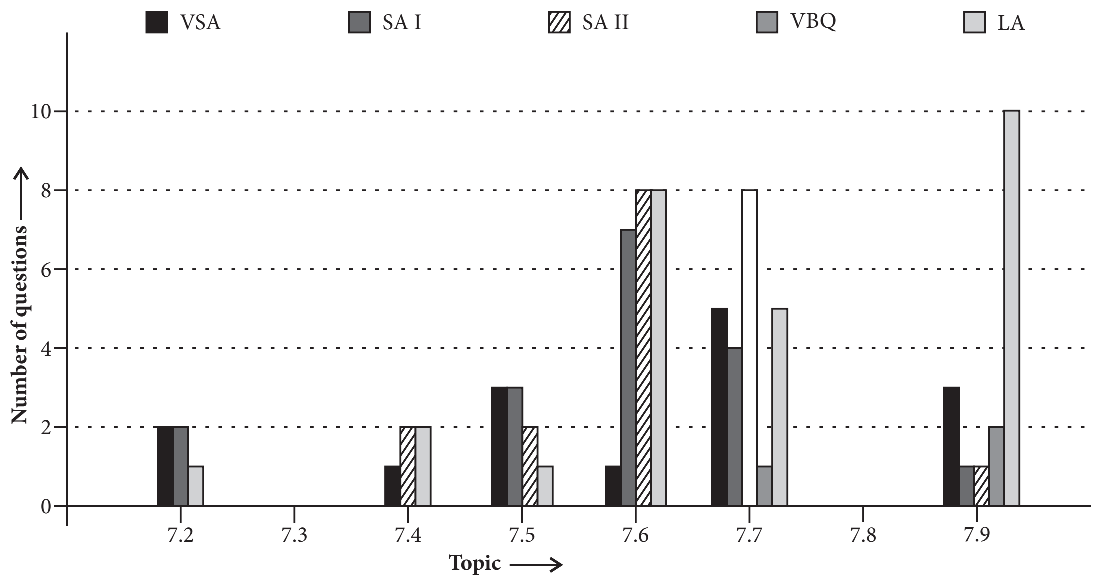
07 Alternating Current

Topicwise Analysis of Last 10 Years' CBSE Board Questions



- Maximum weightage is of *AC Voltage Applied to a Series LCR Circuit.*
- Maximum VSA type questions were asked from *Power in AC Circuit : The power factor*
- Maximum SA I and SA II type questions were asked from *AC Voltage Applied to a Series LCR Circuit.*
- Maximum LA type questions were asked from *Transformer.*

QUICK RECAP

- **Alternating current :** It is that current which changes continuously in magnitude and periodically in direction. It can be represented by a sine curve or a cosine curve

$$I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

Here, I_0 is peak value of current and is known as amplitude of ac, I is instantaneous value of alternating current. $\omega = 2\pi/T = 2\pi\upsilon$ where T is period of ac and υ is frequency of ac

- **Mean or average value of alternating current or voltage over one complete cycle :** The mean or average value of alternating current or voltage over one complete cycle is zero.

$$I_m \text{ or } \bar{I} \text{ or } I_{av} = \frac{\int_0^T I_0 \sin\omega t\, dt}{\int_0^T dt} = 0$$

$$V_m \text{ or } \bar{V} \text{ or } V_{av} = \frac{\int_0^T V_0 \sin\omega t\, dt}{\int_0^T dt} = 0$$

- **Mean value or average value of alternating current over any half cycle :** It is that value of steady current, which would send the same amount of charge through a circuit in the time of half cycle *i.e.* $T/2$ as is sent by ac through the same circuit in the same time.

$$I_{av} = \frac{2I_0}{\pi} = 0.637\, I_0$$

Similarly, for alternating voltage

$$V_{av} = \frac{2I_0}{\pi} = 0.637\, V_0$$

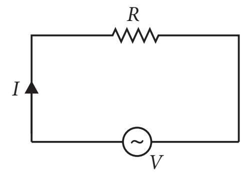
- **Root mean square (rms) value of alternating current :** It is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating current, when passed through the same resistance for the same time. The rms value of ac is also known as effective value or virtual value of ac. It is represented by I_{rms}, I_{eff} or I_v.

$$I_{rms} \text{ or } I_v = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

Similarly, for alternating voltage

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

- **AC circuit containing pure resistance only :**

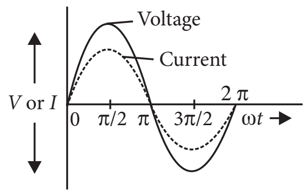


Let $V = V_0 \sin\omega t$

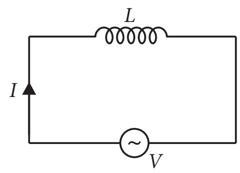
Then, $I = \frac{V}{R} = \frac{V_0}{R} \sin\omega t = I_0 \sin\omega t$

Here the alternating voltage is in phase with current, when ac flows through a resistor.

The instantaneous value of alternating current and alternating voltage in the circuit containing R, follow that both are in same phase as shown in figure.



- **AC circuit containing pure inductor only :**

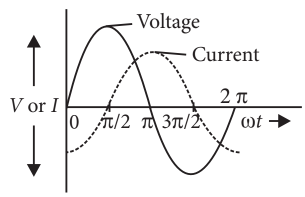


Let $V = V_0 \sin\omega t$

Then, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

where $I_0 = \frac{V_0}{\omega L}$

Thus, the alternating current lags behind the alternating voltage by a phase angle of $\frac{\pi}{2}$ when ac flows through an inductor as shown in figure.

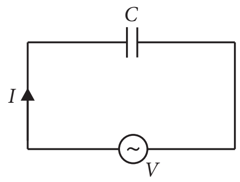


- **Inductive reactance :** It is the opposition offered by the inductor to the flow of alternating current through it.

$X_L = \omega L = 2\pi\upsilon L$

The inductive reactance is zero for dc ($\upsilon = 0$) and has a finite value for ac.

AC circuit containing pure capacitor only :

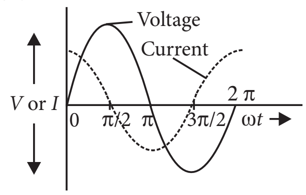


Let $V = V_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $I_0 = (\omega C)V_0$.

Thus, the alternating current leads the voltage by a phase angle of $\frac{\pi}{2}$, when ac flows through a capacitor.

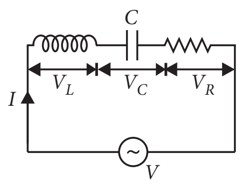


- **Capacitive reactance :** It is the opposition offered by the capacitor to the flow of alternating current through it.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

The capacitive reactance is infinite for dc ($\upsilon = 0$) and has a finite value for ac.

AC through *LCR*-series circuit :

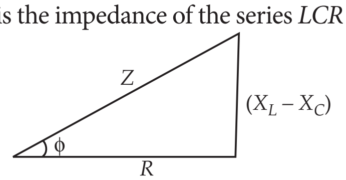


Let $V = V_0 \sin\omega t$

Then, $I = I_0 \sin(\omega t - \phi)$

where $I_0 = \frac{V_0}{Z}$

Here Z is the impedance of the series *LCR* circuit.



$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

- The alternating current lags behind the voltage by a phase angle ϕ

$$\tan\phi = \frac{X_L - X_C}{R}$$

  – When $X_L > X_C$, $\tan\phi$ is positive. Therefore, ϕ is positive. Hence current lags behind the voltage by a phase angle ϕ. The ac circuit is inductance dominated circuit.
  – When $X_L < X_C$, $\tan\phi$ is negative. Therefore, ϕ is negative. Hence current leads the voltage by a phase angle ϕ. The ac circuit is capacitance dominated circuit.

Resonant series *LCR* circuit : When the frequency of ac supply is such that the inductive reactance and capacitive reactance become equal ($X_L = X_C$), the impedance of the series *LCR* circuit is equal to the ohmic resistance in the circuit. As such, the current in the circuit becomes maximum. Such a series *LCR* circuit is known as resonant series *LCR* circuit and the frequency of the ac supply is known as resonant frequency (υ_r). The resonant frequency is

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}, \quad \omega_r = \frac{1}{\sqrt{LC}}$$

- The series resonance circuit is known as acceptor circuit. It is used in radio and TV receivers sets for tuning a particular radio station/TV channel.
- Resonance phenomenon is exhibited by a circuit only if both *L* and *C* are present in the circuit. Then only voltages across *L* and *C* cancel each other. We cannot have resonance in a *RL* or *RC* circuit.
- **Quality factor :** It is a measure of sharpness of resonance. It is defined as the ratio of reactance of either the inductance or capacitance at the resonant angular frequency to the total resistance of the circuit.

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$Q = \frac{X_C}{R} = \frac{1}{\omega_r CR}$$

$$\therefore \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Quality factor is also expressed in terms of bandwidth

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

- **Power in an ac circuit :** In an ac circuit we may define three types of power.
- **Instantaneous power :** The power in the ac circuit at any instant of time is known as instantaneous power. It is equal to the product of values of alternating voltage and alternating current at that time.
- **Average power (P_{av}) :** The power averaged over one full cycle of ac is known as average power. It is also known as true power or average power.

$$P_{av} = V_{rms}\, I_{rms} \cos\phi = \frac{V_0 I_0}{2}\cos\phi$$

- **Apparent power :** The product of virtual voltage (V_{rms}) and virtual current (I_{rms}) in the circuit is known as virtual power or apparent power.

$$P_v = V_{rms}\, I_{rms} = \frac{V_0 I_0}{2}$$

- **Power factor :** It is defined as the ratio of true power to apparent power of an ac circuit

$$\cos\phi = \frac{\text{True power}}{\text{Apparent power}}$$

It is also defined as the ratio of the resistance to the impedance of an ac circuit

$$\cos\phi = \frac{R}{Z}$$

It is unitless and dimensionless quantity.

– In pure resistive circuit,
$\phi = 0°$; $\cos\phi = 1$.

– In pure inductive or capacitive circuit

$$\phi = \frac{\pi}{2};\ \cos\phi = 0.$$

– In RL circuit,

$$Z = \sqrt{R^2 + X_L^2} \text{ and } \cos\phi = \frac{R}{Z}$$

– In RC circuit,

$$Z = \sqrt{R^2 + X_C^2} \text{ and } \cos\phi = \frac{R}{Z}$$

– In series LCR circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \text{ and } \cos\phi = \frac{R}{Z}$$

– At resonance, $X_L = X_C$
$\therefore\ Z = R$ and $\phi = 0°$
$\cos\phi = 1$

- **Wattless current :** The average power associated over a complete cycle with a pure inductor or pure capacitor is zero, even though a current is flowing through them. This current is known as the wattless current or idle current.
- **LC-Oscillations :** When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced, these oscillations are called LC-oscillations.
- **Transformer :** It is a device used for converting a low alternating voltage to a high alternating voltage and vice versa. It is based on phenomenon of mutual induction.

For ideal transformer, $\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = k.$

where k is called transformation ratio.

- For a step-up transformer, $k > 1$. *i.e.* $V_S > V_P$, $I_S < I_P$ and $N_S > N_P$.
- For a step-down transformer, $k < 1$. *i.e.* $V_S < V_P$, $I_S > I_P$ and $N_S < N_P$.
- Efficiency of a transformer,

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_S I_S}{V_P I_P}.$$

Previous Years' CBSE Board Questions

7.2 AC Voltage Applied to a Resistor

VSA (1 mark)

1. The peak value of emf in ac is E_0. Write its (i) rms (ii) average value over a complete cycle. *(Foreign 2011)*

2. Define the term 'rms value of the current'. How is it related to the peak value? *(AI 2010C)*

SA I (2 marks)

3. An alternating voltage given by $V = 140 \sin 314t$ is connected across a pure resistor of 50 Ω. Find
(i) the frequency of the source.
(ii) the rms current through the resistor. *(AI 2012)*

4. Distinguish between the term 'average value' and 'rms value' of an alternating current. The instantaneous current from an ac source is $I = 5 \sin (314t)$ ampere. What are the average and rms values of the current? *(Delhi 2007)*

LA (5 marks)

5. Derive the relationship between the peak and the rms value of current in ac circuit. *(3/5, Delhi 2009)*

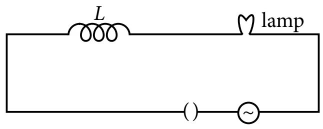
7.4 AC Voltage Applied to an Inductor

VSA (1 mark)

6. When an ac source is connected across an inductor, show on a graph the nature of variation of the voltage and the current over one complete cycle. *(Delhi 2012C)*

SA II (3 marks)

7. (i) When an AC source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero.
(ii) A lamp is connected in series with an inductor and an AC source. What happens to the brightness of the lamp when the key is plugged in and an iron rod is inserted inside the inductor? Explain.



(AI 2016)

8. An ac voltage, $V = V_0 \sin \omega t$, is applied across a pure inductor L. Obtain an expression for the current I in the circuit and hence obtain the
(i) inductive reactance of the circuit, and
(ii) the 'phase', of the current flowing, with respect to the applied voltage. *(AI 2010C)*

LA (5 marks)

9. Show that in an ac circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase. *(2/5, AI 2011)*

10. (i) Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage.
(ii) An ac voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance L. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\pi/2$. *(AI 2007)*

7.5 AC Voltage Applied to a Capacitor

VSA (1 mark)

11. Define capacitive reactance. Write its S.I. units. *(Delhi 2015)*

12. Plot a graph showing variation of capacitive reactance with the change in the frequency of the ac source. *(AI 2015C)*

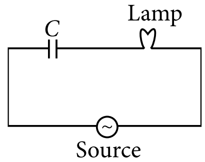
13. A reactive element, in an a.c. circuit, causes the current flowing (i) to lead in phase by $\pi/2$, (ii) to lag in phase by $\pi/2$ with respect to the applied voltage. Identify the element in each case. *(Delhi 2010C)*

SA I (2 marks)

14. Show that the current leads the voltage in phase by $\pi/2$ in an ac circuit containing an ideal capacitor. *(Foreign 2014)*

15. A lamp is connected in series with a capacitor. Predict your observation when this combination is connected in turn across (i) ac source and (ii) a dc battery. What change would you notice in each case if the capacitance of the capacitor is increased? *(Delhi 2012C)*

16. An electric lamp having coil of negligible inductance connected in series with a capacitor and an ac source is glowing with certain brightness. How does the brightness of the lamp change on reducing the (i) capacitance, and (ii) the frequency? Justify your answer.

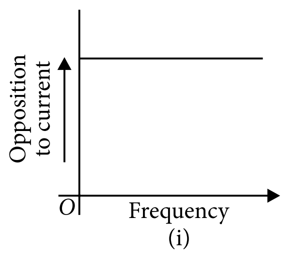


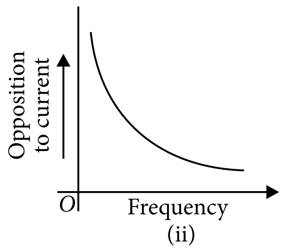
(Delhi 2010)

SA II (3 marks)

17. A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a dc and then an ac source. What happens in each case if the capacitance of the capacitor is reduced? *(Delhi 2013C)*

18. The graphs (i) and (ii) shown in the figure represent variation of opposition offered by the circuit elements, X and Y, respectively to the flow of alternating current *vs* the frequency of the applied emf. Identity the elements X and Y.





(2/3, AI 2012C)

LA (5 marks)

19. Explain the term 'capacitive reactance'. Show graphically the variation of capacitive reactance with frequency of the applied alternating voltage.
An ac voltage $E = E_0 \sin \omega t$ is applied across a pure capacitor of capacitance C. show mathematically that the current flowing through it leads the applied voltage by a phase angle of $\pi/2$. *(AI 2007)*

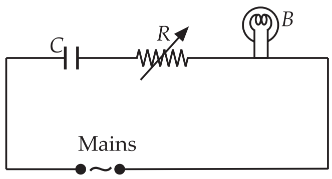
7.6 AC Voltage Applied to a Series *LCR* Circuit

VSA (1 mark)

20. In a series LCR circuit, the voltages across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is the phase difference between the applied voltage and the current in the circuit? *(AI 2007)*

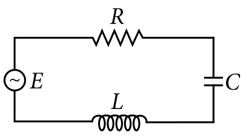
SA I (2 marks)

21. A capacitor 'C', a variable resistor 'R' and a bulb 'B' are connected in series to the ac mains in circuit as shown. The bulb glows with some brightness.



How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be the same; (ii) the resistance R is increased keeping the same capacitance? *(Delhi 2014)*

22. The figure shows a series *LCR* circuit connected to a variable frequency 200 V source with $L = 50$ mH, $C = 80$ μF and $R = 40$ Ω.
Determine
(i) the source frequency which derives the circuit in resonance;
(ii) the quality factor (*Q*) of the circuit.



(AI 2014C)

23. A series *LCR* circuit is connected to an ac source (200 V, 50 Hz). The voltages across the resistor, capacitor and inductor are respectively 200 V, 250 V and 250 V.
(i) The algebraic sum of the voltages across the three elements is greater than the voltage of the source. How is this paradox resolved?
(ii) Given the value of the resistance of *R* is 40 Ω, calculate the current in the circuit. *(Foreign 2013)*

24. Calculate the quality factor of a series LCR circuit with $L = 2.0$ H, $C = 2$ μF and $R = 10$ Ω. Mention the significance of quality factor in *LCR* circuit. *(Foreign 2012)*

25. Write the expression for the impedance offered by the series combination of resistor, inductor and capacitor connected to an ac source of voltage $V = V_0 \sin \omega t$.

Show on a graph the variation of the voltage and the current with 'ωt' in the circuit.

(AI 2012C)

26. Derive an expression for the impedance of an ac circuit consisting of an inductor and a resistor.

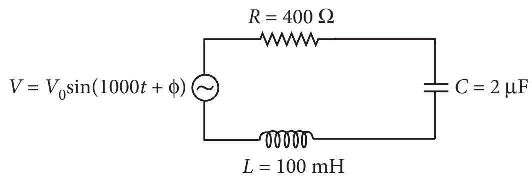
(Delhi 2008)

27. An ac voltage of 100 V, 50 Hz is connected across a 20 ohm resistor and 2 mH inductor in series. Calculate (i) impedance of the circuit, (ii) rms current in the circuit. *(AI 2007)*

SA II (3 marks)

28. An inductor *L* of inductive reactance X_L is connected in series with a bulb *B* and an ac source. How would brightness of the bulb change when (i) number of turn in the inductor is reduced, (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your answer in each case. *(Delhi 2015)*

29. Determine the value of phase difference between the current and the voltage in the given series *LCR* circuit.

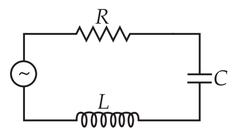


(2/3, AI 2015)

30. A source of ac voltage $V = V_0 \sin\omega t$ is connected to a series combination of a resistor '*R*' and a capacitor '*C*'. Draw the phasor diagram and use it to obtain the expression for (i) impedance of the circuit and (ii) phase angle. *(AI 2015C)*

31. In a series *LCR* circuit connected to an ac source of variable frequency and voltage $V = V_m \sin\omega t$, draw a plot showing the variation of current (*I*) with angular frequency (ω) for two different values of resistance R_1 and $R_2 (R_1 > R_2)$. Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define *Q*-factor of the circuit and give its significance. *(Delhi 2013)*

32. The figure shows a series *LCR* circuit with $L = 10.0$ H, $C = 40$ μF, $R = 60$ Ω connected to a variable frequency 240 V source, calculate
(i) the angular frequency of the source which drives the circuit at resonance,
(ii) the current at the resonating frequency,
(iii) the rms potential drop across the inductor at resonance.



(Delhi 2012)

33. A series *LCR* circuit is connected to an ac source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation. *(AI 2012)*

34. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 100 V, 60 Hz supply. Calculate (i) the reactance, (ii) the impedance and (iii) maximum current in the circuit. *(AI 2011C, AI 2008)*

35. An inductor 200 mH, capacitor 500 μF, resistor 10 Ω are connected in series with a 100 V variable frequency a.c. source. Calculate the
(i) frequency at which the power factor of the circuit is unity
(ii) current amplitude at this frequency
(iii) *Q*-factor *(Delhi 2008)*

LA (5 marks)

36. An ac source of voltage $V = V_0 \sin\omega t$ is connected to a series combination of *L*, *C* and *R*. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called ? *(3/5, Delhi 2016)*

37. A 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an ac source

(i) What should be the frequency of the source such that current drawn in the circuit is maximum? What is this frequency called?

(ii) If the peak value of emf of the source is 200 V, find the maximum current.

(iii) Draw a graph showing variation of amplitude of circuit current with changing frequency of applied voltage in a series *LCR* circuit for two different values of resistance R_1 and R_2 ($R_1 > R_2$).

(iv) Define the term 'Sharpness of Resonance'. Under what condition, does a circuit become more selective? *(Foreign 2016)*

38. (a) A series *LCR* circuit is connected to an ac source of variable frequency. Draw a suitable phasor diagram to deduce the expressions for the amplitude of the current and phase angle.

(b) Obtain the condition of resonance. Draw a plot showing the variation of current with the frequency of a.c. source for two resistances R_1 and R_2 ($R_1 > R_2$). Hence define the quality factor, *Q* and write its role in the tuning of the circuit. *(Delhi 2014C)*

39. (a) Using phasor diagram for a series *LCR* circuit connected to an ac source of voltage $V = V_0 \sin \omega t$, derive the relation for the current flowing in the circuit and the phase angle between the voltage across the resistor and the net voltage in the circuit.
(b) Draw a plot showing the variation of the current *I* as a function of angular frequency 'ω' of the applied ac source for the two cases of a series combination of (i) inductance L_1, capacitance C_1 and and resistance R_1 and (ii) inductance L_2, capacitance C_2 and resistance R_2 where $R_2 > R_1$. Write the relation between L_1, C_1 and L_2, C_2 at resonance. Which one, of the two, would be better suited for fine tuning in a receiver set? Give reason. *(Foreign 2013)*

40. (a) An ac source of voltage $V = V_0 \sin \omega t$ is connected across a series combination of an inductor, a capacitor and a resistor. Use the phasor diagram to obtain the expression for (i) impedance of the circuit and (ii) phase angle between the voltage and the current.
(b) A capacitor of unknown capacitance, a resistor of 100 Ω and an inductor of self inductance $L = (4/\pi^2)$ henry are in series connected to an ac source of 200 V and 50 Hz, Calculate the value of the capacitance and the current that flows in the circuit when the current is in phase with the voltage. *(AI 2013C)*

41. Derive an expression for the impedance of a series *LCR* circuit connected to an ac supply of variable frequency.
Plot a graph showing variation of current with the frequency of the applied voltage.
Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set. *(Delhi 2011)*

42. Explain
(i) Resistance,
(ii) Reactance and
(iii) Impedance *(Delhi 2011C)*

43. An ac source generating a voltage $V = V_m \sin \omega t$ is connected to a capacitor of capacitance *C*.

Find the expression for the current i, flowing through it. Plot a graph of v and i versus ωt to show that the current is $\pi/2$ ahead of the voltage.

A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source. Calculate the current in the circuit and the rms voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage ? If yes, resolve the paradox.

(AI 2008)

7.7 Power in AC Circuit: The Power Factor

VSA (1 mark)

44. Define 'quality factor' of resonance in series *LCR* circuit. What is its SI unit? *(Delhi 2016)*

45. The power factor of an ac circuit is 0.5. What is the phase difference between voltage and current the circuit? *(Foreign 2016)*

46. Why is the use of ac voltage preferred over dc voltage? Give two reasons. *(AI 2014)*

47. Define the term wattless current. *(Delhi 2011)*

48. The instantaneous current and voltage of an ac circuit are given by
$i = 10 \sin 300\, t$ A and $V = 200 \sin 300\, t$ V. What is the power dissipation in the circuit?

(AI 2008)

SA I (2 marks)

49. In series *LCR* circuit obtain the conditions under which (i) the impedance of the circuit is minimum and (ii) wattless current flows in the circuit. *(Foreign 2014)*

50. A resistor '*R*' and an element '*X*' are connected in series to an ac source of voltage. The voltage is found to lead the current in phase by $\pi/4$. If '*X*' is replaced by another element '*Y*', the voltage lags behind the current by $\pi/4$.

(i) Identify elements '*X*' and '*Y*'.

(ii) When both '*X*' and '*Y*' are connected in series with '*R*' to the same source, will the power dissipated in the circuit be maximum or minimum? Justify your answer. *(Foreign 2013)*

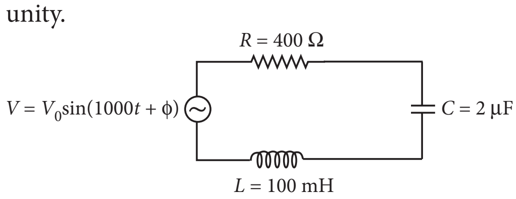
51. A light bulb is rated 100 W for 220 V ac supply of 50 Hz. Calculate
(i) the resistance of the bulb;
(ii) the rms current through the bulb.

(AI 2012)

52. Prove that an ideal capacitor, in an ac circuit does not dissipate power. *(Delhi 2008)*

SA II (3 marks)

53. Calculate the value of the additional capacitor which may be joined suitably to the capacitor *C* that would make the power factor of the circuit unity.



(1/3, AI 2015)

54. A circuit containing an 80 mH inductor and a 250 mF capacitor in series connected to a 240 V, 100 rad/s supply. The resistance of the circuit is negligible.

(i) Obtain rms value of current.

(ii) What is the total average power consumed by the circuit? *(Delhi 2015C)*

55. A voltage $V = V_0 \sin\omega t$ is applied to a series *LCR* circuit. Derive the expression for the average power dissipated over a cycle. Under what condition is
(i) no power dissipated even though the current flows through the circuit,
(ii) maximum power dissipated in the circuit?

(AI 2014)

56. (a) For a given ac $i = i_m \sin\omega t$, show that the average power dissipated in a resistor *R* over a complete cycle is $\frac{1}{2}i_m^2 R$.

(b) A light bulb is rated at 100 W for a 220 V ac supply. Calculate the resistance of the bulb.

(AI 2013)

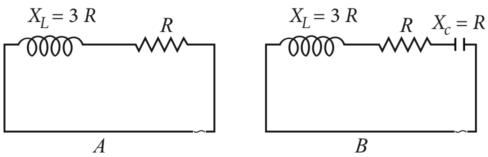
57. When an ac source is connected to an ideal capacitor show that the average power supplied by the source over a complete cycle is zero.

(2/3, Delhi 2013C)

58. Prove that an ideal capacitor, in an ac circuit does not dissipate power. *(AI 2011)*

59. A series *LCR* circuit is connected to a 220 V variable frequency (ac) supply. If $L = 10$ mH, $C = \left(\frac{400}{\pi^2}\right)\mu\text{F}$ and $R = 55\ \Omega$.
(a) Find the frequency of the source, for which the average power absorbed by the circuit is maximum.
(b) Calculate the value of maximum current amplitude. *(Delhi 2010C)*

60. Given below are two electric circuits *A* and *B*



Calculate the ratio of power factor of the circuit *B* to the power factor of circuit *A*.
(Delhi 2007)

VBQ (4 marks)

61. Sushil is in the habit of charging his mobile and then leaving the charger connected through the mains with the switch on. When his sister Asha pointed it out him, he replied there was no harm as the mobile had been disconnected. Asha then explained to him and convinced him, how the energy was still being wasted as the charger was continuously consuming energy. Answer the following questions.
(a) What values did Asha display in convincing her brother?
(b) What measures in your view, should be adopted to minimise the wastage of electric energy in your households?
(c) Imagine an electric appliance of 2 W, left connected to the mains for 20 hours. Estimate the amount of electrical energy wasted.
(Foreign 2015)

LA (5 marks)

62. In series *LR* circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance *C* such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate P_1/P_2.
(2/5 Delhi 2016)

63. A voltage $V = V_0 \sin \omega t$ applied to a series *LCR* circuit drives a current $i = i_0 \sin \omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.
For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.
Define the term 'Wattless current'.
(Delhi 2012C)

64. (a). An alternating voltage $V = V_m \sin \omega t$ applied to a series *LCR* circuit drives a current given by $i = i_m \sin (\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.
(b) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.
(c) Determine the current and quality factor at resonance for a series *LCR* circuit with $L = 1.00$ mH, 1.00 nF and $R = 100\ \Omega$ connected to an ac source having peak voltage of 100 V.
(Foreign 2011)

65. A series *LCR* circuit is connected to an ac source having voltage $V = V_m \sin\omega t$. Derive the expression for the instantaneous current *I* and its phase relationship to the applied voltage. Obtain the condition for resonance to occur. Define 'power factor'. State the conditions under which it is (i) maximum and (ii) minimum.
(Delhi 2010)

66. (a) Derive an expression for the average power consumed in a series *LCR* circuit connected to ac source in which the phase difference between the voltage and the current in the circuit is ϕ.
(b) Define the quality factor in an ac circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends. *(Delhi 2009)*

7.9 Transformers

VSA (1 mark)

67. Why is the core of a transformer laminated ?
(Delhi 2013C)

68. Mention the two characteristic properties of the material suitable for making core of a transformer. *(AI 2012)*

69. What is the function of a step-up transformer ? *(AI 2011C)*

SA I (2 marks)

70. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers? *(AI 2012)*

SA II (3 marks)

71. Explain with the help of a labelled diagram the underlying principle and working of a step-up transformer. Why cannot such a device be used to step-up dc voltage ? *(Delhi 2007)*

VBQ (4 marks)

72. A group of students while coming from the school noticed a box marked "Danger H.T. 2200 V" at a substation in the main street. They did not understand the utility of a such a high voltage, while they argued, the supply was only 220 V. They asked their teacher this question the next day. The teacher thought it to be an important question and therefore explained to the whole class.
Answer the following questions :
(i) What device is used to bring the high voltage down to low voltage of ac current and what is the principle of its working?
(ii) Is it possible to use this device for bringing down the high dc voltage to the low voltage? Explain.
(iii) Write the values displayed by the students and the teacher. *(Delhi 2015)*

73. One morning an old man walked bare-foot to replace the fuse wire in kit kat fitted with the power supply mains for his house. Suddenly he screamed and collapsed on the floor. His wife cried loudly for help. His neighbour's son Anil heard the cries and rushed to the place with shoes on. He took a wooden baton and used it to switch off the main supply.
Answer the following questions:
(i) What is the voltage and frequency of mains supply in India?
(ii) These days most of the electrical devices we use require ac voltage. Why?
(iii) Can a transformer be used to step up dc voltage?
(iv) Write two qualities displayed by Anil by his action. *(AI 2015)*

LA (5 marks)

74. (i) Draw a labelled diagram of a step-down transformer. State the principle of its working.
(ii) Express the turn ratio in terms of voltages.
(iii) Find the ratio of primary and secondary currents in terms of turn ratio in an ideal transformer.
(iv) How much current is drawn by the primary of a transformer connected to 220 V supply when it delivers power to a 110 V – 550 W refrigerator ? *(AI 2016)*

75. (i) Write the function of a transformer. State its principle of working with the help of a diagram. Mention various energy losses in this device.
(ii) The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are respectively 220 V and 1100 W. Calculate
(a) number of turns in secondary
(b) current in primary
(c) voltage across secondary
(d) current in secondary
(e) power in secondary *(Delhi 2016)*

76. (a) Draw a schematic arrangement for winding of primary and secondary coil in a transformer when the two coils are would on top of each other.
(b) State the underlying principle of a transformer and obtain the expression for the ratio of secondary to primary voltage in terms of the
(i) number of secondary and primary windings and
(ii) primary and secondary currents.
(c) Write the main assumption involved in deriving the above relations.
(d) Write any two reasons due to which energy losses may occur in actual transformers. *(AI 2014C)*

77. (a) Explain with the help of a labelled diagram, the principle and working of a transformer. deduce the expression for its working formula.
(b) Name any four causes of energy loss in an actual transformer. *(AI 2013C)*

78. (a) State the principle of a step-up transformer. Explain, with the help of a labelled diagram, its working.

(b) Describe briefly any two energy losses, giving the reasons for their occurrence in actual transformers. *(Foreign 2012)*

79. (i) With the help of a labelled diagram, describe briefly the underlying principle and working of a step up transformer.

(ii) Write any two sources of energy loss in a transformer.

(iii) A step up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy? Explain.

(Delhi 2011)

80. A power transmission line feeds power at 2200 V with a current of 5 A to step down transformer with its primary winding having 4000 turns. Calculate the number of turns and the current in the secondary in order to get output power at 220 V. *(2/5, Foreign 2011)*

81. (a) Draw a schematic diagram of a step-up transformer. Explain its working principle.

Assuming the transformer to be 100% efficient, obtain the relation for (i) the current in the secondary in terms of the current in the primary, and (ii) the number of turns in the primary and secondary windings.

(b) Mention two important energy losses in actual transformers and state how these can be minimized. *(Delhi 2011C)*

82. Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils?

How is the transformer used in large scale transmission and distribution of electrical energy over long distances? *(AI 2010)*

83. A step down transformer operates on a 2.5 kV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, calculate:

(i) the power output,

(ii) the voltage, and

(iii) the current in the secondary.

(3/5, Foreign 2010)

84. Describe briefly, with the help of a labelled diagram, working of a step-up transformer.

A step-up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy ? Explain.

(2/5, Delhi 2009)

Detailed Solutions

1. E_0 = peak value of emf

(i) rms value $[E_{rms}] = \frac{E_0}{\sqrt{2}}$

(ii) average value $[E_{av}]$ = zero

2. Root Mean Square value of the current : The root mean square (rms) value of ac is defined as the value of steady current, which when passed through a resistance for a given time would produce the same amount of heat as is produced by the alternating current in the same resistance in same time. It is denoted by I_{rms}.

$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$

3. (i) $2\pi\upsilon = 314$ rad s^{-1} $\Rightarrow \upsilon = 50$ Hz

(ii) $i_{rms} = \frac{V_{rms}}{R}$ where $V_{rms} = \frac{V}{\sqrt{2}}$

$= \frac{140}{\sqrt{2} \times 50} = 1.98\text{A} \approx 2\text{A}$

4. (a) Average value of ac over any half cycle is that value of steady current, which would send the same amount of charge through a circuit in the time of half cycle as is sent by the ac through the same circuit in the same time.

$I_{av} = \frac{2}{\pi} I_m$

Root mean square (rms) value of ac is that value of steady current, which when passed through a resistance for a given time would produce the same amount of heat as is produced by the alternating current in the same resistance in same time. It is denoted by I_{rms}.

$I_{rms} = \frac{I_m}{\sqrt{2}}$

where I_m is peak value of current.

(b) $I = 5 \sin(314t)$ A

$I_m = 5$ A

$I_{av} = \frac{2}{\pi} \times 5 = 3.18$ A

$I_{rms} = \frac{5}{\sqrt{2}} = 3.54$ A

5. Derivation of rms value of current : The instantaneous value of ac passing through a resistance R is given by

$I = I_0 \sin \omega t$

The alternating current changes continuously with time. Suppose that the current through the resistance constant for an infinitesimally small time dt.

The, small amount of heat produced in the resistance R in time dt is given by

$dH = I^2 R\, dt = (I_0 \sin \omega t)^2 R\, dt = I_0^2 R \sin^2 \omega t\, dt$

The amount of heat produced in the resistance in time $T/2$ is

$$H = \int_0^{T/2} I_0^2 R \sin^2 \omega t\, dt = I_0^2 R \int_0^{T/2} \frac{1 - \cos 2\omega t}{2} dt$$

$$H = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{T/2} = \frac{I_0^2 R}{2}\left[\frac{T}{2} - \frac{\sin 2\omega \cdot \frac{T}{2}}{2\omega} - 0\right]$$

$$H = \frac{I_0^2 R}{2}\left[\frac{T}{2} - \frac{\sin 2 \cdot \frac{2\pi}{T} \cdot \frac{T}{2}}{2\omega}\right] = \frac{I_0^2 R}{2}\left[\frac{T}{2} - \frac{\sin 2\pi}{2\omega}\right]$$

$$H = \frac{I_0^2 R}{2} \cdot \frac{T}{2} \qquad \text{...(i)} \quad [\because \sin 2\pi = 0]$$

If I_{rms} be the rms value of ac they by definition

$$H = I_{rms}^2 R \frac{T}{2} \qquad \text{...(ii)}$$

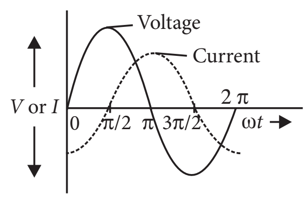
From equation (i) and (ii), we have

$$I_{rms}^2 R \frac{T}{2} = \frac{I_0^2 R}{2} \cdot \frac{T}{2}$$

$$I_{rms}^2 = \frac{I_0^2}{2} \quad \therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0.$$

6. $V = V_0 \sin\omega t$

$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$



7. (i) As $P_{av} = V_{rms} I_{rms} \cos\phi$

In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$.

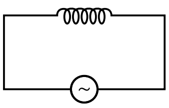
$\therefore \quad \phi = \pi/2 \quad$ so, $\quad P_{av} = V_{rms} I_{rms} \cos \pi/2$

or $\quad P_{av} = V_{rms} I_{rms} \times 0$.

or $\quad P_{av} = 0$.

(ii) Brightness of the lamp decreases. It is because when iron rod is inserted inside the inductor, its inductance L increases, thereby increasing its inductive reactance X_L and hence impedance Z of the circuit. As $I_{rms} = \dfrac{V_{rms}}{Z}$, so, this decreases the current I_{rms} in the circuit and hence the brightness of lamp.

8.



$V = V_0 \sin \omega t$

the instantaneous ac potential difference across the ends of an inductor of inductance.

$V = V_0 \sin \omega t$ ...(i)

If I is the instantaneous current through L at instant t,

$$V = L\frac{dI}{dt} \text{ or } V_0 \sin\omega t = L\frac{dI}{dt}$$

$$\text{or } dI = \frac{V_0}{L}\sin\omega t\, dt$$

Integrating both sides,

$$I = \frac{V_0}{L}\int_0^t \sin\omega t dt = \frac{V_0}{L}\left[\frac{-\cos\omega t}{\omega}\right]_0^t$$

$$\text{or } I = \frac{-V_0}{\omega L}\cos\omega t \text{ or } I = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

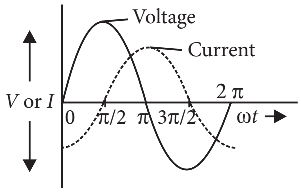
$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

where, $I = \dfrac{V_0}{\omega L}$ is the amplitude of the current.

(i) The quantity ωL in $I = \dfrac{V_0}{\omega L}$ is analogous to the resistance and is called inductive reactance denoted by X_L.

$X_L = \omega L = 2\pi\upsilon L$

(ii) from eqns (i) and (ii), it is clear that, in an ac circuit containing inductance, current lags voltage by $\pi/2$.



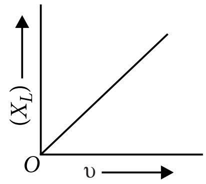
9. *Refer to answer 8(ii).*

10. (i) The measure of the effective resistance or opposition offered by the inductor to flow of ac through it is known as inductive reactance. It is denoted by X_L.

$X_L = \omega L = 2\pi\upsilon L$

where υ is frequency of ac supply. The SI unit of inductive reactance is ohm (Ω). For ac $X_L \propto \upsilon$; for dc, $\upsilon = 0$, so $X_L = 0$

Variation of X_L with frequency : as $X_L \propto \upsilon$, so the graph of X_L versus υ is a straight line with positive slope.



(ii) *Refer to answer 8.*

11. Capacitive reactance is the resistance offered by a capacitor to flow of ac through it. It is denoted by X_C.

Mathematically,

$$X_C = \frac{1}{2\pi\upsilon C}$$

Where υ = frequency of ac source

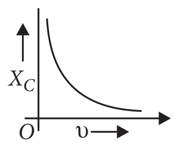
C = capacitance of the capacitor.

Ohm (Ω) is the SI unit of capacitive reactance.

12. Showing variation of capacitive reactance with the change in the frequency of the AC source.

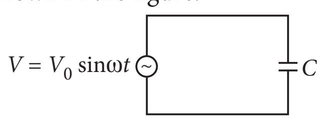
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

$$X_C \propto \frac{1}{\upsilon}$$



13. In case (i), reactive element is capacitor and in case (ii), reactive element is inductor.

14. Let us consider a capacitor C connected to an ac source as shown in the figure.



Let the ac voltage applied be

$V = V_0 \sin\omega t$ ...(i)

$\therefore \quad V = \frac{q}{C}$ or $q = CV$

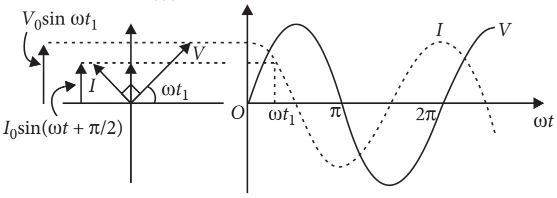
$I = \frac{dq}{dt}$

$I = \frac{d}{dt}(CV_0 \sin \omega t) = \omega C V_0 \cos \omega t = I_0 \cos \omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{...(ii)}$$

where,

$I_0 = \omega C V_0 = \frac{V_0}{\frac{1}{\omega c}}$ = current amplitude.



Hence, the current leads the voltage in phase by $\pi/2$

15. (i) On increasing capacitance, current will increases. It also increases the brightness of bulb.
(ii) There will no flow of current and hence bulb will not glow.

16. When AC source is connected, the capacitor offers capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \upsilon C}$. The current flows in the circuit and the lamp glows.
(i) On reducing capacitance C, X_C increases so current in the circuit reduces. Therefore, the brightness of the bulb reduces.
(ii) On reducing frequency υ, X_C increases so current in the circuit reduces. Therefore, the brightness of the bulb reduces.

17. For dc, capacitor is an open circuit because $X_C = \frac{1}{\omega C} = \infty$, the lamp will not glow at all, even if C is reduced. For ac, the lamp will glow because capacitor conducts ac. If C is reduced, the reactance X_C will increase and the brightness of the lamp will decrease further.

18. (i) Element X is a pure resistor because opposition to current is independent of frequency.
(ii) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \upsilon C}$, graph of X_C and υ is a rectangular hyperbola. Therefore element Y is capacitor.

19. (i) *Refer to answers 11 and 12.*
(ii) *Refer to answer 14.*

20. Phase difference between the applied voltage and the current in the circuit will be zero because $V_L = V_C$, circuit is resistive in nature.

21. For the RC circuit,

Impedance, $Z = \sqrt{R^2 + (1/\omega C)^2}$

$$\text{Current, } I = \frac{\varepsilon_0}{Z} \qquad \text{... (i)}$$

Case I : When a dielectric slab is introduced between the plates of the capacitor, then its capacitance increases. Hence, from equation (i), impedance of the circuit is decreased and the current through it is increased. So, brightness of the bulb will increase.
Case II : The resistance R is increased and capacitance is same. Hence, from equation (i), impedance of the circuit is increased and the current flowing through it is decreased. So, brightness of the bulb will decrease.

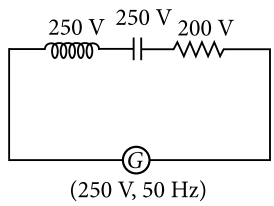
22. (i) $L = 50 \times 10^{-3}$ H, $C = 80 \times 10^{-6}$ F, $R = 40\ \Omega$

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 80 \times 10^{-6}}}$

$\omega = \frac{10^3}{2} = 500 \text{ rad s}^{-1} \Rightarrow \upsilon = \frac{500}{2\pi} = 80 \text{ Hz}$

(ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{40}\sqrt{\frac{50 \times 10^{-3}}{80 \times 10^{-6}}} = \frac{1}{40} \times \sqrt{625} = 0.625$

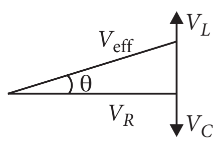
23. (i) From given parameter $V_R = 200$ V,
$V_L = 250$ V and $V_C = 250$ V$\cdot V_{\text{eff}}$ should be given as
$V_{\text{eff}} = V_R + V_L + V_C$
$= 200 \text{ V} + 250 \text{ V} + 250 \text{ V} = 700\text{V}$



(250 V, 50 Hz)

However, $V_{\text{eff}} > 200$ V of the ac source. This paradox can be solved only by using phasor diagram, as given below :

$(V_{\text{eff}}) = \sqrt{V_R^2 + (V_L - V_C)^2}$



Since $V_L = V_C$

so $V_{\text{eff}} = V_R = 200$ V

(ii) Given $R = 40\ \Omega$, so current in the LCR circuit.

$$I_{\text{eff}} = \frac{V_{\text{eff}}}{R} = \frac{200}{40} = 5\text{ A} \qquad [X_L = X_C \text{ or } Z = R]$$

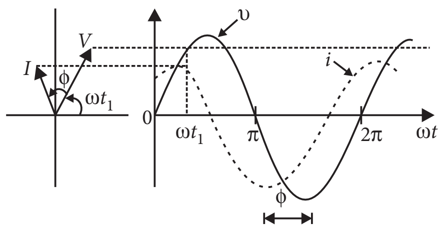
24. We have, $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{2\times10^{-6}}} = 100$

It signifies the sharpness of resonance.

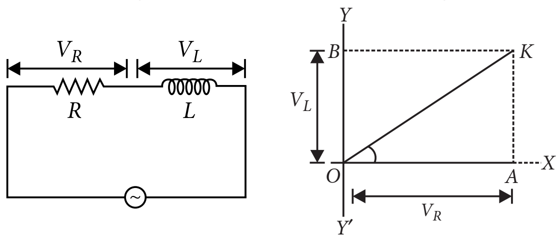
25. Impedance offered by series LCR circuit,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}, \quad V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

As V_C and V_L are the voltages applied across capacitor C and inductor L. V_C or V_L may be greater than V. The situation may be shown in figure, where $V_C > V$.



26. AC circuit containing inductor and resistor : Let an inductor L and resistor R is connected to a source of alternating emf in series as shown in figure (i).



(i) (ii)

The maximum voltage across R is

$V_R = I_0 R$

As V_R is in phase with current, it is represented by the vector $\overrightarrow{OA}$ along OX.

The maximum voltage across L is

$V_L = I_0 X_L$

As voltage across the inductor leads the current by 90°, it is represented by $\overrightarrow{OB}$ along OY, 90° ahead of I_0.

$$\therefore \quad OK = \sqrt{OA^2 + OB^2}$$

$$E_0 = \sqrt{V_R^2 + V_L^2} = \sqrt{I_0^2 R^2 + I_0^2 X_L^2} = I_0\sqrt{R^2 + X_L^2}$$

The impedance of the circuit is given by

$$Z = \frac{E_0}{I_0} = \frac{I_0\sqrt{R^2 + X_L^2}}{I_0} \Rightarrow Z = \sqrt{R^2 + X_L^2}$$

27. Given : $V_{\text{rms}} = 100$ volt; $\upsilon = 50$ Hz; $R = 20\ \Omega$; $L = 2 \times 10^{-3}$ H

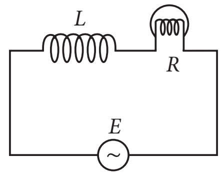
(i) $Z = \sqrt{R^2 + X_L^2}$

$X_L = 2\pi\upsilon L = 100 \times \pi \times 2 \times 10^{-3} = 0.63\ \Omega$

$$Z = \sqrt{(20)^2 + (0.63)^2} = 20\ \Omega$$

(ii) $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{100}{20} = 5\text{ A}$

28. Inductive reactance, $X_L = \omega L$



Impedance of the circuit,

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{\omega^2 L^2 + R^2}$$

(i) When the number of turns in a inductor coil decreases then its inductance L decreases. So, the net impedance of the circuit decreases and current through the bulb (circuit) increases. Hence brightness (I^2R) of bulb increases.

(ii) When an iron rod is inserted in the inductor, then its inductance L increases. So, Z will increase and current through the bulb will decrease. Hence, brightness of the bulb will decrease.

(iii) A capacitor is connected in the series in the circuit, so its impedance,

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$Z = R \quad (\because X_L = X_C)$

This is the case of resonance so maximum current will flow through the circuit. Hence brightness of the bulb will increase.

29. Here, $V = V_0 \sin(1000t + \phi)$

On comparing with $V = V_0 \sin(\omega t + \phi)$

$\omega = 1000$ rad s^{-1}

$X_L = \omega L = 1000 \times 100 \times 10^{-3} = 100\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000\times2\times10^{-6}} = \frac{1}{2\times10^{-3}} = 500\ \Omega$$

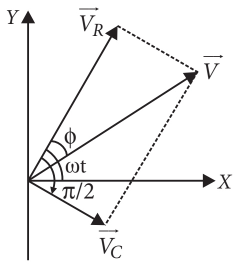
$R = 400\ \Omega$

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{500-100}{400} = \frac{400}{400}$$

$$\phi = \tan^{-1}\left(\frac{400}{400}\right) = \tan^{-1}(1) = 45°$$

30. (i) $V = V_0 \sin\omega t$ ...(i)

From diagram, by parallelogram law of vector addition, $\vec{V_R} + \vec{V_C} = \vec{V}$



Using pythagorean theorem, we get

$V^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2$

$V^2 = I^2 (R^2 + X_C^2)$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

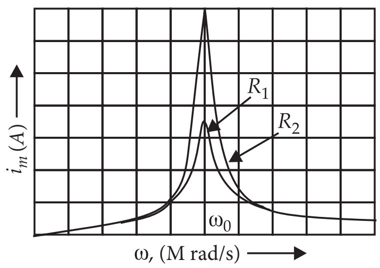
where, $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Z = impedance.

The phase angle ϕ between resultant voltage and current is given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega RC}$$

31. Figure shows the variation of i_m with ω in a LCR series circuit for two values of resistance R_1 and $R_2 (R_1 > R_2)$,



The condition for resonance in the LCR circuit is,

$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

We see that the current amplitude is maximum at the resonant frequency. Since $i_m = V_m / R$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1.

Quality factor or simply the Q-factor of a resonant LCR circuit is defined as the ratio of voltage drop across the resistance at resonance.

$$Q = \frac{V_L}{V_R} = \frac{\omega L}{R}$$

Thus finally, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

The Q factor determines the sharpness at resonance as for higher value of Q factor the tuning of the circuit and its sensitivity to accept resonating frequency signals will be much higher.

32. Here, $L = 10.0$ H, $C = 40\ \mu$F $= 40 \times 10^{-6}$ F

$R = 60\ \Omega$, $V_{rms} = 240$ V

(i) At resonance the angular frequency of the source is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10.0)(40\times10^{-6})}} = \frac{1}{2\times10^{-2}} = 50 \text{ rad s}^{-1}$$

(ii) At resonating frequency

Impedance, $Z = R$ $(\because X_L = X_C)$

The rms current at resonance

$$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{240 \text{ V}}{60\ \Omega} = 4 \text{ A}$$

(iii) The inductive reactance is

$X_L = \omega_r L = 50 \times 10.0 = 500\ \Omega$

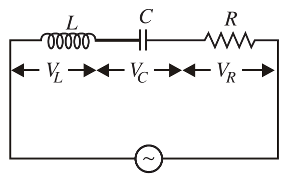
The rms potential drop across inductor at resonance,

$(V_{rms})_L = I_{rms} \times X_L = (4 \text{ A})(500\ \Omega) = 2000$ V

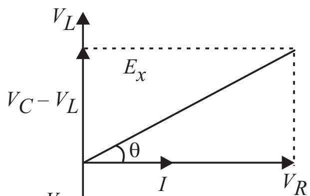
33. AC circuit containing inductor, capacitor and resistor in series [Series LCR circuit]

If I is the current in the circuit containing inductor of inductance L capacitor of capacitance C and resistor of resistance R in series, then the voltage drop across the inductor is

$$V_L = I \times X_L$$



which leads current I by phase angle of $\pi/2$, and voltage drop across the capacitor is $V_C = I \times X_C$



Phasor diagram

which lags behind current I by phase angle of $\pi/2$, and voltage drop across the resistor is

$V_R = I R$

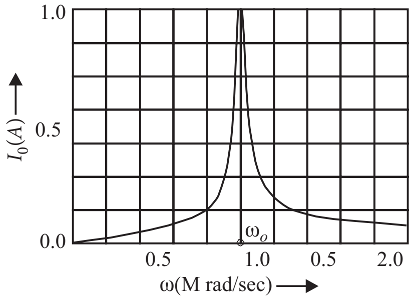
which is in phase with current I. So the net voltage E, across the circuit is

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

or $E = I\sqrt{R^2 + (X_L - X_C)^2}$ or $E = IZ$

where Z is the effective resistance offered by ac circuit containing inductor, capacitor and resistor in series, known as impedance in series LCR circuit. Hence in series LCR circuit, phase difference ϕ between the current I and the voltage E is

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$



With increase in ω, current first increases (up to ω_0) and then decreases.

34. (i) Reactance, $X_C = \frac{1}{2\pi f C}$

$$= \frac{1}{2\times 3.14\times 60\times 10^{-4}} = \frac{1}{.03768} = 26.5\ \Omega$$

(ii) Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{40^2 + 26.5^2}$

$$= \sqrt{1600 + 702.25} = \sqrt{2302.25} = 47.98 = 48\ \Omega$$

(iii) $V_{eff} = 100$ V

$$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2}V_{eff}}{Z} = \frac{1.414\times 100}{48} = 2.95 \text{ A.}$$

35. Here $L = 200$ mH $= 200 \times 10^{-3} = 0.2$ H

$C = 500\ \mu$F $= 500 \times 10^{-6} = 5 \times 10^{-4}$ F

$R = 10\ \Omega$ and $E_\upsilon = 100$ V

(i) Power factor, $\cos\phi = 1$ (given)

$$\frac{R}{Z} = 1$$

$$Z = R$$

$$\sqrt{R^2 + (X_L - X_C)^2} = R$$

$$R^2 + (X_L - X_C)^2 = R^2 \Rightarrow X_L - X_C = 0 \Rightarrow X_L = X_C$$

$$2\pi\upsilon L = \frac{1}{2\pi\upsilon C}$$

$$4\pi^2\upsilon^2 LC = 1$$

$$\therefore \quad \upsilon = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{0.2\times 5\times 10^{-4}}}$$

$$= \frac{1}{2\times 3.14\times 10^{-2}} = \frac{100}{6.28} = 15.92 \approx 16 \text{ Hz}$$

(ii) Current amplitude at resonance

$$I = \frac{E_\upsilon}{Z} = \frac{E_\upsilon}{R} = \frac{100}{10} = 10 \text{ A}$$

(iii) $Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.2}{5\times 10^{-4}}}$

$$= \frac{1}{10}\sqrt{\frac{2}{50\times 10^{-4}}} = \frac{1}{10}\sqrt{\frac{1}{25\times 10^{-4}}} = \frac{1}{10\times 5\times 10^{-2}}$$

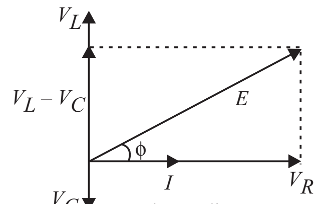
$$= \frac{1}{10\times 5\times 10^{-2}} = \frac{10^2}{50} = \frac{100}{50} = 2.$$

36. If I is the current in the circuit containing inductor of inductance L, capacitor of capacitance C and resistor of resistance R in series, then the voltage drop across the inductor is

$$V_L = I \times X_L$$

which leads current I by phase angle of $\pi/2$, and voltage drop across the capacitor is

$$V_C = I \times X_C$$



Phasor diagram

which lags behind current I by phase angle of $\pi/2$, and voltage drop across the resistor is

$$V_R = IR$$

which in phase with current I. So the net voltage E across the circuit is (using phasor diagram)

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

or $E = I\sqrt{R^2 + (X_L - X_C)^2}$

or $E = IZ$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is known as impedance.

Phase angle between voltage and current, is given by

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

A series LCR circuit has its natural angular frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

and natural (resonating) frequency $\upsilon = \frac{1}{2\pi\sqrt{LC}}$

when the applied ac in the circuit has this frequency the series *LCR* circuit offers minimum impedance *i.e.,* only '*R*' and current at this frequency flows maximum. In the case of resonance, voltage and current are in same phase.

Above mentioned condition is known as condition of resonance. In this condition

(i) Inductive and capacitive reactances are equal

$X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$$\omega = \frac{1}{\sqrt{LC}}, \upsilon = \frac{1}{2\pi\sqrt{LC}}$$

(ii) Potential drop across inductor and capacitor are equal.

$V_L = V_C$

(iii) The series resonant circuit is also called an acceptor circuit because when a number of different frequency currents are fed into the circuit, the circuit offers minimum impedance to natural frequency current.

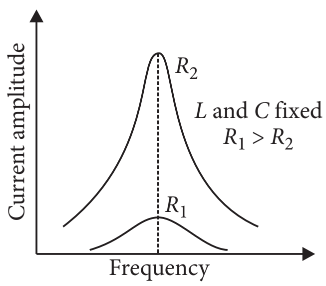
37. (i) To draw maximum current from a series *LCR* circuit, the circuit at particular frequency $X_L = X_C$. The frequency of the source will be

$$\upsilon = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{8\times 2\times 10^{-6}}} = 39.80 \text{ Hz}$$

This frequency is known as the series resonance frequency.

(ii) $I_0 = \frac{E_0}{R} = \frac{200}{100} = 2\text{ A}$

(iii)



(iv) Sharpness of resonance. It is defined as the ratio of the voltage developed across the inductance (*L*) or capacitance (*C*) at resonance to the voltage developed across the resistance (*R*).

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It may also be defined as the ratio of resonance angular frequency to the bandwidth of the circuit

$$Q = \frac{\omega_r}{2\Delta\omega}$$

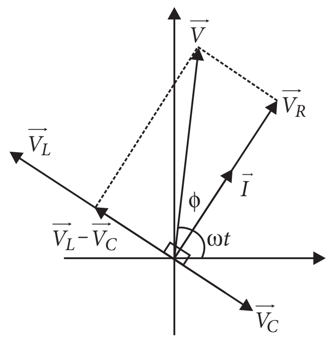
Circuit become more selective if the resonance is more sharp, maximum current is more, the circuit is close to resonance for smaller range of $(2\Delta\omega)$ of frequencies. Thus, the tuning of the circuit will be good.

38. (a) ac source, $V = V_0 \sin\omega t$

Voltage across resistor of resistance *R*, $V_R = IR$

Voltage across inductor of inductance *L*, $V_L = IX_L$

Voltage across capacitor of capacitance *C*, $V_C = IX_C$

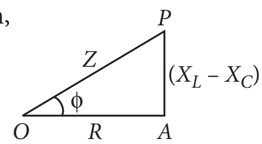


Using Pythagorean theorem,

$V^2 = V_R^2 + (V_L - V_C)^2$

$V^2 = I^2R^2 + I^2(X_L - X_C)^2$

$V^2 = I^2[R^2 + (X_L - X_C)^2]$



$$\therefore \quad I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_0}{Z}$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is called its impedance.

Using impedance triangle the phase angle can be given as $\tan\phi = \frac{X_L - X_C}{R}$

(b) Resonance condition of a series *LCR*-circuit : A series *LCR*-circuit is said to be in the resonance condition when the current through it has its maximum value.

The current amplitude I_0 for a series *LCR*-circuit is given by

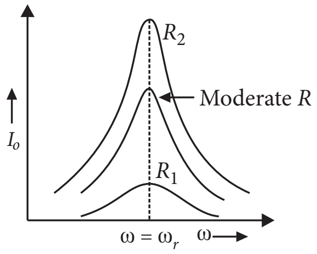
$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Clearly, I_0 becomes zero both for $\omega \to 0$ and $\omega \to \infty$. The value of I_0 is maximum when

$$\omega L - \frac{1}{\omega C} = 0 \text{ or } \omega = \frac{1}{\sqrt{LC}}$$

Then impedance, $Z = \sqrt{R^2 + (\omega L - \omega L)^2} = R$

Clearly, the impedance is minimum. The current and voltage are in the same phase and the current in the circuit is maximum. This condition of the *LCR*-circuit is called resonance condition.



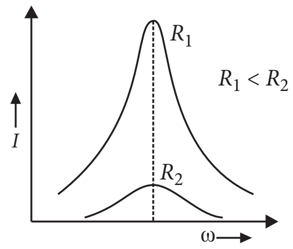
The *Q*-factor of a series resonant circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

39. (a) *Refer to answer 38(a).*

$I = I_0 \sin(\omega t - \phi)$ For $V_L > V_C$ or $X_L > X_C$

$I = I_0 \sin(\omega t + \phi)$ For $V_L < V_C$ or $X_L < X_C$

Variation of the current I as a function of angular frequency ω.



At resonance, when maximum current flows through the circuit.

$$\omega_r = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}} \Rightarrow L_1C_1 = L_2C_2 \Rightarrow \frac{L_1}{L_2} = \frac{C_2}{C_1}$$

For fine tuning in the receiver set, combination L_1C_1 and R_1 is better because maximum current flows through the circuit.

40. (a) *Refer to answer 38(a).*

(b) $L = \frac{4}{\pi^2}$ H, $\upsilon = 50$ Hz, $R = 100\ \Omega$, $V = 200$ V

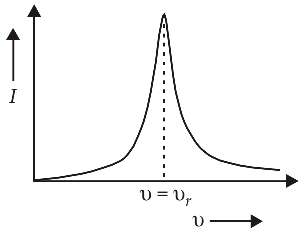
$\because\ X_L = X_C$ or $\omega L = \frac{1}{\omega C}$

$$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2\upsilon^2 \times L} = \frac{1}{4\pi^2 \times 50 \times 50 \times \frac{4}{\pi^2}}$$

$$= \frac{1}{2500 \times 16} = \frac{1}{40000} = 2.5 \times 10^{-5}\text{ F} = 25\ \mu\text{F}$$

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$

41. *Refer to answer 38(a).*



The practical application of series resonance circuit is in radio and T.V. receiver sets. The antenna of a radio/T.V. intercepts signals from many broadcasting stations. To receive one particular radio station/T.V. channel, we tune our receiver set by changing the capacitance of a capacitor in the tuning circuit of the set such that resonance frequency of the circuit becomes equal to the frequency of the desired station. Therefore, resonance occurs. The amplitude of current with the frequency of the signal from the desired station becomes maximum and it is received in our set.

42. (i) Resistance : The property due to which a conductor resists the flow of electrons through it, is called resistance of the conductor. It is measured by the ratio of potential difference between the ends of the conductor to the current flowing through it. If an alternating current is passed through a resistor, the current and voltage are in the same phase.

(ii) Reactance : The opposition offered by an inductor or a capacitor or both to the flow of ac through it, is called reactance.

There are two types of reactance :

(i) Capacitive reactance (X_C)

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} \Rightarrow X_C \propto \frac{1}{\upsilon}$$

(ii) Inductive reactance (X_L)

$X_L = \omega L = 2\pi\upsilon L$ $\therefore$ [$\upsilon \to$ Frequency of ac]

[$L \to$ inductance of the inductor]

$X_L \propto \upsilon$

(iii) Impedance : The total opposition offered by *LCR* circuit to the flow of alternating current is called impedance. It is denoted by *Z* and is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The impedance of an ac circuit plays the same role as resistance in dc circuit.

43. AC source containing capacitor : Let a source of alternating emf $V = V_m \sin \omega t$ be connected to a capacitor of capacitance *C* only

$V = V_m \sin \omega t$ ... (1)

At every instant, the potential *V* is given by

$$V = \frac{q}{C} \Rightarrow V_m \sin\omega t = \frac{q}{C}$$

$$\therefore q = C\, V_m \sin\omega t.$$

If *I* is instantaneous value of current in the circuit at instant *t*, then

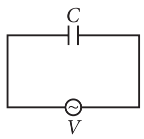
$$I = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t)$$

$$I = C\, V_m (\cos\omega t).\omega = \frac{V_m}{1/\omega C}\sin(\omega t + \frac{\pi}{2})$$

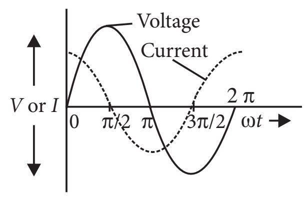
The current will be maximum, when

$$\sin\left(\omega t + \frac{\pi}{2}\right) = 1 \quad \therefore\ I_m = \frac{V_m}{1/\omega C} \times 1 = \frac{V_m}{1/\omega C}$$

$$I = I_m \sin(\omega t + \frac{\pi}{2})$$



Therefore, alternating current *I* leads the alternating voltage by a phase angle of $\frac{\pi}{2}$.



Given, $R = 200\ \Omega$

$C = 15.0\ \mu F = 15 \times 10^{-6}$ F, $V_{rms} = 220$ V,

$\upsilon = 50$ Hz, $I_{rms} = ?$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} = \frac{1}{2 \times \frac{22}{7} \times 50 \times 15 \times 10^{-6}} = \frac{7 \times 10^6}{33000}$$

$$= 212.12 \approx 212\ \Omega$$

$$\therefore\ Z = \sqrt{R^2 + X_c^2} = \sqrt{(200)^2 + (212)^2}$$

$$= \sqrt{40000 + 44944} = \sqrt{84944} = 291.45\ \Omega$$

$$\therefore\ I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{291.45} = 0.75 \text{ A}$$

$\therefore\ V_R = I_{rms} R = 0.75 \times 200 = 150$ V

$V_C = I_{rms} X_C = 0.75 \times 212 = 159$ V

$\therefore\ V_R + V_C = 150 + 159 = 309$

$\therefore\ V_R + V_C > V$

This is the reason, these voltages are not in same phase and they cannot be added like ordinary numbers.

$$\therefore\ V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(150)^2 + (159)^2} = \sqrt{22500 + 25281}$$

$$= \sqrt{47781} = 218.58 \text{ V} \approx 220 \text{ V}$$

44. The quality factor (*Q*) of resonance in series *LCR* circuit is defined as the ratio of voltage drop across inductor (or capacitor) to the applied voltage,

$$i.e.,\ Q = \frac{V_L}{V_R} = \frac{I_0 X_L}{I_0 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

It is an indicator of sharpness of the resonance. Quality factor has no unit.

45. Power factor, $\cos\phi = 0.5$

$\cos\phi = \cos 60° \Rightarrow \phi = 60°$

Phase difference = 60°

46. (i) ac can be transmitted with much lower energy losses as compared to dc.

(ii) ac voltage can be adjusted (stepped up or stepped down) as per requirement.

(iii) ac current in a circuit can be controlled using (almost) wattless devices like the choke coil.

(iv) ac is easier to generate.

47. The current which consumes no power for its maintenance (*i.e.*, power factor is zero) in the circuit is called wattless current.

48. We have given, $i_0 = 10$ A and $V_0 = 200$ V

$$i_{\text{rms}} = \frac{10}{\sqrt{2}}, V_{\text{rms}} = \frac{200}{\sqrt{2}}, \phi = 0°$$

$\therefore$ Average power dissipation = $V_{rms} i_{rms} \cos\phi$

$$= \frac{200}{\sqrt{2}} \times \frac{10}{\sqrt{2}} = 1000 \text{ W}$$

49. (i) The impedance of a series *LCR* circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Z will be minimum when $\omega L = \frac{1}{\omega C}$ *i.e.,* when the circuit is under resonance. Hence, for this condition, Z will be minimum and will be equal to R.

(ii) Average power dissipated through a series LCR circuit is given by

$P_{av} = EI\cos(\phi)$

where E = rms value of alternating voltage

I = rms value of alternating current

ϕ = phase difference between current and voltage

For wattless current, the power dissipated through the circuit should be zero *i.e.,*

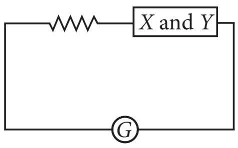
$\cos(\phi) = 0$

$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

Hence, the condition for wattless current is that the phase difference between the current and voltage should be $\pi/2$ and the circuit is purely inductive or purely capacitive.

50. (i) In $R - L$ series combination, voltage leads the current by phase $\phi = \frac{\pi}{4}$. It means element X is an inductor (with reactance equal to R). In RC series combination, voltage lags behind the current by phase $\phi = \frac{\pi}{4}$. So element Y is a capacitor (with reactance equal to R).

(ii) If both elements X and Y are connected in series with R, then power dissipation in the combination can be given as



$P = V_{rms} \cdot I_{rms} \cdot \cos\phi$

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Here, $X_L = X_C = R$. So, $\cos\phi = 1$

Hence, $P = V_{rms}I_{rms}$ (Maximum)

51. (i) $P = \frac{V^2}{R}$

$$100 = \frac{(220)^2}{R} \Rightarrow R = \frac{220 \times 220}{100} = 484\ \Omega$$

(ii) $i_{rms} = \frac{V_{rms}}{R}$ or $\frac{P}{V_{rms}} = \frac{220}{484}$ or $\frac{100}{220} = 0.45$ A

52. In a circuit containing capacitor C, current leads the voltage by a phase angle of $\frac{\pi}{2}$.

$\therefore \quad E = E_0 \sin\omega t$ then $I = I_0\sin(\omega t + \frac{\pi}{2})$

$\therefore \quad I = I_0 \cos\omega t$

Work done in one complete cycle is

$$W = \int_0^T EI\,dt = \int_0^T (E_0\sin\omega t)(I_0\cos\omega t)dt$$

$$= E_0I_0\int_0^T \sin\omega t\cos\omega t\ dt = E_0\ I_0\int_0^T \frac{\sin 2\omega t}{2}\ dt$$

[$\because$ sin2θ = 2 sinθ cosθ]

$$= \frac{E_0I_0}{2}\left[-\frac{\cos 2\omega t}{2\omega}\right]_0^T = -\frac{E_0I_0}{2}\left[\frac{\cos 2\omega T}{2\omega} - \frac{\cos 0}{2\omega}\right]$$

$$= -\frac{E_0I_0}{2}\left[\frac{\cos 2.\frac{2\pi}{T}.T}{2\omega} - \frac{1}{2\omega}\right] = -\frac{E_0I_0}{2}\left[\frac{\cos 4\pi}{2\omega} - \frac{1}{2\omega}\right]$$

$$= -\frac{E_0I_0}{2}\left[\frac{1}{2\omega} - \frac{1}{2\omega}\right] = 0 \quad [\because \cos 4\pi = 1]$$

$$\therefore \quad \text{Average power} = \frac{W}{T} = \frac{0}{T} = 0$$

Hence, average power supplied to an ideal capacitor by the source over a complete cycle of ac is zero.

53. For unity power factor, $X_L = X_C$

$$\omega L = \frac{1}{\omega C'} \qquad [\because C' = C + C'']$$

$$C' = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2 \times 100 \times 10^{-3}} = 10^{-5}\,\text{F} = 10\,\mu\text{F}$$

$\because \quad C' = C + C''$

$\quad\ C'' = C' - C = 10 - 2 = 8\ \mu$F

So required capacitor is 8 μF which is added in parallel with the given capacitor.

54. (i) Here, $L = 80$ mH, $C = 250$ mF, $\omega = 100$ rad/sec, $V_{rms} = 240$ V

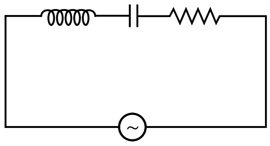
$$\text{Reactance} = \left|\omega L - \frac{1}{\omega C}\right|$$

$$= \left|100 \times 80 \times 10^{-3} - \frac{1}{100 \times 250 \times 10^{-3}}\right| = \left|8 - \frac{1}{25}\right| = 7.96$$

$$I_{rms} = \frac{V_{rms}}{\text{Reactance}} = \frac{240}{7.96} = 30.15\ \text{A}$$

(ii) The total average power consumed by circuit is zero.

55. The rate at which electrical energy is consumed in an electric circuit is called its power.



$V = V_0 \sin \omega t$

Suppose in an ac circuit, voltage and current are having a phase difference ϕ.

$V = V_0 \sin \omega t$

$I = I_0 \sin (\omega t - \phi)$

Work done by source of emf in a small time dt with negligible change in current.

$dW = VI\,dt$

$dW = V_0 I_0 \sin \omega t \sin (\omega t - \phi) dt$

where $\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$

$dW = V_0 I_0\, [\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi] dt$

$$dW = V_0 I_0 \left[\left(\frac{1 - \cos 2\omega t}{2}\right)\cos\phi - \frac{\sin 2\omega t}{2}\sin\phi\right]dt$$

Now total work done in a complete cycle

$$W = \frac{V_0 I_0}{2} \times \left[\int_0^T \cos\phi\, dt - \cos\phi \int_0^T \cos 2\omega t\, dt - \sin\phi \int_0^T \sin 2\omega t\, dt\right]$$

we can solve $\int_0^T \cos 2\omega t\, dt = \int_0^T \sin 2\omega t\, dt = 0$

$$W = \frac{V_0 I_0}{2} \int_0^T \cos\phi\, dt = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi T$$

Thus power consumed over a cycle,

$$\text{P} = \frac{W}{T} = V_{rms} I_{rms} \cos \phi$$

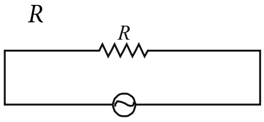
(i) Minimum power : In an ac circuit containing pure L only current I lags behind the applied voltage V by phase angle $\pi/2$. So average power consumed by pure inductor 'L' in complete cycle of ac is then given by

$P = V_{rms}\, I_{rms} \cos \pi/2 = 0$

(ii) Maximum power : In ac circuit containing R only both applied voltage V and current I are in same phase, so average power consumed by resistor R in complete cycle of a.c. is then given by

or $P = V_{rms}\, I_{rms} \cos 0° = V_{rms}\, I_{rms}$

$$P = \frac{V_{rms}^2}{R}$$

56. 

Average power in one cycle, $P = \dfrac{W}{t} = \dfrac{\int_0^T Vi dt}{\int_0^T dt}$

where current and voltage are in same phase across resistance R.

If $i = i_m \sin\omega t$ then $V = V_m \sin\omega t$

$$\text{Hence, } P = \frac{V_m i_m \int_0^T \sin^2 \omega t dt}{\int_0^T dt}$$

$$P = \frac{V_m i_m}{T} \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$P = \frac{V_m i_m}{2T}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right]$$

$$P = \frac{V_m i_m}{2T}[T - 0] = \frac{V_m i_m}{2}$$

Also, $i_m = \dfrac{V_m}{R}$

So, $P = \dfrac{i_m^2 R}{2}$

(b) Bulb is rated at 100 W, 220 V ac supply.

$$P = \frac{V^2}{R}$$

Hence, $R = \dfrac{V^2}{P} = \dfrac{220 \times 220}{100} = 484\ \Omega$

57. *Refer to answer 52.*

58. *Refer to answer 52.*

59. Here $L = 10$ mH $= 10 \times 10^{-3} = 10^{-2}$ H

$$C = \left(\frac{400}{\pi^2}\right)\mu\text{F} = \frac{400}{\pi^2} \times 10^{-6}\,\text{F} = \frac{4}{\pi^2} \times 10^{-4}\ \text{F}$$

$R = 55\ \Omega$

(a) The average power absorbed by the circuit is maximum at resonance.

$\therefore\ X_L = X_C$

$$2\pi\upsilon_0 L = \frac{1}{2\pi\upsilon_0 C}$$

$4\pi^2\upsilon_0^2\, LC = 1$

$$\upsilon_0^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore\ \upsilon_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-2} \times \frac{4}{\pi^2} \times 10^{-4}}}$$

$$= \frac{1}{2\pi \times \frac{2}{\pi} \times 10^{-3}} = \frac{10^3}{4} = \frac{1000}{4} = 250\ \text{Hz}$$

(b) Maximum current amplitude is,

$$i_m = \frac{V_m}{R} = \frac{V_{rms}\sqrt{2}}{R} = \frac{220\sqrt{2}}{55} = 4\sqrt{2}\ \text{A}$$

60. We know $\cos\phi = \frac{R}{Z}$

Power factor of circuit A is

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}}$$

As, $X_L = 3R$

$$\cos\phi = \frac{R}{\sqrt{R^2 + 9R^2}} = \frac{R}{R\sqrt{10}} = \frac{1}{\sqrt{10}}$$

Power factor of circuit B is

$$\cos\phi' = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

As $X_L = 3R$; $X_C = R$

$$\cos\phi' = \frac{R}{\sqrt{R^2 + 4R^2}} = \frac{R}{R\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\frac{\cos\phi'}{\cos\phi} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{10}}{1} = \sqrt{2} : 1$$

61. (a) Asha displayed the values of awareness towards energy saving and her concern towards wastage of energy.

(b) Following measures should be adopted to minimise the wastage of electric energy in our households :

(i) We should switch off the electrical appliances which are not in use.

(ii) The old electrical appliances should be repaired time to time, as they often consume more electricity.

(iii) Condensers of electrical appliances should be replaced time to time.

(iv) New electrical and electronic appliances must be purchased in accordance with their efficiency to consume less energy.

(v) Use of solar devices like solar heater and cookers must be encouraged rather than the use of conventional electrical appliances.

(c) Electrical energy dissipated by an appliance is given by

$E = P \times t$

where P is the power and t is the time.

$P = 2$ W, $t = 20$ h $= 20 \times 60 \times 60$ s

So, $E = 2 \times 20 \times 60 \times 60 \Rightarrow E = 1.44 \times 10^5$ J

62. For LR circuit, $X_L = R$

Power factor, $P_1 = \cos\phi$

$$= \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}}$$

For LCR circuit, as C is put in series with LR circuit and, $X_L = X_C$

Power factor, $P_2 = \cos\phi$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (X_L - X_L)^2}} = \frac{R}{R} = 1$$

Required ratio $= \frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

63. (a) $V = V_0 \sin\omega t$ and $I = I_0 \sin\omega t$

Work done in small time dt will be

$dW = P\,dt = VI\,dt = V_0 I_0 \sin^2 \omega t$ dt

$$= \frac{V_0 I_0}{2}(1 - \cos 2\omega t)dt$$

The average power dissipated per cycle in the resistor will be

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW$$

$$= \frac{V_0 I_0}{2T}\int_0^T (1 - \cos 2\omega t) = \frac{V_0 I_0}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{V_0 I_0}{2T}[(T-0) - 0] = \frac{V_0 I_0}{2} = \frac{V_0^2}{2R}$$

or $P_{av} = \frac{V_0 I_0}{\sqrt{2}\sqrt{2}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^2}{R}$ $\left[\because \frac{V_0}{2} = V_{\text{rms}}\right]$

(b) The power is $P = V_{rms} I_{rms} \cos\phi$. If $\cos\phi$ is small, then current considerably increases when voltage is constant. Power loss, is $I^2 R$. Hence, power loss increases.

(c) *Refer to answer 47.*

64. (a) $V = V_m \sin\omega t$, $i = i_m \sin(\omega t + \phi)$

and instantaneous power, $P = Vi$

$= V_m \sin\omega t . i_0 \sin(\omega t + \phi) = V_m i_m \sin\omega t \sin(\omega t + \phi)$

$= \frac{1}{2} V_m i_m 2 \sin\omega t . \sin(\omega t + \phi)$

From trigonometric formula

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$\therefore$ Instantaneous power, $P = \frac{1}{2} V_m i_m [\cos(\omega t + \phi - \omega t) - \cos(\omega t + \phi + \omega t)]$

$$P = \frac{1}{2} V_m i_m [\cos\phi - \cos(2\omega t + \phi)] \quad \ldots (i)$$

Average power for complete cycle

$$\overline{P}=\frac{1}{2}V_m i_m[\cos\phi-\overline{\cos(2\omega t+\phi)}]$$

For a complete cycle, $\overline{\cos(2\omega t+\phi)}=0$

∴ Average power,

$$\overline{P}=\frac{1}{2}V_m i_m\cos\phi=\frac{V_0}{\sqrt{2}}\frac{i_0}{\sqrt{2}}\cos\phi=V_{rms}i_{rms}\cos\phi$$

(b) *Refer to answer 63(b).*

(c) Given, $L = 1.00$ mH $= 1\times10^{-3}$ H,

$C = 1.00$ nF $= 1\times10^{-9}$ F

$R = 100\ \Omega$, $E_0 = 100$ V

$$I_0=\frac{E_0}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}=\frac{E_0}{Z}\quad\left\{\begin{array}{l}\text{At resonance } \omega L=\frac{1}{\omega C}\\ \text{Hence } Z=R\end{array}\right\}$$

$$\therefore\ I_0=\frac{V}{R}=\frac{100}{100}, I_0=1\text{ A}$$

$$I_v=\frac{I_0}{\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\frac{1.414}{2}=0.707\text{ A}$$

$I_v = 0.707$ A

$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{100}\sqrt{\frac{1.0\times10^{-3}}{1.0\times10^{-9}}}=\frac{1}{100}\times10^3=10$$

$Q = 10$

65. *Refer to answer 38.*

Power factor : Power factor is defined as the ratio of true power to apparent power. It is denoted by $\cos\phi$

$$\therefore\quad \text{Power factor} = \cos\phi=\frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

(i) Power factor is maximum when the circuit contains only R *i.e.* at resonance.

(ii) Power factor is minimum for purely inductive or capacitive circuit.

66. (a) *Refer to answer 55.*

(b) The Q-factor of a resonant LCR circuit is defined as ratio of the voltage drop across inductor or capacitor to the applied voltage.

$$\therefore\ Q=\frac{\text{Voltage across } L \text{ or } C}{\text{applied voltage}}$$

Since $V_L = IX_L$ and $V = IR$

$$\therefore\ Q=\frac{IX_L}{IR}=\frac{\omega_0 L}{R}$$

Hence at high frequencies, the Q-factor $\frac{\omega_o L}{R}$ is quite large.

∴ The voltage drop across the inductor will be quite large as compared to the applied voltage.

Also $Q=\frac{1}{\sqrt{LC}}\cdot\frac{L}{R}\qquad\left[\because \omega_o=\frac{1}{\sqrt{LC}}\right]$

$$\therefore\ Q=\frac{1}{R}\sqrt{\frac{L}{C}}$$

The Q-factor of LCR-series circuit depends on L, C and R.

67. The core of transformer is laminated to reduce the energy losses due to eddy currents, so that its efficiency may remain nearly 100%.

68. Characteristic properties :

(a) Low coercivity/Low retentivity.

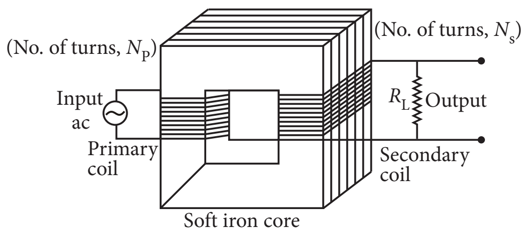
(b) Low hysteresis loss

69. A step-up transformer is used to convert a low voltage at high current into a high voltage at low current.

70. A transformer is based on principle of mutual induction which states that due to continuous change in the current in the primary coil, an emf gets induced across the secondary coil.

Electric power generated at the power station, is stepped up to very high voltages by means of a step-up transformer and transmitted to a distant place. At receiving end, it is stepped down by a step down transformer.

71. Step-up transformer (or transformer) is based on the principle of mutual induction.



An alternating potential (V_p) when applied to the primary coil induced an emf in it.

$$\varepsilon_p=-N_p\frac{d\phi}{dt}$$

If resistance of primary coil is low $V_p = e_p$.

$$i.e.,\ V_p=-N_p\frac{d\phi}{dt}$$

As same flux is linked with the secondary coil with the help of soft iron core due to mutual induction emf is induced in it.

$$\varepsilon_s = -N_s \frac{d\phi}{dt}$$

If output circuit is open $V_s = \varepsilon_s$

$$V_s = -N_s \frac{d\phi}{dt}$$

Thus $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$

For an ideal transformer, $P_{out} = P_{in}$

$\Rightarrow \quad I_s V_s = I_p V_p$

$\therefore \quad \dfrac{V_s}{V_p} = \dfrac{I_p}{I_s} = \dfrac{N_s}{N_p}$

For step-up transformer $\dfrac{N_s}{N_p} > 1$

In case of dc voltage flux does not change. Thus no emf is induced in the circuit.

72. (i) Transformer is a device which is used to bring the high voltage down to low voltage of ac current. It works on the principle of mutual induction of two coils in a transformer.

(ii) Transformer does not work for dc voltage. A dc current gives constant magnetic field and constant magnetic flux through the coil of fixed area of cross section. As there is no change in magnetic flux so there is no induced emf in the coil.

(iii) The values displayed by the students are gaining knowledge and curiosity to learn new things. The values displayed by the teacher are providing good education and helpful.

73. (i) Voltage of house supply is E_V = 220 V and frequency 50 Hz.

(ii) (a) ac is cheap to produce.

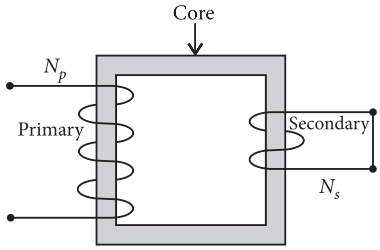
(b) Due to step-up and step-down of ac by transformers, the ac can be supplied with less power wastage.

(c) We have electrical elements as capacitors, inductors to control ac with no power loss.

(iii) No, a transformer can't be used to step up dc voltage.

(iv) Anil has scientific aptitude and is helpful and has a good presence of mind.

74. (i) Step down transformer :



Principle : When the current flowing through the primary coil changes, an emf is induced in the secondary coil due to the change in magnetic flux linked with it *i.e.*, it works on the principle of mutual induction.

For step down transformer,

$N_S < N_P$, hence $\varepsilon_s < \varepsilon_p$.

(ii) $\dfrac{\varepsilon_s}{\varepsilon_p} = \dfrac{N_s}{N_p}$

(iii) For an ideal transformer,

$P_{in} = P_{out}$

or $\quad \varepsilon_p I_p = \varepsilon_s I_s \quad \therefore \quad \dfrac{I_p}{I_s} = \dfrac{\varepsilon_s}{\varepsilon_p} = \dfrac{N_s}{N_p}$

(iv) $P_{in} = P_{out} = 550$ W or $\varepsilon_p I_p = 550$ or $220 \times I_p = 550$

$I_p = \dfrac{550}{220} = \dfrac{5}{2} = 2.5$ A

75. (i) *Refer to answer 71.*

There are number of energy losses in a transformer.

(a) Copper losses due to Joule's heating produced across the resistances of primary and secondary coils. It can be reduced by using copper wires.

(b) Hysteresis losses due to repeated magnetization and demagnetization of the core of transformer. It is minimized by using soft iron core, as area of hysteresis loop for soft iron is small and hence energy loss also becomes small.

(c) Iron losses due to eddy currents produced in soft iron core. It is minimized by using laminated iron core.

(d) Flux losses due to flux leakage or incomplete flux linkage and can be minimised by proper coupling of primary and secondary coils.

(ii) Here $N_P = 100$, $\dfrac{N_S}{N_P} = 100$

$\varepsilon_i = \varepsilon_P = 220$ V, $P_I = 1100$ W

(a) $N_P = 100 \therefore\ N_S = 10000$

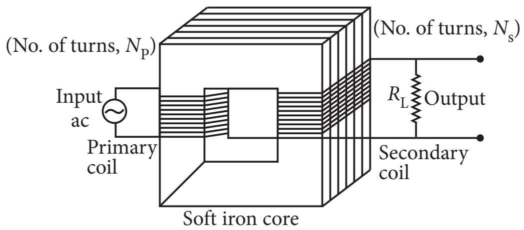
(b) $I_P = \frac{P_I}{\varepsilon_P} = \frac{1100}{220} = 5$ A

(c) $\varepsilon_S = \frac{N_S}{N_P} \times \varepsilon_P = 100 \times 220 = 22000$ V

(d) $I_S = \frac{P_O}{\varepsilon_S} = \frac{1100}{22000} = \frac{1}{20}$ A $\quad (\because P_O = P_I)$

(e) $P_S = P_O = P_I = 1100$ W.

76. (a)



(b) *Refer to answer 71.*
(c) The following three assumptions are involved
(i) The primary resistance and current are small.
(ii) The same flux links both with the primary and secondary windings as flux leakage from the core is negligibly small.
(iii) The terminals of the secondary are open or the current taken from it is small.
(d) *Refer to answer 75(i).*

77. *Refer to answer 75(i).*

78. *Refer to answer 75(i).*

79. *Refer to answer 75(i).*
(iii) Input power = Output power

$V_p I_p = V_s I_s$

when output voltage increases the output current automatically decreases to keep the power same. Thus, there is no violation of conservation of energy in a step up transformer.

80. $V_p = 2200$ V, $I_p = 5$ A, $N_p = 4000$
$V_s = 220$ V, $N_s = ?\ I_s = ?$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$\frac{220}{2200} = \frac{5}{I_s} = \frac{N_s}{4000}$$

$$\frac{220}{2200} = \frac{5}{I_s} \Rightarrow \frac{1}{10} = \frac{5}{I_s} \therefore\ I_s = 50 \text{ A}$$

$$\frac{5}{I_s} = \frac{N_s}{4000} \Rightarrow \frac{5}{50} = \frac{N_s}{4000} \therefore\ N_s = 400$$

81. *Refer to answer 76(i).*

82. *Refer to answer 72.*
Transformer is mainly used in long distance transmission of electrical energy. At the electric power producing station, a step-up transformer is used which increases the alternating voltage upto several kilo volts, thereby decreasing the electric current flowing through transmission wires. As Joule's heating is proportional to square of current, so this decreases the loss of electrical energy across transmission wires. Further a step-down transformer is used to decrease the alternating voltage at substation before distributing electrical energy for domestic use.

83. Given $V_p = 2.5$ kV $= 2.5 \times 10^3$ V, $I_p = 20$ A
Input power, $P_{in} = V_p I_p = 2.5 \times 10^3 \times 20$
$= 50 \times 10^3$ W $= 50$ kW

(i) $\eta = \frac{P_{out}}{P_{in}}$

Power output, $P_{out} = \eta P_{in} = 0.9 \times 50$ kW $= 45$ kW
(ii) Voltage across secondary

$$V_s = \frac{N_S}{N_P} V_p = \frac{1}{10} \times 2.5 \times 10^3 \text{ V} = 250 \text{ V}$$

(iii) Current in secondary

$$I_s = \frac{P_{out}}{V_s} = \frac{45 \times 10^3}{250} = 180 \text{ A}$$

84. (a) *Refer to answer 71.*
(b) *Refer to answer 79(iii).*