\cos\theta\,(2cos^2\theta - 1)}$

$= \frac{\sin\theta(1 - 2 + 2cos^2\theta)}{\cos\theta(2cos^2\theta - 1)}$

$= \frac{\sin\theta(2cos^2\theta - 1)}{\cos\theta(2\,cos^2\theta - 1)}$

As tanθ = sinθ/cosθ= tanθ = R.H.SHence, Proved.**(viii)**
To Prove: $(\sin A + cosec\ A)^2(\cos A + \sec A)^2 = 7 + tan^2A + cot^2A$

Proof:

LHS: = $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

Use the formula $(a+b)^2 = a^2 + b^2 + 2ab$ to get,

= $(\sin^2 A + \text{cosec}^2 A + 2\sin A \text{ cosec } A) + (\cos^2 A + \sec^2 A + 2 \cos A \sec A)$

Since sinθ=1/cosecθ and cosθ = 1/secθ

$= \left(sec^2A + cosec^2A + 2\sin A\ \frac{1}{\sin A}\right) + \left(cos^2A + sec^2A + 2\cos A\frac{1}{\cos A}\right)$

= $\sin^2A$ + $cosec^2A$ + 2 + $\cos^2A$ + $\sec^2A$ + 2

=($\sin^2A$ + $\cos^2A$)+$cosec^2A$+$\sec^2A$+2+2

Use the identities $\sin^2A + \cos^2A = 1$, $\sec^2A = 1 + \tan^2A$ and $cosec^2A = 1 + \cot^2A$ to get

= 1+ 1 + $\tan^2A$ + 1 + $\cot^2A$ + 2 + 2

= 1 + 2 + 2 + 2 + $\tan^2A$ + $\cot^2A$

= 7 + $\tan^2A$ + $\cot^2A$

= RHS

(viii) Not available in ncert solution

(ix) To prove: $(cosec\ A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

[Hint: Simplify LHS and EHS separately]

Proof: **LHS** = (cosec A – sin A) (sec A – cos A)

Use the formula sinθ = 1/cosecθ and cosθ = 1/secθ

$= \left(\frac{1}{sin\ A} - sin\ A\right)\left(\frac{1}{cos\ A} - cos\ A\right)$

$= \frac{(1-sin^2A)}{sin\ A} \times \frac{(1-cos^2A)}{sin\ A}$

$= \frac{cos^2A}{sin\ A} \times \frac{sin^2A}{cos\ A}$ $[(1 - sin^2A) = cos^2A]$

$[(1 - cos^2A) = cos^2A]$

= cos A sin A

RHS

$\frac{1}{tan\ \theta + cot\ \theta}$

use the formula tanθ=sinθ/cosθ and cotθ=cosθ/sinθ $= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$$= \frac{1}{\frac{sin^2 A + cos^2 A}{\cos A \sin A}}$$

$$= \frac{\cos A \sin A}{sin^2 A + cos^2 A}$$

Use the identity $\sin^2 A + \cos^2 A = 1$

$$= \frac{\cos A \sin A}{1}$$

= cos A sin A

LHS = RHS

(x) To Prove: $\left(\frac{1+tan^2 A}{1+cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = tan^2 A$

Proof

Taking left most term Since, cot A is the reciprocal of tan A, we have

$$\frac{1+tan^2 A}{1+cot^2 A} = \frac{1+tan^2 A}{1+\frac{1}{tan^2 A}}$$

$$= \frac{1}{\frac{1}{tan^2 A}}$$

$$= tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$$

= right most part Taking middle part: $= \left(\frac{1}{\frac{-1}{\tan A}}\right)^2$

$$= (-\tan A)^2$$

$$= tan^2 A$$

= right most part Hence, Proved.

Testing

Q. 4 Draw the graph of 2x – 3y = 4. From the graph, find whether x = -1, y = -2 is a solution or not. (Final)

Answer: Given equation, 2x – 3y = 4

⇒ 3y = 2x- 4

$y = \frac{2x-4}{3}$

When x=-4, then,

$y = \frac{2x-4}{3}$

$\Rightarrow \frac{2\times(-4)-4}{3}$

$\Rightarrow y = \frac{-8-4}{3}$

$\Rightarrow y = \frac{-12}{3}$

⇒ y = -4

When x=-1, then,

$y = \frac{2x-4}{3}$

$y = \frac{2\times(-1)(-4)}{3}$

$\Rightarrow y = \frac{-2-4}{3}$

$\Rightarrow y = \frac{-6}{3}$

⇒ y = -2

When x=2, then,

$y = \frac{2x-4}{3}$

$\Rightarrow y = \frac{2\times2-4}{3}$

$\Rightarrow y = \frac{4-4}{3}$

$\Rightarrow$ y = 0

When x=5, then,

$y = \frac{2x-4}{3}$

$\Rightarrow y = \frac{2\times5-4}{3}$

$\Rightarrow y = \frac{10-4}{3}$

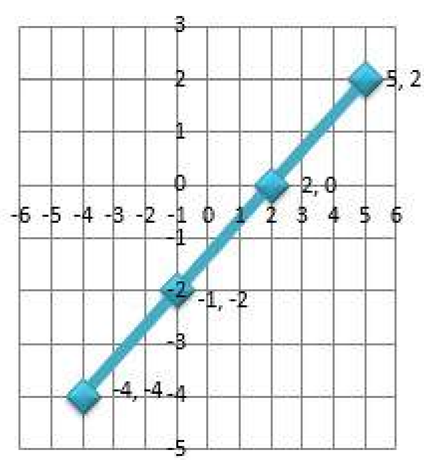
$\Rightarrow y = \frac{6}{3}$

$\Rightarrow$ y = 2

Thus we have the following table,

X	-4	-1	2	5
Y	-4	-2	0	2

On plotting these points we have the following graph,



Clearly, from the graph (-1, -2) is the solution of the line 2x – 3y = 4