Chapter -4

क्रमचय एवं संचय
PERMUTATION AND COMBINATION

बहुविकल्पीय प्रश्न (Multiple choice question)

(1) यदि ${}^{n}p_3 = 60$ तो n का मान है।
If ${}^{n}P_3 = 60$, then value of n is.
(a) 3 (b) 5
(c) 9 (d) 20

(2) ${}^{9}P_3$ का मान बताइए :
Find the value of ${}^{9}p_3$:
(a) 500 (b) 504
(c) 502 (c) 510

(3) ${}^{10}C_6$ का मान है
The value of ${}^{10}C_6$ is :
(a) 210 (b) 100
(c) 1024 (d) 60

(4) ${}^{7}P_4$ का मान है
The value of ${}^{7}P_4$ is
(a) $4 \times 3 \times 2 \times 1$ (b) 7×4
(c) $7 \times 6 \times 5 \times 4$ (d) $\llcorner 7 \times \llcorner 4$

(5) ${}^{n}P_r$ समान होगा
${}^{n}P_r$ is qual to
(a) $\frac{\lfloor n}{\lfloor n \lfloor n-r}$ (b) $\frac{\lfloor n}{\lfloor r-n}$

(c) $\frac{\lfloor n}{\lfloor n-r}$

(d) इनमें से कोई नहीं ।
(none of these)

(6) CAT शब्द के अक्षरों में 2-2 अक्षरों के कितने विभिन्न शब्द बन सकते हैं ?
How many different word of 2 letter can be formed out of the letters of the word CAT ?
(a) 8 (b) 6
(c) 3 (d) 2

(7) यदि ${}^{n}C_{12} = {}^{n}C_8$ तब n बराबर है:
If ${}^{n}C_{12} = {}^{n}C_8$ then n is equal to :
(a) 20 (b) 12
(c) 30 (d) 6

(8) 'SUNDAY' शब्द में एक बार में दो अक्षर लेकर केवल कितने शब्द बनाये जा सकते हैं ?
How many words can be formed with the letters of the word 'SUNDAY' taken two at the time ?
(a) 50 (b) 30
(c) 40 (d) 60

(9) 'PRODUCT' शब्द के अक्षरों से तीन-तीन अक्षरों के कितने भिन्न-भिन्न शब्द बन सकते हैं ?
How many different words of 3 letters can be formed out of the letters of the word 'PRODUCT' ?

(a) 210 (b) 200
(c) 250 (d) 240

(10) निम्नलिखित शब्दों के अक्षरों से कितने विभिन्न शब्द बनाए जा सकते हैं ?
How many different words can be formed out of the letters of the following words ?
(A) STATISTICS
(a) 50,000 (b) 50,400
(c) 50,500 (d) 60,000
(B) ACCOUNTANCY
(a) 16,00,000
(B) 16,50,000
(C) 16,63,000
(D) 16,40,200

(11) 'Vice-chancellor' शब्द के सभी अक्षरों को लेकर कितने शब्द बनाये जा सकते हैं ?
How many word can be formed with all the letters of the word vice - chancellor ?

(a) $\frac{\lfloor 14}{\lfloor 3 \lfloor 2}$ (b) $\frac{\lfloor 14}{\lfloor 2 \lfloor 2}$

(c) $\frac{\lfloor 14}{\lfloor 3 \lfloor 2 \lfloor 2}$ (d) $\frac{\lfloor 14}{\lfloor 2}$

ANSWER

(1)	b	(2)	b	(3)	a
(4)	c	(5)	c	(6)	b
(7)	a	(8)	b	(9)	a
(10-A)	b	(10 - B)	c	(11)	c

लघु उत्तरीय प्रश्न
(Short Answer type question)

(1) यदि ${}^9P_r = 3024$, तो r का मान निकालें ।

If ${}^9Pr = 3024$, find the value of r.

उत्तर : Given

दिया है ${}^9P_r = 3024$

$$\Rightarrow \frac{9!}{(9-r)!} = 3024$$

$$\Rightarrow \frac{9!}{3024} = (9-r)!$$

$\Rightarrow$ (9 - r)! = 120
$\Rightarrow$ (9 - r)! = 5!
$\Rightarrow$ 9 - r = 5
$\therefore$ r = 4

(2) यदि ${}^nP_5 = 20 \times {}^nP_3$ तो n का मान लिकालिए।

If ${}^nP_5 = 20 \times {}^nP_3$ find the value of n .

उत्तर : $\because {}^nP_5 = 20 \times {}^nP_3$

$$\therefore \frac{\lfloor n}{\lfloor n-5} = 20 \times \frac{\lfloor n}{\lfloor n-3}$$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-5} = \frac{20 \times \lfloor n}{(n-3)(n-4)\lfloor n-5}$$

$\Rightarrow$ (n-3) (n-4) = 5 × 4
$\Rightarrow$ n - 3 = 5
n = 8

(3) यदि ${}^nC_{10} = {}^nC_{12}$ तो ${}^nC_5$ निकालिए ।

(if ${}^nC_{10} = {}^nC_{12}$ then find ${}^nC_5$)

${}^nC_{10} = {}^nC_{12}$
$\because 10 \neq 12$
$\therefore$ 10 + 12 = n
$\Rightarrow$ n=22

$$\Rightarrow {}^nC_5 = {}^{22}C_5 = \frac{\lfloor 22}{\lfloor 5 \lfloor 17}$$

$$= \frac{22 \times 21 \times 20 \times 19 \times 18 \times \lfloor 17}{5 \times 4 \times 3 \times 2 \times 1 \times \lfloor 17} = 26,334$$

(4) यदि ${}^nP_r = 720$ तथा ${}^nC_r = 120$ तो r का मान ज्ञात कीजिए।

Find the value of r if ${}^nP_r = 720$ and ${}^nC_r = 120$

${}^nC_r = \frac{{}^nP_r}{\lfloor r}$

$$\Rightarrow 120 = \frac{720}{\lfloor r}$$

$$\Rightarrow \lfloor r = 6 = (3 \times 2 \times 1)$$

$$\Rightarrow \lfloor r = \lfloor 3$$

$\therefore$ r = 3

(5) किसी कॉलेज के एक क्लास में 50 विद्यार्थी हैं। वे कितने प्रकार से 3 प्रतिनिधि कॉलेज यूनियन के लिए चुन सकते हैं ?

There are 50 students in a class of a collage. in how many ways can they select 3 representa tives for the collage union ?

उत्तरः अभीष्ट तरीका $= {}^{50}C_3$

$$= \frac{\lfloor 50}{\lfloor 3 \lfloor 47}$$

$$= \frac{50 \times 49 \times 48 \times \lfloor 47}{3 \times 2 \times \lfloor 47} = \frac{50 \times 49 \times 48}{3 \times 2}$$

= 19,600

(6) कितने तरीकों से 5 व्यक्ति एक टेबल के चारों ओर बैठ सकते हैं ?

In how many ways can 5 men sit around a table?

उत्तरः व्यक्तियों की संख्या
(Number of men) = 5

इसलिए उनके वृत्तीय क्रमचय की संख्या
so noumber of their cicular permutation
= (n - 1) !
= (5 -1)!
= 4 !
= 24

दीर्घ उत्तरीय प्रश्न (Long Answer Type Question)

(1) यदि ${}^{2n}C_3 : {}^nC_{3:} :: 11:1$ तो n का मान निकालिए।

$\Rightarrow$ If ${}^{2n}C_3 : {}^nC_{3:} :: 11:1$ find n .

उत्तरः ${}^{2n}C_3 : {}^nC_{3:} :: 11:1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^nc_3} = \frac{11}{1}$$

$$\frac{\lfloor 2n}{\lfloor 3 \lfloor 2n-3} \times \frac{\lfloor 3 \lfloor n-3}{\lfloor n} = 11$$

$$\Rightarrow \frac{2n(2n-1)(2n-2\lfloor 2n-3 \times \lfloor 3 \lfloor n-3)}{\lfloor (2n-3) \lfloor 3 (n-1)(n-2) \lfloor n-3} = 11$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = 11$$

$\Rightarrow$ 11n - 22 = 8n - 4

$\Rightarrow$ 11n - 8n = 22 - 4

$\Rightarrow$ 3n = 18

$\therefore$ n = 6

(2) यदि ${}^{n}P_{r} = 336$ और ${}^{n}C_{r} = 56$, तो r का मान ज्ञात करें।

If ${}^{n}P_{r} = 336$ and ${}^{n}C_{r} = 56$, find the value of r.

उत्तर: ${}^{n}P_{r} = 336$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-r} = 336.....(i)$$

$\Rightarrow {}^{n}C_{r} = 56$

$$\frac{\lfloor n}{\lfloor n-r \lfloor n} = 56....(ii)$$

सभी (i) तथा (ii) को हल करने पर

by solving equ. (i) and equ (ii)

$$\Rightarrow \frac{\lfloor n}{\lfloor n-r} \times \frac{\lfloor n-r \lfloor r}{\lfloor n} = \frac{336}{56}$$

$$\Rightarrow \lfloor r = \lfloor 6$$

$\Rightarrow$ r = 6, Hence यहाँ, r = 6

(3) यदि ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11:52$, तो r का मान निकालिए।

If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11:52$ then find the value of r .

उत्तर:

$$\frac{{}^{22}p_{r+1}}{{}^{22}p_{r+2}} = \frac{11}{52}$$

$$\Rightarrow \frac{\frac{\lfloor 22}{\lfloor 21-r}}{\frac{\lfloor 20}{\lfloor 18-r}} = \frac{11}{52}$$

$$\Rightarrow \frac{22 \times 21 \times \lfloor 20}{\lfloor 20} \times \frac{18-r}{(21-r)(20-r)(19-r)\lfloor 18-r} = \frac{11}{52}$$

$$\Rightarrow \frac{22 \times 21}{(21-r)(20-r)(19-r)} = \frac{11}{52}$$

$\Rightarrow$ (21-r) (20-r) (19-r) = 2×21×52×=12×13×14

$\Rightarrow$ 21 -r = 14

$\therefore$ r = 7

(4) विभिन्न रंगों के 6 झण्डे से कितने तरह के संकेत दिए जा सकते हैं जबकि एक बार मे कितने भी झण्डे फहराए जा सकते हैं ?

How many different signals can be made by hoisting 6 differently coloured flags one above the other when any number of then may be hoisted at once ?

उत्तर: कुल संकेतों की संख्या

Total number of signals =

$${}^{6}P_{1}+{}^{6}P_{2}+{}^{6}P_{3}+{}^{6}P_{4}+{}^{6}P_{5}+{}^{6}P_{6} =$$

$$\frac{\lfloor 6}{\lfloor 6-1} + \frac{\lfloor 6}{\lfloor 6-2} + \frac{\lfloor 6}{\lfloor 6-3} + \frac{\lfloor 6}{\lfloor 6-4} + \frac{\lfloor 6}{\lfloor 6-5} + \frac{\lfloor 6}{\lfloor 6-6}$$

$$= \frac{\lfloor 6}{\lfloor 5} + \frac{\lfloor 6}{\lfloor 4} + \frac{\lfloor 6}{\lfloor 3} + \frac{\lfloor 6}{\lfloor 2} + \frac{\lfloor 6}{\lfloor 1} + \frac{\lfloor 6}{\lfloor 0}$$

= 6+6×5+6×5×4+6×5×4×3+6×5×4×3×2+6×5×4×3×2×1

= 6 + 30 + 120 + 360 + 720 + 720

= 1956 तरीके ways .