Chapter 4 The Method of Deduction

For as one may feel sure that a chain will hold when he is assured that each separate link is of good material and that it clasps the two neighbouring links, viz: the one preceding and the one following it, so we may be sure of the accuracy of the reasoning when the matter is good, that is to say, when nothing doubtful enters into it and when the form consists into perpetual concatenation of truths which allow no gap - Gottfried Leibniz

4.1 DEDUCTIVE PROOF

Logic aims at distinguishing between good and bad reasoning. One of the basic problems in logic, therefore is to decide whether a given argument is valid. Another important task of logicians is to find out whether the given statement form is a tautology, contradiction or contingency. Various methods are used by logicians to deal with these. The methods are of two types : (1) Decision procedure (2) Methods that are not Decision procedures.

Truth table as we have seen is a decision procedure whereas Deductive proof is another important method used in logic which is **not a decision procedure** as all the three conditions of an effective decision procedure are not satisfied by the deductive proof. The Deductive proof is reliable, finite but not mechanical as intelligence is required to use the method. Unlike decision procedure, deductive proof is used to prove the validity of arguments and not to decide whether it is valid or invalid and it is also used to prove that the statement form is a tautology and not to decide whether it is a tautology, contradiction or contingency.

The method of deductive proof consists in deducing the conclusion of an argument from its premises by a sequence of (valid) elementary arguments. These elementary arguments are known to be valid. They are substitution instances of elementary valid argument forms which are called rules of inference.

The method of deductive proof can be used to prove the validity of deductive arguments only. In a valid deductive argument the conclusion is a logical consequence of the premises i.e. in a valid deductive argument premises imply the conclusion. Therefore, if one is able to deduce the conclusion from the premises by using valid elementary arguments, the argument is proved to be valid. The proof constructed to establish the validity of an argument by deductive proof is called formal proof of validity.

Deductive proof is of three types - 1. Direct Proof 2. Conditional Proof 3. Indirect Proof. In this chapter we will study Direct proof.

Direct proof can be used only to prove validity of arguments whereas Conditionl proof and Indirect proof can be used for proving the validity of arguments as well as tautologies.

4.2 DIRECT PROOF

The method of direct proof consists in deducing the conclusion of an argument directly from its premises by a sequence of (valid) elementary arguments. This method is called diret proof because it does not involve an assumption at any step before arriving at the conclusion

Construction of formal proof of validity involves the following steps :

1. Write down the premises in order and number them.

2. Write the conclusion on the line where the last premise is written. Separate it from the premise by a slanting line as shown below :

    1. Premise
    2. Premise
    3. Premise / ∴ Conclusion

3. Deduce the conclusion from the premises by applying rules of Inference along with rule of Replacement. Before arriving at the conclusion one may have to derive some statements. These statements can be taken as additional premises for further proof. These statements are to be numbered and the justification for each statement should be written on the right side of the statement. The justification for a statement consists in stating the number of step/steps from which the statement is derived and the rule applied to derive it. It is advised to use only one rule at a time while constructing the proof.

4. Once the conclusion is derived from the premises the proof is complete and the validity of the argument is established.

4.3 RULES OF INFERENCE AND RULE OF REPLACEMENT

For constructing formal proof of validity by deductive proof, nineteen rules are used. These nineteen rules are of two types. First nine rules of Inference form one group and are different in nature from remaining ten rules which are based on the rule of Replacement. To begin with let us study the first nine rules of inference and their application.

First nine rules of Inference are elementary valid forms of argument. Any argument which is a substitution instance of such form is also valid. With the help of these valid forms of inference one can deduce the conclusion from the premises and show that it is a logical consequence of the premises.

It should be noted that these rules can be applied only to the whole statement and not to a part of the statement. The first nine rules of inference are as follows.

(1) Modus Ponens (M. P.)

This rule is based on the nature of conditional statement. In a conditional statement the antecedent implies the consequent, which means if a conditional statement is true and its antecedent is also true, its consequent must be true, it cannot be false. The **form** of the rule is as follows -

$p \supset q$
p
$\therefore \quad q$

The following argument illustrates the rule :

(a) If you study Logic then your reasoning skill improves.
You study Logic.
Therefore, your reasoning skill improves.

(b) If a student is intelligent then he will pass.
The student is intelligent.
Therefore, he will pass.

Application of the rule ---

If in an argument, a conditional statement is given as one of the premises and antecedent of the same statement is also given as another premise then by applying the rule of M. P. one can validly infer the consequent of the same conditional statement.

For example ---

(1)	$B \supset M$	
(2)	B	
(3)	$M \supset A$	$/ \therefore A$
(4)	M	1, 2, M.P.
(5)	A	3, 4, M.P.

TRY this :

(1)	$M \supset R$	
(2)	M	
(3)	$R \supset S$	
(4)	$S \supset T$	$/ \therefore T$
(5)	________	1, 2, M.P.
(6)	S	________
(7)	________	4, 6 M.P.

(2) Modus Tollens (M.T.)

The rule of Modus Tollens is also based on the nature of conditional statement. A conditional statement is false only when the antecedent is true and the consequent is false. Therefore if a conditional statement is true and the consequent is false then the antecedent must be false. The

form of the rule is as follows -

$p \supset q$
$\sim q$
$\therefore \sim p$

The following argument illustrates the rule :

If Karan is hardworking then he will get a scholarship.
Karan did not get a scholarship.
Therefore, Karan is not hardworking.

Application of the rule ---

If in an argument a conditional statement is given as one of the premises and negation of its consequent is also given then from these two premises one can infer negation of the antecedent of that conditional statement.

For example --

(1)	$M \supset \sim T$	
(2)	$S \supset T$	
(3)	M	/ $\therefore \sim S$
(4)	$\sim T$	1, 3 M.P.
(5)	$\sim S$	2, 4 M.T.

TRY this :

(1)	$R \supset T$	
(2)	$\sim T$	
(3)	$\sim R \supset K$	/ $\therefore K$
(4)	________	1, 2, M.T.
(5)	K	________

(3) Hypothetical Syllogism (H.S.)

For this rule we need two conditional statements such that, consequent of one statement is the antecedent of the other. From such two statements we can deduce a conditional statement whose antecedent is the antecedent of the first conditional statement and consequent is the consequent of the second conditional statement. The **form** of Hypothetical Syllogism is as follows -

$p \supset q$
$q \supset r$
$\therefore p \supset r$

The following argument illustrates the rule :

If it rains then the harvest is good.
If the harvest is good then the farmers are happy.
Therefore, if it rains then the farmers are happy.

Application of the rule ---

(1)	$A \supset S$	
(2)	$\sim R \supset K$	
(3)	$S \supset \sim R$	/ $\therefore A \supset K$
(4)	$A \supset \sim R$	1, 3, H.S.
(5)	$A \supset K$	4, 2, H.S.

TRY this :

(1)	$K \supset R$	
(2)	$S \supset K$	
(3)	$R \supset M$	/ $\therefore S \supset M$
(4)	$S \supset R$	________
(5)	________	4, 3, H.S.

(4) Disjunctive Syllogism (D.S.)

This rule states that if a disjunctive statement is given and its first disjunct is denied then one can affirm the second disjunct in the conclusion. This rule is based on the nature of disjunctive statement. Disjunctive statement is true when at least one of the disjuncts is true. The **form** of Disjunctive syllogism is as follows-

$p \vee q$
$\sim p$
$\therefore q$

The following argument illustrates the rule :

Either Nilraj will learn to play the guitar or the piano.
Nilraj did not learn to play the guitar.
Therefore, Nilraj will learn to play the piano.

Application of the rule ---

(1)	$T \supset B$	
(2)	$\sim B$	
(3)	$T \vee R$	/ $\therefore R$
(4)	$\sim T$	1, 2, M.T.
(5)	R	3, 4, D.S.

TRY this :

(1) $R \supset T$
(2) $\sim T$
(3) $R \vee \sim S$ / $\therefore \sim S$
(4) ________ 1, 2, M.T.
(5) $\sim S$ ________

(5) Constructive Dilemma (C.D.)

To apply this rule we need two statements such that, one statement is a conjunction of two conditional statements and the second statement is a disjunctive statement which affirms antecedents of the conditional statements. From such two statements we can infer a disjunctive statement which affirms consequents of the conditional statements. The **form** of Constructive Dilemma is as follows -

$(p \supset q) \bullet (r \supset s)$

$p \vee r$

$\therefore$ $q \vee s$

The following argument illustrates the rule :

If you exercise then you become healthy and if you eat fast food then you become unhealthy.

Either you exercise or you eat fast food.

Therefore, either you become healthy or unhealthy.

Application of the rule ---

(1) $A \supset (J \vee K)$
(2) A
(3) $(J \supset R) \cdot (K \supset T)$ / $\therefore R \vee T$
(4) $J \vee K$ 1, 2, M.P.
(5) $R \vee T$ 3, 4, C.D.

TRY this :

(1) $(A \supset B) \cdot (R \supset S)$
(2) $M \supset (A \vee R)$
(3) M
(4) $\sim B$ / $\therefore S$
(5) $A \vee R$ ________
(6) ________ 1, 5, C.D.
(7) S ________

(6) Destructive Dilemma (D.D.)

For this rule we need two statements such that, one statement is a conjunction of two conditional statements and the second statement is a disjunctive statement which denies consequents of the conditional statements. From such two statements we can infer a disjunctive statement which denies antecedents of the conditional statements. The **form** of Destructive Dilemma is as follows ---

$(p \supset q) \cdot (r \supset s)$
$\sim q \vee \sim s$
$\therefore$ $\sim p \vee \sim r$

The following argument illustrates the rule :

If you use solar power then it reduces pollution and if you use dustbins then you keep the city clean.

Either pollution is not reduced or you do not keep the city clean.

Therefore, either you do not use solar power or you do not use dustbins.

Application of the rule ---

(1) A
(2) $A \supset \sim P$
(3) $P \vee (\sim S \vee \sim R)$
(4) $(T \supset S) \cdot (B \supset R)$ / $\therefore \sim T \vee \sim B$
(5) $\sim P$ 2, 1, M.P.
(6) $\sim S \vee \sim R$ 3, 5, D.S.
(7) $\sim T \vee \sim B$ 4, 6, D.D.

TRY this :

(1) $M \supset \sim R$
(2) $R \vee (\sim S \vee \sim T)$
(3) M
(4) $(J \supset S) \cdot (K \supset T)$
(5) $\sim \sim J$ / $\therefore \sim K$
(6) $\sim R$ ________
(7) ________ 2, 6, D.S.
(8) $\sim J \vee \sim K$ ________
(9) $\sim K$ ________

(7) Simplification (Simp.)

The rule of Simplification states that, if a conjunctive statement is given as one of the premises then one can validly infer the first conjunct. This rule is based on the nature of conjunct. This rule is based on the nature of conjunctive statement. A conjunctive statement is true only when both the conjuncts are true, therefore, from a conjunctive statement one can derive the first conjunct. The **form** of rule of Simplification is as given below ---

$p \bullet q$

$\therefore$ p

The following argument illustrates the rule :

Ishita practices yoga and Ishita is flexible.
Therefore, Ishita practices yoga.

Application of the rule ---

(1)	$(M \supset N) \cdot (R \supset S)$	
(2)	$(M \vee R) \cdot D$	$/ \therefore N \vee S$
(3)	$M \vee R$	2, Simp.
(4)	$N \vee S$	1, 3, C.D.

TRY this :

(1)	$\sim\sim M \cdot A$	
(2)	$\sim M \vee \sim S$	
(3)	$(A \supset S) \cdot (P \supset T)$	$/ \therefore \sim A$
(4)	$\sim\sim M$	________
(5)	________	2, 4, D.S.
(6)	________	3, Simp.
(7)	$\sim A$	________

(8) Conjunction (Conj.)

The rule of Conjunction is also based on the nature of conjunctive statement. It states that if two statements are true seperately then the conjunction of these two statements is also true. Thus from two different statements, their conjunction can validly be inferred. The **form** of rule of conjunction is as given below -

p
q
$\therefore$ $p \cdot q$

The following argument illustrates the rule :

Radhika loves reading.
She writes poems.
Therefore, Radhika loves reading and she writes poems.

Application of the rule ---

(1)	$F \vee T$	
(2)	$A \supset K$	
(3)	A	
(4)	$\sim F$	$/ \therefore T \cdot K$
(5)	K	2, 3, M.P.
(6)	T	1, 4, D.S.
(7)	$T \cdot K$	6, 5, Conj.

TRY this :

(1)	$S \supset T$	
(2)	$A \supset B$	
(3)	$S \vee A$	
(4)	M	$/ \therefore (T \vee B) \cdot M$
(5)	________	1, 2, Conj.
(6)	$T \vee B$	________
(7)	________	6,4, Conj.

(9) Addition (Add.)

As per the rule of Addition, from any given statement, we can infer a disjunctive statement whose first disjunct is the statement itself and the second disjunct is any other statement. This rule is based on the nature of disjunctive statement. Such type of inference is valid because a disjunctive statement is true when at least one of the disjuncts is true. So, if 'p' is true then its disjunction with any other statement irrespective of its truth value must also be true.

The **form** of the rule is as follows -

p

$\therefore$ $p \vee q$

The following argument illustrates the rule :

Tejas plays football.
Therefore, Tejas plays football or Rohan plays hockey.

Application of the rule ---

(1)	S	
(2)	$(S \cdot T) \supset A$	
(3)	T	$/ \therefore A \vee K$
(4)	$S \cdot T$	1, 3, Conj.
(5)	A	2, 4, M.P.
(6)	$A \vee K$	5, Add.

TRY this :

(1)	A	
(2)	$(A \vee S) \supset \sim T$	
(3)	$T \vee \sim M$	$/ \therefore \sim M \vee \sim S$
(4)	$A \vee S$	__________
(5)	__________	2, 4 M.P.
(6)	$\sim M$	__________
(7)	$\sim M \vee \sim S$	__________

THE RULE OF REPLACEMENT

The nine rules of Inference, cannot prove the validity of all arguments.

For example, to prove the validity of the argument- $A \cdot D \;\; / \therefore D$, nine rules are not sufficient. The Rule of replacement is therefore accepted in addition to the nine rules of Inference. The rule of replacement is also called the Principle of Extensionality.

It is based on the fact that, if any compound statement is replaced by an expression which is logically equivalent to that statement, the truth value of the resulting statement is the same as that of the original statement.

When the rule of replacement is adopted as an additional rule of inference, it allows us to infer a statement from any given statement which is logically equivalent to it. This rule can be applied to the whole as well as part of a statement. Since these rules are logically equivalent statements they can be applied in both the ways i.e. left hand expression can be replaced by right hand expression and vice versa. Based on the rule of replacement, ten logical equivalences are added to the list of rules of inference and are numbered after the nine rules. They are as follows -

(10) De Morgan's Laws (De M.)

The De Morgan's Laws are as follows -

$$\sim (p \cdot q) \equiv (\sim p \vee \sim q)$$
$$\sim (p \vee q) \equiv (\sim p \cdot \sim q)$$

The first De Morgan's law is based on the nature of conjunctive statement. Conjunctive statement is false when at least one of the conjuncts is false. So, the first De Morgan's law states that, the denial of the conjunctive statement '$\sim (p \cdot q)$' is the same as saying that either 'p' is false or 'q' is false.

The following argument illustrates the rule :

The statement, 'It is not true that Niraj is hardworking and lazy' is logically equivalent to the statement - 'Either Niraj is not hardworking or Niraj is not lazy'.

The second De Morgan's law is based on the nature of disjunctive statement. Disjunctive statement is false when both the disjuncts are false. So, the second De Morgan's law states that, the denial of the disjunctive statement '$\sim (p \vee q)$' is the same as saying that 'p' is false and 'q' is false.

The following argument illustrates the rule :

The statement, 'It is false that plastic bags are either eco friendly or are degradable' is logically equivalent to the statement - 'Plastic bags are not eco friendly and are not degradable.'

Application of the rule ---

(1)	$\sim (A \vee M)$	
(2)	$\sim (S \cdot T)$	
(3)	$A \vee J$	
(4)	$\sim \sim S$	$/ \therefore \sim T \cdot J$
(5)	$\sim A \cdot \sim M$	1, De M.
(6)	$\sim S \vee \sim T$	2, De M.
(7)	$\sim T$	6, 4, D.S.
(8)	$\sim A$	5, Simp.
(9)	J	3, 8, D.S.
(10)	$\sim T \cdot J$	7, 9, Conj.

TRY this :

(1)	$S \supset T$	
(2)	$\sim (T \vee K)$	
(3)	$S \vee M$	$/ \therefore M \vee \sim R$
(4)	________	2, De M.
(5)	$\sim T$	________
(6)	$\sim S$	________
(7)	________	3, 6, D.S.
(8)	$M \vee \sim R$	________

(11) Commutation (Com.)

The Commutative Laws are as follows -

$(p \cdot q) \equiv (q \cdot p)$

$(p \vee q) \equiv (q \vee p)$

Commutation means changing the place of components. The first commutative law which deals with conjunctive statement states that '$p \cdot q$' is logically equivalent to '$q \cdot p$'.

Changing the place of conjuncts makes no difference to the truth value of a statement.

The following argument illustrates the rule :

The statement, 'I like to study logic and philosophy is logically equivalent to the statement' I like to study philosophy and logic.'

The second commutative law deals with disjunctive statement and allows us to change the order of disjuncts. Changing the place of disjuncts makes no difference to the truth value of a statement.

The following argument illustrates the rule :

The statement, 'Either I will use cloth bags or paper bags' is logically equivalent to the statement 'Either I will use paper bags or cloth bags.'

Application of the rule ---

(1)	$\sim (A \vee K)$	
(2)	$T \cdot K$	$/ \therefore K \cdot \sim K$
(3)	$\sim A \cdot \sim K$	1, De M.
(4)	$\sim K \cdot \sim A$	3, Com.
(5)	$K \cdot T$	2, Com.
(6)	$\sim K$	4, Simp.
(7)	K	5, Simp.
(8)	$K \cdot \sim K$	7, 6, Conj.

TRY this :

(1)	$\sim S \cdot T$	
(2)	$(T \supset R) \cdot (A \supset B)$	
(3)	A	$/ \therefore R \cdot B$
(4)	________	1, Com.
(5)	T	________
(6)	$T \supset R$	________
(7)	________	6, 5, M.P.
(8)	________	2, Com.
(9)	$A \supset B$	________
(10)	________	9, 3 M.P.
(11)	$R \cdot B$	________

(12) Association (Assoc.)

The Association Laws are as follows -

$[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$

$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$

The Associative Laws state that in case of conjunctive and disjunctive statements if there are three components joined with the same connective i.e. either by dot or by wedge, then, whichever way you group them makes no difference to their truth value.

The following argument illustrates the first rule:

The truth value of the statement, 'Rutuja is beautiful and (hardworking and successful)' remains the same even when expressed as, '(Rutuja is beautiful and hardworking) and successful.'

The following argument illustrates the second rule :

The truth value of the statement, 'Shreyas will either eat a burger or (a sandwich or a pizza) remains the same even when expressed as, '(Shreyas will either eat a burger or a sandwich) or a pizza.'

Application of the rule ---

(1)	$(S \cdot B) \cdot T$	
(2)	$A \lor (K \lor T)$	
(3)	$\sim T$	$/ \therefore S \cdot (A \lor K)$
(4)	$S \cdot (B \cdot T)$	1, Assoc.
(5)	S	4, Simp.
(6)	$(A \lor K) \lor T$	2, Assoc.
(7)	$T \lor (A \lor K)$	6, Com.
(8)	$A \lor K$	7, 3, D.S.
(9)	$S \cdot (A \lor K)$	5,8, Conj

TRY this :

(1)	$P \lor (Q \lor M)$	
(2)	$\sim (P \lor Q)$	
(3)	$S \cdot (R \cdot A)$	$/ \therefore A \cdot M$
(4)	________	1, Assoc.
(5)	M	________
(6)	$(S \cdot R) \cdot A$	________
(7)	________	6,Com.
(8)	A	________
(9)	$A \cdot M$	________

(13) Distribution (Dist.)

The Distributive Laws are as follows -

$$[p \cdot (q \lor r)] \equiv [(p \cdot q) \lor (p \cdot r)]$$
$$[p \lor (q \cdot r)] \equiv [(p \lor q) \cdot (p \lor r)]$$

In the first distributive law, conjunction is distributed over disjunction. If a statement is conjoined with a disjunctive statement then it is the same as saying that, either it is conjoined with the first disjunct or it is conjoined with the second disjunct.

The following argument illustrates the rule :

The statement,

'Anuja is an actor and she is either a singer or a dancer' is logically equivalent to the statement 'Either Anuja is an actor and a singer or Anuja is an actor and a dancer.'

In the second distributive law, disjunction is distributed over conjunction. If a statement is in disjunction with a conjunctive statement then it is the same as saying that, it is in disjunction with the first conjunct and it is in disjunction with the second conjunct.

The following argument illustrates the rule :

The statement,

'Either Vikas plays cricket or he sings and paints' is logically equivalent to the statement 'Either Vikas plays cricket or he sings and either Vikas plays cricket or he paints'.

Application of the rule ---

(1)	$\sim (S \cdot A)$	
(2)	$S \cdot (A \lor B)$	
(3)	$K \lor (P \cdot D)$	$/ \therefore (S \cdot B) \cdot (K \lor D)$
(4)	$(S \cdot A) \lor (S \cdot B)$	2, Dist.
(5)	$S \cdot B$	4, 1, D. S.
(6)	$(K \lor P) \cdot (K \lor D)$	3, Dist.
(7)	$(K \lor D) \cdot (K \lor P)$	6, Com.
(8)	$K \lor D$	7, Simp.
(9)	$(S \cdot B) \cdot (K \lor D)$	5, 8, Conj.

TRY this :

(1)	$P \lor (R \cdot S)$	
(2)	$\sim R$	
(3)	$\sim (P \lor M)$	$/ \therefore \sim M \cdot P$
(4)	________	1, Dist.
(5)	$P \lor R$	________
(6)	________	5, Com.
(7)	P	________
(8)	________	3, DeM.
(9)	$\sim M \cdot \sim P$	________
(10)	________	9, Simp.
(11)	$\sim M \cdot P$	________

(14) Double Negation (D. N.)

The form of this rule is as follows -

$$p \equiv \sim \sim p$$

The rule of Double Negation states that a statement is equivalent to the negation of its contradictory.

The following argument illustrates the rule :

To say that, 'Global warming is a current world crisis' is logically equivalent to saying, 'It is not the case that global warming is not a current world crisis.'

Application of the rule ---

(1)	$\sim R \vee (S \vee B)$	
(2)	R	
(3)	$\sim S$	$/ \therefore \sim\sim B$
(4)	$\sim\sim R$	2, D. N.
(5)	$S \vee B$	1, 4, D. S.
(6)	B	5, 3, D. S.
(7)	$\sim\sim B$	6, D. N.

TYR this :

(1)	$\sim A \supset B$	
(2)	$\sim B$	
(3)	$\sim(\sim M \vee R)$	$/ \therefore A \cdot M$
(4)	________	1, 2, M. T.
(5)	A	________
(6)	________	3, DeM.
(7)	________	6, D.N.
(8)	M	________
(9)	$A \cdot M$	________

(15) Transposition (Trans.)

The rule of Transposition is expresssed as follows :

$$(p \supset q) \equiv (\sim q \supset \sim p)$$

Like commutative laws this rule allows us to change the places of components. However, when we interchange the antecedent and consequent, we have to negate both of them so that the truth value remains the same.

The following argument illustrates the rule :

To say that, 'If people take efforts then environmental pollution can be controlled' is logically equivalent to saying that, 'If environmental pollution is not controlled then people have not taken efforts.'

Application of the rule ---

(1)	$\sim\sim K$	
(2)	$K \supset A$	$/ \therefore \sim\sim A$
(3)	$\sim A \supset \sim K$	2, Trans.
(4)	$\sim\sim A$	3, 1, M. T.

TRY this :

(1)	$T \supset A$	
(2)	$\sim S \supset R$	
(3)	$(\sim A \supset \sim T) \supset \sim R$	$/ \therefore S \vee (B \cdot Q)$
(4)	$\sim A \supset \sim T$	________
(5)	________	3,4 M.P.
(6)	$\sim\sim S$	________
(7)	________	6, D.N.
(8)	$S \vee (B \cdot Q)$	________

(16) Material Implication (Impl.)

The rule is stated as follows -

$$(p \supset q) \equiv (\sim p \vee q)$$

This rule is based on the nature of conditional statement. A conditional statement is false only when its antecedent is true and consequet is false. But if antecedent is false then whatever may be the truth value of consequent the conditional statement is true or if consequent is true then whatever may be the truth value of antecedent the conditional statement is true. Therefore the rule of implication states that, if '$p \supset q$' is true then either 'p' is false or 'q' is true.

The following argument illustrates the rule :

To say that, 'If you litter on streets then you are irresponsible.' is logically equivalent to the statement, 'Either you do not litter on streets or you are irresponsible.'

Application of the rule ---

(1)	$(A \supset B) \vee S$	
(2)	A	
(3)	$\sim B$	/ $\therefore S$
(4)	$(\sim A \vee B) \vee S$	1, Impl.
(5)	$\sim A \vee (B \vee S)$	4, Assoc.
(6)	$\sim\sim A$	2, D.N.
(7)	$B \vee S$	5, 6, D.S.
(8)	S	7, 3, D.S.

TRY this :

(1)	$Q \supset T$	
(2)	$(\sim Q \vee T) \supset M$	
(3)	$T \supset S$	/ $\therefore M \cdot (\sim Q \vee S)$
(4)	$\sim Q \vee T$	__________
(5)	__________	2,4 M.P.
(6)	$Q \supset S$	__________
(7)	__________	6, Impl.
(8)	$M \cdot (\sim Q \vee S)$	__________

(17) Material Equivalence - (Equiv.)

The two rules are as given below -

$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$

$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$

The first rule states the nature of bi-conditional statement i.e. in a bi-conditional statement both the components imply each other. The truth condition of a materially equivalent statement is expressed in the second rule i.e. a materially equivalent statement is true either when both the components are true or when both are false.

The following argument illustrates this rule :

According to the first rule, the statement, 'If and only if you pursue your passion then you will succeed,' is logically equivalent to the statement, 'If you pursue your passion then you will succeed and if you succeed then you have pursued your passion.'

As per the second rule, the same statement is logically equivalent to the statement,' Either you pursue your passion and succeed or you do not pursue your passion and you do not succeed.'

Application of the rule ---

(I)

(1)	$S \equiv M$	
(2)	$\sim S$	/ $\therefore \sim M$
(3)	$(S \supset M) \cdot (M \supset S)$	1, Equiv.
(4)	$(M \supset S) \cdot (S \supset M)$	3, Com.
(5)	$M \supset S$	4, Simp.
(6)	$\sim M$	5, 2, M.T.

(II)

(1)	$S \equiv M$	
(2)	$\sim S$	/ $\therefore \sim M$
(3)	$(S \cdot M) \vee (\sim S \cdot \sim M)$	1, Equiv.
(4)	$\sim S \vee \sim M$	2, Add.
(5)	$\sim (S \cdot M)$	4, DeM.
(6)	$\sim S \cdot \sim M$	3, 5, D.S.
(7)	$\sim M \cdot \sim S$	6, Com.
(8)	$\sim M$	7, Simp.

TRY this :

(1)	$A \equiv S$	
(2)	S	
(3)	$(K \cdot T) \vee (\sim K \cdot \sim T)$	
(4)	$(K \equiv T) \supset \sim P$	
(5)	$P \vee M$	/ $\therefore M \cdot A$
(6)	$(A \supset S) \cdot (S \supset A)$	__________
(7)	__________	6, Com.
(8)	$S \supset A$	__________
(9)	__________	8, 2, M.P.
(10)	__________	3, Equiv.
(11)	$\sim P$	__________
(12)	__________	5, 11 D.S.
(13)	$M \cdot A$	__________

(18) Exportation (Exp.)

The rule is as follows -

$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$

This rule is applied when we have a conditional statement having three components.

In such a case it is the same as saying that, first and second components both imply the third one. First implying the second and second implying the third.

The following argument illustrates the rules:

'If you drink and drive then an accident can take place'is logically equivalent to the statement,'If you drink then if you drive then an accident can take place.'

Application of the rule ---

(1)	B	
(2)	(B · S) ⊃ T	
(3)	T ⊃ R	/ ∴ S ⊃ R
(4)	B ⊃ (S ⊃ T)	2, Exp.
(5)	S ⊃ T	4, 1, M. P.
(6)	S ⊃ R	5, 3, H.S.

TRY this :

(1)	~ P ⊃ (Q ⊃ ~ S)	
(2)	~ P · Q	/ ∴ S ⊃ S
(3)	_________	1, Exp.
(4)	~ S	_________
(5)	_________	4, Add.
(6)	S ⊃ S	_________

(19) Tautology (Taut.)

The rule is as follows-

p ≡ (p · p)

p ≡ (p ∨ p)

This rule states that any statement is equivalent to an expression where the statement is in conjunction with itself or the statement is in disjunction with the statement itself.

The following argument illustrates the rule:

According to the first rule, the statement, 'The weather is pleasant' is logically equivalent to the statement,' The weather is pleasant and the weather is pleasant' and as per the second rule, the statement, 'The weather is pleasant' is logically equivalent to the statement, 'The weather is pleasant or the weather is pleasant.'

Application of the rule ---

(1)	(S ⊃ R) · (B ⊃ R)	
(2)	(~ K · ~ K) ⊃ M	
(3)	~ M	
(4)	S ∨ B	/ ∴ R · K
(5)	R ∨ R	1, 4, C. D.
(6)	R	5, Taut.
(7)	~ K ⊃ M	2, Taut.
(8)	~ ~ K	7, 3, M. T.
(9)	K	8, D. N.
(10)	R · K	6, 9, Conj.

TRY this :

(1)	(A ⊃ B) • (M ⊃ N)	
(2)	~ B ∨ ~ B	
(3)	A ∨ M	
(4)	(~ N ∨ S) ∨ (~ N ∨ S)	/ ∴ ~ S ⊃ ~ R
(5)	_________	1, 3 C.D.
(6)	~ B	_________
(7)	_________	5, 6, D.S.
(8)	_________	4, Taut.
(9)	~ ~ N	_________
(10)	_________	8, 9, D.S.
(11)	S ∨ ~ R	_________
(12)	_________	11, Impl.

Rules of Inference :

(1) Modus Ponens (M.P.)

$p \supset q$

p

$\therefore \quad q$

(2) Modus Tollens (M. T.)

$p \supset q$

$\sim q$

$\therefore \quad \sim p$

(3) Hypothetical Syllogism (H. S.)

$p \supset q$

$q \supset r$

$\therefore \quad p \supset r$

(4) Disjunctive Syllogism (D. S.)

$p \vee q$

$\sim p$

$\therefore \quad q$

(5) Construtive Dilemma (C. D.)

$(p \supset q) \cdot (r \supset s)$

$p \vee r$

$\therefore \quad q \vee s$

(6) Destrutive Dilemma (D.D.)

$(p \supset q) \cdot (r \supset s)$

$\sim q \vee \sim s$

$\therefore \quad \sim p \vee \sim r$

(7) Simplification (Simp.)

$p \cdot q$

$\therefore \quad p$

(8) Conjunction (Conj.)

p

q

$\therefore \quad p \cdot q$

(9) Addition (Add.)

p

$\therefore \quad p \vee q$

Rule of Replacement :

(10) De Morgan's Laws (De M.)

$\sim (p \cdot q) \equiv (\sim p \vee \sim q)$

$\sim (p \vee q) \equiv (\sim p \cdot \sim q)$

(11) Commutation (Com.)

$(p \cdot q) \equiv (q \cdot p)$

$(p \vee q) \equiv (q \vee p)$

(12) Association (Assoc.)

$[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$

$[p \vee (q \ r)] \equiv [(p \vee q) \vee r]$

(13) Distribution Laws (Dist.)

$[p \cdot (q \vee r)] \equiv [(p \cdot q) \vee (p \cdot r)]$

$[p \vee (q \cdot r)] \equiv [(p \vee q) \cdot (p \vee r)]$

(14) Double Negation (D.N.)

$p \equiv \sim \sim p$

(15) Transposition (Trans.)

$(p \supset q) \equiv (\sim q \supset \sim p)$

(16) Material Implication - (Impl.)

$(p \supset q) \equiv (\sim p \vee q)$

(17) Material Equivalence - (Equiv.)

$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$

$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$

(18) Exportation (Exp.)

$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$

(19) Tautology (Taut.)

$p \equiv (p \cdot p)$

$p \equiv (p \vee p)$

Summary

- The method of deductive proof is used for proving the validity of arguments. It consists in deducing the conclusion of an argument from its premises by a sequence of valid elementary arguments.
- The method of deductive proof is not a decision procedure, as it is not mechanical.
- The method of direct proof consists in deducing the conclusion of an argument directly from its premises by a sequence of (valid) elementary arguments.
- In the method of deductive proof, nineteen rules are used for constructing formal proof of validity.
- The first nine rules of inference are elementary valid forms of arguments. Remaining ten rules are logically equivalent statements, based on the rule of replacement.
- Rules of inference can be applied only to the whole statement. Rules based on the rule of Replacement can be applied to the whole as well as part of the statement.

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets :

1. According to De Morgan's Law (DeM.), $\sim (S \cdot \sim R) \equiv$ ....

   [$(S \lor R)$ / $(\sim S \lor \sim\sim R)$]

2. The rule involved in $(A \lor M) \equiv (M \lor A)$ is .... *[Commutation / Transposition]*

3. Thr rule of Simplification (Simp.) is based on the nature of .... statement. *[Disjunctive / Conjunctive]*

4. $(B \supset \sim R) \equiv$ .... is by the rule of Material Implication (Impl.) *[$(\sim B \lor \sim R)$ / $(B \lor R)$]*

5. The rule used in $\sim T \equiv (\sim T \lor \sim T)$ is .... *[Tautology / Commutation]*

6. $[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$ is by the rule of .... *[Association / Exportation]*

7. $(K \supset T) \equiv$ .... is by the rule of Transposition (Transp.)

   [$(T \supset \sim K)$ / $(\sim T \supset \sim K)$]

8. The rule of Modus Tollens is based on the nature of .... statement. *[Conjunctive / Conditional]*

9. $[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$ is by the rule of .... *[Distribution / Exportation]*

10. The rule of replacement can be applied to .... of the statement. *[Whole / Whole as well as part]*

Q. 2. State whether the following statements are True or False :

1. Rules of inference can be applied to the part of the statement.
2. The method of deductive proof is a decision procedure.
3. The rule of Disjunctive Syllogism (D.S.) can be applied to the part of the statement.
4. The method of direct proof consists in deducing the conclusion directly from the premises.
5. p / $\therefore$ $p \lor q$ is the rule of simplification (Simp.)

6. $[(p \supset q) \cdot p] \supset q$ is the rule of Modus Ponens (M.P.)
7. In the rule of Transposition (Trans.), places of antecedent and consequent are changed and both of them are negated.
8. The method of deductive proof is a mechanical method.
9. The rule of Hypothetical syllogism (H.S.) is based on the nature of disjunctive statement.
10. $p, q \ / \therefore p \cdot q$ is the rule of Addition (Add.)

Q. 3. Match the columns :

	(A)		(B)
1.	p	1.	$(\sim p \vee q)$
2.	$(p \supset q)$	2.	$(\sim p \vee \sim q)$
3.	$(p \equiv q)$	3.	$[(p \vee q) \cdot (p \vee r)]$
4.	$\sim (p \cdot q)$	4.	$\sim \sim p$
5.	$[p \vee (q \cdot r)]$	5.	$[(p \supset q) \cdot (q \supset p)]$

Q. 4. Give reasons for the following :

1. The method of deductive proof is not a decision procedure.
2. The nine rules of inference can be applied to the whole statement only.
3. The rules based on the rule of replacement can be applied to the whole as well as part of the statement.

Q. 5. Explain the following :

1. Rule of Association.
2. Rule of Distribution.
3. Rule of Constructive Dilemma
4. Rule of Destructive Dilemma.
5. Rule of Addition.
6. Rule on De Morgan's Laws.
7. Rule of Double Negation.
8. Rule of Material Implication.
9. Rule of Material Equivalence.
10. Rule of Exportation.
11. Rule of Tautology.

Q. 6. Answer the following questions :

1. Explain the method of Deductive proof.
2. Explain the method of Direct Deductive proof.
3. Distinguish between rules of Inference and Rule of Replacement.
4. Distinguish between rule of Modus Ponens and rule of Modus Tollens.
5. Distinguish between rule of Hypothetical Syllogism and rule of Disjunctive Syllogism.
6. Distinguish between rule of Simplification and rule of Conjunction.
7. Distinguish between rule of Commutation and rule of Transposition.

Q. 7. State whether the following arguments are valid or invalid :

(1) $(A \supset B) \supset \sim C$

$A \supset B$

$\therefore C$

(2) $(M \bullet N) \vee (T \equiv S)$

$M \bullet N$

$\therefore T \equiv S$

(3) $L \supset (K \vee L)$

$\sim L$

$\therefore K \vee L$

(4) $\sim R \supset (T \bullet W)$

$\sim (T \bullet W)$

$\therefore R$

(5) $(S \supset \sim T) \bullet (R \supset W)$

$S \vee R$

$\therefore \sim T \vee W$

(6) $(H \supset L) \bullet (K \supset J)$

$\sim L \vee \sim J$

$\therefore \sim H \vee \sim K$

(7) $(R \equiv S) \bullet (M \supset N)$

$R \vee M$

$\therefore S \vee N$

(8) $(T \supset W) \bullet L$

$\therefore T \supset W$

(9) $S \vee \sim L$

$\sim T \supset W$

$\therefore (S \vee \sim L) \bullet (\sim T \supset W)$

(10) $J \supset L$

$\sim L \supset K$

$\therefore J \supset K$

Q. 8. State whether the following equivalances are correct or incorrect :

(1) $\sim (p \vee \sim q) \equiv (\sim p \bullet q)$

(2) $\sim \sim R \equiv R$

(3) $(\sim K \vee \sim K) \equiv K$

(4) $[(R \bullet \sim S) \bullet \sim T] \equiv [R \vee (\sim S \vee \sim T)]$

(5) $[\sim A \bullet (B \vee C)] \equiv [(\sim A \bullet B) \vee (\sim A \bullet C)]$

(6) $(\sim p \supset \sim q) \equiv (q \supset p)$

(7) $(\sim S \bullet \sim T) \equiv (T \bullet S)$

(8) $(\sim p \supset q) \equiv (p \vee q)$

(9) $[(p \bullet q) \vee (q \bullet p)] \equiv (p \equiv q)$

(10) $[(p \supset q) \supset r] \equiv [p \bullet (q \supset r)$

Q. 9. State the justification for each step of the following arguments :

(1) 1 $(K \vee S) \cdot (K \vee \sim T)$

2 $S \supset T$ / $\therefore K$

3 $K \vee (S \cdot \sim T)$

4 $\sim S \vee \sim \sim T$

5 $\sim S \vee T$

6 $\sim (S \cdot \sim T)$

7 $(S \cdot \sim T) \vee K$

8 K

(2) 1 $(W \supset L) \cdot (W \supset K)$

2 $(L \cdot K) \supset Z$

3 $\sim Z$ / $\therefore \sim W$

4 $\sim (L \cdot K)$

5 $\sim L \vee \sim K$

6 $\sim W \vee \sim W$

7 $\sim W$

(3) 1 $(X \supset \sim Y) \cdot (Z \supset A)$

2 $\sim (\sim X \cdot \sim Z)$ / $\therefore Y \supset A$

3 $\sim \sim X \vee \sim \sim Z$

4 $X \vee Z$

5 $\sim Y \vee A$

6 $Y \supset A$

(4) 1 $(A \vee B) \supset \sim C$

2 C / $\therefore \sim B$

3 $\sim \sim C$

4 $\sim (A \vee B)$

5 $\sim A \cdot \sim B$

6 $\sim B \cdot \sim A$

7 $\sim B$

(5) 1 $\sim L \supset K$

2 $(L \vee M) \supset (U \cdot W)$

3 $\sim K$ / $\therefore U \vee U$

4 $\sim \sim L$

5 L

6 $L \vee M$

7 $U \cdot W$

8 U

9 $U \vee U$

(6) 1 $W \vee S$

2 $\sim S$

3 $(W \cdot X) \supset Y$ / $\therefore \sim X \vee Y$

4 $S \vee W$

5 W

6 $W \supset (X \supset Y)$

7 $X \supset Y$

8 $\sim X \vee Y$

(7) 1 $(A \cdot B) \cdot C$

2 $A \supset (D \vee K)$

3 $\sim D$ / $\therefore K$

4 $A \cdot (B \cdot C)$

5 A

6 $D \vee K$

7 K

(8) 1 K ∨ L
2 (L · M) ⊃ (O · P)
3 ~ K
4 M / ∴ G ⊃ O
5 L
6 L · M
7 O · P
8 O
9 O ∨ ~ G
10 ~ G ∨ O
11 G ⊃ O

(9) 1 ~ D ∨ E
2 E ⊃ G
3 (~ G ⊃ ~ D) ⊃ H / ∴ H ∨ K
4 D ⊃ E
5 D ⊃ G
6 ~ G ⊃ ~ D
7 H
8 H ∨ K

(10) 1 A ⊃ B
2 C ⊃ D
3 ~ (B · D) / ∴ ~ A ∨ ~ C
4 (A ⊃ B) · (C ⊃ D)
5 ~ B ∨ ~ D
6 ~ A ∨ ~ C

Q. 10. Construct formal proof of validity for the following arguments using nine rules of Inference :

(1) 1 P ⊃ Q
2 P ⊃ R
3 P / ∴ Q · R

(2) 1 T ⊃ P
2 ~ P
3 ~ T ⊃ ~ R / ∴ ~ R ∨ S

(3) 1 M ⊃ N
2 N ⊃ O
3 (M ⊃ O) ⊃ (N · P) / ∴ N ∨ R

(4) 1 A ∨ B
2 ~ A
3 M · D / ∴ B · M

(5) 1 M ∨ ~ S
2 ~ M
3 P ⊃ S / ∴ ~ P ∨ R

(6) 1 ~ A
2 ~ B
3 (~ A · ~ B) ⊃ R / ∴ R

(7) 1 A · S
2 A ⊃ ~ B
3 B ∨ T / ∴ T ∨ ~ M

(8) 1 W ∨ T
2 (W ∨ T) ⊃ (L · ~ S) / ∴ L

(9) 1(P ⊃ Q) · R
2 (Q ⊃ R) · S / ∴ P ⊃ R

(10) 1 (A · B) ⊃ S
2 S ⊃ R
3 A
4 B / ∴ R

(11) 1 (T ∨ S) ⊃ P
2 P ⊃ Q
3 T / ∴ Q

(12) 1 Q ⊃ S
2 P ⊃ T
3 Q ∨ P
4 ~ S / ∴ T

(13) 1 (M ∨ O) ⊃ (A · M)
2 (A · M) ⊃ (D · E)
3 M / ∴ D

(14) 1 P ⊃ T
2 T ⊃ ~ D
3 ~ D ⊃ M / ∴ P ⊃ M

(15) 1 H ⊃ K
2 T ∨ F
3 H
4 ~ T / ∴ F · K

(16) 1 $A \supset (B \vee S)$

2 $\sim (B \vee S)$

3 $D \supset L$

4 $A \vee D$ / $\therefore L$

(17) 1 $A \vee B$

2 $B \supset M$

3 $A \supset D$

4 $\sim D$ / $\therefore B \cdot (A \vee B)$

(18) 1 $A \supset B$

2 $\sim A \supset \sim C$

3 $C \vee (D \cdot E)$

4 $\sim B$ / $\therefore D \vee (S \equiv \sim R)$

(19) 1 $\sim S \supset (P \supset T)$

2 $\sim (P \supset T)$

3 $A \supset M$

4 $\sim S \vee A$ / $\therefore M \vee (R \cdot Q)$

(20) 1 $\sim S \cdot (A \vee B)$

2 $(M \supset S) \cdot R$

3 $M \vee \sim T$ / $\therefore \sim T \vee \sim K$

(21) 1 $A \supset M$

2 $P \supset T$

3 $P \vee A$

4 $\sim T$ / $\therefore M$

(22) 1 $S \supset M$

2 $P \supset A$

3 $\sim A \vee \sim M$

4 $K \cdot S$ / $\therefore (\sim P \vee \sim S) \cdot K$

(23) 1 $R \supset S$

2 $A \supset B$

3 $\sim T$

4 $\sim S \vee \sim B$ / $\therefore (\sim R \vee \sim A) \cdot \sim T$

(24) 1 $A \supset (\sim B \vee \sim D)$

2 $D \supset A$

3 D

4 $A \supset B$

5 $M \supset D$ / $\therefore \sim A \vee \sim M$

(25) 1 $R \supset T$

2 $S \supset B$

3 $R \cdot M$

4 $\sim T$ / $\therefore B \vee \sim A$

(26) 1 $R \vee S$

2 $[(R \vee S) \vee K] \supset \sim L$

3 T / $\therefore \sim L \cdot T$

(27) 1 $\sim K \cdot \sim S$

2 $M \vee T$

3 $M \supset K$ / $\therefore T \vee (S \supset R)$

(28) 1 $\sim A \supset R$

2 $S \supset \sim A$

3 $\sim R$

4 $S \vee \sim P$ / $\therefore \sim P$

(29) 1 $L \vee \sim S$

2 $\sim A$

3 $(\sim A \vee \sim M) \supset \sim L$

4 $P \cdot B$ / $\therefore \sim S \cdot (P \cdot B)$

(30) 1 $A \supset \sim B$

2 $A \cdot \sim R$

3 $B \vee (S \vee \sim M)$

4 $\sim S \cdot \sim T$ / $\therefore A \cdot \sim M$

Q.11. Construct formal proof of validity for the following arguments using the rule of Inference and Replacement :

(1) 1 $\sim (M \cdot R)$

2 M

3 $(\sim R \supset B) \cdot (A \supset K)$ / $\therefore B \vee K$

(2) 1 $B \cdot A$

2 $\sim A \vee S$

3 $S \supset T$ / $\therefore T \vee (\sim R \supset M)$

(3) 1 $A \vee (B \vee M)$

2 $\sim B$ / $\therefore A \vee M$

(4) 1 $M \supset N$

2 $A \supset N$

3 $M \vee A$ / $\therefore N$

(5) 1 $R \lor (S \cdot T)$
2 $\sim T$
3 $\sim S$ / $\therefore R$

(6) 1 $\sim (S \lor T)$
2 $\sim S \supset \sim P$
3 $P \lor R$ / $\therefore R \lor \sim M$

(7) 1 $A \supset \sim B$
2 $A \cdot S$
3 $B \lor R$ / $\therefore R \cdot S$

(8) 1 $T \supset \sim S$
2 $T \lor T$
3 $S \lor \sim K$ / $\therefore \sim K \lor \sim K$

(9) 1 $\sim K \supset \sim T$
2 $\sim K \cdot S$
3 $\sim T \supset R$
4 $(R \cdot S) \supset M$ / $\therefore M \lor M$

(10) 1 $S \supset T$
2 $T \supset M$ / $\therefore M \lor \sim S$

(11) 1 $A \supset M$
2 $(\sim A \lor M) \supset R$
3 $\sim S \lor T$ / $\therefore (S \supset T) \cdot R$

(12) 1 $A \supset (B \supset M)$
2 $A \cdot B$ / $\therefore M \cdot [(A \cdot B) \supset M]$

(13) 1 $P \equiv S$
2 $\sim P$ / $\therefore \sim S \lor \sim M$

(14) 1 $A \lor (R \lor \sim P)$
2 P / $\therefore A \lor R$

(15) 1 $W \lor B$
2 $W \supset \sim S$
3 $B \supset \sim S$
4 $T \supset S$ / $\therefore \sim T$

(16) 1 $\sim B \lor M$
2 $M \supset R$ / $\therefore \sim R \supset \sim B$

(17) 1 $(S \cdot T) \supset P$
2 $P \supset F$
3 $\sim F$ / $\therefore \sim S \lor \sim T$

(18) 1 $(R \supset Q) \cdot (Q \supset R)$
2 $(B \lor M) \lor S$
3 $\sim B$
4 $\sim S$ / $\therefore (R \equiv Q) \cdot M$

(19) 1 $\sim (S \lor M)$
2 $P \supset M$
3 $M \lor \sim N$ / $\therefore \sim (P \lor N)$

(20) 1 $S \lor T$
2 $(S \lor M) \supset (Q \cdot B)$
3 $\sim B$ / $\therefore T$

(21) 1 $\sim (\sim A \lor R)$
2 R / $\therefore T \cdot A$

(22) 1 $(R \cdot M) \supset S$
2 R / $\therefore \sim S \supset \sim M$

(23) 1 $(S \cdot T) \lor (\sim S \cdot \sim T)$
2 $\sim S \lor \sim R$
3 $\sim (\sim S \cdot \sim T)$ / $\therefore \sim (R \cdot B) \cdot (S \equiv T)$

(24) 1 $\sim A \lor B$
2 $S \supset T$
3 $A \lor S$ / $\therefore \sim B \supset T$

(25) 1 $\sim (A \lor M)$
2 $S \supset A$
3 $M \lor \sim R$ / $\therefore \sim (S \lor R)$

(26) 1 $R \lor (S \cdot T)$
2 $(R \lor T) \supset \sim M$ / $\therefore M \supset F$

(27) 1 $S \supset A$
2 $B \supset S$
3 $\sim T \cdot \sim A$ / $\therefore \sim B \cdot \sim T$

(28) 1 $S \supset T$
2 $R \lor S$ / $\therefore \sim T \supset R$

(29) 1 $(R \supset S) \cdot (R \supset M)$
2 $\sim S \lor \sim M$ / $\therefore \sim (T \cdot R)$

(30) 1 $B \supset K$
2 $\sim B \supset S$ / $\therefore (K \lor S) \lor \sim A$