ALL INDIA TEST SERIES

FULL TEST – 6

JEE (Main)

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 75 questions.
- Each subject (PCM) has 25 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each part has only three sections: **Section-A**, **Section-B and Section-C**.

  Section-A (01 – 20, 26 – 45, 51 – 70) *contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.*

  Section-B (21 – 22, 46 – 47, 71 – 72) *contains 6 Numerical based questions with answer as numerical value from **0 to 9** and each question carries **+4 marks** for correct answer. There is no negative marking.*

  Section-C (23 – 25, 48 – 50, 73 – 75) *contains 9 Numerical answer type questions with answer 3XXXXX.XX and each question carries **+4 marks** for correct answer. There is no negative marking.*

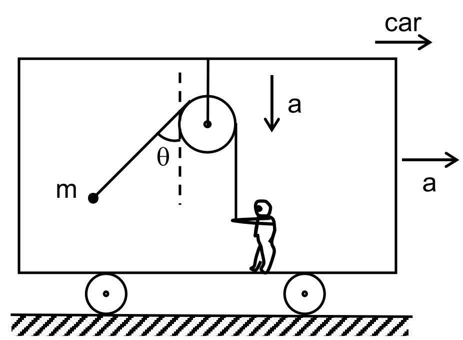
Physics

PART – I

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with a constant acceleration a directed horizontally as shown in the figure. Other end of the string is pulled down vertically with a constant acceleration a relative to car. The tension in the string is equal to (where tan θ = a/g)



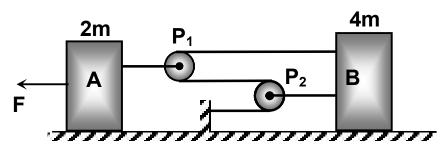
(A) $m\sqrt{g^2+a^2}$

(B) $m\sqrt{g^2+a^2}-ma$

(C) $m\sqrt{g^2+a^2}+ma$

(D) $m(g+a)$

2. The acceleration of the block B shown in the figure will be (Assuming the surfaces and the light pulleys P_1 and P_2 all are smooth)



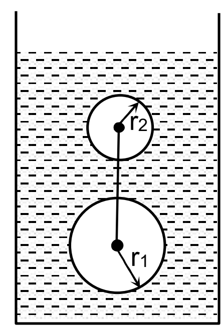
(A) $\frac{F}{4m}$

(B) $\frac{F}{6m}$

(C) $\frac{F}{2m}$

(D) $\frac{3F}{17m}$

3. Two solid spherical balls of radius r_1 and r_2 ($<r_1$) and of density σ are tied up with a long string and released in a viscous liquid column of lesser density ρ with the string taut as shown in the figure. The tension in the string when the terminal velocity is attained will be



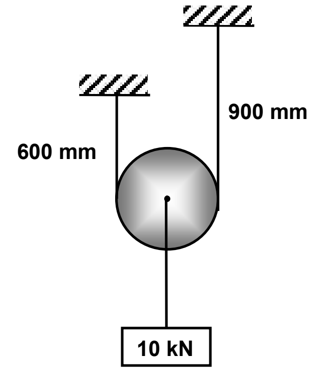
(A) $\frac{4}{3}\pi\left[\frac{r_2^4-r_1^4}{r_2-r_1}\right](\sigma-\rho)g$

(B) $\frac{4}{3}\pi r_1 r_2\left[r_1-r_2\right](\sigma-\rho)g$

(C) $\frac{4}{3}\pi\left[r_1^3-r_2^3\right](\sigma-\rho)g$

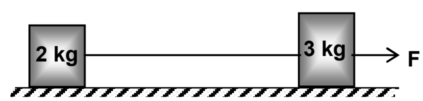
(D) none of these

4. A load of 10 kN is supported from a light pulley which in turn is supported by a rope of cross-sectional area 1×10^3 mm^2 and modulus of elasticity 10^3 N/mm^2 as shown in the figure. Neglect the friction at the pulley, determine the deflection of load in equilibrium.
(A) 2.75 mm
(B) 3.75 mm
(C) 5.25 mm
(D) 6.50 mm



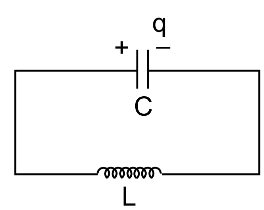
5. A balloon of volume V, contains a gas whose density is σ and the density of the air at the earth's surface is 15σ. If the envelope of the balloon be of weight w but of negligible volume. Find the acceleration with which it will begin to ascend.
(A) $\left[\dfrac{7Vg\sigma - w}{Vg\sigma + w}\right] \times g$
(B) $\left[\dfrac{2Vg\sigma - w}{Vg\sigma + w}\right] \times g$
(C) $\left[\dfrac{14Vg\sigma - w}{Vg\sigma + w}\right] \times g$
(D) $\left[\dfrac{7Vg\sigma + w}{Vg\sigma - w}\right] \times g$

6. Two bodies of masses 2 kg and 3 kg are connected by a metal wire of cross sectional area 0.4 mm^2. Breaking stress of metal wire is 2.5×10^8 N/m^2. The maximum force F that can be applied to 3 kg block so that the wire does not break.



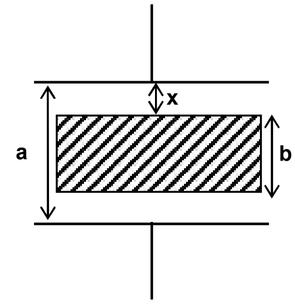
(A) 100 N
(B) 150 N
(C) 200 N
(D) 250 N

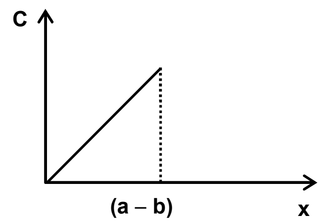
7. In an LC circuit shown in the figure, C = 1F, L = 4H. At time t = 0, charge on the capacitor is 4C and it is decreasing at a rate of $\sqrt{5}$ C/s. Choose the correct statement.

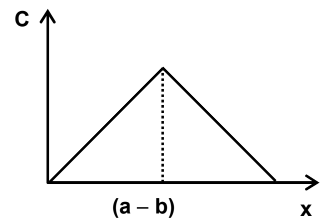


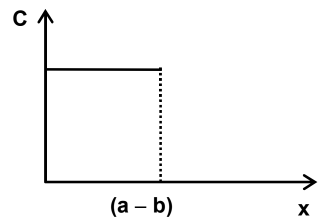
(A) Maximum charge on the capacitor will be 6C.
(B) Maximum charge on the capacitor will be 8C.
(C) Charge on the capacitor will be maximum after time $2\sin^{-1}\left(\dfrac{2}{3}\right)$ seconds.
(D) None of these

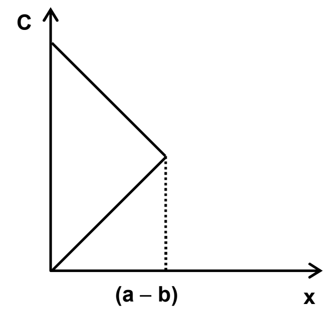
8. The distance between two parallel plates of a capacitor is a. A conductor of thickness b (b < a) is inserted between the plates as shown in the figure. The variation of effective capacitance between the plates of the capacitor as a function of the distance (x) is best represented by



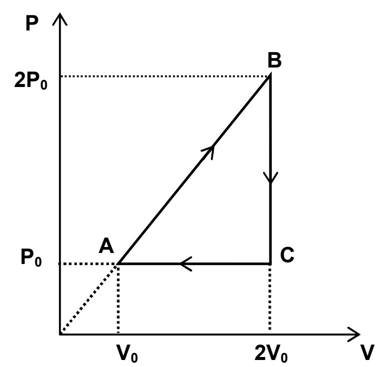
(A) 

(B) 

(C) 

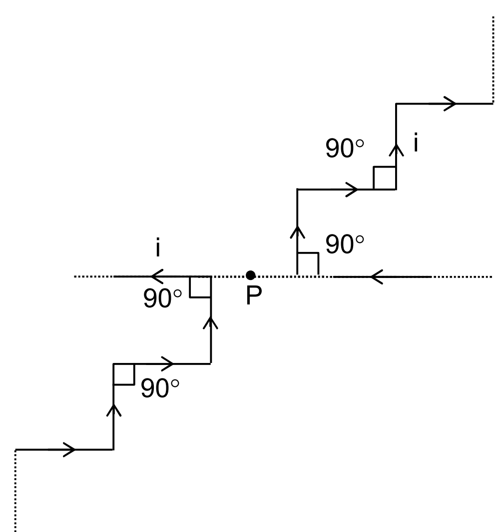
(D) 

9. One mole of an ideal monoatomic gas is taken through a cyclic process ABCA as shown in the P-V diagram. The efficiency (in percentage) of the cyclic process is



(A) 8.33
(B) 12.33
(C) 16.33
(D) 20.33

10. Two infinitely long conductors carrying equal currents are shaped as shown. The short sections are all of equal lengths. The point P is located symmetrically with respect to the two conductors. The magnetic field at point P due to any one conductor is B. The total magnetic field at point P is



(A) zero
(B) B
(C) $\sqrt{2}$B
(D) 2B

11. Three polaroids are kept coaxially. Angle between the transmission axes of first and third polaroid is 90°. Angle between the transmission axes of first and second polaroid is 60°. If the intensity of the unpolarized light incident normally on the first polaroid is I_0. The intensity of the light that emerges out from the system is :
(A) zero

(B) $\frac{3I_0}{32}$

(C) $\frac{3I_0}{16}$

(D) $\frac{\sqrt{3}I_0}{8}$

12 In a Coolidge tube experiment, the minimum wavelength of the continuous x-ray spectrum is equal to 66.3 pm.
(A) electrons accelerate through the potential difference of 12.75 kV in the coolidge tube.
(B) electrons accelerate through a potential difference of 18.70 kV in the coolidge tube.
(C) de-Broglie wavelength of the electrons reaching the anticathode is 10 Å.
(D) de-Broglie wavelength of the electrons reaching the anticathode is 0.01 Å.

13. The ratio of magnetic moment to the angular momentum is a universal constant for hydrogen like atoms and ions. The value of this constant is

(A) $\frac{e}{8\varepsilon_0 m}$

(B) $\frac{e}{2m}$

(C) $\frac{\pi m}{2e}$

(D) $\frac{m^2}{2e\varepsilon_0}$

14. The counting rate observed from a radioactive source at t = 0, was N_0 counts per second and at 4t seconds $\frac{N_0}{16}$ counts per second. The counting rate observed, as counts per second, at 5t seconds will be

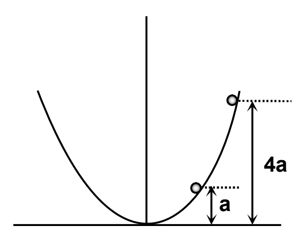
(A) $\frac{N_0}{128}$

(B) $\frac{N_0}{64}$

(C) $\frac{N_0}{32}$

(D) $\frac{N_0}{256}$

15. A bead of mass m is located on a parabolic wire with its axis vertical and vertex at the origin as shown in the figure whose equation is $x^2 = 4ay$. The wire frame is fixed and the bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when the bead reaches the position given by y = a is



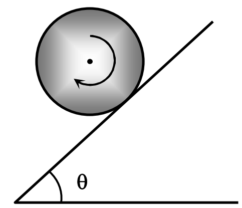
(A) $\frac{g}{2}$

(B) $\frac{\sqrt{3}}{2}g$

(C) $\frac{g}{\sqrt{2}}$

(D) $\dfrac{g}{\sqrt{5}}$

16. A solid cylinder of mass m and radius R is spinned to a clockwise angular velocity ω_0 and then gently placed on an inclined plane for which coefficient of friction $\mu = \tan\theta$, where θ is the angle made by inclined plane with horizontal. The centre of mass of the cylinder will remain stationary for a time interval



(A) $\dfrac{\omega_0 R}{g\sin\theta}$

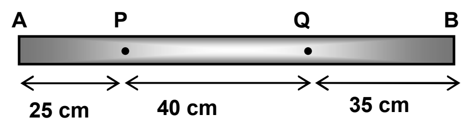
(B) $\dfrac{2\omega_0 R}{5g\sin\theta}$

(C) $\dfrac{2\omega_0 R}{3g\sin\theta}$

(D) $\dfrac{\omega_0 R}{2g\sin\theta}$

17. A heavy nucleus X having mass number 200 gets disintegrated into two small fragments Y and Z of mass number 80 and 120 respectively. If binding energy per nucleon for the parent atom X is 6.5 MeV and for daughter nuclei Y and Z are 7 MeV and 8 MeV respectively. Energy released in the decay will be
(A) 200 MeV
(B) 240 MeV
(C) 220 MeV
(D) 180 MeV

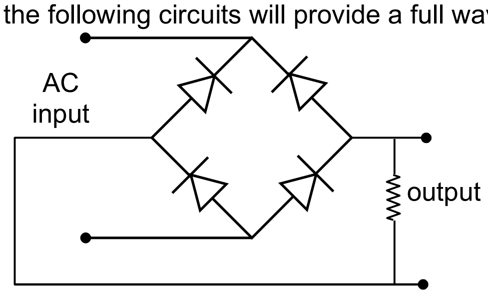
18. A metallic rod AB of length 1 m is rigidly clamped at two points P and Q as shown in the figure. The minimum frequency of longitudinal vibration of the rod is (Young's modulus of the rod, $Y = 2 \times 10^{11}$ N/m^2, density of the rod $\rho = 8000\ kg/m^3$)



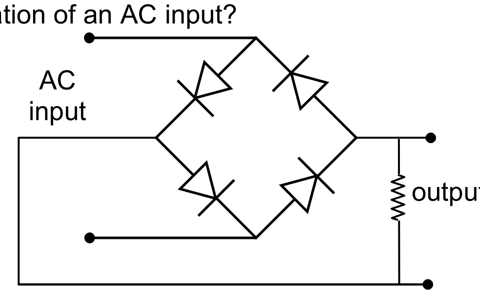
(A) 25 kHz
(B) 50 kHz
(C) 75 kHz
(D) 100 kHz

19. Which of the following circuits will provide a full wave rectification of an AC input?

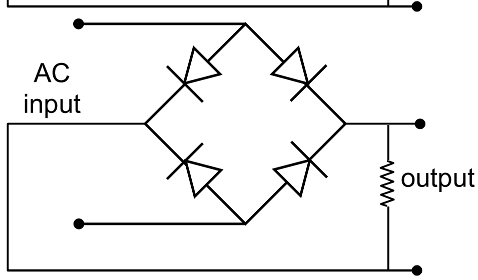
(A)



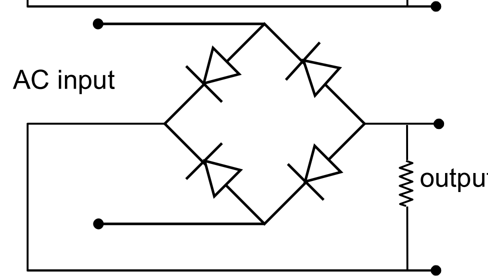
(B)



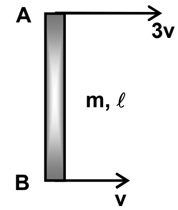
(C)



(D)



20. Two ends of a thin uniform rod of mass 'm' and length 'ℓ' move with the velocities 3v and v perpendicular to the length of the rod and in the same direction as shown in the figure. Then the kinetic energy of the rod is



(A) $\frac{13}{6}mv^2$

(B) $\frac{7}{6}mv^2$

(C) $\frac{10}{3}mv^2$

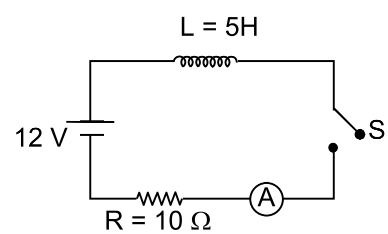
(D) $\frac{5}{3}mv^2$

SECTION – B
(Single digit integer type)

This section contains **02** questions. The answer to each question is a **single Digit integer** ranging from **0 to 9, both inclusive**.

21. A radioactive sample contains two radioactive nucleus A and B having decay constant λ hr^{-1} and 2λ hr^{-1}. Initially 20% of decay comes from A. How long (in hr) will it take before 50% of decay comes from A. (Take $\lambda = \ell n2$)

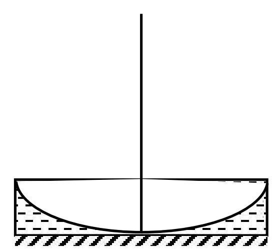
22. For an ideal L-R circuit shown, the resistance is R = 10Ω, the inductance is L = 5H and the battery has voltage ε = 12 volts. At some time after the switch S in the circuit is closed, the ammeter in the circuit reads 0.40 A. If the rate at which energy is being stored by the inductor at this instant is $\frac{16}{x}$ watts. Find the value of x.



SECTION – C
(Numerical Answer Type)

This section contains **03** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

23. A thin plano-convex lens $\left(\mu = \frac{3}{2}\right)$ of focal length 12.5 cm is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is filled with water $\left(\mu_w = \frac{4}{3}\right)$. Find the distance (in cm) above the lens at which a point object is placed on its principal axis so that the object coincides with its own image.



24. A container whose bottom has a round hole with a diameter of 0.1 mm is filled with water. The maximum height (in cm) upto which the water can be filled in the container will be (surface tension of water = 73×10^{-3} N/m, density of water = 10^3 kg/m^3 and g = 10 m/s^2)

25. A thin rod of negligible mass and area of cross-section 2×10^{-6} m^2, suspended vertically from one end, has a length of 0.5 m at 200°C. The rod is cooled to 0°C, but prevented from contracting by attaching a mass at the lower end. Calculate the mass (in kg) attached to the lower end of the rod. (Young's modulus' of rod, Y = 1.2×10^{11} N/m^2, coefficient of linear expansion, $\alpha = 1.4 \times 10^{-6}$ per°C and g =10 m/s^2)

Chemistry PART – II

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

26. The electron are identified by the quantum numbers 'n' and 'ℓ'

I. $n = 4,\ \ell = 1$

II. $n = 4,\ \ell = 0$

III. $n = 3,\ \ell = 2$

IV. $n = 3,\ \ell = 1$

Can be placed in order of increasing energy as

(A) III < IV < II < I
(B) IV < II < III < I
(C) II < IV < I < III
(D) I < III < II < IV

27. A metal M can give following observable changes in a sequence of reactions

$$M \xrightarrow[HNO_3]{\text{dilute}} \text{colourless solution} \xrightarrow{\text{aq. NaOH}} \text{White precipitate}$$

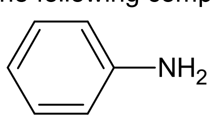
$$\text{White precipitate} \xrightarrow{\text{Excess NaOH}} \text{Colourless solution}$$

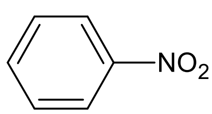
$$\text{Colourless solution} \xrightarrow{H_2S} \text{White precipitate}$$

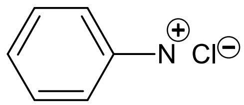
The metal M is

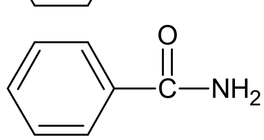
(A) Mg
(B) Pb
(C) Zn
(D) Sn

28. Which of the following compounds does not perform Lassigne's test?

(A) 

(B) 

(C) 

(D) 

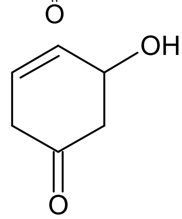
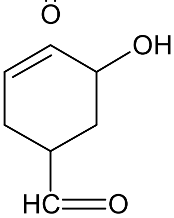
29. What is the number of chiral centres present in penicillin?
(A) 2
(B) 3
(C) 4
(D) 1

30. pH of a saturated solution of $Ba(OH)_2$ is 12. Hence, the K_{sp} of the $Ba(OH)_2$
(A) 5×10^{-7}
(B) 5×10^{-4}
(C) 10^{-6}
(D) 4×10^{-5}

31. A gas occupies 2 litre volume at STP. It is provided 300 Joule heat so that its volume becomes 2.5 litre at 1 atm. Change in internal energy would be
(A) 239 Joule
(B) 205 Joule
(C) 249.37 Joule
(D) 220.37 Joule

32. The compound A gives the following reactions

$$A(C_6H_8O_2) \xrightarrow{Na} H_2(g)\uparrow$$
$$A(C_6H_8O_2) \xrightarrow{2,4\ DNP} \text{Yellow orange precipitate}$$
$$A(C_6H_8O_2) \xrightarrow[Zn/H_2O]{O_3} B(C_6H_8O_4)$$

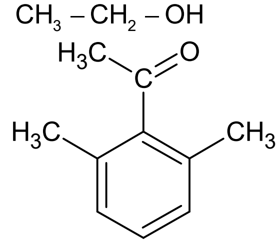
The structure of A can be
(A) $CH_2 = CH-(CH_2)_2-\underset{\underset{O}{\parallel}}{C}-CH_2OH$
(B) $H-\underset{\underset{O}{\parallel}}{C}-(CH_2)_2-CH = CH-COOH$
(C)

(D)


33. $E^{o}_{BrO_3^-/BrO^-} = 0.54$ volt
$E^{o}_{BrO^-/\frac{1}{2}Br_2} = 0.41$ volt
Find out $E^{o}_{BrO_3^-/\frac{1}{2}Br_2}$

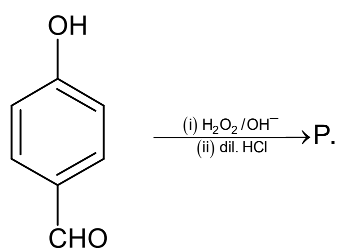
(A) 0.35 volt
(B) 0.52 volt
(C) 0.71 volt
(D) 0.66 volt

34. 50 ml of 1 M oxalic acid is shaken with 0.5 gm wood charcoal. The final concentration of the solution after adsorption is 0.5 M. What is the amount of oxalic acid adsorbed per gram of carbon?
(A) 3.15 gm
(B) 3.45 gm
(C) 6.3 gm
(D) None of these

35. Cu^{2+} on reaction with $K_4\left[Fe(CN)_6\right]$ gives reddish brown precipitate. The formula of reddish brown precipitate is
(A) $Cu_4\left[Fe(CN)_6\right]$
(B) $Cu_2\left[Fe(CN)_6\right]$
(C) $Cu_3\left[Fe(CN)_6\right]$
(D) $Cu_3\left[Fe(CN)_6\right]_3$

36. Which of the following does not undergo haloform reaciton
(A) $Ph-\underset{OH}{\underset{|}{CH}}-CH_3$
(B) CH_3-CH_2-OH
(C) 

(D) $H_3C-\overset{O}{\overset{\|}{C}}-CH_3$

37. 

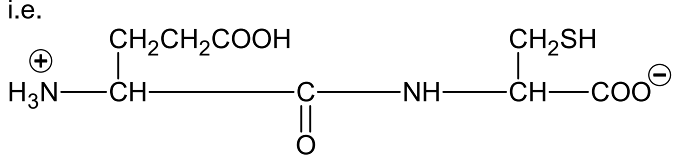
Which of the following statement is wrong regarding the product?
(A) The product gives violet colouration with $FeCl_3$
(B) The product participates in Reimer Tiemann reaction
(C) The product reacts with aq. $NaHCO_3$
(D) The product reacts with aq. NaOH

38. The degree of dissociation of $Ca(NO_3)_2$ in a diluted aqueous solution containing 7 gm of salt per 100 gm of water at $100^{\circ}C$ is 70%. Calculate the vapour pressure of solution?
(A) 750.32 mm
(B) 746.27 mm
(C) 732.65 mm
(D) 767.15 mm

39. $CrO_4^{2-} \xrightarrow{PH=x} Cr_2O_7^{2-} \xrightarrow{PH=y} CrO_4^{2-}$
x and y are respectively
(A) 3 and 5
(B) 3 and 8
(C) 8 and 3
(D) 8 and 11

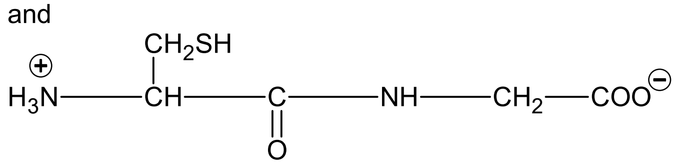
40. In which of the following reactions, HNO_3 does not behave as an oxidizing agent?
(A) $HNO_3 + FeSO_4 + H_2SO_4 \longrightarrow$
(B) $HNO_3 + H_2SO_4 \longrightarrow$
(C) $KI + HNO_3 \longrightarrow$
(D) $Cu + HNO_3 \longrightarrow$

41. A tripeptide X on partial hydrolysis gave two dipeptide cys-gly and glu-cys
i.e.



glu-cys

and

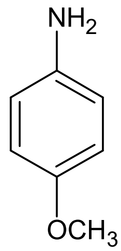


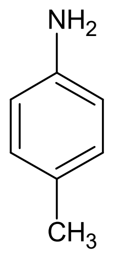
cys-gly

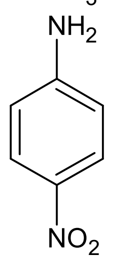
Identify the tripeptide
(A) glu-cys-gly
(B) glu-glu-cys
(C) cys-gly-glu
(D) cys-glu-gly

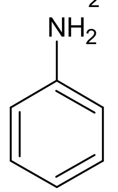
42. Which of the following is a cyclic oxoacid?
(A) $H_4P_2O_7$
(B) $H_4P_2O_6$
(C) $H_3P_3O_8$
(D) $H_4P_2O_5$

43. Which of the following compounds responds in the carbyl amine test in the fastest rate?

(A) 

(B) 

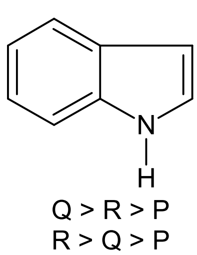
(C) 

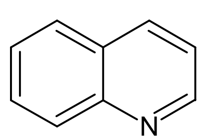
(D) 

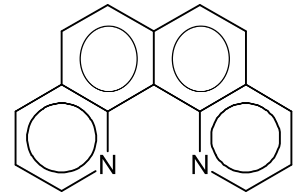
44. The V – C bond distances in $V(CO)_6$ and $\left[V(CO)_6\right]^-$ are respectively in pm

(A) 200, 200
(B) 193, 200
(C) 200, 193
(D) 193, 193

45. Which is correct order of basic nature?

P. 

Q. 

R. 

(A) Q > R > P
(B) R > Q > P
(C) R > P > Q
(D) Q > P > R

SECTION – B
(Single digit integer type)

This section contains **02** questions. The answer to each question is a **single Digit integer** ranging from **0 to 9, both inclusive**.

46. $\text{Galena} \xrightarrow{\text{Roasting}} \text{Metal oxide} + \text{gas}$

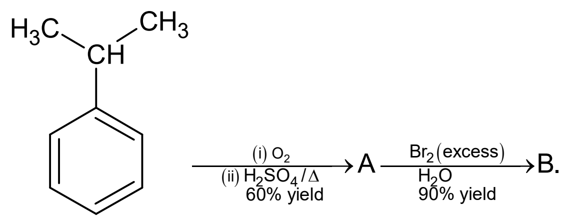
What is the difference between the oxidation state of sulphur in product and reactant?

47. If the structure of Mg is similar to NaCl, what is the coordination number of Mg?

SECTION – C
(Numerical Answer Type)

This section contains **03** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

48.



Find the amount of B formed in gram, if 1 mole of cumene was taken earlier.

49. $CdCl_2(aq) + Fe(s) \longrightarrow Cd(s) + FeCl_2(aq)$

$\left(\frac{dE}{dT}\right)_P = 1.5 \times 10^{-4}\ V\ K^{-1}$ at 298 K

Find out the $\Delta S(\text{in J / K mol})$ during this reaction

50. At 1000 K, a sample of pure NO_2 decomposed as

$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$

The equilibrium constant K_P = 156.25 atm. If partial pressure of O_2 is 0.25 atm at equilibrium, what is the partial pressure of NO_2 at equilibrium?

Mathematics PART – III

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

51. The value of $\sum_{r=1}^{\infty} \cot^{-1}\left(3r^2 - r - \frac{1}{3}\right)$ is equal to
(A) $\cot^{-1}(1)$
(B) $\cot^{-1}(\sqrt{3})$
(C) $\cot^{-1}(0)$
(D) $\cot^{-1}(-1)$

52. The sum of the series $1 + \left(1+\frac{1}{2}\right)\frac{1}{3} + \left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2\right)\left(\frac{1}{3}\right)^2 + ...$ to infinite terms is
(A) $\frac{9}{5}$
(B) $\frac{12}{5}$
(C) $\frac{9}{10}$
(D) $\frac{19}{10}$

53. Let R = {(a, b) : $a^2 + b^2 = 4$, a, b $\in$ R}, then relation R is
(A) Anti-symmetric
(B) Reflexive
(C) Symmetric
(D) Transitive

54. If $f(x) = \frac{e^{2x-1}}{e^{2x-1}+1}$, then $\sum_{r=1}^{2018} f\left(\frac{r}{2019}\right)$ is equal to
(A) 2019
(B) 2018
(C) 1009.5
(D) 1009

55. From an unlimited number of red, white, blue and green balls, a selection of 18 balls is to be made, so that there are at least two of each colour. If the number of selection is k, then k is equal to
(A) 455
(B) 680
(C) 286
(D) 1001

56. If $\frac{\cos x+\cos y+\cos z}{\cos(x+y+z)} = 5050$ and $\frac{\sin x+\sin y+\sin z}{\sin(x+y+z)} = 5050$, then the value of $\cos(x + y) + \cos(y + z) + \cos(z + x)$ is equal to (where $x, y, z \in R$)
(A) 5151
(B) 101
(C) 5050
(D) 100

57. Let f be a differentiable function such that $\lim_{x\to 1}\frac{f(1+x^3-x)-f(x)}{\sin(x-1)} = \lim_{x\to 0}\frac{f(1-x)-f(1)}{x}+10$, then $f'(1)$ is equal to
(A) 1
(B) 2
(C) 4
(D) 5

58. A straight line through A(6, 8) meets the curve $2x^2 + y^2 = 2$ at B and C, P is a point on BC such that AB, AP, AC are in H.P., then the minimum distance of locus of P from origin is
(A) $\frac{5}{\sqrt{52}}$
(B) $\frac{10}{\sqrt{52}}$
(C) $\frac{15}{\sqrt{52}}$
(D) $\frac{1}{\sqrt{52}}$

59. If (h, k) lies on $x^2 + y^2 = 25$, then the minimum value of $\frac{225}{h^2}+\frac{25}{k^2}$ is equal to
(A) 8
(B) $\frac{1}{4}$
(C) 4
(D) 16

60. If $a^2 + b^2 + c^2 + 6 = 2a + 2b + 4c$ and quadratic equations $ax^2 + bx + c = 0$ and $2px^2 + qx + 4r = 0$ have at least one root in common (where $p, q, r \in N$), then the minimum value of $(2p + q + 3r)$ is
(A) 10
(B) 7
(C) 8
(D) 6

61. If angles of right angle triangle ABC are in A.P. and greatest angle ($\angle C$) is three times the least angle ($\angle A$), then the algebraic sum of the coefficients in the expansion of $\left(\left(\frac{b-a}{b+a}\right)x+\sqrt{3}y\right)^{10}$ is (where a, b, c denote sides of the triangle)
(A) 2^{20}
(B) 2
(C) 3
(D) 2^{10}

62. Let $A = \{x \mid x^3 + x^2 - px + q = 0,\ p,\ q \in R\}$ and $B = \{x \mid x^2 - qx + 2 = 0,\ q \in R\}$ be the sets. If $n(A \cap B) = 2$ and $x_0 \in (A - B)$, then the value of $|p - q + x_0|$ is
(A) 2
(B) 3
(C) 6
(D) 10

63. Let A be a 3 × 3 matrix with real elements such that det(adj A) = 2, then det(adj(adj A)) is
(A) 2
(B) 4
(C) 8
(D) 16

64. If roots of the equation $iz^3 + z^2 - z + i = 0$ are vertices of a triangle, then area of the triangle (in sq. units) is
(A) $\frac{1}{\sqrt{2}}$
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{3}{2}$

65. If $f : R \to R$ be a function defined as $f(x) = x^3 + x + 3$ and $g(x) = f^{-1}(x)$, then $g''(5)$ is
(A) $-\frac{1}{16}$
(B) $-\frac{1}{30}$
(C) $-\frac{3}{32}$
(D) $\frac{1}{30}$

66. If $I_1 = \int_0^1 e^{-x} \cos^2 x\, dx$; $I_2 = \int_0^1 e^{-x^2} \cos^2 x\, dx$; $I_3 = \int_0^1 e^{-x^2} dx$ and $I_4 = \int_0^1 e^{-\frac{x^2}{2}} dx$, then the largest integral among I_1, I_2, I_3, I_4 is
(A) I_4
(B) I_2
(C) I_3
(D) I_1

67. Let α, β, γ, δ be four real numbers such that $\alpha^2 + \gamma^2 = 9$ and $\beta^2 + \delta^2 = 1$, then the maximum value of $3(\alpha\beta - \gamma\delta) + 4(\alpha\delta + \beta\gamma)$ is
(A) 20
(B) 10
(C) 5
(D) 15

68. The differential equation of the curve $\frac{x}{A-1}+\frac{y}{A+1}=1$, is given by

(A) $(y'-1)(y+xy')=2y'$
(B) $(y'+1)(y+xy')=y'$
(C) $(y'+1)(y-xy')=2y'$
(D) none of these

69. The standard deviation of 5 scores 1, 2, 3, 4, 5 is

(A) $\frac{3}{5}$
(B) $\sqrt{2}$
(C) $\sqrt{3}$
(D) $\frac{2}{5}$

70. If α, β, γ be the roots of the equation $x^3 - 9x^2 + 15x + 2 = 0$. If v be the volume of parallelepiped (in cu. units) with non-parallel sides $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$, then |v| is

(A) 225
(B) 324
(C) 144
(D) 244

SECTION – B
(Single digit integer type)

This section contains **02** questions. The answer to each question is a **single Digit integer** ranging from **0 to 9, both inclusive**.

71. The number of roots common to the equation $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{2017} + x^{2018} + 1 = 0$ is/are

72. The shortest distance between the curves $x^2 + y^2 = 8$ and $2[(x-3)^2 + (y+3)^2] = (x-y-2)^2$ is k unit, then k is equal to

SECTION – C
(Numerical Answer Type)

This section contains **03** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

73. The eccentricity of conjugate hyperbola of the hyperbola
$\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$, is e′, then 4e′ is equal to

74. If a and b are chosen randomly from the set consisting of numbers {1, 2, 3, 4, 5, 6} with replacement. If p be the probability that $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\left(\frac{2}{x}\right)}=6$, then $\frac{9}{2}p$ is

75. If g(x) be a continuous non-constant function for all x, such that $(g(x))^2=\int_0^x g(t)\cdot\frac{2\sec^2 t}{4+\tan t}dt$ and g(0) = 0, then $\left(\frac{8}{5}\right)\left(\frac{g\left(\frac{\pi}{4}\right)}{\ln\left(\frac{5}{4}\right)}\right)$ is

ALL INDIA TEST SERIES

FULL TEST – 6

JEE (Main)

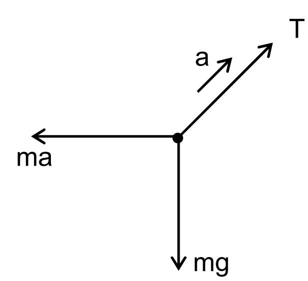
ANSWERS, HINTS & SOLUTIONS

Physics **PART – I**

SECTION – A

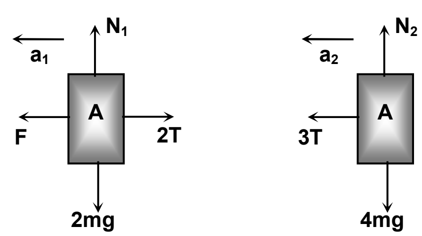
1. C

Sol. $T - m\sqrt{g^2 + a^2} = ma$



2. D

Sol. $F - 2T = 2ma_1$
$3T = 4ma_2$
$2a_1 = 3a_2$
Solving we get
$a_2 = \frac{3F}{17m}$



3. B

Sol. $(m_1 + m_2)g\left(1 - \frac{\rho}{\sigma}\right) - 6\pi\eta(r_1 + r_2)V = 0$

$m_1g - \frac{m_1\rho}{\sigma}g - T - 6\pi\eta r_1V = 0$

$$T = m_1 g\left(\frac{\sigma-\rho}{\sigma}\right) - \frac{6\pi\eta r_1(m_1+m_2)g\left(\frac{\sigma-\rho}{\sigma}\right)}{6\pi\eta(r_1+r_2)}$$

$$T = \frac{4}{3}\pi r_1 r_2 (r_1 - r_2) g(\sigma-\rho)$$

4. B

Sol. $y = \frac{\Delta\ell_1 + \Delta\ell_2}{2} = \frac{T}{2AY}(\ell_1 + \ell_2)$

$= \frac{5\times10^3(600+900)\times10^{-3}}{2\times10^{-3}\times10^9} = 3.75 \text{ mm}$

5. C

Sol. $15\sigma Vg - \sigma Vg - w = \left(\frac{w}{g} + \sigma V\right)a$

$a = \left(\frac{14\sigma Vg - w}{w + \sigma Vg}\right)g$

6. D

Sol. T = ma

$T = 2\times\frac{F}{5}$

$0.25\times10^9\times0.4\times10^{-6} = \frac{2F}{5}$

$10^2 = \frac{2F}{5}$

$F_{max} = \frac{5}{2}\times10^2 = 250 \text{ N}$

7. A

Sol. $Q = Q_0 \sin(\omega t + \phi)$

$i = \frac{dQ}{dt} = Q_0\omega\cos(\omega t + \phi)$

At t = 0

$4 = Q_0 \sin\phi$

$-\sqrt{5} = \frac{Q_0}{2}\cos\phi$

16 + 20 = Q_0^2

Maximum charge, Q_0 = 6C

$\phi = \pi - \sin^{-1}\left(\frac{2}{3}\right)$

8. C

Sol. $C = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{A\varepsilon_0}{x}\times\frac{A\varepsilon_0}{(a-b-x)}}{\frac{A\varepsilon_0}{x}+\frac{A\varepsilon_0}{(a-b-x)}}$

$C = \frac{A\varepsilon_0}{(a-b)}$ independent of x

9. A

Sol. $\Delta W_{cycle} = \frac{1}{2}V_0P_0 = \frac{P_0V_0}{2}$

For the process AB, P= KV $\Rightarrow$ PV^{-1} = constant

Molar heat capacity of the gas in the process AB,

$$C = C_V + \frac{R}{(1-x)} = \frac{3R}{2} + \frac{R}{2} = 2R$$

$\Delta Q_{AB} = nC\Delta T = n2R(4T_0 - T_0) = 6nRT_0 = 6P_0V_0$

$\Delta Q_{BC} < 0$ and $\Delta Q_{CA} < 0$

The efficiency of the cyclic process, $\eta = \frac{\Delta W_{cycle}}{\Delta Q_{supplied}} \times 100 = \frac{P_0V_0}{2 \times 6P_0V_0} \times 100$

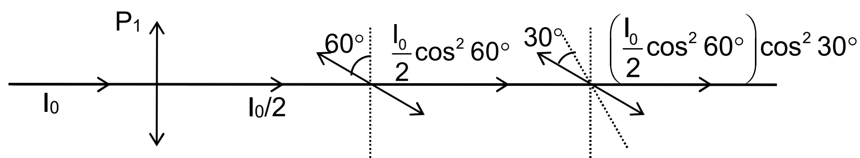
$$\eta = \frac{25}{3} = 8.33\%$$

10. A

Sol. By the symmetry, $\vec{B}_{total} = 0$

11. B

Sol. $I' = \frac{I_0}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3I_0}{32}$



12. B

Sol. λ_{min} = 66.3 pm

$$V = \frac{1240}{66.3 \times 10^{-3}} = 18.70 \text{ kV}$$

de-Broglie wavelength for electron = $\frac{h}{\sqrt{2meV}} = 0.09$ Å

13. B

Sol. $\mu = \frac{e}{T}\pi r^2 = \frac{e\omega\pi r^2}{2\pi} = \frac{er^2\omega}{2}$

$L = mr^2\omega$

$\frac{\mu}{L} = \frac{e}{2m}$

14. C

Sol. At 4t its $\left(\frac{1}{2}\right)^4 N_0$

So, at 5t it will be $\left(\frac{1}{2}\right)^5 N_0$

i.e., t = $T_{1/2}$

15. C

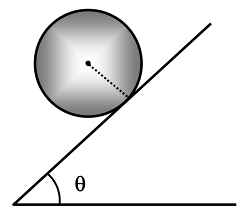
Sol. $y = \frac{x^2}{4a}$

$$\frac{dy}{dx} = \frac{2x}{4a}$$

When y = a, $\frac{dy}{dx} = 1 = \tan\theta$

Tangential acceleration = g sin θ = $\frac{g}{\sqrt{2}}$

16. D



Sol. $\because$ $\mu = \tan\theta$

Till the cylinder rotates it will remain stationary.
as mg sinθ = μmg cosθ

$\mu mg \cos\theta R = \frac{mR^2}{2}\alpha$

$$\alpha = \frac{mg\sin\theta R \cdot 2}{mR^2} = \frac{2g\sin\theta}{R}$$

$\omega_0 = \alpha t \Rightarrow t = \frac{\omega_0}{\alpha} = \frac{\omega_0 R}{2g\sin\theta}$

17. C

Sol. Q = (80 × 7) + (120 × 8) – (200 × 6.5) MeV = 220 MeV

18. A

Sol. Velocity of longitudinal wave in the rod,

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times10^{11}}{8\times10^{3}}} = 5000\ \text{m/s}$$

Now,

$n_1 : n_2 : n_3 = \ell_1 : \ell_2 : \ell_3$

= 25 : 40 : 35 = 2.5 : 4 : 3.5

The minimum no of loops formed

n = 2.5 + 4 + 3.5 = 10 loops

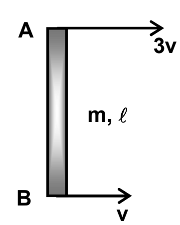
$$\frac{10\lambda}{2} = \ell \Rightarrow \lambda = 0.20\ \text{m}$$

The minimum frequency of vibration, $f = \frac{v}{\lambda} = \frac{5000}{0.2} = 25\ \text{kHz}$

19. D

Sol. By the property of full wave rectifier

20. A



Sol. The angular velocity of the rod, $\omega = \frac{3v - v}{\ell} = \frac{2v}{\ell}$

The velocity of centre of mass of the rod,

$$v_{CM} = \frac{\omega\ell}{2} + v = 2v$$

The kinetic energy of the rod, $K = \frac{1}{2}\frac{m\ell^2}{12}\omega^2 + \frac{1}{2}mv_{CM}^2$

$$K = \frac{mv^2}{6} + \frac{1}{2}m(2v)^2 = \frac{13}{6}mv^2$$

SECTION – B

21. 2

Sol. $-\frac{dN_A}{dt} = \lambda N_0 e^{-\lambda t}$

$-\frac{dN_B}{dt} = 2\lambda N_0' e^{-2\lambda t}$

At t = 0, $4\frac{dN_A}{dt} = \frac{dN_B}{dt}$ (20% and 80%)

$N_0 \lambda e^{-\lambda t} = 2N_0 \cdot 2\lambda e^{-2\lambda t}$

$t = \frac{2\ell n2}{\lambda} = 2$ hours

22. 5

Sol. iR = 4V

$L\frac{di}{dt} = 8V$

$L\frac{di}{dt}i = \frac{16}{x}$

$x = \frac{16}{3.2} = 5$

SECTION – C

23. 00037.50

Sol. $\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} + \frac{1}{R}\right) = \frac{1}{2R}$

$2R = f_1 \Rightarrow R = \frac{f_1}{2} = \frac{25}{4}$ cm

$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{1}{3R}$

$f_2 = -3R = -\frac{75}{4}$ cm

Now, $-\frac{1}{F} = \frac{2}{f_1} + \frac{2}{f_2} - \frac{1}{f_m}$

$-\frac{1}{F} = \frac{4}{25} - \frac{8}{75} - \frac{1}{\infty}$

$-\frac{1}{F} = \frac{4}{75}$

$\Rightarrow F = -\frac{75}{4}$ cm

The point object is placed at a distance,

$d = \frac{75}{2} = 37.50$ cm

24. 00029.20

Sol. $\frac{2T}{r} = \rho g h$

$$h = \frac{73 \times 10^{-3} \times 2}{\frac{0.1}{2} \times 10 \times 10^{3} \times 10^{-3}} = 29.2 \text{ cm}$$

25. 00006.72

Sol. $\frac{F}{A} = \frac{Y\Delta\ell}{\ell}$

$mg = YA\alpha\Delta t$

$mg = 1.2 \times 10^{11} \times 2 \times 10^{-6} \times 1.4 \times 10^{-6} \times 200$

$mg = 67.2 \Rightarrow m = 6.72$ kg

SECTION – A

26. B

Sol. I. $n+\ell=4+1=5$

II. $n+\ell=4+0=4$

III. $n+\ell=3+2=5$

IV. $n+\ell=3+1=4$

I is having greater n value as compared to III.
II is having greater n value as compared to IV.

27. C

Sol.

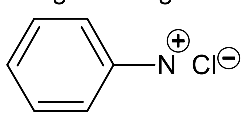
$$M \xrightarrow[HNO_3]{\text{dilute}} Zn(NO_3)_2 \xrightarrow{\text{aq. NaOH}} \underset{\text{White precipitate}}{Zn(OH)_2}$$

$$Zn(OH)_2 \xrightarrow{\text{Excess NaOH}} Na_2[Zn(OH)_4]$$

$$Na_2[Zn(OH)_4] \xrightarrow{H_2S} \underset{\text{White precipitate}}{ZnS\downarrow}$$

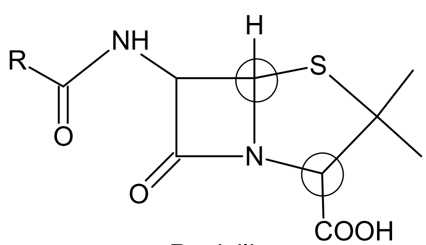
28. C

Sol. As it gives N_2 gas on heating, which escapes from reaction medium.



29. A

Sol.



Penicilin

30. A

Sol. $Ba(OH)_2(s) \rightleftharpoons Ba^{2+}(aq)+2OH^-(aq)$

$pH=12 \qquad \therefore pOH=2$

i.e. $[OH^-]=10^{-2}$

$$[Ba^{2+}]=\frac{10^{-2}}{2}=5\times10^{-3}$$

$K_{sp}=[Ba^{2+}][OH^-]^2=5\times10^{-3}\times(10^{-2})^2$

$=5\times10^{-7}$

31. C

Sol. Work done $= -P\Delta V = -1(2.5-2) = -1\times 0.5$

$= -0.5$ litre atm

$= -50.63$ J

$Q = \Delta U - W$

$300 = \Delta U - (-50.63)$

$\Delta U = 249.37$ J

32. C

Sol. The compound must be cyclic and contains a – OH and – CHO group. It must contain six C atoms.

33. B

Sol. $$\underbrace{BrO_3^- \xrightarrow[n_1=5]{E_1^o} BrO^- \xrightarrow[n_2=2]{E_2^o} \frac{1}{2}Br_2}_{E^o}$$

$$E^o = \frac{n_1E_1^o + n_2E_2^o}{n_1+n_2} = \frac{5\times 0.54 + 2\times 0.41}{5+2}$$

= 0.52 V

34. A

Sol. Amount of oxalic acid initially present $= \frac{1\times 50}{1000}\times 63 = 3.15$ gm

Amount of oxalic acid after adsorption $= \frac{50}{1000}\times\frac{63}{2} = 1.575$ gm

Amount adsorbed = 3.15 – 1.575 = 1.575 gm

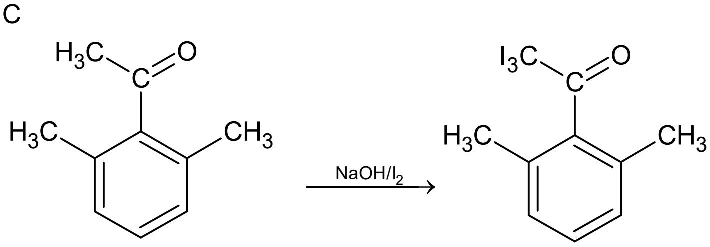
Amount adsorbed per gram of charcoal = 1.575 × 2 = 3.15 gm.

35. B

Sol. $2Cu^{2+} + K_4[Fe(CN)_6] \longrightarrow 4K^+ + Cu_2[Fe(CN)_6]$

36. C

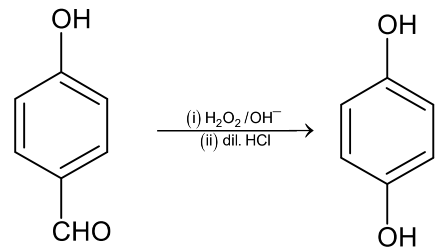
Sol.



Nucleophilic attack by OH^- in the next step is not possible due to steric hindrance.

37. C

Sol.



P does not react with aq. $NaHCO_3$ as it is a weak acid.

38. B

Sol.

$$Ca(NO_3)_2 \rightleftharpoons Ca^{2+} + 2NO_3^-$$

Before dissociation	1	0	0
After dissociation	$1-\alpha$	α	2α

Total mole of at equilibrium $= 1-\alpha+\alpha+2\alpha = 1+2\alpha = 1+2\times 0.7 = 2.4$

$[\alpha = 0.7]$

$$\frac{m_{calc.}}{m_{exp}} = 1+2\alpha$$

$$m_{exp} = \frac{m_N}{1+2\alpha} = \frac{164}{2.4}$$

Also at $100^\circ C$, P of $H_2O P^o_{H_2O} = 760$ mm

$W_{solute} = 7$ gm $\quad W_{H_2O} = 100$ gm

$$\frac{P^o - P_s}{P_s} = \frac{7\times 18}{68.33\times 100} = 0.0184$$

$$P_s = \frac{760}{1.0184} = 746.27 \text{ mm}$$

39. B

Sol. In alkaline medium, chromate ions are present while in acidic medium, dichromate ions are present.

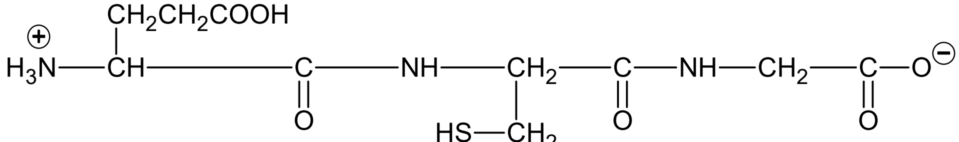
$$2CrO_4^{2-} \xrightarrow[PH=3]{H^+} Cr_2O_7^{2-} \xrightarrow[PH=8]{OH^-} 2CrO_4^{2-}$$

40. B

Sol. In this reaction HNO_3 behaves as H^+ acceptor and H_2SO_4 behaves as H^+ donor.

41. C

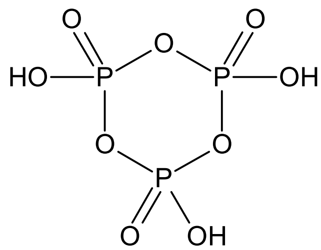
Sol. Since the tripeptide on hydrolysis gave two dipeptides glu-cys and cys-gly. Hence, cystine must be in between glutamic acid and glycine as given below



glu-cys-gly

42. C

Sol.



43. A

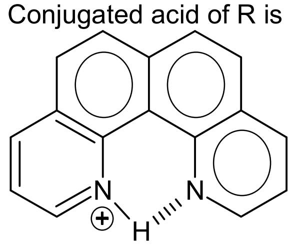
Sol. Due to +R effect of $-OCH_3$ group, it reacts with electrophile dichlorocarbene in fastest rate.

44. C

Sol. In $\left[V(CO)_6\right]^-$, the tendency of back donation of electron in π^* orbital of CO is greater, thus, V – C bond length decreases.

45. B

Sol. Conjugated acid of R is stabilized through intramolecular H bonding.



SECTION – B

46. 6

Sol. $Pb\overset{(-2)}{S} \xrightarrow{\text{Roasting}} PbO + \overset{+4}{S}O_2$

Change of oxidation state = (4 – (–2)) = 6

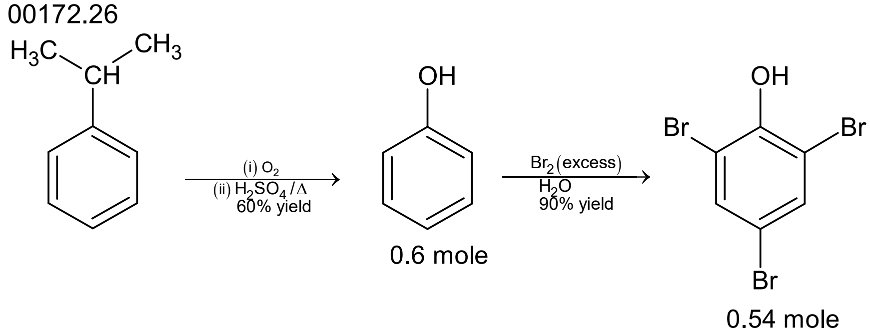
47. 6

Sol. Factual.

SECTION – C

48. 00172.26

Sol.



Mass of 5.4 mole 2,4, 6-Tribromo phenol = 172.26

49. 00028.95

Sol. $\Delta S = nF\left(\frac{dE}{dT}\right)_P = 2 \times 96500 \times 1.5 \times 10^{-4} = 28.95\ J/K - mol$

50. 00000.02

Sol. If P = initial partial pressure of NO_2

$$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$$

$t = 0$	P		
t_{eqbm}	$P - 2x$	$2x$	x

$x = 0.25$ $\therefore 2x = 0.5$

$$K_P = \frac{(2x)^2(x)}{(P-2x)^2} \Rightarrow 156.25 = \frac{(0.5)^2(0.25)}{P_{NO_2}^2}$$

$\therefore P_{NO_2} = 0.02\ atm$

Mathematics PART – III

SECTION – A

51. A

Sol. $T_r = \tan^{-1}\left(\frac{3}{9r^2 - 3r - 1}\right) = \tan^{-1}\left(\frac{(3r+1)-(3r-2)}{1+(3r+1)(3r-2)}\right)$

$$\sum_{r=1}^{n} T_r = \tan^{-1}(3n+1) - \tan^{-1}(1)$$

$$\sum_{r=1}^{\infty} T_r = \tan^{-1}(1)$$

52. A

Sol. $S = \left(\frac{1-\frac{1}{2}}{1-\frac{1}{2}}\right) + \left(\frac{1-\frac{1}{2^2}}{1-\frac{1}{2}}\right)\left(\frac{1}{3}\right) + \left(\frac{1-\frac{1}{2^3}}{1-\frac{1}{2}}\right)\left(\frac{1}{3^2}\right) + ...$

$= 2\left\{\left(1+\frac{1}{3}+\frac{1}{3^2}+.....\right) - \frac{1}{2}\left(1+\frac{1}{2\cdot 3}+\frac{1}{(2\cdot 3)^2}+.....\right)\right\}$

$= 2\left\{\frac{1}{\left(1-\frac{1}{3}\right)} - \frac{1}{2}\cdot\frac{1}{\left(1-\frac{1}{6}\right)}\right\} = 2\left\{\frac{3}{2} - \frac{1}{2}\times\frac{6}{5}\right\} = 3 - \frac{6}{5} = \frac{9}{5}$

53. C

Sol. Both (a, b) and (b, a) lie on circle

54. D

Sol. $f(x) + f(1 - x) = 1$

55. C

Sol. Let number of red, white, blue and green balls are a, b, c and d respectively

$\therefore$ a + b + c + d = 18

a, b, c, d $\geq$ 2

Required number of way = ${}^{13}C_3 = 286$

56. C

Sol. Let, x + y + z = α and λ = 5050

cos x + cos y + cos z = λ cos α

sin x + sin y + sin z = λ sin α

cos(x + y) + cos(y + z) + cos(z + x) = cos(α – z) + cos(α – x) + cos(α – y)

= cos α(λ cos α) + sin α(λ sin α) = λ = 5050

57. D

Sol. $\lim\limits_{x\to 1} \frac{f'(1+x^3-x)(3x^2-1) - f'(x)}{1} = \lim\limits_{x\to 0} \frac{-f'(1-x)-0}{1} + 10$

$\Rightarrow f'(1) = 5$

58. D

Sol. Equation of line is $\frac{x-6}{\cos\theta} = \frac{y-8}{\sin\theta} = r$

$\therefore$ $2(r \cos \theta + 6)^2 + (r \sin \theta + 8)^2 = 2$

$\Rightarrow (2 \cos^2 \theta + \sin^2 \theta)r^2 + 2(12 \cos \theta + 8 \sin \theta)r + 134 = 0$

AB and AC are roots

AB, AP, AC are in H.P.

$\frac{2}{AP} = \frac{AB+AC}{AB \cdot AC} \Rightarrow \frac{1}{r} = \frac{-(6\cos\theta + 4\sin\theta)}{67}$

$\Rightarrow -6r \cos \theta - 4r \sin \theta = 67 \Rightarrow -6(x-6) - 4(y-8) - 67 = 0$

$\Rightarrow 6(x-6) + 4(y-8) + 67 = 0 \Rightarrow 6x + 4y - 1 = 0$

Required distance = $\frac{1}{\sqrt{52}}$

59. D

Sol. Let $\lambda = \frac{225}{h^2} + \frac{25}{k^2}$

$h = 5 \cos \theta$, $k = 5 \sin \theta$

$\lambda = 9 \sec^2 \theta + \operatorname{cosec}^2 \theta$

$\lambda_{min} = 16$

60. B

Sol. $(a-1)^2 + (b-1)^2 + (c-2)^2 = 0$

$a = b = 1$, $c = 2$

$x^2 + x + 2 = 0$

($D < 0$, therefore both the roots are common)

$\therefore \frac{2p}{1} = \frac{q}{1} = \frac{4r}{2} = \lambda(\text{say})$

$p = \frac{\lambda}{2}$, $q = \lambda$

$r = \frac{\lambda}{2}$

$2p + q + 3r = \frac{7}{2}\lambda$

$(2p + q + 3r)_{min} = \frac{7}{2} \times 2 = 7$

61. D

Sol. Clearly $C = 90°$, $A = 30°$, $B = 60°$

$\frac{b-a}{b+a} = \frac{\sin B - \sin A}{\sin B + \sin A} = 2 - \sqrt{3}$

$\left(\left(\frac{b-a}{b+a}\right)x + \sqrt{3}y\right)^{10} = \left((2-\sqrt{3})x + \sqrt{3}y\right)^{10}$

$x = y = 1$

$\left(2 - \sqrt{3} + \sqrt{3}\right)^{10} = 2^{10}$

62. B

Sol. $(x - x_0)(x^2 - qx + 2) = x^3 + x^2 - px + q$

$-x_0 - q = 1$

$2 + qx_0 = -p$

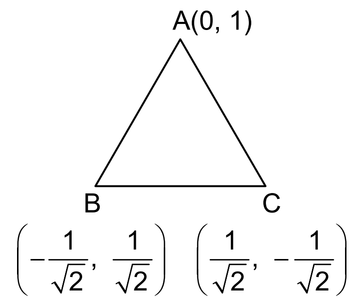
$-2x_0 = q$
$x_0 = 1$
$q = -2$
$p = 0$
$|p - q + x_0| = |0 + 2 + 1| = 3$

63. B
Sol. $|\text{adj}(\text{adj } A)| = |\text{adj } A|^2 = (2)^2 = 4$

64. A
Sol. $(z^2 + i)(iz + 1) = 0$

$z = i, \sqrt{-i}, -\sqrt{-i}$

Area $= \frac{1}{2} \times 2 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$



65. C
Sol. $f(g(x)) = x$ and $g(5) = 1$

$$g'(x) = \frac{1}{f'(g(x))} \Rightarrow g'(5) = \frac{1}{f'(1)} = \frac{1}{4}$$

$$g''(x) = \frac{-f''(g(x)) \cdot g'(x)}{\left(f'(g(x))\right)^2}$$

$$g''(5) = \frac{-f''(1) \cdot \frac{1}{4}}{\left(f'(1)\right)^2} = -\frac{3}{32}$$

66. A
Sol. $e^{-\frac{x^2}{2}} > e^{-x^2}$. I_4 is largest

67. D
Sol. $\alpha = 3 \cos \theta, \gamma = 3 \sin \theta, \beta = \cos \phi, \delta = \sin \phi$
$3(\alpha\beta - \gamma\delta) + 4(\alpha\delta + \beta\gamma) = 3(3 \cos \theta \cdot \cos \phi - 3 \sin \theta \cdot \sin \phi) + 4(3 \cos \theta \cdot \sin \phi + \cos \phi \cdot 3 \sin \theta)$
$= 3(3 \cos (\theta + \phi) + 4 \sin(\theta + \phi))$

68. C
Sol. $\frac{1}{A-1} + \frac{y'}{A+1} = 0$, $A = \frac{y'-1}{y'+1}$

69. B
Sol. $\bar{x} = \frac{1+2+3+4+5}{5} = 3$

$$\sum (x - \bar{x})^2 = 4 + 1 + 0 + 1 + 4 = 10$$

Standard deviation $= \sqrt{\frac{\sum (x-\bar{x})^2}{N}} = \sqrt{\frac{10}{5}} = \sqrt{2}$

70. B

Sol. Volume = $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 3\alpha\beta\gamma - \alpha^3 - \beta^3 - \gamma^3$

$|v| = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$

$= 9(81 - 3 \times 15) = 324$

SECTION – B

71. 2

Sol. $\because\ x^3 + 2x^2 + 2x + 1 = (x + 1)(x^2 + x + 1) = 0$

$x = -1, \omega, \omega^2$

72. 0

Sol. Second curve is parabola with focus (3, –3) and directrix $x - y - 2 = 0$ equation of axis is $x + y = 0$ and foot of directrix (1, –1)

Vertex = (2, –2)

$\therefore$ Shortest distance = 0

SECTION – C

73. 00005.00

Sol. Clearly foci are (1, 2) and (5, 5) distance between foci = 5 and transverse axis = 3

$\therefore$ Eccentricity of hyperbola $e = \frac{5}{3}$

$\therefore\ \frac{1}{e^2} + \frac{1}{(e')^2} = 1 \Rightarrow \frac{9}{25} + \frac{1}{(e')^2} = 1 \Rightarrow \frac{1}{(e')^2} = 1 - \frac{9}{25} = \frac{16}{25}$

$e' = \frac{5}{4} \Rightarrow 4e' = 5$

74. 00000.50

Sol. $\lim\limits_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{\left(\frac{2}{x}\right)} = e^{\lim\limits_{x\to 0}\left(\frac{2}{x}\right)\left(\frac{a^x+b^x}{2}-1\right)} = e^{\ln a + \ln b} = ab = 6$

(a, b) can be {(1, 6), (6, 1), (2, 3), (3, 2)}

$p = \frac{4}{36} = \frac{1}{9}$; $\frac{9}{2}p = \frac{1}{2} = 0.5$

75. 00001.60

Sol. $2g(x) \cdot g'(x) = g(x) \cdot \frac{2\sec^2 x}{4 + \tan x} \Rightarrow \int g'(x)dx = \int \frac{\sec^2 x}{4 + \tan x}dx$

$\Rightarrow g(x) = \ln(4 + \tan x) + c; 0 = \ln 4 + c \Rightarrow c = -\ln 4$

$g(x) = \ln(4 + \tan x) - \ln 4 = \ln\left(1 + \left(\frac{\tan x}{4}\right)\right)$

$g\left(\frac{\pi}{4}\right) = \ln\left(\frac{5}{4}\right)$