Thermodynamics

Temperature

Temperature is that property of a body which helps us to decide the degree of its hotness. When two bodies at different temperatures are kept in contact, in absence of any other effect, heat flows from the body at higher temperature to the one at lower temperature always. This flow occurs till the temperature of the two bodies become equal. This state is also referred to as **thermal equilibrium.**

Zeroth Law of Thermodynamics

If two systems A and B are each in thermal equilibrium with a third system C, then A and B will be in thermal equilibrium with each other.

Heat

Heat is the energy transferred between a system and its environment because of a temperature difference that exists between them. It is a scalar quantity. Its SI unit is joule. It is path dependent.

Work

The work done by the system is

$$W = \int dW = \int_{V_i}^{V_f} P\,dV$$

Here V_i and V_f referred to the change in volume from state i to state f.

During the change in volume, the pressure and temperature of the system also change. It is a scalar quantity. Its SI unit is joule. It is path dependent. In P-V diagram (also called indicator diagram) the area under P-V curve represents work done.

Internal Energy

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

Change in internal energy is path independent and depends only on the initial and final states of the system.

The internal energy of an ideal gas is only dependent upon temperature whereas that of a real gas it is a function of both temperature and volume.

Thermodynamic Variables

These are macroscopic physical quantities like pressure (P), volume (V) and temperature (T) etc. which are used to describe the state of the system. The relation between these variables is called **equation of state**. Heat and work are not thermodynamic variables.

Thermodynamic variables are of two kinds :

- Extensive
- Intensive

Internal energy U, volume V, total mass M are extensive variables. Pressure P, temperature T and density ρ are intensive variables.

First law of Thermodynamics

First law of thermodynamics is simply a restatement of principle of conservation of energy. Imagine a gaseous system for which P(pressure), V(volume) and T(temperature) are related by certain equation of state and is undertaken through a process. If ΔQ, ΔU and ΔW represent the heat given to the system, change in internal energy and work done by the system respectively. During the process the first law of thermodynamic states that

$$\Delta Q = \Delta U + \Delta W$$

Sign Convention

- Heat absorbed by the system is taken as positive while the heat lost by the system is taken as negative.
- Work done by the system is taken as positive while work done on a system is taken as negative.
- The increase in internal energy of the system is taken as positive while decrease in internal energy is taken as negative.

Isothermal Process

For such a process temperature remains constant throughout the process. Hence PV = constant as T is constant. Work is done at the same rate as heat is supplied, hence there is no increase of internal energy. Due to this, $\Delta U = 0$ and thus $\Delta Q = \Delta W$.

Work done during an isothermal process

$$W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$$

$$W = \mu RT \ln\left(\frac{P_1}{P_2}\right)$$

The slope of isothermal curve on a P-V diagram at any point on the curve is given by

$$\frac{dP}{dV} = \frac{-P}{V}$$

In an isothermal expansion the gas absorbs heat and does work and in an isothermal compression work is done on the gas by the environment and heat is released.

Adiabatic Process

An adiabatic process is one that occurs so rapidly or occurs in a system that is so well insulated that no transfer of energy as heat occurs between the system and its environment.

Equation of adiabatic process,

PV^{γ} = constant

$TV^{\gamma-1}$ = constant

where, $\gamma = \left(\frac{C_P}{C_V}\right)$ is called adiabatic exponent.

Work done during an adiabatic process

$$W = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} = \mu R \frac{(T_1 - T_2)}{\gamma - 1}$$

The slope of adiabatic curve on a P-V diagram at any point on the curve is given by

$$\frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$$

In an adiabatic expansion temperature of the gas will fall ($T_2 < T_1$) while in adiabatic compression temperature of the gas will rise ($T_2 > T_1$).

Second Law of Thermodynamics

Kelvin-Planck statement : No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.

Clausius statement : No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

Reversible and Irreversible Processes

Reversible process : A reversible process is one which can be retraced in the opposite direction.

A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is an example of a reversible process.

Irreversible process : An irreversible process is one which cannot be retraced back in the opposite direction.

All spontaneous processes of nature are irreversible processes. *e.g.* transfer of heat from a hot body to a cold body, diffusion of gases, etc. are all irreversible processes.

Carnot Engine

Carnot engine is a reversible heat engine operating between two temperatures T_1 (source) and T_2 (sink).

Carnot cycle : Carnot engine works in series of operations. The operations consist of an isothermal expansion and then adiabatic expansion. Further operations are isothermal compression and adiabatic compression so that the working substance is back at the initial state at the end of each cycle. This cycle of operations is called Carnot cycle.

The efficiency of a Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

The efficiency of Carnot engine depends on the temperature of source (T_1) and temperature of the sink (T_2), but does not depend upon the nature of the working substance.

Carnot theorem : No heat engine operating between two given temperatures can be more efficient than a Carnot engine operating between the same two temperatures.

EXAM DRILL

1. A gas at pressure 6×10^5 N m^{-2} and volume 1 m^3 and its pressure falls to 4×10^5 N m^{-2}. When its volume is 3 m^3. Given that the indicator diagram is a straight line, work done by the system is
(a) 6×10^5 J (b) 3×10^5 J
(c) 4×10^5 J (d) 10×10^5 J

2. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is (R = 8.3 J mol^{-1} K^{-1})
(a) diatomic
(b) triatomic
(c) a mixture of monoatomic and diatomic
(d) monoatomic

3. The temperatures of inside and outside of a refrigerator are 273 K and 303 K respectively. Assuming that the refrigerator cycle is reversible, for every joule of work done, the heat delivered to the surroundings will be nearly.
(a) 10 J (b) 20 J
(c) 30 J (d) 50 J

4. In a given process of an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas
(a) the temperature will decrease
(b) the volume will increase
(c) the pressure will remain constant
(d) the temperature will increase

5. An ideal gas at a pressure of 1 atmosphere and temperature of 27°C is compressed adiabatically until its pressure becomes 8 times the initial pressure. Then the final temperature is $\left(\gamma = \frac{3}{2}\right)$
(a) 627°C (b) 527°C
(c) 427°C (d) 327°C

6. An ideal Carnot engine whose efficiency is 40% receives heat at 500 K. If the efficiency is to be 50%, the in take temperature for the same exhaust temperature is
(a) 600 K (b) 900 K
(c) 700 K (d) 800 K

7. A gas expands with temperature according to the relation $V = KT^{2/3}$. Calculate work done when the temperature changes by 60 K?
(a) $10R$ (b) $30R$
(c) $40R$ (d) $20R$

8. A Carnot engine whose sink is at 300 K has an efficiency of 40%. By how much should the temperature of source be increased so as to increase its efficiency by 50% of original efficiency?
(a) 380 K (b) 275 K
(c) 325 K (d) 250 K

9. An ideal gas heat engine operates in Carnot cycle between 227°C and 127°C. It absorbs 6×10^4 cal of heat at higher temperature. Amount of heat converted to work is
(a) 2.4×10^4 cal (b) 6×10^4 cal
(c) 1.2×10^4 cal (d) 4.8×10^4 cal

10. A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature rise from 27°C to 97°C. The heat produced in the gas (γ = 1.5) is
(a) 250.6 cal (b) 276.7 cal
(c) 298.5 cal (d) 320 cal

11. An electric heater supplies heat to a system at a rate of 120 W. If system performs work at a rate of 80 J s^{-1}, the rate of increase in internal energy is
(a) 30 J s^{-1} (b) 40 J s^{-1}
(c) 50 J s^{-1} (d) 60 J s^{-1}

12. One mole of an ideal gas undergoes an isothermal change at temperature T so that its volume V is doubled. R is the molar gas constant. Work done by the gas during this change is
(a) $RT\ln 4$ (b) $RT\ln 2$
(c) $RT\ln 1$ (d) $RT\ln 3$

13. Two spheres A and B have diameters in the ratio 1 : 2, densities in the ratio 2 : 1 and specific heats in the ratio 1 : 3, the ratio of their thermal capacities is
(a) 1 : 6 (b) 1 : 12
(c) 1 : 3 (d) 1 : 4

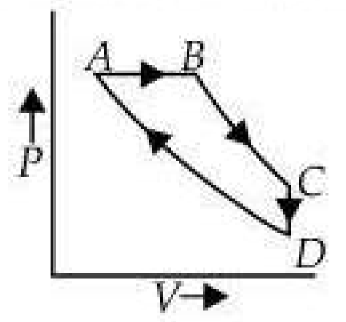
14. How much heat energy in joules must be supplied to 14 g of nitrogen at room temperature to raise its temperature by 40°C at constant pressure?
(Mol. wt. of N_2 = 28 g)
(a) $50R$ (b) $60R$
(c) $70R$ (d) $80R$

15. If R is universal gas constant, the amount of heat needed to raise the temperature of 2 moles of an ideal monoatomic gas from 273 K to 373 K when no work is done is
(a) $100R$ (b) $150R$
(c) $300R$ (d) $500R$

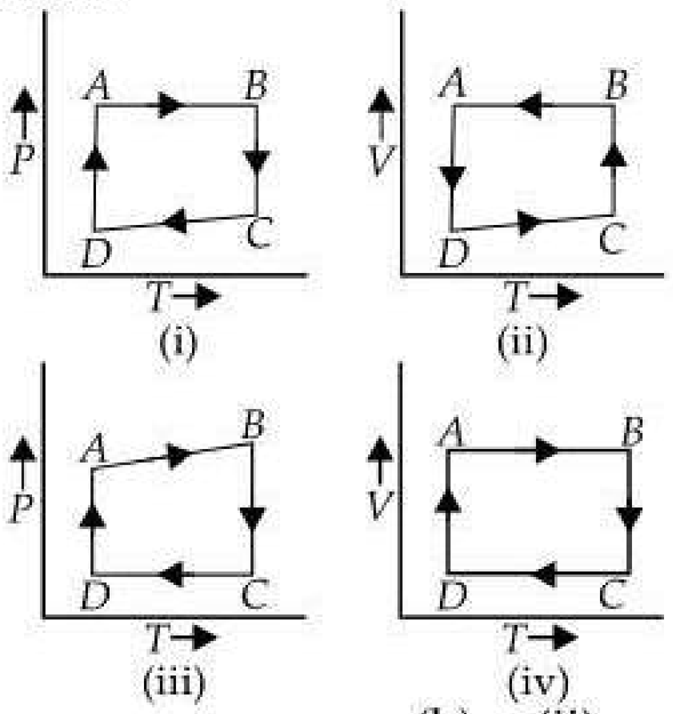
16. When heat energy of 1500 J is supplied to a gas at constant pressure, 2.1×10^5 N m^{-2}, there was an increase in its volume equal to 2.5×10^{-3} m^3. The increase in its internal energy is
(a) 450 J (b) 525 J
(c) 975 J (d) 2025 J

17. A monoatomic gas is compressed adiabatically to $\frac{1}{4}$th of its original volume, the final pressure of gas in terms of initial pressure P is
(a) $7.08\,P$ (b) $8.08\,P$
(c) $9.08\,P$ (d) $10.08\,P$

18. An ideal gas is subjected to a cyclic process *ABCD* as depicted in the *P*-*V* diagram given below.



Which of the following curves represent the equivalent cyclic process?



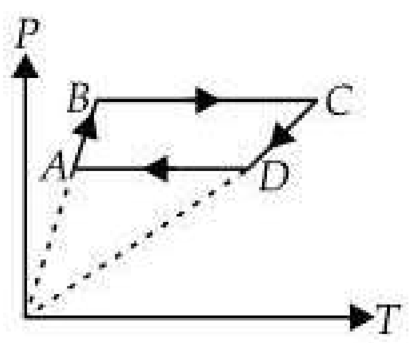
(a) (i) (b) (ii)
(c) (iii) (d) (iv)

19. What amount of heat must be supplied to 35 g of oxygen at room temperature to raise its temperature by 80 °C at constant volume (molecular mass of oxygen is 32 and $R = 8.3$ J mol^{-1} k^{-1})
(a) 1.52 kJ (b) 3.23 kJ
(c) 1.81 kJ (d) 1.62 kJ

20. A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees should be temperature of the high temperature reservoir be increased?
(a) 933 K (b) 432 K
(c) 373 K (d) 267 K

21. The inside and outside temperatures of a refrigerator are 273 K and 303 K respectively. Assuming that refrigerator cycle is reversible for every joule of work done, the heat delivered to the surroundings will be
(a) 10 J (b) 20 J
(c) 30 J (d) 50 J

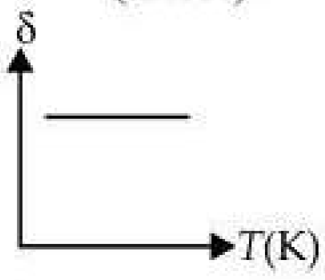
22. Six moles of an ideal gas performs a cycle shown in figure. If the temperature are $T_A = 600$ K, $T_B = 800$ K, $T_C = 2200$ K and $T_D = 1200$ K, the work done per cycle is approximately

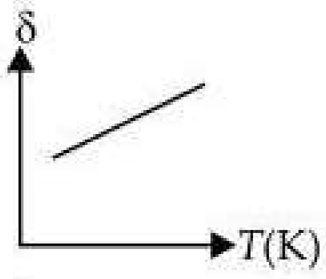


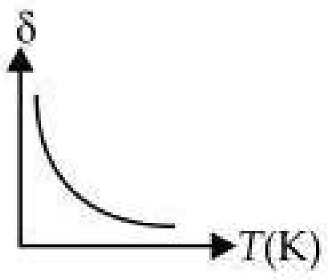
(a) 20 kJ (b) 30 kJ
(c) 40 kJ (d) 60 kJ

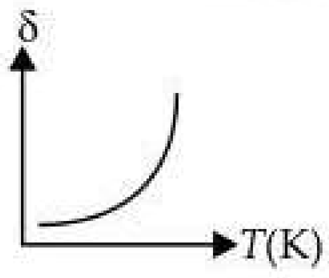
23. An ideal gas with pressure *P*, volume *V* and temperature *T* is expanded isothermally to a volume 2*V* and a final pressure P_I. The same gas is expanded adiabatically to a volume 2*V*, the final pressure is P_A. In terms of the ratio of the two specific heats for the gas γ, the ratio P_I/P_A is
(a) $2^{\gamma-1}$ (b) $2^{1-\gamma}$
(c) 2^{γ} (d) 2γ

24. An ideal gas is initially at temperature *T* and volume *V*. Its volume is increased by Δ*V*, due to an increase in temperature Δ*T*, pressure remaining constant. The quantity $\delta\left(\dfrac{\Delta V}{V\Delta T}\right)$ varies with temperature as

(a) 

(b) 

(c) 

(d) 

25. An engine has an efficiency of 1/6. When the temperature of sink is reduced by 62°C, its efficiency is doubled. Temperature of the source is
(a) 37°C (b) 62°C
(c) 124°C (d) 99°C

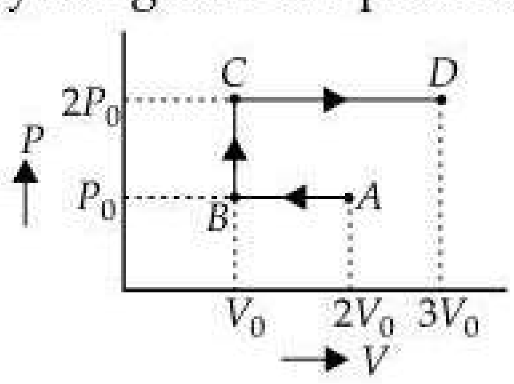
26. The pressure and density of a diatomic gas $\left(\gamma = \dfrac{7}{5}\right)$ change adiabatically from (*P*, ρ) to (*P*′, ρ′). If $\dfrac{\rho'}{\rho} = 32$, then $\dfrac{P'}{P}$ is
(a) $\dfrac{1}{128}$ (b) 32
(c) 128 (d) 256

27. *P*–*V* diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be
(where *R* is universal gas constant)
(a) 4*R* (b) 3*R*
(c) $\dfrac{4}{3}R$ (d) 2.5*R*

28. A Carnot engine whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be
(a) 1200 K (b) 750 K
(c) 600 K (d) 800 K

29. An ideal gas is taken through a cyclic thermodynamical process through four steps. The amounts of heat involved in these steps are: $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J, $Q_4 = 3645$ J; respectively. The corresponding works involved are: $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively. The value of W_4 is
(a) 1315 J (b) 275 J
(c) 765 J (d) 675 J

30. *P*–*V* diagram of an ideal gas is as shown in figure. Work done by the gas in the process *ABCD* is



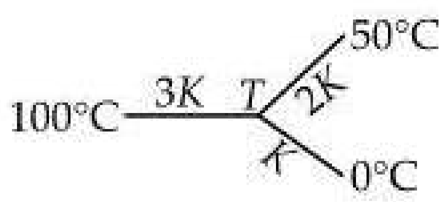
(a) $4P_0V_0$ (b) $2P_0V_0$
(c) $3P_0V_0$ (d) P_0V_0

31. 1 g of ice is mixed with 1 g of steam. After thermal equilibrium is reached, the temperature of mixture is
(a) 100°C (b) 55°C
(c) 75°C (d) 0°C

32. Two cylinders A and B, fitted with pistons. A and B contain equal moles of an ideal monoatomic gas at 400 K. Piston in A is free to move, while piston in B is held fixed. Same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 42 K, then the increase in the temperature of the gas in B is, $\left(\gamma = \frac{5}{3}\right)$
(a) 35 K (b) 42 K
(c) 70 K (d) 21 K

33. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle, the volume of the gas increases from V to $32V$, the efficiency of the engine is
(a) 0.75 (b) 0.99
(c) 0.25 (d) 0.5

34. Three rods of same dimensions have thermal conductivities $3K$, $2K$ and K respectively. They are arranged as shown below



What will be the temperature T of the junction?
(a) $\frac{200}{3}$°C (b) $\frac{100}{3}$°C
(c) 75°C (d) $\frac{50}{3}$°C

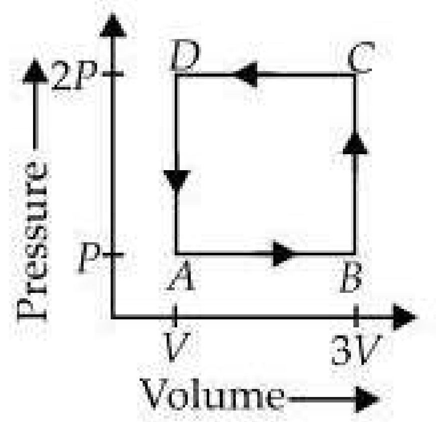
35. 5 moles of hydrogen $\left(\gamma = \frac{7}{5}\right)$ initially at STP are compressed adiabatically so that its temperature becomes 400°C. The increase in the internal energy of the gas is
($R = 8.30$ J mol^{-1} K^{-1})
(a) 21.5 kJ (b) 41.5 kJ
(c) 65.5 kJ (d) 80.5 kJ

36. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is
(a) $-\frac{9}{8}RT_1$ (b) $\frac{3}{2}RT_1$
(c) $\frac{15}{8}RT_1$ (d) $\frac{9}{2}RT_1$

37. A clock pendulum made of invar has a period of 0.5 s at 20°C. If the clock is used in a climate where the temperature averages 30°C, how much time does the clock lose in each oscillation? (For invar, $\alpha = 9 \times 10^{-7}$°C^{-1} and g = constant)
(a) 2.25×10^{-6} s (b) 2.5×10^{-7} s
(c) 5×10^{-7} s (d) 1.125×10^{-6} s

38. One mole of an ideal gas ($\gamma = 1.4$) is adiabatically compressed so that its temperature rises from 27°C to 35°C. The change in the internal energy of the gas is (given $R = 8.3$ J^{-1}mole^{-1} K^{-1})
(a) – 166 J (b) 166 J
(c) – 168 J (d) 168 J

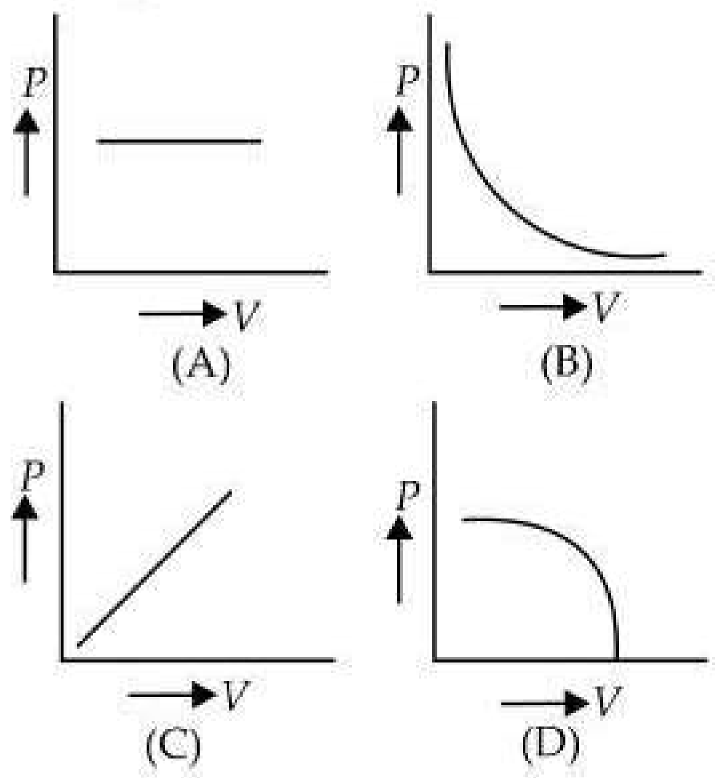
39. A thermodynamic system is taken through the cycle $ABCD$ as shown in figure. Heat rejected by the gas during the cycle is



(a) $2PV$
(b) $4PV$
(c) $\frac{1}{2}PV$
(d) PV

40. When an ideal gas with pressure P and volume V is compressed isothermally to one fourth of its volume, the pressure is P_1. When the same gas is compressed polytropically according to the equation $PV^{1.5}$ = constant to one fourth of its initial volume, the pressure is P_2. The ratio $\frac{P_1}{P_2}$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{2^{1.5}}$
(c) 2 (d) $2^{1.5}$

41. Which of the following P-V diagrams, best represents an isothermal process?



(a) A (b) B
(c) C (d) D

42. One mole of an ideal monoatomic gas at temperature T_0 expands slowly according to the law $\frac{P}{V}$. If the final temperature is $2T_0$ heat supplied the gas is
(a) $2RT_0$ (b) RT_0
(c) $\frac{3}{2}RT_0$ (d) $\frac{1}{2}RT_0$

43. What is the change in internal energy of one mole of a gas, when volume changes from V to $2V$ at constant pressure P?
(a) $\frac{R}{(\gamma - 1)}$ (b) PV
(c) $\frac{PV}{(\gamma - 1)}$ (d) $\frac{\gamma PV}{(\gamma - 1)}$

44. The efficiency of Carnot's heat engine is 0.5 when the temperature of the source is T_1 and that of sink is T_2. The efficiency of another Carnot's heat engine is also 0.5. The temperature of source and sink of the second engine are respectively

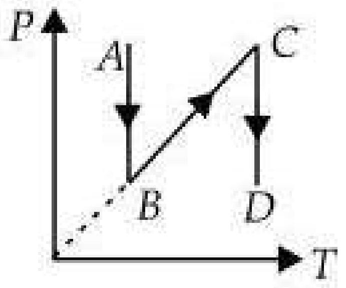
(a) $2T_1, 2T_2$

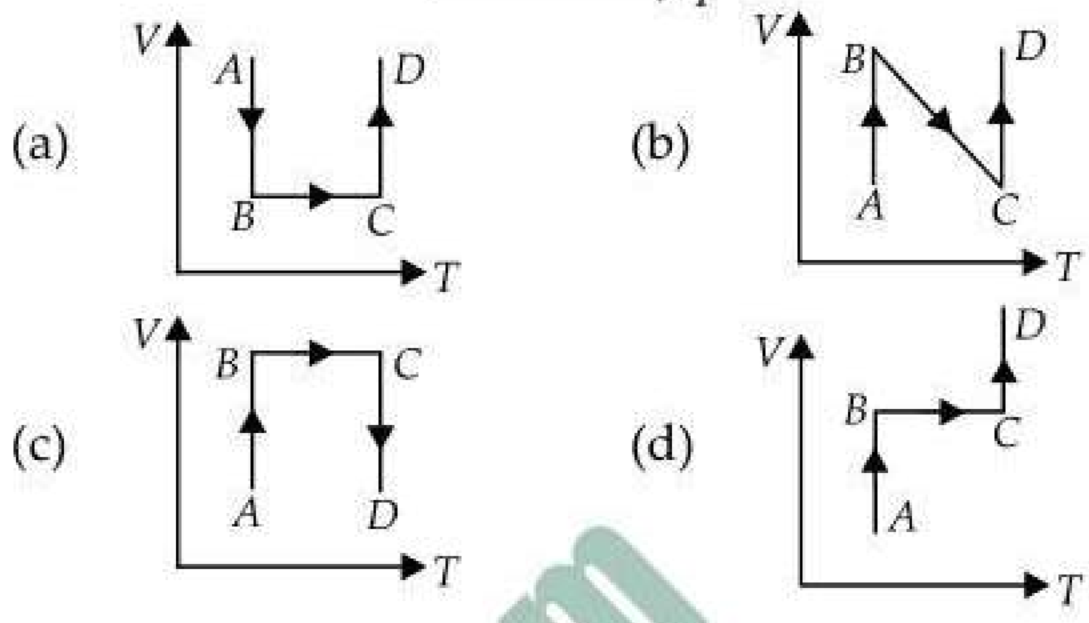
(b) $2T_1, \frac{T_2}{2}$

(c) $T_1 + 5, T_2 - 5$

(d) $T_1 + 10, T_2 - 10$

45. P-T diagram is shown in figure. Choose the corresponding V-T diagram.





DAY 11 OMR SHEET

Time : 45 min

INSTRUCTIONS

- Use HB pencil only and darken each circle completely.
- If you wish to change your answer, erase the already darkened circle completely and then darken the appropriate circle.
- Mark only one choice for each question as indicated.

Correct marking ● ⓑ ⓒ ⓓ

Wrong marking ⊗ ⓑ ◐ ⊙

1. ⓐ ⓑ ⓒ ⓓ	10. ⓐ ⓑ ⓒ ⓓ	19. ⓐ ⓑ ⓒ ⓓ	28. ⓐ ⓑ ⓒ ⓓ	37. ⓐ ⓑ ⓒ ⓓ
2. ⓐ ⓑ ⓒ ⓓ	11. ⓐ ⓑ ⓒ ⓓ	20. ⓐ ⓑ ⓒ ⓓ	29. ⓐ ⓑ ⓒ ⓓ	38. ⓐ ⓑ ⓒ ⓓ
3. ⓐ ⓑ ⓒ ⓓ	12. ⓐ ⓑ ⓒ ⓓ	21. ⓐ ⓑ ⓒ ⓓ	30. ⓐ ⓑ ⓒ ⓓ	39. ⓐ ⓑ ⓒ ⓓ
4. ⓐ ⓑ ⓒ ⓓ	13. ⓐ ⓑ ⓒ ⓓ	22. ⓐ ⓑ ⓒ ⓓ	31. ⓐ ⓑ ⓒ ⓓ	40. ⓐ ⓑ ⓒ ⓓ
5. ⓐ ⓑ ⓒ ⓓ	14. ⓐ ⓑ ⓒ ⓓ	23. ⓐ ⓑ ⓒ ⓓ	32. ⓐ ⓑ ⓒ ⓓ	41. ⓐ ⓑ ⓒ ⓓ
6. ⓐ ⓑ ⓒ ⓓ	15. ⓐ ⓑ ⓒ ⓓ	24. ⓐ ⓑ ⓒ ⓓ	33. ⓐ ⓑ ⓒ ⓓ	42. ⓐ ⓑ ⓒ ⓓ
7. ⓐ ⓑ ⓒ ⓓ	16. ⓐ ⓑ ⓒ ⓓ	25. ⓐ ⓑ ⓒ ⓓ	34. ⓐ ⓑ ⓒ ⓓ	43. ⓐ ⓑ ⓒ ⓓ
8. ⓐ ⓑ ⓒ ⓓ	17. ⓐ ⓑ ⓒ ⓓ	26. ⓐ ⓑ ⓒ ⓓ	35. ⓐ ⓑ ⓒ ⓓ	44. ⓐ ⓑ ⓒ ⓓ
9. ⓐ ⓑ ⓒ ⓓ	18. ⓐ ⓑ ⓒ ⓓ	27. ⓐ ⓑ ⓒ ⓓ	36. ⓐ ⓑ ⓒ ⓓ	45. ⓐ ⓑ ⓒ ⓓ

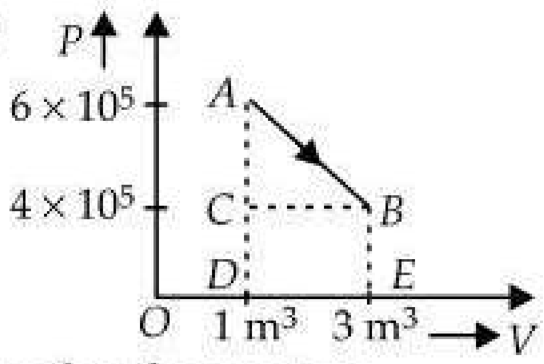
(1) Number of questions attempted : ____________ (3) Marks scored : ____________

(2) Number of questions correct : ____________

For every correct answer award yourself 4 marks. For every incorrect answer deduct 1 mark.

HINTS & SOLUTIONS

1. **(d) :**



Work done by the system

= area under $P-V$ diagram

= area of rectangle $BCDE$ + area of ΔABC

$$= 4\times10^5\times2+\frac{2\times10^5\times2}{2}$$

$W = 10\times10^5$ J

2. **(a) :** For adiabatic process,

$dQ = 0$

So, $dU = -\Delta W$

or $\mu C_V dT = +146\times10^3$ J

or $\frac{\mu fR}{2}\times7 = 146\times10^3 \quad \left(\because C_V = \frac{fR}{2}\right)$

($f\rightarrow$ Degree of freedom)

$\Rightarrow \frac{10^3\times f\times8.3\times7}{2} = 146\times10^3$

$f = 5.02 \approx 5$

Therefore, gas is diatomic.

3. **(a) :** Using $\beta = \frac{Q_2}{W} = \frac{T_L}{T_H - T_L}$

$T_L = 273$ K, $T_H = 303$ K and $W = 1$ J

$\therefore \quad Q_2 = \frac{273}{303-273}\times1 = \frac{273}{30} \cong 9$ J

Hence, heat delivered to surroundings,

$Q_1 = Q_2 + W = 9 + 1 = 10$ J

4. **(a) :** From first law of thermodynamics

$dQ = dU + dW$

$dQ = dU \quad$ (If $dW = 0$)

Since, $dQ < 0$

$\therefore \quad dU < 0$

or $U_{final} < U_{initial}$

Hence, temperature will decrease.

5. **(d) :** Here, $P_1 = 1$ atm, $T_1 = 27°C$

$= 27 + 273 = 300$ K

$P_2 = 8P_1$, $T_2 = ?$, $\gamma = \frac{3}{2}$

As changes are adiabatic,

$\therefore \quad P_1^{\gamma-1}T_1^{-\gamma} = P_2^{\gamma-1}T_2^{-\gamma}$

$$\left(\frac{T_2}{T_1}\right)^{-\gamma} = \left(\frac{P_1}{P_2}\right)^{\gamma-1}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} = 300(8)^{(1.5-1)/1.5} = 300(8)^{1/3}$$

$T_2 = 600\ K = (600-273)°C = 327°C$

6. **(a) :** Here,

$T_1 = 500$ K, As $\eta = 1-\frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = 1-\eta$

$\therefore \quad \frac{T_2}{500} = 1-\frac{40}{100} = \frac{3}{5}$ or $T_2 = 300$ K

Now $\quad \frac{T_2}{T_1'} = 1-\eta' = 1-\frac{50}{100} = \frac{1}{2}$

$\therefore \quad T_1' = 2T_2 = 2\times300 = 600$ K

7. **(c) :** $dW = PdV = \frac{RT}{V}dV \quad$ ...(i)

As $V = KT^{2/3} \quad \therefore \quad dV = K\frac{2}{3}T^{-1/3}\,dT$

$\therefore \quad \frac{dV}{V} = \frac{K\frac{2}{3}T^{-1/3}dT}{KT^{2/3}} = \frac{2}{3}\frac{dT}{T}$

From (i), $W = \int_{T_1}^{T_2} RT\frac{dV}{V} = \int_{T_1}^{T_2} RT\frac{2}{3}\frac{dT}{T}$

$W = \frac{2}{3}R(T_2-T_1) = \frac{2}{3}R\times60 = 40R$

8. **(d) :** Efficiency of a Carnot engine,

$\eta = 1-\frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = 1-\eta = 1-\frac{40}{100} = \frac{3}{5}$

$\therefore \quad T_1 = \frac{5}{3}\times T_2 = \frac{5}{3}\times300 = 500$ K

Increase in efficiency = 50% of 40% = 20%

$\therefore$ New efficiency, $\eta' = 40\% + 20\% = 60\%$

$\therefore \quad \frac{T_2}{T_1'} = 1-\frac{60}{100} = \frac{2}{5}$

$\therefore \quad T_1' = \frac{5}{2}\times T_2 = \frac{5}{2}\times300 = 750$ K

Increase in temperature of source

$= T_1' - T_1 = 750 - 500 = 250$ K

9. **(c) :** Efficiency, $\eta = \frac{T_1-T_2}{T_1} = \frac{W}{Q}$

$$W = \frac{Q(T_1-T_2)}{T_1} = \frac{6\times10^4[(227+273)-(127+273)]}{(227+273)}$$

$$= \frac{6\times10^4\times100}{500} = 1.2\times10^4 \text{ cal}$$

10. **(b) :** Here, initial temperature,

$T_1 = 27°C = 273 + 27 = 300$ K

Final temperature, $T_2 = 97° C = 273 + 97 = 370$ K
When a gas is compressed adiabatically, work done on the gas is given by

$$W = \frac{R}{(1-\gamma)}(T_2 - T_1) = \frac{8.3 \times (370-300)}{1-1.5}$$

or $W = -11.62 \times 10^2$ J

$\therefore$ Heat produced,

$$H = \frac{W}{J} = \frac{11.62 \times 10^2}{4.2} = 276.7 \text{ cal}$$

11. (b) : According to first Law of thermodynamic

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad \frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

Here, $\frac{\Delta Q}{\Delta t} = 120\,W$,

$$\frac{\Delta W}{\Delta t} = 80 \text{ J s}^{-1}$$

$$\therefore \quad \frac{\Delta U}{\Delta t} = 120 - 80 = 40 \text{ J s}^{-1}$$

12. (b) : Work done by the gas during the isothermal process

$$W = nRT \ln\frac{V_2}{V_1}$$

$$= (1)RT \ln\left(\frac{2V}{V}\right) = RT\ln 2$$

13. (b) : Thermal capacity, S = Specific heat (s) × Mass (m)

$$\therefore \frac{S_A}{S_B} = \frac{m_A s_A}{m_B s_B} = \frac{\left(\frac{4}{3}\right)\pi r_A^3 \rho_A s_A}{\left(\frac{4}{3}\right)\pi r_B^3 \rho_B s_B}$$

$$= \left(\frac{r_A}{r_B}\right)^3 \frac{\rho_A s_A}{\rho_B s_B} = \left(\frac{1}{2}\right)^3 \times \left(\frac{2}{1}\right) \times \left(\frac{1}{3}\right) = \frac{1}{12}$$

14. (c) : As nitrogen is diatomic, its molar specific heat at constant pressure is

$$C_P = \frac{7}{2}R$$

No. of moles, $n = \frac{14}{28} = \frac{1}{2}$

$$\therefore \quad \Delta Q = nC_P \Delta T$$

$$= \frac{1}{2} \times \frac{7}{2} R \times 40 = 70R$$

15. (c) : As no work is done, therefore, $\Delta W = 0$
According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W = \Delta U = nC_V \Delta T = n\left(\frac{R}{\gamma - 1}\right)\Delta T$$

Here, $n = 2, \gamma = \frac{5}{3}$

$\Delta T = T_2 - T_1 = (373 - 273) = 100$ K

$$\Delta Q = 2 \times \frac{R}{\left(\frac{5}{3} - 1\right)} \times 100 = 300\,R$$

16. (c) : Heat supplied to a gas,

ΔQ = 1500 J

Work done by the gas,

$\Delta W = P\Delta V = 2.1 \times 10^5 \times \text{N m}^{-2} \times 2.5 \times 10^{-3} \text{ m}^3$
$= 5.25 \times 10^2$ N m = 525 J

According to first law of thermodynamics,

$\Delta Q = \Delta W + \Delta U$
$\Delta U = \Delta Q - \Delta W$
= 1500 J – 525 J = 975 J

17. (d) : The initial pressure and volume of gas are P and V.

Now the gas is compressed adiabatically to 1/4^th^ of its original volume, *i.e*

$$V_2 = \frac{V}{4} \quad \dots \text{(i)}$$

As $P_1 V_1^\gamma = P_2 V_2^\gamma \quad \dots$ (ii)

or $PV^\gamma = P_2\left(\frac{V}{4}\right)^\gamma = \frac{P_2 V^\gamma}{4^\gamma}$ (using (i))

$$P_2 = 4^\gamma P$$

Now for monoatomic gases $\gamma = \frac{5}{3}$,

$\therefore \quad P_2 = 4^{5/3} P = 10.08\, P$

18. (a)

19. (c) : Here mass of oxygen (m) = 35 g,
molar mass of O_2 (M) = 32 g mol^{-1}
rise in temperature, ΔT = 80 °C

$\therefore$ number of moles $n = \frac{m}{M} = \frac{35}{32} = 1.09$ mol

As oxygen is a diatomic gas, then molar specific heat at constant volume is $C_V = \frac{5}{2}R$

and amount of heat supplied to gas

$$\Delta Q = nC_V \Delta T$$

$$= 1.09 \times \frac{5}{2} R \times 80 = 1.09 \times \frac{5}{2} \times 8.3 \times 80$$

= 1809.4 J = 1.8094 kJ 1.81 ≃ 1.81 kJ

20. (a) : Initially, $\eta = \frac{T_1 - T_2}{T_2}$

i.e. $0.5 = \frac{T_1 - (273+7)}{T_1}$ or $\frac{1}{2} = \frac{T_1 - 280}{T_1}$

$\therefore \quad T_1 = 560$ K

Finally, $\eta_1' = \frac{T_1' - T_2}{T_1'}$ or $0.7 = \frac{T_1' - (273+7)}{T_1'}$

$\therefore \quad T_1' = 933$ K

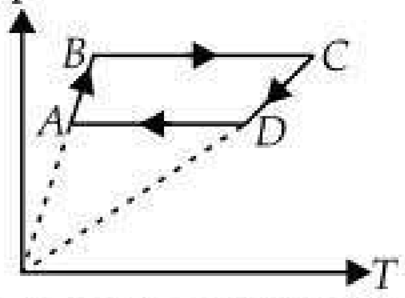
21. (a) : $\beta = \frac{Q_2}{W} = \frac{T_L}{T_H - T_L}$

$Q_2 = \frac{273}{303-273} = \frac{273}{30} \simeq 9$ J

Heat delivered to the surroundings

$Q_1 = Q_2 + W = 9 + 1 = 10$ J

22. (c) :



Processes A to B and C to D are parts of straight line graphs passing through origin $P \propto T$.

So, volume remains constant for the graph AB and CD

So, no work is done during processes for A to B and C to D

i.e., $W_{AB} = W_{CD} = 0$

and $W_{BC} = P_2(V_C - V_B) = nR(T_C - T_B)$

$= 6R(2200 - 800) = 6R \times 1400$ J

Also, $W_{DA} = P_1(V_A - V_D) = nR(T_A - T_D)$

$= 6R(600 - 1200) = -6R \times 600$ J

Hence, work done in complete cycle

$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$= 0 + 6R \times 1400 + 0 - 6R \times 600$

$= 6R \times 800 = 6 \times 8.3 \times 800 = 40$ kJ

23. (a) : For an isothermal process

$PV =$ constant $\quad \therefore \quad PV = P_I\, 2V$

$P_I = \frac{P}{2}$ ...(i)

For an adiabatic process

$PV^\gamma =$ constant $\therefore$ $PV^\gamma = P_A(2V)^\gamma$

or $P_A = \frac{P}{2^\gamma}$ ...(ii)

Divide (i) by (ii), we get

$\frac{P_I}{P_A} = \frac{2^\gamma}{2} = 2^{\gamma - 1}$

24. (c)

25. (d) : Here $\eta_1 = \frac{1}{6}, \eta_2 = 2 \times \frac{1}{6}$

From, $\eta_1 = 1 - \frac{T_2}{T_1}$ or $\frac{1}{6} = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = \frac{5}{6}$

or $T_2 = \frac{5}{6}T_1$ ...(i)

Again, $\eta_2 = 1 - \frac{(T_2 - 62)}{T_1}$ or $2 \times \frac{1}{6} = 1 - \frac{(T_2 - 62)}{T_1}$

$\frac{(T_2 - 62)}{T_1} = 1 - \frac{1}{3} = \frac{2}{3}$ or $3T_2 - 186 = 2T_1$

or $3 \times \frac{5}{6}T_1 - 186 = 2T_1$ (Using (i))

$0.5T_1 = 186$

$\therefore \quad T_1 = 372$ K

$T_1 = (372 - 273) = 99°$C

26. (c) : Here, $\gamma = \frac{7}{5}, P_1 = P, \rho_1 = \rho; P_2 = P', \rho_2 = \rho'$

For an adiabatic process, $PV^\gamma =$ constant

$\therefore \quad P_1V_1^\gamma = P_2V_2^\gamma$

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{\rho'}{\rho}\right)^{7/5} = (32)^{7/5}$

$\frac{P'}{P} = (2^5)^{7/5} = 2^7 = 128$

27. (b) : As P–V diagram is a straight line passing through origin, therefore,

$P \propto V$ or $PV^{-1} =$ constant

In the process, $PV^x =$ constant, molar heat capacity is given by

$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$

where $x = -1$ here and $\gamma = 1.4$ for diatomic gas.

$\therefore \quad C = \frac{R}{1.4 - 1} + \frac{R}{1 - (-1)} = \frac{5}{2}R + \frac{R}{2}$

$C = 3R$

28. (b) : Efficiency of Carnot engine,

$\eta = 1 - \frac{T_2}{T_1}$

where T_1 is the temperature of the source and T_2 is the temperature of the sink

For the 1st case

$\eta = 40\%, T_1 = 500$ K

$\therefore \quad \frac{40}{100} = 1 - \frac{T_2}{500}$

$\frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$

$T_2 = \frac{3}{5} \times 500 = 300$ K

For the 2nd case

$\eta = 60\%, T_2 = 300$ K

$\therefore \quad \frac{60}{100} = 1 - \frac{300}{T_1}$

$\frac{300}{T_1} = 1 - \frac{60}{100} = \frac{2}{5}$

$T_1 = \frac{5}{2} \times 300 = 750$ K

29. (c) : $\Delta Q = Q_1 + Q_2 + Q_3 + Q_4$

$= 5960\text{ J} - 5585\text{ J} - 2980\text{ J} + 3645\text{ J} = 1040$ J

$\Delta W = W_1 + W_2 + W_3 + W_4$

$= 2200\text{ J} - 825\text{ J} - 1100\text{ J} + W_4 = 275\text{ J} + W_4$

For a cyclic process, $U_f = U_i$

$\Delta U = U_f - U_i = 0$

From first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$1040\text{ J} = 0 + 275\text{ J} + W_4$ or $W_4 = 765$ J

30. (c) : $W_{AB} = -P_0V_0$,

$W_{BC} = 0$

$W_{CD} = 4P_0V_0$

$W_{ABCD} = W_{AB} + W_{BC} + W_{CD}$

$= -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

31. (a) : Heat required by ice to convert totally into water at 100°C,

$Q_1 = 1 \times 80 + 1 \times 1 \times 100 = 180$ cal

Heat supplied by steam if it was to condense totally and convert into water at 100°C,

$Q_2 = 1 \times 540 = 540$ cal

As $Q_2 > Q_1$, entire steam will not condense and final temperature = 100°C.

Both water and steam will exist together in equilibrium at 100°C.

32. (c) : In cylinder A, heat is supplied at constant pressure while in cylinder B, heat is supplied at constant volume.

$\therefore (\Delta Q)_A = nC_P(\Delta T)_A$

and $(\Delta Q)_B = nC_V(\Delta T)_B$

Given: $(\Delta Q)_A = (\Delta Q)_B$

$$\therefore \quad (\Delta T)_B = \frac{C_P}{C_V}(\Delta T)_A = \gamma(\Delta T)_A \quad \left(\because \ \gamma = \frac{C_P}{C_V}\right)$$

$$= \left(\frac{5}{3}\right)(42\text{ K}) = 70\text{ K}$$

33. (a) : Let T_1 be the temperature of diatomic gas at volume V and T_2 is the temperature at volume $32V$.

For an adiabatic process,

$TV^{\gamma-1}$ = constant

$\therefore \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\text{or} \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V}{32V}\right)^{\gamma-1} = \frac{1}{(32)^{\gamma-1}}$$

For diatomic gas, $\gamma = \frac{7}{5}$

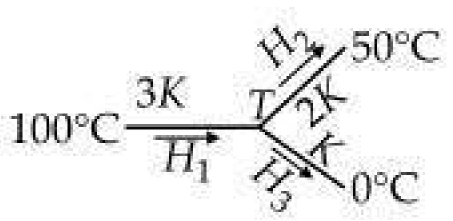
$$\therefore \quad \frac{T_2}{T_1} = \frac{1}{(32)^{\frac{7}{5}-1}} = \frac{1}{(2^5)^{\frac{7}{5}-1}} = \frac{1}{(2^5)^{2/5}} = \frac{1}{4} \quad \text{...(i)}$$

Efficiency of engine, $\eta = 1 - \frac{T_2}{T_1}$

$= 1 - \frac{1}{4}$ (Using (i))

$= \frac{3}{4} = 0.75$

34. (a) : Let L and A be length and area of cross-section of each rod.



At steady state,

$H_1 = H_2 + H_3$

$$\frac{(100-T)(3K)A}{L} = \frac{(T-50)2KA}{L} + \frac{(T-0)KA}{L}$$

$3(100 - T) = 2(T - 50) + T$

$300 - 3T = 2T - 100 + T$

$6T = 400$

$$\text{or} \quad T = \frac{400}{6} = \frac{200}{3}\,°\text{C}$$

35. (b) : Here, $\gamma = \frac{7}{5}$

$$\text{Work done} = \frac{nR}{\gamma - 1}\Delta T$$

$$= \frac{5 \times 8.3 \times 400}{\frac{7}{5} - 1} = 41500 \text{ J} = 41.5 \times 10^3 \text{ J} = 41.5 \text{ kJ}$$

Work done = Change in internal energy

($\because$ $\Delta Q = 0$ for adiabatic process)

$\therefore$ Change or increase in internal energy

= 41.5 kJ

36. (a) : Here, $V_1 = 5.6$ litre, $V_2 = 0.7$ litre, $W = ?$

Helium is a monoatomic gas.

For helium, $\gamma = \frac{5}{3}$

For an adiabatic process,

$TV^{\gamma-1}$ = constant

$$\therefore \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \text{ or } T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\text{or} \quad T_2 = T_1\left(\frac{5.6}{0.7}\right)^{\left(\frac{5}{3}-1\right)} = T_1(8)^{2/3} = 4T_1$$

No. of moles of helium, $n = \frac{5.6 \text{ litre}}{22.4 \text{ litre}} = \frac{1}{4}$

Work done during an adiabatic process is

$$W = \frac{nR[T_1 - T_2]}{(\gamma - 1)} = \frac{\frac{1}{4}R[T_1 - 4T_1]}{\left(\frac{5}{3} - 1\right)} = -\frac{9}{8}RT_1$$

Here –ve sign shows that work is done on the gas.

37. (a) : $\Delta t = \frac{1}{2}\alpha t \Delta T$

$$= \frac{1}{2} \times 9 \times 10^{-7}\,°\text{C}^{-1} \times 0.5 \text{ s} \times (30°\text{C} - 20°\text{C})$$

$= 22.5 \times 10^{-7}$ s $= 2.25 \times 10^{-6}$ s

38. (b) : $\Delta U = C_V \Delta T$.

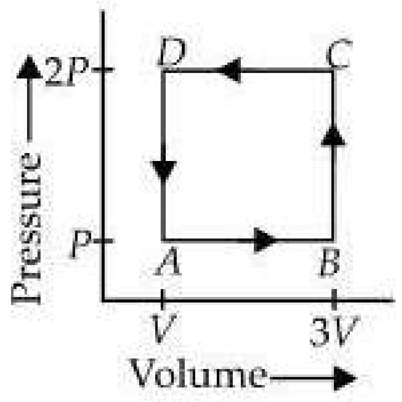
Now

$$C_P - C_V = R \text{ or } \frac{C_P}{C_V} - 1 = \frac{R}{C_V}$$

$$\text{or} \quad C_V = \frac{R}{\gamma - 1}, \left(\because \ \gamma = \frac{C_P}{C_V}\right).$$

$$\text{Hence} \quad \Delta U = \frac{R\Delta T}{(\gamma - 1)} = \frac{8.3 \times 8}{(1.4 - 1)} = 166 \text{ J}$$

39. **(a) :** In a cyclic process, $\Delta U = 0$, and work done is equal to the area under the cycle and is positive if the cycle is clockwise and negative if anticlockwise.



$\therefore \Delta W = -$ Area of rectangle $ABCD$

$= -P(2V) = -2PV$

According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$ or $\Delta Q = \Delta W$ (As $\Delta U = 0$)

i.e., heat supplied to the system is equal to the work done

So heat absorbed, $\Delta Q = \Delta W = -2PV$

$\therefore$ Heat rejected by the gas $= 2PV$

40. **(a) :** For isothermal process : PV = constant

or $PV = P_1 \dfrac{V}{4} \Rightarrow P_1 = 4P$

For polytropic process: $PV^{1.5}$ = constant

$$PV^{1.5} = P_2\left(\frac{V}{4}\right)^{1.5}$$

$$\Rightarrow P_2 = (2^2)^{3/2}\ P = 8P$$

$$\therefore \frac{P_1}{P_2} = \frac{1}{2}$$

41. **(b) :** For an isothermal process, PV = constant

or $P \propto \dfrac{1}{V}$

42. **(a) :** In a process PV^x = constant, molar heat capacity is given by $C = \dfrac{R}{\gamma - 1} + \dfrac{R}{1 - x}$

As the process is $\dfrac{P}{V}$ = constant,

i.e., PV^{-1} = constant, therefore, $x = -1$.

For an ideal monoatomic gas, $\gamma = \dfrac{5}{3}$

$$\therefore C = \frac{R}{\frac{5}{3} - 1} + \frac{R}{1 - (-1)} = \frac{3}{2}R + \frac{R}{2} = 2R$$

$\Delta Q = nC(\Delta T) = 1(2R)(2T_0 - T_0) = 2RT_0$

43. **(c) :** According to ideal gas equation, $PV = nRT$

When pressure is constant, $PdV = nRdT$

$P(2V - V) = nRdT$

$PV = nRdT$ ...(i)

As $\gamma = \dfrac{C_P}{C_V}$

$$\therefore \gamma - 1 = \frac{C_P - C_V}{C_V}$$

or $C_V = \dfrac{C_P - C_V}{\gamma - 1} = \dfrac{R}{\gamma - 1}$ $(\because C_P - C_V = R)$

$$\Delta U = nC_V dT = \frac{nRdT}{\gamma - 1}$$

$\Delta U = \dfrac{PV}{\gamma - 1}$ (Using (i))

44. **(a) :** Efficiency of Carnot's heat engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

For first Carnot's heat engine, $0.5 = 1 - \dfrac{T_2}{T_1}$

For second heat engine, $0.5 = 1 - \dfrac{T_2'}{T_1'}$

Efficiency remains the same when both T_1 and T_2 are increased by same factor

Therefore, the temperature of source and sink of second engine are

$T_1' = 2T_1,\ T_2' = 2T_2$

45. **(d)**