Logarithms

Exercise 9.1

Question 1.
Convert the following to logarithmic form:
(i) $5^2 = 25$
(ii) $a^5 = 64$
(iii) $7^x = 100$
(iv) $9^0 = 1$
(v) $6^1 = 6$
(vi) $3^{-2} = \frac{1}{9}$
(vii) $10^{-2} = 0.01$
(viii) $(81)^{\frac{3}{4}} = 27$
Solution:

(i) $5^2 = 25 \Rightarrow \log_5 25 = 2$

(ii) $a^5 = 64 \Rightarrow \log_a 64 = 5$

(iii) $7^x = 100 \Rightarrow \log_7 100 = x$

(iv) $9^0 = 1 \Rightarrow \log_9 1 = 0$

(v) $6^1 = 6 \Rightarrow \log_6 6 = 1$

(vi) $3^{-2} = \frac{1}{9} \Rightarrow \log_3 \frac{1}{9} = -2$

(vii) $10^{-2} = 0.01 \Rightarrow \log_{10} 0.01 = -2$

(viii) $(81)^{\frac{3}{4}} = 27 \Rightarrow \log_{81} 27 = \frac{3}{4}$

Question 2.
Convert the following into exponential form:
(i) $\log_2 32 = 5$
(ii) $\log_3 81 = 4$
(iii) $\log_3 \frac{1}{3} = -1$
(iv) $\log_3 4 = \frac{2}{3}$
(v) $\log_8 32 = \frac{5}{3}$
(vi) $\log_{10} (0.001) = -3$
(Vii) $\log_2 0.25 = -2$
(viii) $\log_a (\frac{1}{a}) = -1$
Solution:

(i) $\log_2 32 = 5 \Rightarrow 2^5 = 32$

(ii) $\log_3 81 = 4 \Rightarrow 3^4 = 81$

(iii) $\log_3 \frac{1}{3} = -1 \Rightarrow 3^{-1} = \frac{1}{3}$

(iv) $\log_8 4 = \frac{2}{3} \Rightarrow (8)^{\frac{2}{3}} = 4$

(v) $\log_8 32 = \frac{5}{3} \Rightarrow (8)^{\frac{5}{3}} = 32$

(vi) $\log_{10}(0.001) = -3 \Rightarrow 10^{-3} = 0.001$

(vii) $\log_2 0.25 = -2 \Rightarrow 2^{-2} = 0.25$

(viii) $\log_a \frac{1}{a} = -1 \Rightarrow a^{-1} = \frac{1}{a}$

Question 3.

By converting to exponential form, find the values of:

(i) $\log_2 16$
(ii) $\log_5 125$
(iii) $\log_4 8$
(iv) $\log_9 27$
(v) $\log_{10}(.01)$
(vi) $\log_7 \frac{1}{7}$
(vii) $\log_5 256$
(Viii) $\log_2 0.25$

Solution:

(*i*) Let, $\log_2 16 = x$

$\Rightarrow \quad (2)^x = 16$

$\Rightarrow \quad (2)^x = 2 \times 2 \times 2 \times 2$

$\Rightarrow \quad (2)^x = (2)^4$

$\therefore \quad x = 4$

(*ii*) Let, $\log_5 125 = x \quad \Rightarrow \quad (5)^x = 125$

$\Rightarrow \quad (5)^x = 5 \times 5 \times 5 \quad \Rightarrow \quad (5)^x = (5)^3$

$\therefore x = 3$

(*iii*) Let, $\log_4 8 = x \quad \Rightarrow \quad (4)^x = 8$

$\Rightarrow \quad (2 \times 2)^x = 2 \times 2 \times 2 \quad \Rightarrow \quad (2)^{2x} = (2)^3$

$\Rightarrow \quad 2x = 3$

$\therefore \quad x = \dfrac{3}{2}$

(*iv*) $\log_9 27 = x$

$\Rightarrow \quad (9)^x = 27$

$\Rightarrow \quad (3 \times 3)^x = 3 \times 3 \times 3 \quad \Rightarrow \quad (3)^{2x} = (3)^3$

$\Rightarrow \quad 2x = 3$

$\therefore \quad x = \dfrac{3}{2}$

(*v*) $\log_{10} (.01) = x \quad \Rightarrow \quad (10)^x = .01$

$\Rightarrow \quad (10)^x = \dfrac{1}{100} \quad \Rightarrow \quad (10)^x = \dfrac{1}{10} \times \dfrac{1}{10}$

$\Rightarrow \quad (10)^x = \dfrac{1}{(10)^2} \quad \Rightarrow \quad (10)^x = (10)^{-2}$

$\therefore \quad x = -2$

(*vi*) $\log_7 \dfrac{1}{7} = x \quad \Rightarrow \quad (7)^x = \dfrac{1}{7}$

$\Rightarrow \quad (7)^x = (7)^{-1}$

$\therefore \quad x = -1$

(*vii*) Let, $\log_{.5} 256 = x$

$\Rightarrow (.5)^x = 256 \quad \Rightarrow \left(\dfrac{5}{10}\right)^x = 256$

(vii) $\log_{81} x = \dfrac{3}{2}$

$\therefore\ x = 81^{\frac{3}{2}} = \left(3^4\right)^{\frac{3}{2}} = 3^{4\times\frac{3}{2}} = 3^6$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

(viii) $\log_9 x = 2.5 = \dfrac{5}{2}$

$\therefore\ x = (9)^{\frac{5}{2}} = \left(3^2\right)^{\frac{5}{2}} = 3^{2\times\frac{5}{2}} = 3^5$

$= 3 \times 3 \times 3 \times 3 \times 3 = 243$

(ix) $\log_4 x = -1.5 = \dfrac{-3}{2}$

$\therefore\ x = (4)^{\frac{-3}{2}} = \left(2^2\right)^{\frac{-3}{2}} = 2^{2\times\left(\frac{-3}{2}\right)}$

$= 2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{2\times2\times2} = \dfrac{1}{8}$

(x) $\log_{\sqrt{5}} x = 2 \Rightarrow \left(\sqrt{5}\right)^2 = x$

$\Rightarrow\ (5)^{2\times\frac{1}{2}} = x \Rightarrow (5)^1 = x \quad \therefore\ x = 5$

(xi) $\log_x 0.001 = -3 \Rightarrow (x)^{-3} = \dfrac{1}{1000}$

$\Rightarrow\ (x)^{-3} = \dfrac{1}{(10)^3} \quad \therefore\ (x)^{-3} = 10^{-3} \quad \therefore\ x = 10$

(xii) $\log_{\sqrt{3}}(x+1) = 2 \Rightarrow \left(\sqrt{3}\right)^2 = x + 1$

$\Rightarrow\ 3 = x + 1 \Rightarrow x + 1 = 3$

$\Rightarrow\ x = 3 - 1 \quad \therefore \quad x = 2$

(xiii) $\log_4 (2x + 3) = \dfrac{3}{2} \Rightarrow (4)^{\frac{3}{2}} = 2x + 3$

$\Rightarrow\ (2\times2)^{\frac{3}{2}} = 2x + 3 \Rightarrow (2)^{2\times\frac{3}{2}} = 2x + 3$

$\Rightarrow\ (2)^3 = 2x + 3 \Rightarrow 2 \times 2 \times 2 = 2x + 3$

$\Rightarrow\ 8 = 2x + 3 \Rightarrow 2x = 8 - 3$

$\therefore\ 2x = 5 \quad \therefore\ x = \dfrac{5}{2}$

(xiv) $\log_{\sqrt[3]{2}} x = 3 \Rightarrow \left(\sqrt[3]{2}\right)^3 = x \Rightarrow \left[(2)^{\frac{1}{3}}\right]^3 = x$

$\Rightarrow \quad (2)^{\frac{1}{3}\times 3} = x \Rightarrow (2)^1 = x \qquad \therefore x = 2$

(xv) $\log_2(x^2-1) = 3 \Rightarrow (2)^3 = x^2 - 1$

$\Rightarrow \quad 2 \times 2 \times 2 = x^2 - 1 \Rightarrow 8 = x^2 - 1$

$\Rightarrow \quad x^2 - 1 = 8 \Rightarrow x^2 = 8 + 1 \Rightarrow x^2 = 9$

$\therefore x = \pm 3$

(xvi) $\log x = -1 \Rightarrow (10)^{-1} = x \Rightarrow x = 10^{-1}$

$\therefore \quad x = \frac{1}{10}$

(xvii) $\log(2x - 3) = 1 \Rightarrow (10)^1 = 2x - 3$

$\Rightarrow \quad 10 = 2x - 3 \Rightarrow 2x = 10 + 3 \Rightarrow 2x = 13$

$\therefore \quad x = \frac{13}{2} = 6\frac{1}{2}$

(xviii) $\log x = -2, 0, \frac{1}{3}$

$\log x = -2 \Rightarrow (10)^{-2} = x \Rightarrow \frac{1}{100} = x \Rightarrow x = \frac{1}{100}$

when, $\log x = 0 \Rightarrow (10)^0 = x \Rightarrow x = 1$

when $\log x = \frac{1}{3} \Rightarrow (10)^{\frac{1}{3}} = x \Rightarrow x = \sqrt[3]{10}$

Hence, $x = \frac{1}{100}, 1 \sqrt[3]{10}$

Question 5.
Given $\log_{10} a = b$, express 10^{2b-3} in terms of a.
Solution:

Given $\log_{10} a = b \Rightarrow (10)^b = a$

Now $10^{2b-3} = \frac{(10)^{2b}}{(10)^3} = \frac{(10^b)^2}{10\times10\times10} = \frac{(10^b)^2}{1000}$

$= \frac{a^2}{1000}$.

Question 6.

Given $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$,
(i) write down 10^{2a-3} in terms of x.
(ii) write down 10^{3b-1} in terms of y.
(iii) if $\log_{10} P = 2a + \frac{b}{2} - 3c$, express P in terms of x, y and z.

Solution:

Given that $\log_{10} x = a \Rightarrow (10)^a = x$ ....(1)

$\log_{10} y = b \Rightarrow (10)^b = y$ ....(2)

$\log_{10} z = c \Rightarrow (10)^c = z$ ....(3)

$$(i)\ 10^{2a-3} = \frac{(10)^{2a}}{(10)^3} = \frac{(10^a)^2}{10 \times 10 \times 10} = \frac{(x)^2}{1000} = \frac{x^2}{1000}$$

$$(ii)\ 10^{3b-1} = \frac{(10)^{3b}}{(10)^1} = \frac{(10^b)^3}{10} = \frac{(y)^3}{10} = \frac{y^3}{10}$$

$$(iii)\ \text{Log}_{10} P = 2a + \frac{b}{2} - 3c$$

Substituting the value of a, b and c from equation (1), (2) and (3) we get

$$\text{Log}_{10} P = 2 \log_{10} x + \frac{1}{2} \log_{10} y - 3 \log_{10} z$$

$$\Rightarrow \text{Log}_{10} P = \log_{10} (x)^2 + \log_{10} (y)^{\frac{1}{2}} - \log_{10} (2)^3$$

$$\Rightarrow \text{Log}_{10} P = \log_{10} (x^2 \times y^{\frac{1}{2}}) - \log_{10} z^3$$

$$\Rightarrow \text{Log}_{10} P = \log_{10} \left(\frac{x^2 \sqrt{y}}{z^3} \right) \Rightarrow P = \frac{x^2 \sqrt{y}}{z^3}$$

Question 7.

If $\log_{10} x = a$ and $\log_{10} y = b$, find the value of xy.

Solution:

Given that $\log_{10} x = a$ and $\log_{10} y = b$

$\Rightarrow (10)^a = x$ and $(10)^b = y$

Then $xy = (10)^a \times (10)^b = (10)^{a+b}$

Question 8.

Given $\log_{10} a = m$ and $\log_{10} b = n$, express $\frac{a^3}{b^2}$ in terms of m and n.

Solution:

Given $\log_{10} a = m$ and $\log_{10} b = n$

Then $(10)^m = a$ and $(10)^n = b$

$$\frac{a^3}{b^2} = \frac{(10^m)^3}{(10^n)^2} = \frac{(10)^{3m}}{(10)^{2n}} = (10)^{3m-2n}$$

Question 9.

Given $\log_{10}a = 2a$ and $\log_{10}y = -\frac{b}{2}$

(i) write 10^a in terms of x.

(ii) write 10^{2b+1} in terms of y.

(iii) if $\log_{10}P = 3a - 2b$, express P in terms of x and y .

Solution:

Given that $\log_{10} x = 2a \Rightarrow (10)^{2a} = x$

and $\log_{10} y = \frac{b}{2}, \Rightarrow (10)^{\frac{b}{2}} = y$

(*i*) $10^a = \left(10^{2a}\right)^{\frac{1}{2}} = (x)^{\frac{1}{2}} = \sqrt{x}$

(*ii*) $10^{2b+1} = 10^{2b} \times 10^1 = 10^{4\left(\frac{b}{2}\right)} \times 10^1$

$= \left(10^{\frac{b}{2}}\right)^4 \times 10 = y^4 \times 10 = 10y^4$

(*iii*) $\log_{10} P = 3a - 2b$

$\Rightarrow \log_{10} P = \frac{3}{2}(2a) - 4\left(\frac{b}{2}\right)$

$\Rightarrow \log_{10} P = \frac{3}{2}(\log_{10} x) - 4 = (\log_{10} y)$

$\Rightarrow \log_{10} P = \log_{10} (x)^{3/2} - \log_{10} y^4$

$\Rightarrow \log_{10} P = \log_{10}\left(\frac{(x)^{3/2}}{y^4}\right) \Rightarrow P = \frac{(x)^{3/2}}{y^4}$

Question 10.
If $\log_2 y = x$ and $\log_3 z = x$, find 72^x in terms of y and z.
Solution:

$\log_2 y = x, \log_3 z = x$

$y = 2^x$ and $z = 3^x$ ....(i)

$72^x = (2 \times 2 \times 2 \times 3 \times 3)^x = (2^3 \times 3^2)^x$

$= 2^{3x} \times 3^{2x} = (2^x)^3 \times (3^x)^2 = y^3.z^2$ [From (i)]

Hence $72^x = y^3 . z^2$

Question 11.
If $\log_2 x = a$ and $\log_5 y = a$, write 100^{2a-1} in terms of x and y.
Solution:

$\log_2 x = a$ and $\log_5 y = a$

$\therefore\ x = 2^a$ and $y = 5^a$

$100^{2a-1} = (2 \times 2 \times 5 \times 5)^{2a-1}$
$= (2^2 \times 5^2)^{2a-1} = 2^{4a-2} \times 5^{4a-2}$

$$= \frac{2^{4a}}{2^2} \times \frac{5^{4a}}{5^2} = \frac{(2^a)^4 \times (5^a)^4}{4 \times 25} = \frac{x^4 \times y^4}{100}$$

$$= \frac{x^4 y^4}{100}$$

Exercise 9.2

Question 1.
Simplify the following :

(i) $\log a^3 - \log a^2$ **(ii)** $\log a^3 \div \log a^2$

(iii) $\frac{\log 4}{\log 2}$ **(iv)** $\frac{\log 8 \log 9}{\log 27}$

(v) $\frac{\log 27}{\log \sqrt{3}}$ **(vi)** $\frac{\log 9 - \log 3}{\log 27}$

Solution:

(i) $\log a^3 - \log a^2 = \log\left(\frac{a^3}{a^2}\right)$ (Quotient Law)

$= \log a$

(ii) $\log a^3 \div \log a^2 = 3 \log a \div 2 \log a$ (Power Law)

$$= \frac{3 \log a}{2 \log a} = \frac{3}{2}$$

(iii) $\frac{\log 4}{\log 2} = \frac{\log(2\times 2)}{\log 2} = \frac{\log 2}{\log 2}$

$= \frac{2\log 2}{\log 2}$ (Power Law) $= 2\,(1) = 2$

(iv) $\frac{\log 8 \log 9}{\log 27} = \frac{\log 2^3 . \log 3^2}{\log 3^3}$

$= \frac{(3 \log 2).(2 \log 3)}{(3\log 3)}$ (Power Law)

$= \frac{(\log 2).(2)}{1} = 2 \log 2 = \log 2^2 = \log 4.$

(v) $\frac{\log 27}{\log\sqrt{3}} = \frac{\log(3\times 3\times 3)}{\log(3)^{1/2}}$

$= \frac{\log(3)^3}{\log(3)^{1/2}} = \frac{3 \log 3}{\frac{1}{2}\log 3}$ (Power Law)

$$= \frac{3\times 2}{1}\left(\frac{\log 3}{\log 3}\right) = 6\,(1) = 6$$

(vi) $\frac{\log 9 - \log 3}{\log 27} = \frac{\log(3\times 3) - \log 3}{\log(3\times 3\times 3)}$

$= \frac{\log 3^2 - \log 3}{\log 3^3} = \frac{2\log 3 - \log 3}{3 \log 3}$ (Power Law)

$$= \frac{\log 3}{3\log 3} = \frac{1}{3}$$

Question 2.

Evaluate the following:

(i) $\log\left(10 \div \sqrt[3]{10}\right)$ (ii) $2 + \frac{1}{2}\log(10^{-3})$

(iii) $2\log 5 + \log 8 - \frac{1}{2}\log 4.$

(iv) $2\log 10^3 + 3\log 10^{-2} - \frac{1}{3}\log 5^{-3} + \frac{1}{2}\log 4$

(v) $2\log 2 + \log 5 - \frac{1}{2}\log 36 - \log\frac{1}{30}$

(vi) $2\log 5 + \log 3 + 3\log 2 - \frac{1}{2}\log 36 - 2\log 10$

(vii) $\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80}.$

(viii) $2\log_{10}5 + \log_{10}8 - \frac{1}{2}\log_{10}4.$

Solution:

$$(i)\ \log\left(10 \div \sqrt[3]{10}\right) = \log\left(10 \div (10)^{\frac{1}{3}}\right)$$

$$= \log\left((10)^1 \div (10)^{\frac{1}{3}}\right) = \log\left(10^{1-\frac{1}{3}}\right) = \log\left(10^{\frac{2}{3}}\right)$$

$$= \frac{2}{3}\log 10 = \frac{2}{3}\ (1) = \frac{2}{3}$$

$$(ii)\ 2 + \frac{1}{2}\ \log(10^{-3}) = 2 + \frac{1}{2} \times (-3) \log 10$$

$$= 2 - \frac{3}{2}\ \log 10 = 2 - \frac{3}{2}\ (1) = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2}$$

$$(iii)\ 2 \log 5 + \log 8 - \frac{1}{2}\ \log 4$$

$$= \log (5)^2 + \log 8 - \frac{1}{2}\ \log (2)^2$$

$$= \log 25 + \log 8 - \frac{1}{2} \times 2 \log 2$$

$$= \log 25 + \log 8 - \log 2 = \log\left(\frac{25 \times 8}{2}\right)$$

$$= \log\left(\frac{25 \times 4}{1}\right) = \log(100) = \log(10)^2$$

$$= 2 \log 10 = 2\ (1) = 2$$

$$(iv)\ 2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2}\ \log 4$$

$$= 2 \times 3 \log 10 + 3\ (-2) \log 10 - \frac{1}{3}\ (-3) \log 5 + \frac{1}{2}$$

$$\log (2)^2$$

$= 6 \log 10 - 6 \log 10 + \frac{3}{3} \log 5 + \frac{1}{2} \times 2 \log 2$

$= 0 + 1 \log 5 + \log 2 = \log 5 + \log 2 = \log (5 \times 2)$

$= \log (10) = 1$

(v) $2 \log 2 + \log 5 - \frac{1}{2} \log 36 - \log \frac{1}{30}$

$= \log (2)^2 + \log 5 - \frac{1}{2} \log (6)^2 - \log \left(\frac{1}{30}\right)$

$= \log 4 + \log 5 - \frac{1}{2} \times 2 \log 6 - \log \frac{1}{30}$

$= \log 4 + \log 5 - \log 6 - (\log 1 - \log 30)$

$= \log 4 + \log 5 - \log 6 - \log 1 + \log 30$

$= (\log 4 + \log 5 + \log 30) - (\log 6 + \log 1)$

$= \log (4 \times 5 \times 30) - \log (6 \times 1)$

$= \log \frac{4 \times 5 \times 30}{6 \times 1} = \log \frac{4 \times 5 \times 5}{1 \times 1} = \log 100$

$= \log (10)^2 = 2 \log 10 = 2(1) = 2$

(vi) $2 \log 5 + \log 3 + 3 \log 2 - \frac{1}{2} \log 36 - 2 \log 10$

$= \log (5)^2 + \log 3 + \log (2)^3 - \frac{1}{2} \log (6)^2 - 2 \log 10$

$= \log 25 + \log 3 + \log 8 - \frac{1}{2} \times 2 \log 6 - 2 \log 10$

$= \log 25 + \log 3 + \log 8 - \log 6 - \log (10)^2$

$= \log (25 \times 3 \times 8) - \log 6 - \log 100$

$= \log \left(\frac{25 \times 3 \times 8}{6 \times 100}\right) = \log \left(\frac{1 \times 3 \times 8}{6 \times 4}\right)$

$= \log \left(\frac{24}{24}\right) = \log 1 = 0.$

(*vii*) $\log 2 + 16 \log \dfrac{16}{15} + 12 \log \dfrac{25}{24} + 7 \log \dfrac{81}{80}$

$= \log 2 + 16 (\log 16 - \log 15) + 12 (\log 25 - \log 24) + 7 (\log 81 - \log 80)$

$= \log 2 + 16 [\log (2)^4 - \log (3 \times 5)] + 12 [\log (5)^2 - \log (3 \times 2 \times 2 \times 2) + 7 [\log (3 \times 3 \times 3 \times 3) - \log (2)^4 \times 5]$

$= \log 2 + 16 [4 \log 2 - (\log 3 + \log 5)] + 12 [2 \log 5 - \log (3 \times 2^3)] + 7 [\log (3)^4 - (\log 4 + \log 5)$

$= \log 2 + 16 [4 \log 4 - \log 3 - \log 5] + 12 [2 \text{ logs } 5 -$

$(\log 3 + \log 2^3)] + 7 [4 \log 3 - \log 4 - \log 5]$

$= \log 2 + 64 \log 2 - 16 \log 3 - 16 \log 5 + 24 \log 5 - 12 \log 3 - 12 \log 2^3 + 28 \log 3 - 7 \log 2^4 - 7 \log 5$

$= \log 2 + 64 \log 2 - 16 \log 3 - 16 \log 5 + 24 \log 5 - 12 \log 3 - 36 \log 2 + 28 \log 3 - 28 \log 2 - 7 \log 5$

$= (\log 2 + 64 \log 2 - 36 \log 2 - 28 \log 2) + (-16 \log 3 - 12 \log 3 + 28 \log 3) + (-16 \log 5 + 24 \log 5 - 7 \log 5) + 28 \log 3) + (-16 \log 5 + 24 \log 5 - 7 \log 5)$

$= (65 \log 2 - 64 \log 2) + (-28 \log 3 + 28 \log 3) + (-23 \log 5 + 24 \log 5)$

$= \log 2 + 0 + \log 5 = \log 2 + \log 5$

$= \log (2 \times 5) = \log 10 = 1$

(*viii*) $2 \log_{10} 5 + \log_{10} 8 - \dfrac{1}{2} \log_{10} 4$

$= \log_{10}(5)^2 + \log_{10} 8 - \log_{10} (4)^{\frac{1}{2}}$

$= \log_{10} 25 + \log_{10} 8 - \log_{10} (2)^{2 \times \frac{1}{2}}$

$= \log_{10} 25 + \log_{10} 8 - \log_{10} 2$

$= \log_{10} \left(\dfrac{25 \times 8}{2} \right) = \log_{10} (25 \times 4)$

$= \log_{10} 100 = \log_{10}(10)^2 = 2 \log_{10} 10 = 2 (1) = 2.$

Question 3.
Express each of the following as a single logarithm:

(i) $2\log 3 - \frac{1}{2}\log 16 + \log 12$

(ii) $2\log_{10}5 - \log_{10}2 + 3\log_{10}4 + 1.$

(iii) $\frac{1}{2}\log 36 + 2\log 8 - \log 1.5$

(iv) $\frac{1}{2}\log 25 - 2\log 3 + 1$

(v) $\frac{1}{2}\log 9 + 2\log 3 - \log 6 + \log 2 - 2.$

Solution:

(i) $2\log 3 - \frac{1}{2}\log 16 + \log 12$

$= 2\log 3 - \frac{1}{2}\log (4)^2 + \log 12$

$= 2 \log 3 - \frac{1}{2} \times 2 \log 4 + \log 12$

$= 2 \log 3 - \log 4 + \log 12 \; = \log (3)^2 - \log 4 + \log 12$

$= \log 9 - \log 4 + \log 12 = \log \frac{9 \times 12}{4} = \log \left(\frac{9 \times 3}{1}\right)$

$= \log 27.$

(ii) $2 \log_{10} 5 - \log_{10} 2 + 3 \log_{10} 4 + 1$

$= \log_{10}(5)^2 - \log_{10} 2 + \log_{10} (4)^3 + \log_{10} 10$

$(\therefore \; \log_{10} 10 = 1)$

$= \log_{10} 25 - \log_{10} 2 + \log_{10} 64 + \log_{10} 10$

$= \log_{10}(25 \times 64 \times 10) - \log_{10} 2 = \log_{10}(16000) - \log_{10} 2$

$= \log_{10} \left(\frac{16000}{2}\right) = \log_{10} 8000$

(iii) $\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5$

$= \log (36)^{\frac{1}{2}} + \log(8)^2 - \log 1.5$

$= \log (6)^{2 \times \frac{1}{2}} + \log 64 - \log \left(\frac{15}{10}\right)$

$= \log 6 + \log 64 - (\log 15 - \log 10)$

$= \log 6 \times 64 - \log 15 + \log 10$

$= \log (6 \times 64 \times 10) - \log 15$

$= \log \left(\frac{60 \times 64}{15}\right) = \log (4 \times 64) = \log 256$

(iv) $\frac{1}{2} \log 25 - 2 \log 3 + 1$

$= \; \log (25)^{\frac{1}{2}} - \log (3)^2 + \log 10 \qquad (\because \; \log 10 = 1)$

$= \; \log (5)^{2 \times \frac{1}{2}} - \log 9 + \log 10$

$= \; \log 5 - \log 9 + \log 10 = \log (5 \times 10) - \log 9$

$= \; \log \frac{5 \times 10}{9} = \log \frac{50}{9}$

(v) $\frac{1}{2}\log 9 + 2\log 3 - \log 6 + \log 2 - 2.$

$\frac{1}{2}\log 9 + 2\log 3 - \log 6 + \log 2 - 2$

$= \log 9^{\frac{1}{2}} + \log 3^2 - \log 6 + \log 2 - \log 100$

$= \log 3 + \log 9 - \log 6 + \log 2 - \log 100$

$= \log \frac{3 \times 9 \times 2}{6 \times 100}$

$= \log \frac{9}{100}$

Question 4.
Prove the following :
(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$
(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$
Solution:

(i) L.H. S $= \log_{10} 4 \div \log_{10} 2$

$= \log_{10} (2)^2 \div \log_{10} 2 = 2\log_{10} 2 \div \log_{10} 2$

$= \frac{2\log_{10} 2}{\log_{10} 2} = 2(1) = 2$

R.H.S. $= \log_3 9 = \log_3 (3)^2 = 2\log_3 3 = 2(1) = 2$

Hence, Proved. L.H.S. = R.H.S.

(ii) L.H.S. $= \log_{10} 25 + \log_{10} 4 = \log_{10} 25 \times 4$

$= \log_{10} 100 = \log_{10} 10^2$

$= 2\log_{10} 10 = 2 \times 1$

$= 2 \qquad (\because \log_a a = 1)$

R.H.S. $= \log_5 25 = \log_5 (5)^2$

$= 2\log_5 5 = 2 \times 1 = 2$

$(\because \log_a a = 1)$

Hence L.H.S. = R.H.S.

Question 5.

If x = 100)^a , y = (10000)^b and z = (10)^c, express

$\log \dfrac{10\sqrt{y}}{x^2z^3}$ **in terms of *a*, *b*, *c*.**

Solution:

Given that $x = (100)^a = [(10)^2]^a = (10)^{2a}$

$y = (10000)^b = [(10)^4]^b = (10)^{4b}$

$z = (10)^c = (10)^c$

Now, $\log \dfrac{10\sqrt{y}}{x^2z^3}$

$= (\log 10 + \log \sqrt{y}) - (\log x^2 + \log z^3)$

$= \left(1 + \log(y)^{\frac{1}{2}}\right) - (\log (x)^2 + \log (z)^3)$ $[\because \log 10 = 1]$

$= \left(1 + \dfrac{1}{2}\log y\right) - (2 \log x + 3 \log z)$

Substituting the value of x, y and z, we get

$= \left(1 + \dfrac{1}{2}\log(10)^{4b}\right) - (2 \log(10)^{2a} + 3 \log (10)^c)$

$= \left(1 + \dfrac{1}{2} \times 4b \log 10\right) - (2 \times 2a \log 10 + 3 \times c \log 10)$

$= \left(1 + \dfrac{1}{2} \times 4b \times 1\right) - (2 \times 2a \times 1 + 3 \times c \times 1)$

$[\because \log 10 = 1]$

$= (1 + 2b) - (4a + 3c) = 1 + 2b - 4a - 3c$

$= 1 - 4a + 2b - 3c$

Question 6.

If a = log$_{10}$x, find the following in terms of a :

(i) x

(ii) log$_{10}\sqrt[5]{x^2}$

(iii) log$_{10}$5x

Solution:

(*i*) Given that,

$a = \log_{10} x \Rightarrow (10)^a = x \Rightarrow x = (10)^a$

(*ii*) $\log_{10} \sqrt[5]{x^2} = \log_{10} (x^2)^{\frac{1}{5}} = \log_{10}(x)^{\frac{2}{5}}$

$= \frac{2}{5} \log_{10} x = \frac{2}{5}(a) = \frac{2}{5} a.$

(*iii*) $x = (10)^a = \log_{10} 5x = \log_{10} 5\,(10)^a$

$= \log_{10} 5 + \log_{10} 10 = \log_{10} 5 + a\,(1)$

$= a + \log_{10} 5$

Question 7.

If $a = \log \frac{2}{3}$, $b = \log \frac{3}{5}$ and $c = 2 \log \sqrt{\frac{5}{2}}$,

find the value of

(*i*) $a + b + c$ (*ii*) 5^{a+b+c}.

Solution:

Given that

$a = \log \frac{2}{3}, b = \log \frac{3}{5}, c = 2 \log \sqrt{\frac{5}{2}}$

(*i*) $a + b + c = \log \frac{2}{3} + \log \frac{3}{5} + 2 \log \sqrt{\frac{5}{2}}$

$= (\log 2 - \log 3) + (\log 3 - \log 5) + 2 \log \left(\frac{5}{2}\right)^{\frac{1}{2}}$

$= \log 2 - \log 3 + \log 3 - \log 5 + 2 \times \frac{1}{2} \log \left(\frac{5}{2}\right)$

$= \log 2 - \log 3 + \log 3 - \log 5 + \log \frac{5}{2}$

$= \log 2 + (-\log 3 + \log 3) - \log 5 + \log 5 - \log 2$

$= (\log 2 - \log 2) + 0 + (\log 5 - \log 5)$

$= 0 + 0 + 0 = 0$

(*ii*) $5^{a+b+c} = 5^0 = 1$

Question 8.

If $x = \log \frac{3}{5}$, $y = \log \frac{5}{4}$ and $z = 2 \log \frac{\sqrt{3}}{2}$,
find the values of
(i) $x + y - z$, (ii) 3^{x+y-z}

Solution:

$$x = \log \frac{3}{5}, y = \log \frac{5}{4}, z = 2 \log \frac{\sqrt{3}}{2}$$

$\therefore x = \log 3 - \log 5,$
$y = \log 5 - \log 4$

$$z = \log \left(\frac{\sqrt{3}}{2}\right)^2 = \log \frac{3}{4} = \log 3 - \log 4$$

(i) Now, $x + y - z = \log 3 - \log 5 + \log 5 - \log 4$
$- \log 3 + \log 4$
$= 0$

(ii) $3^{x+y-3} = 3^0 = 1$

Question 9.

If $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find the values of
(i)x-y-z
(ii) 7^{x-y-z}

Solution:

$x = \log_{10} 12, y = \log_4 2 \times \log_{10} 9,$
$z = \log_{10} 0{\cdot}4$

(*i*) $x - y - z = \log_{10} 12 - \log_4 2 \times \log_{10} 9 - \log_{10} 0{\cdot}4$

$$= \log_{10}(3 \times 4) - \log_4 4^{\frac{1}{2}} \times \log_{10} 3^2 - \log_{10} \frac{4}{10}$$

$$= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \log_4 4 \times 2 \log_{10} 3 - (\log_{10} 4 - \log_{10} 10)$$

$$= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} \times 1 \times 2 \log_{10} 3 - \log_{10} 4 + 1$$

$$= \log_{10} 3 + \log_{10} 4 - \log_{10} 3 - \log_{10} 4 + 1$$

$$= 1$$

(*ii*) $7^{x-y-z} = 7^1 = 7$

Question 10.

If log V + log3 = log π + log4 + 3 log r, find V in terns of other quantities.

Solution:

$\Rightarrow \quad \log V + \log 3 = \log \pi + \log 4 + \log (r)^3$

$\Rightarrow \quad \log (V \times 3) = \log (\pi \times 4 \times r^3)$

$\Rightarrow \quad \log (3V) = \log 4\pi r^3 \quad \Rightarrow \quad 3V = 4\pi r^3$

$\Rightarrow \quad V = \frac{4}{3} \pi r^3$

Question 11.

Given 3 (log 5 – log3) – (log 5-2 log 6) = 2 – log n , find n.

Solution:

Given that $3(\log 5 - \log 3) - (\log 5 - 2\log 6)$
$= 2 - \log n,$

$\Rightarrow \quad 3\log 5 - 3\log 3 - \log 5 + 2\log 6 = 2 - \log n$

$\Rightarrow \quad 2\log 5 - 3\log 3 + 2\log 6 = 2 - \log n$

$\Rightarrow \quad \log (5)^2 - \log (3)^3 + \log (6)^2 = 2(1) - \log n$

$\Rightarrow \quad \log 25 - \log 27 + \log 36 = 2\log 10 - \log n$

$[\because \log 10 = 1]$

$\Rightarrow \quad \log n = 2\log 10 - \log 25 + \log 27 - \log 36$

$\Rightarrow \quad \log n = \log (10)^2 - \log 25 + \log 27 - \log 36$

$\Rightarrow \quad \log n = \log 100 - \log 25 + \log 27 - \log 36$

$\Rightarrow \quad \log n = (\log 100 + \log 27) - (\log 25 + \log 36)$

$\Rightarrow \quad \log n = \log (100 \times 27) - \log (25 \times 36)$

$$\Rightarrow \quad \log n = \log \left(\frac{100 \times 27}{25 \times 36}\right)$$

$$\Rightarrow \quad \log n = \log \left(\frac{4 \times 27}{1 \times 36}\right)$$

$$\Rightarrow \quad \log n = \log \left(\frac{1 \times 27}{1 \times 9}\right) \quad \Rightarrow \quad \log n = \log 3$$

$\Rightarrow \quad n = 3.$

Question 12.

Given that $\log_{10}y + 2\log_{10}x = 2$, express y in terms of x.

Solution:

$\log_{10} y + 2\log_{10} x = 2$

$\Rightarrow \quad \log_{10} y + \log_{10} x^2 = 2 \quad \Rightarrow \quad \log_{10}(yx^2) = 2$

$\Rightarrow \quad \log_{10}(yx^2) = 2\log_{10} 10$

$\Rightarrow \quad \log_{10}(yx^2) = \log_{10}(10)^2 \quad \Rightarrow \quad yx^2 = (10)^2$

$$\Rightarrow \quad yx^2 = 100 \quad \Rightarrow \quad y = \frac{100}{x^2}$$

Question 13.

Express $\log_{10}2+1$ in the from $\log_{10}x$.

Solution:

$$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10 \quad (\because \log_{10} 10 = 1)$$

$$= \log_{10} 2\times 10 = \log_{10} 20$$

Question 14.

If $a^2 = \log_{10} x$, $b^3 = \log_{10} y$ and $\frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$

express z in terms of x and y.

Solution:

Given that

$a^2 = \log_{10} x$, $b^3 = \log_{10} y$

we have, $\frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$

$$\Rightarrow \frac{1}{2}(\log_{10} x) - \frac{1}{3}(\log_{10} y) = \log_{10} z$$

$$\Rightarrow \log_{10}(x)^{\frac{1}{2}} - \log_{10}(y)^{\frac{1}{3}} = \log_{10} z$$

$$\Rightarrow \log_{10}\sqrt{x} - \log_{10}\sqrt[3]{y} = \log_{10} z$$

$$\Rightarrow \log_{10}\frac{\sqrt{x}}{\sqrt[3]{y}} = \log_{10} z \Rightarrow \frac{\sqrt{x}}{\sqrt[3]{y}} = z$$

Hence, $z = \frac{\sqrt{x}}{\sqrt[3]{y}}$

Question 15.

Given that log m = x + y and log n = x-y, express the value of log m^2n in terms of x and y.

Solution:

. Given that $\log m = x + y$ and $\log n = x - y$

$\log m^2 n = \log m^2 + \log n = 2 \log m + \log n$

$= 2(x + y) + x - y = 2x + 2y + x - y \quad = 3x + y$

Question 16.

Given that $\log x = m+n$ and $\log y = m - n$, express the value of $\log\left(\frac{10x}{y^2}\right)$ in terms of m and n.

Solution:

Given that $\log n = m + n$ and $\log y = m - n$

Then $\log\left(\frac{10x}{y^2}\right) = \log 10x - \log y^2$

$= \log 10 + \log x - 2\log y = 1 + \log x - 2\log y$

$= 1 + (m+n) - 2(m-n) = 1 + m + n - 2m + 2n$

$= 1 - m + 3n$

Question 17.

If $\frac{\log x}{2} = \frac{\log y}{3}$, find the value of $\frac{y^4}{x^6}$.

Solution:

$\frac{\log x}{2} = \frac{\log y}{3} \Rightarrow 3\log x = 2\log y$

$\Rightarrow \log x^3 = \log y^2 \Rightarrow x^3 = y^2$

Squaring both sides, we get

$x^6 = y^4 \Rightarrow y^4 = x^6 \Rightarrow \frac{y^4}{x^6} = 1$

Hence, value of $\frac{y^4}{x^6} = 1$

Question 18.

Solve for x:

(*i*) $\log x + \log 5 = 2\log 3$ (*ii*) $\log_3 x - \log_3 2 = 1$

(*iii*) $x = \frac{\log 125}{\log 25}$ (*iv*) $\frac{\log 8}{\log 2} \times \frac{\log 3}{\log\sqrt{3}} = 2\log x$

Solution:

(i) $\log x + \log 5 = 2 \log 3$

$\Rightarrow \log x = 2 \log 3 - \log 5 \Rightarrow \log x = \log (3)^2 - \log 5$

$\Rightarrow \log x = \log 9 - \log 5 \Rightarrow \log x = \log \left(\frac{9}{5}\right)$

$\therefore x = \frac{9}{5}$

(ii) $\log_3 x - \log_3 2 = 1 \Rightarrow \log_3 x = \log_3 2 + 1$

$\Rightarrow \log_3 x = \log_3 2 + \log_3 3 \quad (\because \log_3 3 = 1)$

$\Rightarrow \log_3 x = \log_3 (2 \times 3) \Rightarrow \log_3 x = \log_3 6$

$\therefore x = 6$

(iii) $x = \frac{\log 125}{\log 25} \Rightarrow x = \frac{\log (5)^3}{\log (5)^2}$

$\Rightarrow x = \frac{3 \log 5}{2 \log 5} = \frac{3}{2} \quad \therefore x = \frac{3}{2}$

(iv) $\frac{\log 8}{\log 2} \times \frac{\log 3}{\log \sqrt{3}} = 2 \log x$

$\Rightarrow \frac{\log (2)^3}{\log 2} \times \frac{\log 3}{\log (3)^{\frac{1}{2}}} = 2 \log x$

$\Rightarrow \frac{3 \log 2}{\log 2} \times \frac{\log 3}{\frac{1}{2} \log 3} = 2 \log x$

$\Rightarrow 3(1) \times \frac{1}{\left(\frac{1}{2}\right)} = 2 \log x \Rightarrow 3 \times \frac{2}{1} = 2 \log x$

$\Rightarrow 2 \log x = 6 \Rightarrow \log x = \frac{6}{2} \Rightarrow \log x = 3$

$\Rightarrow x = (10)^3 \Rightarrow x = 1000$

Question 19.

Given $2 \log_{10} x + 1 = \log_{10} 250$, find

(i) x

(ii) $\log_{10} 2x$

Solution:

(i) $2 \log_{10} x + 1 = \log_{10} 250$

$\Rightarrow \quad \log_{10} x^2 + 1 = \log_{10} 250 \qquad [\log_a m^n = n \log m]$

$\Rightarrow \quad \log_{10} x^2 \times 10 = \log_{10} 250 \quad [\log_{10} 10 = 1]$

$\Rightarrow \quad \log_{10} x^2 \times \log_{10} 10 = \log_{10} 250$

$\Rightarrow \quad x^2 \times 10 = 250 \quad \Rightarrow \quad x^2 = \dfrac{250}{10} \quad \Rightarrow \quad x^2 = 25$

$\Rightarrow \quad (x)^2 = (5)^2 \quad \therefore \quad x = 5$

(ii) $x = 5$ (proved in (i) above)

$\log_{10} 2x = \log_{10} 2 \times 5$ [Putting $x = 5$]

$= \log_{10} 10 = 1 \qquad [\log_{10} 10 = 1]$

Question 20.

If $\dfrac{\log x}{\log 5} = \dfrac{\log y^2}{\log 2} = \dfrac{\log 9}{\log \frac{1}{3}}$, find x and y.

Solution:

$$\frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{\log 9}{\log \frac{1}{3}}$$

Taking first and third terms,

$$\frac{\log x}{\log 5} = \frac{\log 9}{\log \frac{1}{3}} \quad \Rightarrow \quad \log x = \frac{\log 9}{\log \frac{1}{3}} \times \log 5$$

$$\Rightarrow \quad \log x = \frac{\log(3 \times 3)}{\log 1 - \log 3} \times \log 5$$

$$\Rightarrow \quad \log x = \frac{\log(3)^2}{0 - \log 3} \times \log 5 \qquad [\log 1 = 0]$$

$$\Rightarrow \quad \log x = \frac{2\log 3}{-\log 3} \times \log 5$$

$$\Rightarrow \quad \log x = \frac{2\log 3}{\log 3} \times \log 5$$

$$\Rightarrow \quad \log x = -2\,(1) \times \log 5 \quad \Rightarrow \quad \log x = -2\log 5$$

$$\Rightarrow \quad \log x = \log (5)^{-2} \quad \Rightarrow \quad x = (5)^{-2}$$

$$\Rightarrow \quad x = \frac{1}{(5)^2} \quad \Rightarrow \quad x = \frac{1}{25}$$

taking second and third terms,

$$\frac{\log y^2}{\log 2} = \frac{\log 9}{\log\left(\frac{1}{3}\right)} \quad \Rightarrow \quad \log y^2 = \frac{\log 9}{\log\left(\frac{1}{3}\right)} \times \log 2$$

$$\Rightarrow \quad \log y^2 = \frac{\log(3)^2}{\log 1 - \log 3} \times \log 2$$

$$\Rightarrow \quad \log y^2 = \frac{2\log 3}{0 - \log 3} \times \log 2 \quad [\log 1 = 0]$$

$$\Rightarrow \quad \log y^2 = \frac{2\log 3}{-\log 3} \times \log 2$$

$$\Rightarrow \quad \log y^2 = \frac{-2\log 3}{\log 3} \times \log 2$$

$$\Rightarrow \quad \log y^2 = -2 \times \log 2 \quad \Rightarrow \quad \log y^2 = \log(2)^{-2}$$

$$\Rightarrow \quad y^2 = (2)^{-2} \quad \Rightarrow \quad y = (2)^{-2\times\frac{1}{2}} \quad \Rightarrow \quad y = (2)^{-1}$$

$$\Rightarrow \quad y = \frac{1}{2}$$

Question 21.

Prove the following :

(i) $3_{\log 4} = 4_{\log 3}$

(ii) $27_{\log 2} = 8_{\log 3}$

Solution:

• (i) $3^{\log 4} = 4^{\log 3}$ is true

if $\log 3^{\log 4} = \log 4^{\log 3}$

(Taking log both sides)

if $\log 4 \cdot \log 3 = \log 3 \cdot \log 4$

if $\log_2 2 \cdot \log 3 = \log 3 \cdot \log 2^2$

if $2 \log 2 \times \log 3 = \log 3 \times 2 \log 2$

if $2 \log 2 \log 3 = 2 \log 2 \log 3$

which is true

Hence proved

(ii) $27^{\log 2} = 8^{\log 3}$ is true

if $\log 27^{\log 2} = \log 8^{\log 3}$

(Taking log both sides)

if $\log 2 \log 27 = \log 3 \log 8$

if $\log 2 \log 3^3 = \log 3 \log 2^3$

if $\log 2 \cdot 3 \log 3 = \log 3 \cdot 3 \log 2$

if $3 \log 2 \cdot \log 3 = 3 \cdot \log 2 \log 3$

which is true

Hence proved

Question 22.

Solve the following equations :

(i) log (2x + 3) = log 7

(ii) log (x +1) + log (x – 1) = log 24

(iii) log (10x + 5) – log (x – 4) = 2

(iv) $\log_{10}5 + \log_{10}(5x+1) = \log_{10}(x + 5) + 1$

(v) log (4y – 3) = log (2y + 1) – log3

(vi) $\log_{10}(x + 2) + \log_{10}(x - 2) = \log_{10}3 + 31og_{10}4$.

(vii) log(3x + 2) + log(3x – 2) = 5 log 2.

Solution:

(*i*) $\log(2x+3) = \log 7$

$\Rightarrow \quad 2x + 3 = 7 \quad \Rightarrow \quad 2x = 7 - 3 \quad \Rightarrow \quad 2x = 4$

$\Rightarrow \quad x = \dfrac{4}{2} \quad \therefore \quad x = 2$

(*ii*) $\log(x+1) + \log(x-1) = \log 24$

$\Rightarrow \quad \log(x+1)(x-1) = \log 24$

$\Rightarrow \quad \log(x^2 - 1) = \log 24 \quad \Rightarrow \quad x^2 - 1 = 24$

$\Rightarrow \quad x^2 = 24 + 1 \quad \Rightarrow \quad x^2 = 25 \quad \Rightarrow \quad x^2 = (5)^2$

$\therefore \quad x^2 = 5$

(*iii*) $\log(10x+5) - \log(x-4) = 2$

$\Rightarrow \quad \log\dfrac{(10x+5)}{(x-4)} = 2(\log 10) \qquad [\therefore \ \log 10 = 1]$

$\Rightarrow \quad \log\dfrac{10x+5}{x-4} = \log(10)^2$

$\Rightarrow \quad \log\left(\dfrac{10x+5}{x-4}\right) = \log 100 \quad \Rightarrow \quad \dfrac{10x+5}{(x-4)} = 100$

$\Rightarrow \quad 10x + 5 = 100(x-4)$

$\Rightarrow \quad 10x + 5 = 100x - 400 \Rightarrow 10x - 100x = -400 - 5$

$\Rightarrow \quad -90x = -405 \quad \Rightarrow \quad x = \dfrac{-405}{-90}$

$\Rightarrow \quad x = \dfrac{405}{90} = \dfrac{81}{18} = \dfrac{9}{2} \quad \therefore \quad x = 4.5$

(iv) $\log_{10}5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$

$\Rightarrow \quad \log_{10}5 \times (5x + 1) = \log_{10}(x + 5) + \log_{10}10$

$[\log_{10}10 = 1]$

$\Rightarrow \quad \log_{10}5\,(5x + 1) = \log_{10}[10 \times (x + 5)]$

$\Rightarrow \quad 5\,(5x + 1) = 10\,(x + 5) \quad \Rightarrow \quad 25x + 5 = 10x + 5$

$\Rightarrow \quad 25x - 10x = 50 - 5 \quad \Rightarrow \quad 15x = 45$

$\Rightarrow \quad x = \dfrac{45}{15} \quad \therefore \quad x = 3$

(v) $\log(4y - 3) = \log(2y + 1) - \log 3$

$\Rightarrow \quad \log 4y - 3 = \log \dfrac{(2y+1)}{3}$

$\Rightarrow \quad 4y - 3 = \dfrac{2y+1}{3} \quad \Rightarrow \quad 3(4y - 3) = 2y + 1$

$\Rightarrow \quad 12y - 9 = 2y + 1 \quad \Rightarrow \quad 12y - 2y = 1 + 9$

$\Rightarrow \quad 10y = 10 \quad \Rightarrow \quad y = \dfrac{10}{10} = 1$

$\therefore \quad y = 1$

(vi) $\log_{10}(x + 2) + \log_{10}(x - 2) = \log_{10}3 + 3\log_{10}4$

$\Rightarrow \quad \log_{10}(x + 2)(x - 2) = \log_{10}3 + \log_{10}(4)^3$

$\Rightarrow \quad \log_{10}(x^2 - 2^2) = \log_{10}3 + \log_{10}(4 \times 4 \times 4)$

$\Rightarrow \quad \log_{10}(x^2 - 4) = \log_{10}3 + \log_{10}64$

$\Rightarrow \quad \log_{10}(x^2 - 4) = \log_{10}3 \times 64$

$\Rightarrow \quad x^2 - 4 = 3 \times 64 \quad \Rightarrow \quad x^2 - 4 = 192$

$\Rightarrow \quad x^2 = 192 + 4 \quad \Rightarrow \quad x^2 = 196$

$\Rightarrow \quad x^2 = (14)^2$

$\therefore \quad x = 14$

Hence proved

(vii) $\log(3x + 2) + \log(3x - 2) = 5\log 2$

$\Rightarrow \log(3x + 2)(3x - 2) = \log 2^5$

$\Rightarrow \log(9x^2 - 4) = \log 32$

Comparing both sides

$9x^2 - 4 = 32 \quad \Rightarrow 9x^2 = 32 + 4 = 36$

$x^2 = \dfrac{36}{9} = 4 = (\pm 2)^2$

$\therefore x = \pm 2$

$x = 2$

Question 23.

Solve for x :

$\log_3 (x + 1) - 1 = 3 + \log_3 (x - 1)$

Solution:

$\log_3 (x + 1) - 1 = 3 + \log_3 (x - 1)$

$\Rightarrow \log_3 (x + 1) - 3 \log (x - 1) = 3 + 1$

$$\Rightarrow \log_3 \frac{x+1}{x-1} = 4 = 4 \times 1 = 4 \log_3 3$$

$(\because \log_a a = 1)$

$$\Rightarrow \log_3 \frac{x+1}{x-1} = \log_3 3^4 = \log_3 81$$

$$\therefore \frac{x+1}{x-1} = \frac{81}{1}$$

$\Rightarrow 81x - 81 = x + 1$

$\Rightarrow 81x - x = 1 + 81 \Rightarrow 80x = 82$

$$\therefore x = \frac{82}{80} = \frac{41}{40} = 1\frac{1}{40}$$

Question 24.

Solve for x : $5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$.

Solution:

$$5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$$

$$5^{\log x} + 3^{\log x} = 3^{\log x} \cdot 3^{1} - 5^{\log x} . 5^{-1}$$

$$5^{\log x} + 3^{\log x} = 3.3^{\log x} - \frac{1}{5} . 5^{\log x}$$

$$5^{\log x} + \frac{1}{5} . 5^{\log x} = 3.3^{\log x} - 3^{\log x}$$

$$\Rightarrow \left(1+\frac{1}{5}\right) (5^{\log x}) = (3 - 1)(3^{\log x})$$

$$\Rightarrow \frac{6}{5} (5^{\log x}) = 2 \times 3^{\log x}$$

$$\Rightarrow \frac{5^{\log x}}{3^{\log x}} = \frac{2 \times 5}{6} = \left(\frac{5}{3}\right)^{1} \Rightarrow \left(\frac{5}{3}\right)^{\log x} = \left(\frac{5}{3}\right)^{1}$$

Comparing, we get

$\log x = 1 = \log 10$

$\therefore\ x = 10$

Question 25.

If $\log \left(\frac{x-y}{2}\right) = \frac{1}{2} (\log x + \log y)$, prove that $x^2 + y^2 = 6xy$.

Solution:

$$\log\left(\frac{x-y}{2}\right) = \frac{1}{2}(\log x + \log y)$$

$$\Rightarrow \quad \log\left(\frac{x-y}{2}\right) = \frac{1}{2}\log xy$$

$$[\because \ \log m + \log n = \log mn]$$

$$\Rightarrow \quad \log\left(\frac{x-y}{2}\right) = \log (xy)^{\frac{1}{2}} \quad \Rightarrow \quad \frac{x-y}{2} = (xy)^{\frac{1}{2}}$$

Squaring both sides, we get

$$\Rightarrow \quad \left(\frac{x-y}{2}\right)^2 = \left[(xy)^{\frac{1}{2}}\right]^2 \Rightarrow \frac{(x-y)^2}{4} = (xy)^{\frac{1}{2}\times 2}$$

$$\Rightarrow \quad (x-y)^2 = 4 \times xy \quad \Rightarrow \quad x^2 + y^2 - 2xy = 4xy$$

$[\because (A-B)^2 = A^2 + B^2 - 2AB]$

$$\Rightarrow \quad x^2 + y^2 = 4xy + 2xy$$

$$\Rightarrow \quad x^2 + y^2 = 6xy$$

Proved.

Question 26.

If $x^2 + y^2 = 23xy$, Prove that

$$\log \frac{x+y}{5} = \frac{1}{2}(\log x + \log y)$$

Solution:

Given $x^2 + y^2 = 23xy \Rightarrow x^2 + y^2 = 25xy - 2xy$

$\Rightarrow x^2 + y^2 + 2xy = 25xy$

$\Rightarrow (x)^2 + (y)^2 + 2 \times x \times y = 25xy$

$\Rightarrow (x + y)^2 = 25xy \Rightarrow \frac{(x+y)^2}{25} = xy$

taking log on both sides, we get

$\Rightarrow \log \frac{(x+y)^2}{25} = \log xy$

$\Rightarrow \log \left(\frac{x+y}{5}\right)^2 = \log x + \log y$

$\Rightarrow 2 \log \frac{x+y}{5} = \log x + \log y \Rightarrow \log \frac{x+y}{5}$

$= \frac{1}{2} (\log x + \log y)$ Proved.

Question 27.

If $p = \log_{10}20$ and $q = \log_{10}25$, find the value of x if $2 \log_{10} (x + 1) = 2p - q$.

Solution:

Given that $p = \log_{10}20$ and $q = \log_{10}25$

Then, $2\log_{10}(x+1) = 2p - q$

Substituting the value of p and q, we get

$\Rightarrow \quad 2\log_{10}(x+1) = 2\log_{10}20 - \log_{10}25$

$\Rightarrow \quad 2\log_{10}(x+1) = 2\log_{10}20 - \log_{10}(5)^2$

$\Rightarrow \quad 2\log_{10}(x+1) = 2\log_{10}20 - 2\log_{10}5$

$\Rightarrow \quad 2\log_{10}(x+1) = 2(\log_{10}20 - \log_{10}5)$

$\Rightarrow \quad \log_{10}(x+1) = 2\dfrac{(\log_{10}20 - \log_{10}5)}{2}$

$\Rightarrow \quad \log_{10}(x+1) = \log_{10}20 - \log_{10}5$

$\Rightarrow \quad \log_{10}(x+1) = \log_{10}\left(\dfrac{20}{5}\right)$

$\Rightarrow \quad \log_{10}(x+1) = \log_{10}4 \quad \Rightarrow \quad x+1 = 4$

$\Rightarrow \quad x = 4 - 1 \quad \therefore\ x = 3$

Question 28.

Show that:

(i) $\dfrac{1}{\log_2 42} + \dfrac{1}{\log_3 42} + \dfrac{1}{\log_7 42} = 1$

(ii) $\dfrac{1}{\log_8 36} + \dfrac{1}{\log_9 36} + \dfrac{1}{\log_{18} 36} = 2$

Solution:

(*i*) $\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$

$$\text{L.H.S.} = \frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42}$$

$$\left\{\because \log_n m = \frac{\log_m}{\log_n}\right\}$$

$$= \frac{1}{\frac{\log 42}{\log_2}} + \frac{1}{\frac{\log 42}{\log_3}} + \frac{1}{\frac{\log 42}{\log_7}}$$

$$= \frac{\log_2}{\log 42} + \frac{\log_3}{\log 42} + \frac{\log_7}{\log 42}$$

$$= \frac{\log_2 + \log_3 + \log_7}{\log 42} = \frac{\log_{(2\times3\times7)}}{\log 42}$$

$$\left\{\begin{array}{l}\because \log_m + \log_n + \log_p \\ \quad = \log_{mnp}\end{array}\right\}$$

$$= \frac{\log 42}{\log 42} = 1 = \text{R.H.S.}$$

(*ii*) $\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$

$$\text{L.H.S.} = \frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36}$$

$$= \frac{1}{\frac{\log 36}{\log_8}} + \frac{1}{\frac{\log 36}{\log_9}} + \frac{1}{\frac{\log 36}{\log_{18}}}$$

$$= \frac{\log_8}{\log 36} + \frac{\log_9}{\log 36} + \frac{\log_{18}}{\log 36}$$

$$= \frac{\log_8 + \log_9 + \log_{18}}{\log 36}$$

$$= \frac{\log(8 \times 9 \times 18)}{\log 36} = \frac{\log(36)^2}{\log 36}$$

$$= \frac{2\log 36}{\log 36} = 2 = \text{R.H.S.}$$

Question 29.

Prove the following identities:

(*i*) $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

(*ii*) $\log_b a \,.\, \log_c b \,.\, \log_d c = \log_d a$

Solution:

(*i*) $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

$$\text{L.H.S.} = \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$= \frac{1}{\frac{\log abc}{\log_a}} + \frac{1}{\frac{\log abc}{\log_b}} + \frac{1}{\frac{\log abc}{\log_c}}$$

$$\left\{\because \log_n m = \frac{\log_m}{\log_n}\right\}$$

$$= \frac{\log_a}{\log abc} + \frac{\log_b}{\log abc} + \frac{\log_c}{\log abc}$$

$$= \frac{\log_a + \log_b + \log_c}{\log abc}$$

$$= \frac{\log abc}{\log abc} = 1 = \text{R.H.S.}$$

$$\left\{\begin{array}{l}\because \log mnp \\ \quad = \log_m + \log_n + \log_p\end{array}\right\}$$

(*ii*) $\log_b a \,.\, \log_c b \,.\, \log_d c = \log_d a$

L.H.S. $= \log_b a \times \log_c b \times \log_d c$

$$= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log d} = \frac{\log a}{\log d}$$

$= \log_d a =$ R.H.S.

Question 30.

. Given that $\log_a x = \frac{1}{\alpha}$, $\log_b x = \frac{1}{\beta}$, $\log_c x = \frac{1}{\gamma}$, find $\log_{abc} x$.

Solution:

$$\log_a x = \frac{1}{\alpha},\ \log_b x = \frac{1}{\beta},\ \log_c x = \frac{1}{\gamma}$$

$$\log_a x = \frac{1}{\alpha} \Rightarrow \frac{\log x}{\log_a} = \frac{1}{\alpha} \Rightarrow \log_a = \alpha \log x$$

$$\log_b x = \frac{1}{\beta} \Rightarrow \frac{\log x}{\log_b} = \frac{1}{\beta} \Rightarrow \log_b = \beta \log x$$

$$\log_c x = \frac{1}{\gamma} \Rightarrow \frac{\log x}{\log_c} = \frac{1}{\gamma} \Rightarrow \log_c = \gamma \log x$$

$$\text{Now } \log_{abc} x = \frac{\log x}{\log abc}$$

$$= \frac{\log x}{\log a + \log b + \log c}$$

$$= \frac{\log x}{\alpha \log x + \beta \log x + \gamma \log x}$$

$$= \frac{\log x}{\log x\,(\alpha + \beta + \gamma)} = \frac{1}{\alpha + \beta + \gamma}$$

Question 31.

Solve for x :

(*i*) $\log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$

(*ii*) $\log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$

Solution:

(i) $\log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$

$$\Rightarrow \frac{1}{\log_x 3} + \frac{1}{\log_x 9} + \frac{1}{\log_x 81} = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3^1} + \frac{1}{\log_x 3^2} + \frac{1}{\log_x 3^4} = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3} + \frac{1}{2\log_x 3} + \frac{1}{4\log_x 3} = \frac{7}{4}$$

$$\Rightarrow \frac{1}{\log_x 3}\left[1+\frac{1}{2}+\frac{1}{4}\right] = \frac{7}{4}$$

$$\Rightarrow \log_x 3 \times \frac{7}{4} = \frac{7}{4}$$

$$\Rightarrow \log_x 3 = \frac{7}{4} \times \frac{4}{7} = 1 = \log_3 3$$

$\{\because \log_a a = 1\}$

Comparing, we get

$\therefore\ x = 3$

(ii) $\log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$

$$\Rightarrow \frac{1}{\log_x 2} + \frac{1}{\log_x 8} + \frac{1}{\log_x 32} = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2^1} + \frac{1}{\log_x 2^3} + \frac{1}{\log_x 2^5} = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2} + \frac{1}{3\log_x 2} + \frac{1}{5\log_x 2} = \frac{23}{15}$$

$$\Rightarrow \frac{1}{\log_x 2}\left[-1+\frac{1}{3}+\frac{1}{5}\right] = \frac{23}{15}$$

$$\Rightarrow \frac{23}{15}\left[\log_x 2\right] = \frac{23}{15}$$

$$\log_x 2 = \frac{23}{15} \times \frac{15}{23} = 1 = \log_2 2$$

$$\{\because \log_a a = 1\}$$

Comparing,

$x = 2$

Multiple Choice Questions

correct Solution from the given four options (1 to 7):

Question 1.

If log√3 27 = x, then the value of x is

(a) 3

(b) 4

(c) 6

(d) 9

Solution:

$$\log_{\sqrt{3}} 27 = x$$

$$(\sqrt{3})^x = 27$$

$$\Rightarrow (3)^{\frac{1}{2}\times x} = 3^3$$

$$\Rightarrow 3^{\frac{x}{2}} = 3^3 \Rightarrow \frac{x}{2} = 3$$

$$\Rightarrow x = 6 \qquad \text{(c)}$$

Question 2.
If $\log_5 (0.04) = x$, then the vlaue of x is
(a) 2
(b) 4
(c) -4
(d) -2
Solution:

$$\log_5 (0.04) = x$$

$$5^x = 0.04 = \frac{4}{100} = \frac{1}{25} = 5^{-2}$$

$\therefore x = -2$ (d)

Question 3.
If $\log_{0.5} 64 = x$, then the value of x is
(a) -4
(b) -6
(c) 4
(d) 6
Solution:

$$\log_{0.5} 64 = x \Rightarrow 0.5^x = 64$$

$$= \left(\frac{1}{2}\right)^x = 2^6 \Rightarrow 2^{-x} = 2^6$$

$\therefore -x = 6 \Rightarrow x = -6$ (b)

Question 4.
If $\log_{10}\sqrt[3]{5}$ x = -3, then the value of x is

(a) $\frac{1}{5}$ **(b)** $-\frac{1}{5}$

(c) -1 **(d)** 5

Solution:

$$\log_{\sqrt[3]{5}} x = -3, \quad (\sqrt[3]{5})^{-3} = x$$

$$x = (5^{\frac{1}{3}})^{-3} = 5^{\frac{1}{3}(-3)} = 5^{-1}$$

$x = \frac{1}{5}$ (b)

Question 5.
If log (3x + 1) = 2, then the value of x is

(a) $\frac{1}{3}$
(b) 99
(c) 33
(d) $\frac{19}{3}$

Solution:

$$\log(3x + 1) = 2 = \log 100 \quad (\because \log 100 = 2)$$
$$\therefore\ 3x + 1 = 100 \Rightarrow 3x = 100 - 1 = 99$$
$$\Rightarrow x = \frac{99}{3} = 33 \qquad \text{(c)}$$

Question 6.
The value of 2 + $\log_{10}$ (0.01) is
(a)4
(b)3
(c)1
(d)0

Solution:

$$2 + \log_{10}(0.01)$$
$$= 2 + (-2) = 2 - 2 = 0 \qquad \text{(d)}$$

Question 7.
The value of $\frac{\log 8 - \log 2}{\log 32}$ is

(a) $\frac{2}{5}$
(b) $\frac{1}{4}$
(c) $-\frac{2}{5}$
(d) $\frac{1}{3}$

Solution:

$$\frac{\log 8 - \log 2}{\log 32} = \frac{\log \frac{8}{2}}{\log 2^5}$$

$$= \frac{\log 4}{\log 2^5} = \frac{\log 2^2}{\log 2^5}$$

$$= \frac{2 \log 2}{5 \log 2} = \frac{2}{5} \quad \text{(a)}$$

Chapter Test

Question 1.

Expand $\log_a \sqrt[3]{x^7 y^8 \div \sqrt[4]{z}}$

Solution:

$$\log_a \sqrt[3]{x^7 y^8 \div \sqrt[4]{z}}$$

$$= \log_a \left(x^7 y^8 \div \sqrt[4]{z}\right)^{1/3} = \frac{1}{3} \log_a \left(x^7 y^8 \div \sqrt[4]{z}\right)$$

$$= \frac{1}{3} \left[\log_a x^7 y^8 - \log_a \sqrt[4]{z}\right]$$

$$= \frac{1}{3} \left[7 \log_a x + 8 \log_a y - \log_a (z)^{1/4}\right]$$

$$= \frac{1}{3} \left[7 \log_a x + 8 \log_a y - \frac{1}{4} \log_a z\right]$$

$$= \frac{7}{3} \log_a x + \frac{8}{3} \log_a y - \frac{1}{12} \log_a z$$

Question 2.

Find the value of $\log_{\sqrt{3}} 3\sqrt{3} - \log_5 (0.04)$

Solution:

$\log_{\sqrt{3}} 3\sqrt{3} - \log_5 (0\cdot04)$

$= \log_{\sqrt{3}} 3 + \log_{\sqrt{3}} \sqrt{3} - \log_5 \frac{4}{100}$

$= \log_{\sqrt{3}} 3 + 1 - \log_5 \frac{1}{25}$

$= \log_{\sqrt{3}} 3 + 1 - \log_5 5^{-2}$

$= \log_{\sqrt{3}} 3 + 1 - (-2) \log_5 5$

$= \log_{\sqrt{3}} (\sqrt{3})^2 + 1 + 2 \times 1 = 2 \log_{\sqrt{3}} \sqrt{3} + 1 + 2$

$= 2 \times 1 + 1 + 2 = 2 \times 1 + 2 = 5$

Question 3.
Prove the following:

(i) $(\log x)^2 - (\log y)^2 = \log \frac{x}{y} . \log x y$

(ii) $2 \log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91} = \log 2.$

Solution:

(i) $(\log x)^2 - (\log y)^2 = \log \frac{x}{y} \cdot \log x y$

L.H.S $= (\log x)^2 - (\log y)^2 = (\log x - \log y)(\log x + \log y)$

$[\because \ A^2 - B^2 = (A - B)(A + B)]$

$= \left(\log \frac{x}{y}\right)(\log xy) = \log \frac{x}{y} \cdot \log xy = \text{R.H.S.}$

Result is proved.

(ii) $2 \log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91} = \log 2$

L.H.S. $= 2 \log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91}$

$= 2[\log 11 - \log 13] + [\log 130 - \log 77] - [\log 55 - \log 91]$

$= 2 [\log 11 - \log 13] + [\log 13 \times 10 - \log 11 \times 7] - [\log 11 \times 5 - \log 13 \times 7]$

$= 2 [\log 11 - \log 13] + [(\log 13 + \log 10) - (\log 11 + \log 7)] - [(\log 11 + \log 5) - (\log 13 + \log 7)]$

$= 2 \log 11 - 2 \log 13 + \log 10 - \log 11 - \log 7 - \log 11 - \log 5 + \log 13 + \log 7$

$= (2 \log 11 - \log 11 - \log 11) + (-2 \log 13 + \log 13 + \log 13) + \log 10 - \log 5 + (\log 7 - \log 7)$

$= 0 + 0 + \log 10 - \log 5 + 0 = \log 10 - \log 5$

$= \log \left(\frac{10}{5}\right) = \log 2 = \text{R.H.S}$

Hence, Result is proved.

Question 4.

If log (m + n) = log m + log n, show that n = $\frac{m}{m-1}$

Solution:

Question 5.

If $\log \frac{x+y}{2} = \frac{1}{2}(\log x + \log y)$, prove that $x = y$.

Solution:

$$\log \frac{x+y}{2} = \frac{1}{2}(\log x + \log y)$$

$$\Rightarrow \log \frac{x+y}{2} = \frac{1}{2}\log(x \times y)$$

$$\Rightarrow \log \frac{x+y}{2} = \log (x \times y)^{\frac{1}{2}}$$

Comparing, we get,

$$\therefore \frac{x+y}{2} = (x \times y)^{\frac{1}{2}} = xy^{\frac{1}{2}} \quad \Rightarrow x + y = 2(xy)^{\frac{1}{2}}$$

quaring

$\Rightarrow (x+y)^2 = 4xy \qquad \Rightarrow x^2 + y^2 + 2xy = 4xy$

$\Rightarrow x^2 + y^2 + 2xy - 4xy = 0 \quad \Rightarrow x^2 + y^2 - 2xy = 0$

$\Rightarrow (x-y)^2 = 0 \Rightarrow x - y = 0$

$\therefore x = y$ Hence proved.

Question 6.

If a, b are positive real numbers, $a > b$ and $a^2 + b^2 = 27\,ab$, prove that

$$\log\left(\frac{a-b}{5}\right) = \frac{1}{2}(\log a + \log b)$$

Solution:

$$a^2 + b^2 = 27ab$$

$$\Rightarrow a^2 + b^2 - 2ab = 25ab$$

$$\Rightarrow \quad \frac{a^2 + b^2 - 2ab}{25} = ab \quad \Rightarrow \quad \left(\frac{a-b}{5}\right)^2 = ab$$

Taking log both sides, $\log\left(\frac{a-b}{5}\right)^2 = \log ab$

$$\Rightarrow 2\log\left(\frac{a-b}{5}\right) = \log a + \log b$$

$$\Rightarrow \quad \log\left(\frac{a-b}{5}\right) = \frac{1}{2}(\log a + \log b)$$

Hence proved.

Solve the following equations for x

Question 7.

Solve the following equations for x:

(i) $\log_x \frac{1}{49} = -2$ **(ii)** $\log_x \frac{1}{4\sqrt{2}} = -5$

(iii) $\log_x \frac{1}{243} = 10$ **(iv)** $\log_4 32 = x - 4$

(v) $\log_7 (2x^2 - 1) = 2$ **(vi)** $\log (x^2 - 21) = 2$

(vii) $\log_6 (x - 2)(x + 3) = 1$

(viii) $\log_6 (x - 2) + \log_6 (x + 3) = 1$

(ix) $\log (x + 1) + \log (x - 1) = \log 11 + 2 \log 3.$

Solution:

$(i)\ \log_x \dfrac{1}{49} = -2 \quad \Rightarrow \quad (x)^{-2} = \dfrac{1}{49}$

$\Rightarrow (x)^{-2} = \left(\dfrac{1}{7}\right)^2 \quad \Rightarrow \quad (x)^{-2} = (7)^{-2} \quad \Rightarrow x = 7$

$(ii) \qquad \log_x \dfrac{1}{4\sqrt{2}} = -5$

$\Rightarrow \dfrac{-1}{5}\log_x \dfrac{1}{4\sqrt{2}} = 1 \ \Rightarrow -\dfrac{1}{5}\log_x \dfrac{1}{\sqrt{32}} = 1$

$\Rightarrow \ -\dfrac{1}{5}\log_x \dfrac{1}{2^{\frac{5}{2}}} = 1 \quad \Rightarrow \ -\dfrac{1}{5}\log_x 2^{\frac{-5}{2}} = 1$

$\Rightarrow \ -\dfrac{1}{5} \times \left(\dfrac{-5}{2}\right)\log_x 2 = 1 \Rightarrow \ \dfrac{1}{2}\log_x 2 = 1$

$\Rightarrow \quad \log_x (2^{\frac{1}{2}}) = 1 \ \Rightarrow \ \log_x \sqrt{2} = \log_x x$

$\therefore \quad x = \sqrt{2}$

$(iii)\ \log_x \dfrac{1}{243} = 10$

$$\Rightarrow \frac{1}{10}\log_x \frac{1}{243} = 1 \quad \Rightarrow \frac{1}{10}\log_x \frac{1}{3^5} = 1$$

$$\Rightarrow \frac{1}{10}\log_x (3)^{-5} = 1 \Rightarrow \frac{1}{10} \times (-5) \times \log_x 3 = 1$$

$$\Rightarrow -\frac{1}{2}\log_x 3 = \log_x x \Rightarrow \log_x 3^{-\frac{1}{2}} = \log_x x$$

$$\Rightarrow \log_x \frac{1}{\sqrt{3}} = \log_x x$$

$$\therefore \; x = \frac{1}{\sqrt{3}}$$

(iv) $\log_4 32 = x - 4 \quad \Rightarrow \quad (4)^{x-4} = 32$

$$\Rightarrow \quad [(2)^2]^{x-4} = 2 \times 2 \times 2 \times 2 \times 2 \quad \Rightarrow \quad (2)^{2(x-4)} = (2)^5$$

$$\Rightarrow \quad (2)^{2x-8} = (2)^5 \quad \Rightarrow \quad 2x - 8 = 5 \quad \Rightarrow \quad 2x = 5 + 8$$

$$\Rightarrow \quad 2x = 13 \quad \Rightarrow \quad x = \frac{13}{2} = 6\frac{1}{2}$$

(v) $\log_7 (2x^2 - 1) = 2 \quad \Rightarrow \quad (7)^2 = 2x^2 - 1$

$$\Rightarrow \quad 49 = 2x^2 - 1 \quad \Rightarrow \quad 50 = 2x^2 \quad \Rightarrow \quad 2x^2 = 50$$

$$\Rightarrow \quad x^2 = \frac{50}{2} \quad \Rightarrow \quad x^2 = 25 \quad \Rightarrow \quad x^2 = \pm \sqrt{25}$$

$$\Rightarrow \quad x = +5, -5$$

(vi) $\log(x^2 - 21) = 2$

$\Rightarrow \quad (10)^2 = x^2 - 21 \quad \Rightarrow \quad 100 = x^2 - 21$

$\Rightarrow \quad x^2 - 21 = 100 \quad \Rightarrow \quad x^2 = 100 + 21$

$\Rightarrow \quad x^2 = 121 \quad \Rightarrow \quad x = \pm\sqrt{121} \quad \Rightarrow \quad x = \pm 11$

$\therefore \quad x = 11, -11$

(vii) $\log_6 (x-2)(x+3) = 1 = \log_6 6 \qquad \{\because \log_a a = 1\}$

Comparing,

$(x-2)(x+3) = 6$

$\Rightarrow x^2 + 3x - 2x - 6 = 6$

$\Rightarrow x^2 + x - 6 - 6 = 0$

$\Rightarrow x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x+4) - 3(x+4) = 0$

$\Rightarrow (x+4)(x-3) = 0$

Either $x + 4 = 0$, then $x = -4$

or $x - 3 = 0$, then $x = 3$

Hence $x = 3, -4$

(viii) $\log_6 (x-2) + \log_6 (x+3) = 1$

$\Rightarrow \log_6 (x-2)(x+3) = 1 = \log_6 6 \quad \{\because \log_a a = 1\}$

Comparing,

$(x-2)(x+3) = 6 \Rightarrow x^2 + 3x - 2x - 6 = 6$

$\Rightarrow x^2 + x - 6 - 6 = 0 \Rightarrow x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x+4) - 3(x+4) = 0$

$\Rightarrow (x+4)(x-3) = 0$

Either $x + 4 = 0$, then $x = -4$

or $x - 3 = 0$, then $x = 3$

$\therefore x = 3, -4$

(ix) $\log (x+1) + \log (x-1) = \log 11 + 2 \log 3$

$\Rightarrow \log [(x+1)(x-1] = \log 11 + \log (3)^2$

$\Rightarrow \log (x^2 - 1) = \log 11 + \log 9$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$\Rightarrow \log (x^2 - 1) = \log (11 \times 9) \Rightarrow x^2 - 1 = 11 \times 9$

$\Rightarrow x^2 - 1 = 99 \Rightarrow x^2 = 99 + 1 \Rightarrow x^2 = 100$

$\Rightarrow x^2 = (10)^2 \Rightarrow x = 10$

Question 8.

Solve for x and y:

$$\frac{\log x}{3} = \frac{\log y}{2} \text{ and } \log (xy) = 5$$

Solution:

$$\frac{\log x}{3} = \frac{\log y}{2}$$

$\Rightarrow \quad 2 \log x = 3 \log y$

$\Rightarrow \quad 2 \log x - 3 \log y = 0 \quad ...(i)$

and $\quad \log xy = 5$

$\Rightarrow \quad \log x + \log y = 5 \quad ...(ii)$

Multiply (ii) by 3 and (i) by 1,

$2 \log x - 3 \log y = 0$

$3 \log x + 3 \log y = 15$

Adding, $\quad 5 \log x = 15$

$$\Rightarrow \log x = \frac{15}{5} = 3 \Rightarrow \frac{1}{3}\log x = 1 = \log 10$$

$$\Rightarrow \log x^{\frac{1}{3}} = \log 10$$

$$\therefore \quad x^{\frac{1}{3}} = 10$$

$\Rightarrow x = 10^3 = 1000 \quad (\because \log 10 = 1)$

Hence $\quad x = 1000$

Substituting the value of $\log x = 3$ in (ii)

$3 + \log y = 5$

$$\Rightarrow \log y = 5 - 3 = 2 \quad \Rightarrow \quad \frac{1}{2}\log y = 1$$

$$\Rightarrow \quad \log y^{\frac{1}{2}} = \log 10 \quad (\because \log 10 = 1)$$

$$\therefore \quad y^{\frac{1}{2}} = 10$$

$\Rightarrow \quad y = (10)^2 = 100$

Hence $x = 1000$ and $y = 100$

Question 9.

If $a = 1 + \log_x yz$, $6 = 1 + \log_y zx$ and $c = 1 + \log_z xy$, then show that $ab + bc + ca = abc$.

Solution:

$$a = 1 + \log_x yz$$

$$b = 1 + \log_y zx$$

$$c = 1 + \log_z xy$$

$$a = 1 + \log_x yz = \log_x x + \log_x yz$$

$$\Rightarrow a = \log_x xyz \Rightarrow \frac{1}{a} = \log_{xyz} x$$

Similarly,

$$\frac{1}{b} = \log_{xyz} y \text{ and } \frac{1}{c} = \log_{xyz} z$$

Now $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \log_{xyz} x + \log_{xyz} y + \log_{xyz} z$

$$= \frac{\log x}{\log_{xyz}} + \frac{\log y}{\log_{xyz}} + \frac{\log z}{\log_{xyz}}$$

$$= \frac{\log x + \log y + \log z}{\log_{xyz}}$$

$$= \frac{\log xyz}{\log_{xz}} = 1$$

$$\Rightarrow \frac{bc + ca + ab}{abc} = 1 \Rightarrow ab + bc + ca = abc$$

Hence proved.