CHAPTER 3

Units, Dimensions and Measurements

SYSTEMS OF UNITS

S.No.	MKS	CGS	FPS
1.	Length (m)	Length (cm)	Length (ft)
2.	Mass (kg)	Mass (g)	Mass (pound)
3.	Time (s)	Time (s)	Time (s)

Fundamental Quantities in S.I. System and Their Units

S.No.	Physical Quantity	Name of Unit	Symbol
1.	Mass	kilogram	kg
2.	Length	meter	m
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Luminous intensity	candela	Cd
6.	Electric current	ampere	A
7.	Amount of substance	mole	mol

Dimensional Formula

Relations which express physical quantities in terms of appropriate powers of fundamental units.

Application of Dimensional Analysis

- To check the dimensional consistency of a given physical relationship.
- To derive relationship between various physical quantities.
- To convert units of a physical quantity from one system to another

$$n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c \text{ where } u = M^aL^bT^c$$

Dimensional Formulae of Various Physical Quantities

S.No.	Physical Quantity	Dimensional Formula
	Mechanics	
1.	Area	$L \times L = L^2 = [M^0L^2T^0]$
2.	Volume	$L \times L \times L = [M^0L^3T^0]$
3.	Density	$\frac{M}{L^3} = [ML^{-3}T^0]$
4.	Speed or Velocity	$\frac{L}{T} = [M^0LT^{-1}]$
5.	Acceleration	$\frac{LT^{-1}}{T} = LT^{-2} = [M^0LT^{-2}]$
6.	Momentum	$M \times LT^{-1} = [MLT^{-1}]$
7.	Force	$M \times LT^{-2} = [MLT^{-2}]$
8.	Work	$MLT^{-2} \times L = [ML^2T^{-2}]$
9.	Energy	$[ML^2T^{-2}]$
10.	Power	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$
11.	Pressure	$\frac{ML^1T^{-2}}{L^2} = [ML^{-1}T^{-2}]$
12.	Moment of force or torque	$MLT^{-2} \times L = [ML^2T^{-2}]$
13.	Gravitational constant 'G'	$\frac{[MLT^{-2}][L^2]}{M \times M} = [M^{-1}L^3T^{-2}]$
14.	Impulse of a force	$MLT^{-2} \times T = [MLT^{-1}]$
15.	Stress	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$
16.	Strain	$[M^0L^0T^0]$

S.No.	Physical Quantity	Dimensional Formula
17.	Coefficient of elasticity	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$
18.	Surface tension	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$
19.	Free Surface energy	$\frac{ML^2T^{-2}}{L^2} = [ML^0T^{-2}]$
20.	Coefficient of viscosity	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$
21.	Angle	$\frac{L}{L} = 1 = [M^0L^0T^0]$
22.	Angular velocity	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$
23.	Angular acceleration	$\frac{T^{-1}}{T} = T^{-2} = [M^0L^0T^{-2}]$
24.	Moment of inertia	$ML^2 = [ML^2T^0]$
25.	Radius of gyration	$L = [M^0LT^0]$
26.	Angular momentum	$M \times LT^{-1} \times L = [ML^2T^{-1}]$
27.	T-ratios (sin θ, cos θ, tan θ)	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)
28.	Time period	$T = [M^0L^0T^{-1}]$
29.	Frequency	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$
30.	Planck's constant 'h'	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$
31.	Relative density	$\frac{ML^{-3}}{ML^{-3}} = 1 = [M^0L^0T^0]$ (Dimensionless)

S.No.	Physical Quantity	Dimensional Formula
32.	Velocity gradient	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$
33.	Pressure gradient	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$
34.	Force constant	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$
Thermodynamics		
35.	Heat or enthalpy	$[ML^2T^{-2}]$
36.	Specific heat	$\frac{[ML^2T^{-2}]}{[M][K]} = [M^0L^2T^{-2}K^{-1}]$
37.	Latent heat	$\frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$
38.	Thermal conductivity	$\frac{ML^2T^{-2}.L}{L^2.K.T} = [MLT^{-3}K^{-1}]$
39.	Entropy	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$
40.	Universal Gas Constant	$\frac{ML^{-1}T^{-2}L^3}{mol.K} = [ML^2T^{-2}K^{-1}mol^{-1}]$
41.	Thermal conductivity	$\frac{[ML^2T^2.L]}{[L^2.K.T]} = [MLT^{-3}K^{-1}]$
42.	Universal Gas Constant	$\frac{[ML^{-1}T^{-2}[L^3]}{[mol.K]} = [ML^2T^{-3}K^{-1}mol^{-1}]$
43.	Boltzmann's Constant	$\frac{[ML^2T^{-2}]}{[K]} = [ML^2T^{-2}K^{-1}]$
44.	Stefan's constant	$\frac{[ML^2T^{-2}]}{[L^2.T.K^4]} = [ML^0T^{-3}K^{-4}]$

S.No.	Physical Quantity	Dimensional Formula
45.	Solar constant	$\frac{[ML^2T^{-2}]}{[L^2.T]} = [ML^0T^{-3}]$
46.	Mechanical equivalent of heat	$\frac{[ML^2T^{-2}]}{[ML^2T^{-2}]} = [M^0L^0T^0]$ (Dimensionless)
Electrostats		
47.	Electric Charge	$[T.A] = [M^0L^0TA]$
48.	Electrical potential	$\frac{[ML^2T^{-2}]}{[TA]} = [ML^2T^{-3}A^{-1}]$
49.	Resistance	$\frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$
50.	Capacitance	$\frac{[TA]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$
51.	Inductance	$\frac{[ML^2T^{-3}A^{-1}]}{[AT^{-1}]} = [ML^2T^{-2}A^{-2}]$
52.	Permittivity of free space	$\frac{[AT.AT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$
53.	Relative permittivity or dielectric constant	A pure ratio = $[M^0L^0T^0]$ (dimensionless)
54.	Intensity of electric field	$\frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$
55.	Conductance	$\frac{1}{ML^2T^{-3}A^{-2}} = [M^{-1}L^{-2}T^{-3}A^2]$
56.	Specific resistance or resistivity	$\frac{[ML^2T^{-3}A^{-2}][L^2]}{[L]} = [ML^3T^{-3}A^{-2}]$
57.	Specific conductance of conductivity	$[M^{-1}L^{-3}T^3A^2]$
58.	Electric dipole moment	$[AT][L] = [M^0LTA]$

S.No.	Physical Quantity	Dimensional Formula
Magnetism		
59.	Magnetic field	$\frac{MLT^{-2}}{AT.LT^{-1}.1}=[ML^0T^{-2}A^{-1}]$
60.	Magnetic flux	$[MT^{-2}A^{-1}].[L^2]=[ML^2T^{-2}A^{-1}]$
61.	Permeability of free space	$\frac{[L][MLT^{-2}]}{[A^2.L]}=[MLT^{-2}A^{-2}]$
62.	Magnetic moment	$A.L^2=[M^0L^2T^0A]$
63.	Pole strength	$\frac{AL^2}{L}=[M^0LT^0A]$

- Distance of an object by parallax method, $D=\frac{\text{Basis}}{\text{Parallax angle}}$
- Absolute error = True value – Measured value = $[\Delta a_n]$
- True value = Arithmetic mean of the measured values

$$a_{mean}=\frac{a_1+a_2+\ldots+a_n}{n}$$

- Relative error in the measurement of a quantity = $\frac{\Delta a_{maean}}{a_{mean}}$
- Percentage error = $\frac{\Delta a_{maean}}{a_{mean}}\times 100$
- Maximum permissible error in addition or subtraction of two quantities $(A\pm\Delta A)$ and $(B\pm\Delta B)$: $\Delta A+\Delta B$.
- Maximum permissible relative error in multiplication or division of two quantities $(A\pm\Delta A)$ and $(B\pm AB)$:

$$\frac{\Delta A}{A}+\frac{\Delta B}{B}$$

- When $z=\frac{a^p.b^q}{c^r}$, then maximum relative error in z is

$$\frac{\Delta z}{z}=p\frac{\Delta a}{a}+q\frac{\Delta b}{b}+r\frac{\Delta C}{C}$$

SIGNIFICANT FIGURES

The following rules are observed in counting the number of significant figures in a given measured quantity:

Example: 42.3 has three significant figures.
243.4 has four significant figures.

A zero becomes a significant figure if it appears between two non-zero digits.

Example: 5.03 has three significant figures.
5.604 has four significant figures.

Leading zeros or the zeros placed to the left of the number are never significant.

Example: 0.543 has three significant figures.
0.006 has one significant figure.

Trailing zeros or the zeros placed to the right of the number are significant.

Example: 4.330 has four significant figures.
433.00 has five significant figures.

In exponential notation, the numerical portion gives the number of significant figures.

Example: 1.32×10^{-2} has three significant figures.
1.32×10^{4} has three significant figures.

ROUNDING OFF

While rounding off measurements, we use the following rules by convention:

If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example: x = 7.82 is rounded off to 7.8, again x = 3.94 is rounded off to 3.9.

If the digit to be dropped is more than 5, then the preceding digit is raised by 1.

Example: x = 6.87 is rounded off to 6.9, again x = 12.78 is rounded off to 12.8.

If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by 1.

Example: x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is left unchanged, if it is even.

Example: x = 3.250 becomes 3.2 on rounding off. again x = 12.650 becomes 12.6 on rounding off.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1, if it is odd.

Example: x = 3.750 is rounded off to 3.8. again x = 16.150 is rounded off to 16.2.

Significant Figures in Calculation

The following two rules should be followed to obtain the proper number of significant figures in any calculation.

1. The result of an addition or subtraction in the number having different precisions should be rounded off the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples:

(*a*)

$$\begin{array}{r} 33.3 \\ 3.11 \\ +\ 0.313 \\ \hline 36.723 \\ \hline \end{array}$$

(has only one decimal place)

(answer should be rounded off one decimal place)

Answer = 36.7

(*b*)

$$\begin{array}{r} 3.1421 \\ 0.241 \\ +\ 0.09 \\ \hline 3.4731 \\ \hline \end{array}$$

(has 2 decimal places)

(answer should be rounded off 2 decimal places)

Answer = 3.47

(*c*)

$$\begin{array}{r} 62.831 \\ -\ 24.5492 \\ \hline 38.2818 \\ \hline \end{array}$$

(has 3 decimal places)

(answer should be rounded off 3 decimal places)

Answer = 38.282

2. The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples:

(*a*)

$$\begin{array}{r} 142.06 \\ \times\ 0.23 \\ \hline 32.6738 \\ \hline \end{array}$$

(two significant figures)

(answer should have two significant figures)

Answer = 33

(*b*)

$$\begin{array}{r} 51.028 \\ \times\ 1.31 \\ \hline 66.84668 \\ \hline \end{array}$$

(three significant figures)

Answer = 66.8

(*c*) $\dfrac{0.90}{4.26} = 0.2112676$

Answer = 0.21

Order of Magnitude

In scientific notation, the numbers are expressed as: Number = M × 10^3. Where M is a number that lies between 1 and 10 and x is an integer. The order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off. We ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by 1. For example.

Speed of light in vacuum = 3×10^8 $ms^{-1} \approx 10^8$ ms^{-1} (ignoring 3 < 5)

Mass of electron = 9.1×10^{-31} kg $\approx 10^{-30}$ kg (as 9.1 > 5).

Example 1: Each side of a cube is measured to be 7.203 m. Find the volume of the cube up to appropriate significant figures.

Sol. Volume = $a^3 = (7.023)^3 = 373.715$ m^3

Example 2: The mass of abox is 2.3 kg. Two marbles of masses 2.15 g and 12.39 g are added to it. Find the total mass of the box to the correct number of significant figures.

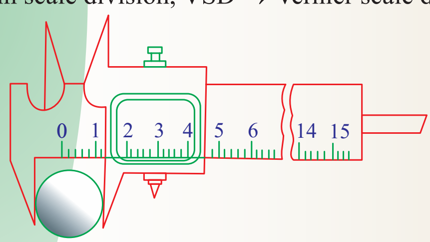
Sol. Total mass = 2.3 + 0.00215 + 0.01239 = 2.31 kg

The total mass in appropriate significant figures will be 2.3 kg.

Vernier scale and screw gauge basics:

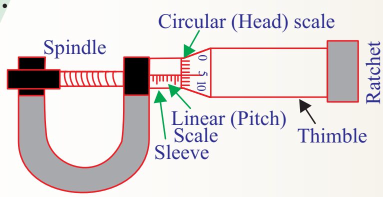
- **Vernier Callipers Least count = 1MSD – 1VSD**

  (MSD → main scale division, VSD → Vernier scale division)



Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each path equal to 1 mm then least count = 1 mm – $\frac{9}{10}$ mm = 0.1 mm [∵ 9 MSD = 10 VSD]

- **Screw Gauge:**



$$\text{Least count} = \frac{\text{pitch}}{\text{total no. of divisions on circular scale}}$$