Chapter 2

Free-body Diagrams—Trusses

CHAPTER HIGHLIGHTS

- Introduction
- Composition and resolution of forces
- 🖙 Equilibrium law
- Internal and external forces

- Superposition and transmissibility
- Equilibrium of concurrent forces in a plane
- 🖙 Lami's theorem
- Analysis of roof trusses

INTRODUCTION

Free-body Diagram

Free-body diagram (FBD) is a sketch of the isolated body, which shows the external forces on the body and the reactions exerted on it by the removed elements. A general procedure for constructing a free-body diagram is as follows:

- **1.** A sketch of the body is drawn by removing the supporting surfaces.
- **2.** Indicate on the sketch all the applied or active forces, which tend to set the body in motion, such as those caused by weight of the body, etc.
- **3.** Also indicate on this sketch all the reactive forces, such as those caused by the constraints or supports that tend to prevent motion.
- 4. All relevant dimensions and angles, reference axes are shown on the sketch. A smooth surface is one whose friction can be neglected. Smooth surface prevents the displacement of a body normal to both the contacting surfaces at their point of contact. The reaction of a smooth surface or support is directed normal to both contacting surfaces at their point of contact and is applied at that point. Some of the examples are shown in the following diagrams.



We isolate the body from its supports and show all forces acting on it by vectors, both active (gravity force) and reactive (support reactions) forces.

We then consider the condition of equilibrium of forces, that is, in order that they will have no resultant.



Beam with roller support at one end



Beam with hinged end and fixed end.



COMPOSITION AND RESOLUTION OF FORCES

The reduction of a given system of forces to the simplest system that will be its equivalent is called 'the problem of composition of forces'. If several forces F_1 , F_2 , F_3 applied to a body at one point, all act in the same plane, then they represent a system of forces that can be reduced to a single resultant force. It then becomes possible to find this resultant by successive application of the parallelogram law. Let us consider, for example, four forces F_1 , F_2 , F_3 , and F_4 acting on a body at point A, as shown in the following figure. To find their resultant, we begin by obtaining the resultant ACof the two forces F_1 and F_2 . Combining this resultant with force F_3 , we obtain the resultant AD which must be equivalent to F_1 , F_2 , and F_3 . Finally, combining the forces AD and F_4 , we obtain the resultant 'R' of the given system of forces F_1, F_2, F_3 , and F_4 . This procedure may be carried on for any number of given forces acting at a single point in a plane.





Resolution of a Force

The replacement of a single force by several components, which will be equivalent in action to the given force, is called 'the problem of resolution of a force'. In the general case of resolution of a force into any number of coplanar components intersecting at one point on the line of action, the problem will be indeterminate unless all, but two of the components are completely specified in both their magnitudes and directions.

EQUILIBRIUM LAW

Two forces acting at a point can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action. Let us consider the equilibrium of a body in the form of a prismatic bar on the ends of which two forces are acting as shown in the figure below.



Neglecting its own weight, it follows from the principle just stated that the bar can be in equilibrium only when the forces are equal in magnitude, opposite in direction and collinear in action, which means that they must act along the line joining the points of application. Considering the equilibrium of a portion of the bar 'AB' to the left of a section *mn*, we conclude that to balance, the external force *S* at *A* the portion to the right must exert on the portion to the left an equal, opposite and collinear force '*S*' as shown in the given figure.

The magnitude of this internal axial force which the one part of a bar in tension exerts on another part is called 'the tensile force in the bar' or simply the force in the bar, since in general it may be either a tensile force or a compressive force. Such an internal force is actually distributed over the crosssectional area of the bar and its intensity, that is, the force per cross-section area is called 'the stress in the bar'.

INTERNAL AND EXTERNAL FORCES

Internal forces are the forces which hold together the particles of a body. For example, if we try to pull a body by applying two equal, opposite and collinear forces, an internal force comes into play to hold the body together. Internal forces always occur in pairs and equal in magnitude, opposite in direction and collinear. Therefore, the resultant of all of these internal forces is zero and does not affect the external motion of the body or its state of equilibrium. External forces or applied forces are the forces that act on the body due to contact with other bodies or attraction forces from other separated bodies. These forces may be surface forces (contact forces) or body forces (gravity forces). Let us consider the equilibrium of a prismatic bar on each end of which two forces are acting as shown as follows.



A force which is equal, opposite, and collinear to the resultant of the two given forces is known as equilibrant of the given two forces.

SUPERPOSITION AND TRANSMISSIBILITY

When two forces are in equilibrium (equal, opposite and collinear), then their resultant is zero, and their combined action on a rigid body is equivalent to that of no force at all. A generalization of this observation gives us the third principle of statics sometimes called 'the law of superposition'.

Law of Superposition

The action of a given system of forces on a rigid body will in no way be changed if we add to or subtract from them another system of forces in equilibrium. Let us consider now a rigid body 'AB' under the action of a force 'P' applied at 'A' and acting along BA as shown in the Figure (a) below. From the principles of superposition we conclude that the application at point 'B' of two oppositely directed forces, each equal to and collinear with P will in no way alter the action of the given force 'P'. That is, the action on the body by the three forces shown in Figure (b) is same as the action on the Figure (a).





effect of the force on any rigid body to which it may be applied. This statement is called 'the theorem of transmissibility of a force'.

EQUILIBRIUM OF CONCURRENT FORCES IN A PLANE

If a body known to be in equilibrium is acted upon by several concurrent coplanar forces, then these forces or rather their free vectors, when geometrically added, must form a closed polygon. This statement represents the condition of equilibrium for any system of concurrent forces in a plane. In Figure (a), we consider a ball supported in a vertical plane by a string '*BC*' and a smooth wall '*AB*'. The free-body diagram in which the ball has been isolated from its supports, and in which all forces acting upon it, both active and reactive, are indicated by vectors as shown in Figure (b).



The three concurrent forces W, S and R_A are a system of forces in equilibrium and, hence their free vectors must build a closed polygon, in this case, a triangle as shown in Figure (c).

If numerical data are not given, we can still sketch the closed triangle of forces, and then express:

$$R_A = W \tan \alpha$$
 and $S = W \sec \alpha$

LAMI'S THEOREM

If three concurrent forces are acting on a body, kept in equilibrium, then each force is proportional to the sine of the angle between the other two forces and the constant of proportionality is the same. Consider forces P, Q and R acting at a point 'O' as shown in Figure (a). Mathematically Lami's theorem is given by the following equation.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = k$$

Since the forces are in equilibrium, the triangle of forces should close. Draw the triangle of forces $\triangle ABC$, as shown in Figure (b), corresponding to forces *P*, *Q*, and *R* acting at a point '*O*'. From the sine rule of the triangle, we get:



 $\sin(\pi - \gamma) = \sin \gamma$

When feasible, the trigonometric solution, or Lami's theorem is preferable to the graphical solution since it is free from the unavoidable small errors associated with the graphical constructions and scaling.

ANALYSIS OF ROOF TRUSSES

Definitions

Truss

A 'truss' or 'frame' or 'braced structure' is the one consisting of a number of straight bars joined together at the extremities. These bars are members of the truss.

Plane Truss

If the centre line of the members of a truss lies in a plane, the truss is called a 'plane truss' or 'frame'. If the centre line is not lying in the same plane, as in the case of a shear leg, the frame is called a 'space frame'.



Strut and Tie

A member under compression is called a 'strut' and a member under tension is called a 'tie'.

Loads

A load is generally defined as a weight or a mass supported. Trusses are designed for permanent, intermittent or varying loads.

Nodes

The joints of a frame are called 'nodes'. A frame is designed to carry loads at the nodes.

Perfect Frame

A pin joined frame which has got just the sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a 'perfect frame'.

Supports

A truss or a framed structure is held on supports which exert reaction on the truss or framed structure that they carry. Reactions are to be considered for finding the stresses in the various members of the structures. The types of supports commonly used are:

- 1. Simple supports
- 2. Pin joint and roller supports
- 3. Smooth surfaces
- 4. Fixed on encaster and fixtures

The reactions of the supports are analytically or graphically evaluated.

- **1.** In a simply supported truss the reactions are always vertical at the supports.
- **2.** At a pin joint support, the reaction passes through the joint.
- **3.** At a roller surface, the support reaction is vertically upwards at the surface.
- **4.** The reaction at a support which is a smooth surface is always normal to the surface.

Assumptions—Analysis of Trusses

- **1.** Each truss is assumed to be composed of rigid members to be all lying in one plane. This means that coplanar force systems are involved.
- **2.** Forces are transmitted from one member to another through smooth pins fitting perfectly in the members. These are called 'two force members'.
- **3.** Weights of the members are neglected because they are negligible in comparison to the loads.



Pin joint and roller support



Pin joint and smooth surface



Fixed support









Free-body diagram of truss as a whole



Free-body diagram of point A















Free-body diagram of joints

Solution by Method of Joints

To use this technique, draw a free-body diagram of any pin in the truss provided no more than two unknown forces act on that pin. This limitation is imposed because the system of forces is a concurrent one for which of course, only two equations are available for a solution. From one pin to another, until the unknown is found out, the procedure can be followed.

Working Rules

- **1.** Depending on the nature of support, provide the reaction components.
 - (a) For hinged support, provide horizontal and vertical reaction components.
 - (b) For roller support, provide vertical reaction components only.
- **2.** Considering the external loads affecting the truss only, apply the laws of statics at equilibrium to evaluate the support reactions.
- **3.** Give the values of the support reactions at appropriate joints.
- **4.** Take the joint which contains the minimum number of members (minimum number of unknowns) and apply the conditions of equilibrium to evaluate the forces in the members.

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Example:



The reaction components at A are A_H and A_V (because A is a hinged joint).

The reaction component at B is B_V only (no horizontal reaction since it is a roller).

Now, evaluate the reaction components considering the external loads only. That is,

$$A_V + B_V = P + Q$$
$$A_H = 0$$

Taking algebraic sum of the moments of all forces about A' and equating to zero, we get two more equations.

These three equations are sufficient to evaluate the support reactions.

Once the support reactions are evaluated, joints can be considered one by one, to evaluate the force in the members.

Method of Members—Analysis of Plane Frames

Frames differ from trusses principally in one aspect, i.e., the action of forces are not limited to their ends only, and so the members are subjected to bending also with tension or compression. In this method, the members are isolated as a free body and analyzed with the forces acting on them by vectors.

Consider the folding stool with the dimensions as shown in the figure (below) resting on a horizontal floor and a force 'F' is acting at a distance of xl form the end point 'A'.



The floor is considered as a smooth floor. Therefore, the reactions at C and D are vertical.

 \therefore By taking moments at *C* and *D*:

$$R_D = (1 - x)F$$
 and $R_c = xF$.

Now, we separate the members *AC* and *BD* and analyze the forces acting on each member.



For the free-body diagram BD, taking moments about B:

$$Y_E \frac{l}{2} + X_E \frac{l}{2} - (1 - x)Fl = 0$$

For the free-body diagram AC, taking the moments about A:

$$Y_E \, \frac{l}{2} - X_E \, \frac{l}{2} + XlF = 0.$$

 $\therefore X_E = F$ and $Y_E = (1 - 2x) F$.

The resultant is $R_E = \sqrt{X_E^2 + Y_E^2} = F\sqrt{1 + (1 - 2x)^2}$

: By knowing the numerical data for l, x, and the load F, the unknowns R_D , R_C and R_E can be found. To find the reactions at A and B, i.e., R_A and R_B take the moments about the point E for both the free-body diagrams and solve for R_A and R_B .

SOLVED EXAMPLES

Example 1

The magnitude and nature of stresses in the member ED of the truss, loaded (shown below), is



Solution



Free-body diagram

Taking moments about (A) for equilibrium, $\Sigma m_A = 0$ $-20 \times 1.5 - 40 \times 4.5 + R_2 \times 6 = 0$ $6R_2 = 30 + 180$ $6R_2 = 210$ $R_2 = 35$ kN But $R_1 + R_2 = 60$ $\therefore R_1 = 25$ kN Take the joint D. Force on the member CD, $F_{CD} = 40.41$ kN because $F_{CD} \sin 60^\circ$

= 35 kN \therefore Force on ED = F_{CD} $\cos 60^\circ = 20.2$ kN (T):

Example 2

All the members of the truss, shown in the following figure, are of equal length and the joints are pinned smooth. It carries a load F at S whose line of action passes through V. The reaction at V is:



(A) Zero

- (B) Vertically upwards and equal to F/4
- (C) Vertically upwards and equal to F/2
- (D) Vertically upwards and equal to F

Solution



Let a =length of one member

From the given figure, $\sin 60^\circ = \frac{SW}{SR}$ $SW = \frac{\sqrt{3}}{2}a (\because SR = a)$ Also, $\cos 60^\circ = \frac{RW}{SR} \implies RW = \frac{a}{2}$, TW = RT - RW $\therefore a - \frac{a}{2} = \frac{a}{2}$ $VW = VT + TW = a + \frac{a}{2} = \frac{3}{2a}$ $\tan \theta = \frac{SW}{VW} = \left[\frac{\sqrt{3}a}{\frac{2}{3a}}\right] = \frac{1}{\sqrt{3}}$ $\theta = 30^\circ$

Taking moments about P for equilibrium, $\Sigma M_P = 0$

$$-F\sin\theta \times \frac{3a}{2} - F\cos\theta \times \frac{\sqrt{3}}{2}a + R_V \times 30 = 0$$
$$F \times \frac{1}{2} \times \frac{3a}{2} + F \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}a = R_V \times 3a$$
$$R_V = \frac{F}{2}$$

Hence, the correct answer is option (C).

Example 3

The force in the member RQ of the truss, as given in the figure below is



- (A) 27 kN (Tensile)
- (B) 15 kN (Compressive)
- (C) 20 kN (Compressive)
- (D) 7 kN (Tensile)

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Solution



Consider the junction *R*. It must be in equilibrium. The force 7 kN can be balanced only by the member Q_R . \therefore The force in the member Q_R FQR = 7 kN (Tensile).

Hence, the correct answer is option (D).

Example 4

The figure is a pin jointed plane truss loaded at point C by hanging a weight of 1200 kN. The member DB of the truss is subjected to a load of



- (A) zero
- (B) 500 kN in compression
- (C) 1200 kN in compression
- (D) 1200 kN in tension

Solution

Member *DB* is perpendicular to *AC*. Resolving the vertical component of the forces at *B*, we observe that no force can be present in member *DB*.

Hence, the correct answer is option (A).

Example 5

Find the force in the member *EC* of the truss shown in the figure.



(A) 8 kN	(B) 4 kN
(C) 3.5 kN	(D) 2 kN

Solution



Equating vertical forces, $V_D = 8$ Taking moment about D, $-2 \times 20 - 4 \times 10 + H_C \times 11.54 = 0$ $80 = 11.54 H_C$:. $H_C = 6.928 \text{ kN}$ Consider the equilibrium of joint A $F_{AE} \sin 30^\circ = 2;$ $F_{AE} = 4 \text{ kN} (\text{T})$ $F_{AF}\cos 30^\circ = F_{BA}$ $4\frac{V3}{2} = 2\sqrt{3}F_{BA}$ $F_{BE} = 0, \ F_{CB} = 2\sqrt{3}$ Consider point C Net horizontal force = $6.928 - 2\sqrt{3}$ = 3.463 kN It is to be balanced by the force on EC. $F_{EC} \cos 30 = 3.463$

 $\therefore F_{EC} = 4 \text{ kN}.$

Hence, the correct answer is option (B).

Example 6

The type of truss as shown in the figure below, is



Solution

The number of joints, J = 6The number of member, n = 10Then, $2j - 3 = 2 \times 6 - 3 = 9$ Since, n > (2j - 3), it is a redundant truss.

Example 7

A weight 200 kN is supported by two cables as shown in the figure below.



The tension in the cable AB will be minimum when the angle θ is: (B) 30°

(A) 0°

(C) 90° (D) 120°

Solution

$$\frac{T_1}{\sin 150} = \frac{T_2}{\sin(90+\theta)}$$
$$= \frac{200 \,\text{kN}}{\sin 180 - (\theta + 60)}$$
$$T_1 = \frac{200 \sin 30}{\sin(120-\theta)}$$
$$= \frac{200 \sin 30}{\sin[120-\theta]} \quad (\because \sin 30^\circ = \sin 150)$$

- T_1 is minimum when $\sin(120 \theta)$ is minimum,
- i.e., when $\sin(120 \theta)$ is maximum

i.e., when $\sin(120 - \theta) = 1$

i.e., when $120 - \theta = 90$

i.e., when $\theta = 30^{\circ}$

Hence, the correct answer is option (B).

Example 8

A 200 kN weight is hung on a string as shown in the figure below. The tension T is:



(A) 200 kN (C) 160 kN

Solution

The three forces T, R_{R} and 200 kN are in equilibrium at point O.

(D) 207.1 kN



$$\frac{T}{\sin 90} = \frac{W}{\sin(90^\circ + 15^\circ)}$$

$$T = W \frac{\sin 90}{\sin 105} = \frac{200 \times 1}{0.965} = 207.1 \text{ kN}$$

Hence, the correct answer is option (D).

Example 9



A mass of 40 N is suspended from a weightless bar PQ which is supported by a cable *QR* and a pin at *P*. At *P*, on the bar, the horizontal one vertical component of the reaction, respectively, are:

(A) 80 N and 0 N (B) 75 N and 0 N (C) 60 N and 80 N (D) 55 N and 80 N

Solution

$$\theta = \cos^{-1} \frac{150}{335} = 63.4^{\circ}$$

The vertical components of the forces at *Q*: $40 = T\cos 63.4^{\circ}$

$$T = \frac{40}{\cos 63.4} = T = 89.336$$
 N

The vertical components of the forces at $P, R_v = 0$ The horizontal component of the forces at

 $RQP, R_H = T \sin 63.4 = 80$ N.

Example 10

A truck of weight Mg is shown in the figure. A force 'F' (pull) is applied. The reaction at the front wheels at location P is:



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(A) $\frac{Mg}{2a} + \frac{Fb}{2}$	(B) $\frac{Mg}{a} + \frac{Fb}{2}$
(C) $\frac{Mg}{2} + \frac{Fb}{2a}$	(D) $\frac{Mg}{2a} + \frac{F}{2}$

Solution

Taking moment about *Q*: $\Sigma M_Q = 0 = R_P \times 2a - Mg \times a - F \times b$ $R_P = \frac{Mg \times a + F \times b}{2a} = \frac{Mg}{2} + \frac{Fb}{2a}.$

Hence, the correct answer is option (C).

Direction for solved examples 11 and 12:

All the forces acting on a particle situated at the point of origin of a two-dimensional reference frame. One force has magnitude of 20 N acting in the positive *x* direction. Whereas the other has a magnitude of 10 N at an angle of 120° with force directed away from the origin with respect to the positive direction to the direction of 20 N.



Example 11

The value of the resultan	t force, <i>R</i> , will be:
(A) 17.32 N	(B) 20 N
(C) 15 N	(D) 21 N

Solution

$R = \sqrt{20^2 + 10^2}$	2 + 2 × 20 × 10 ×	$\cos 120^\circ = 1$	17.32 N
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Hence, the correct answer is option (A).

Example 12

The value of α made by the resultant force with the horizontal force will be:

(A)	30	(B)	13
(C)	14.5	(D)	15

Solution

From triangle OQP:

$$\frac{10}{\sin \alpha} = \frac{R}{\sin 60} = \frac{17.32}{0.866}$$

 $\sin \alpha = 0.5$ $a = 30^{\circ}$.

Hence, the correct answer is option (A).

Exercises

1. The figure shows a pin-jointed plane truss loaded at the point *M* by hanging a mass of 100 kg. The member *LN* of the truss is subjected to a load of



- (A) 0 N
- (B) 490 N in compression
- (C) 981 N in compression
- (D) 981 N in tension
- 2. A truss consists of horizontal members (*PU*, *UT*, *TS*, *QR*) and vertical members (*UQ*, *TR*) all having a length *B* each.



The members PQ, TQ and SR are inclined at 45° to the horizontal. If an uniformly distributed load 'F' per unit length is present on the member QR of the truss shown in the figure above, then the force in the member UT is







The figure shows a pair of pin jointed gripper tongs holding an object weighing 1000 N. The co-efficient of friction (μ) at the gripping surface is 0.1. *XX* is the line of action of the input force *P* and *YY* is the line of application of gripping force. If the pin joint is assumed to be frictionless, then the magnitude of the force *P* required to hold the weight is

(A)	500 N	(B)	1000 N
(C)	2000 N	(D)	2500 N

4. A stone with a mass of 0.2 kg is catapulted as shown in the figure below. The total force F_x (in *N*) exerted by the rubber band as a function of the distance *x* (in m) is given by $F_x = 300 x^2$. If the stone is displaced by 0.2 m from the unstretched position (x = 0) of the rubber band, the energy stored in the rubber band is



5. For the truss shown in the figure, the force (N) in the member BC is



- (A) 0 N (compressive)
- (B) 0.577 W (tensile)
- (C) 0.577 W (compressive)
- (D) 0.866 W (compressive)



A simply supported structure is loaded as shown in the figure. Force in the member AB is

(B) 13.42 kN

(A) 10.26 kN



For the truss loaded as shown in the above figure, force in the member *CD* is

(A) 5 kN (B) 2.5 kN

(C)
$$\frac{5}{\sqrt{2}}$$
 kN (D) $5\sqrt{2}$ kN



A weight of 200 N is hung using a cable as shown in the figure. Tensions in portions of cable AC and BC are respectively

- (A) 59.6, 171.7 N
 (B) 62.4, 176.8 N
 (C) 62.5, 182.7 N
- (D) 68.4, 187.9 N

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Two steel truss members AC and BC with cross-section area 100 mm² is subjected to a horizontal force P kN as shown in figure. Maximum value of P such that axial stress in any of the members does not exceed 50 MPa is (A) 10.15 kN (B) 9.22 kN

- (C) 7.92 kN (D) 6.83 kN
- **10.** A mechanism has 5 numbers of joints and 6 members. The number of additional members needed to make it a perfect frame will be

(A)	4	(B)	3
(C)	2	(D)	1



For the truss loaded as shown in the above figure, force in the member *CD* is

(B) 2.5 kN

(D) $5\sqrt{2}$ kN

(A) 5 kN





Two heavy spheres of equal weight W and radius 50 mm lies in a smooth cup of 150 mm radius and are in equilibrium. The ratio of reaction between sphere and cup to reaction between spheres is

(A) 2 (B)
$$\sqrt{2}$$

(C)
$$\frac{1}{2}$$
 (D) $\frac{1}{\sqrt{2}}$

13. A roller of radius 'r' and weight 'W' is to be rolled over a curb of height 'h' by a horizontal force 'P' applied to the end of a string wound around the circumference of roller. Find the magnitude of 'P' required to start the roller over the curb. Assume that contact at 'B' is frictionless but there is sufficient friction between roller surface and the edge of the curb to present slip at 'A'. Given r/h = 2



In the truss loaded as shown in the figure, tension in the member AB is

(A) 3.66 kN	(B) 3.86 kN
(C) 4.14 kN	(D) 4.92 kN



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Exercis	es								
1. A	2. A	3. D	4. C	5. C	6. B	7. A	8. D	9. D	10. D
11. A	12. A	13. B	14. A						

Previous Years' Questions

1. B **2.** A **3.** C