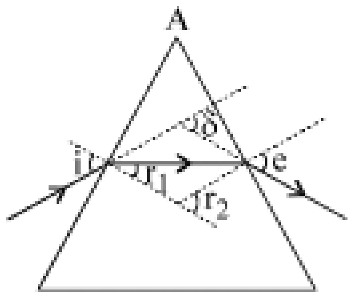
REFRACTION THROUGH PRISM

Prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other.



A prism deviates a light ray

1. Basic Terms

(i) Angle of prism or reflecting angle (A)

The angle between the faces on which light is incident and from which it emerges.

(ii) Angle of deviation (δ)

It is the angle between the emergent and the incident ray. It other words, it is the angle through which incident ray turns in passing through a prism.

$$\delta = (i - r_1) + (e - r_2)$$

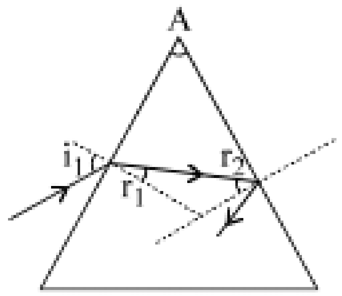
or $$\delta = i + e - (r_1 + r_2)$$

or $$\delta = i + e - A$$

2. Condition of no emergence

A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle.

i.e. $$\mu > \frac{1}{\sin\ (A/2)}$$

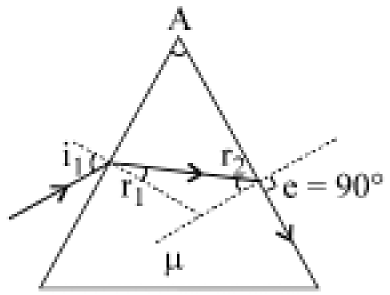


Condition of no emergence

3. Condition of grazing emergence

By the condition of grazing emergence we mean the angle of incidence i at which the angle of emergence becomes e = 90°.

$$i = \sin^{-1}\left[\sqrt{\mu^2 - 1}\ \sin A\ -\ \cos\ A\right]$$



Grazing emerging ray in a prism

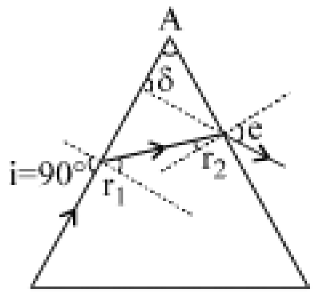
Note : That the light will emerge out of a given prism only if the angle of incidence is greater than the condition of grazing emergence.

4. Condition of maximum deviation

Maximum deviation occurs when the angle of incidence is 90°.

$$\delta_{max} = 90° + e - A$$

where $e = \sin^{-1}[\mu \sin(A - \theta_c)]$



Condition of maximum derivation

5. Condition of minimum deviation

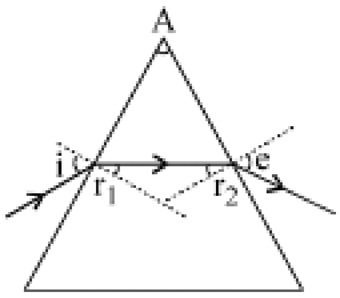
The minimum deviation occurs when the angle of incidence is equal to the angle of emergence, i.e

$i = e$

$\delta_{min} = 2i - A$

Using Snell's law

$$\mu = \frac{\sin\left[\frac{\delta_{min} + A}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$



Light ray passes through a prism symmetrically in the condition of the minimum deviation

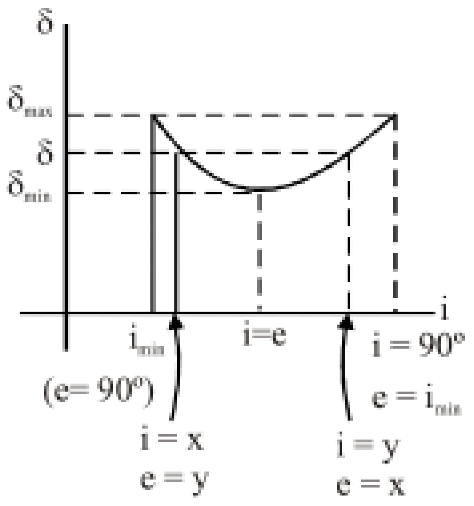
Note : That in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e. the light ray in the prism becomes parallel to its base.

Characteristic of a prism

(a) Variation of δ versus i (shown in diagram).

For one δ (except δ min) there are two values of angle of incidence.

If i and e are interchanged then we get the same value of δ because of reversibility principle of light.



(b) There is one and only one angle of incidence for which the angle of deviation is minimum.

(c) When $\delta = \delta_{min}$, the angle of minimum deviation, then $i = e$ and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that $\delta_{min} = 2i_{min} - A$ where i_{min} = angle of incidence for minimum deviation, and $r = A/2$

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n - 1)A$ (for small values of $\angle A$)

Illustration :

A prism with angle A = 60° produces a minimum deviation of 30°. Find the refractive index of the material.

Sol. *We know that*

$$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here $A = 60°,\ \delta_{min} = 30°$

$$\therefore \quad \mu = \frac{\sin\left(\frac{90+30}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

Thin prisms

In thin prisms the distance between the refracting surfaces is negligible and the angle of prism (A) is very small.

Since $A = r_1 = r_2$, therefore, if A is small then both r_1 and r_2 are also small, and the same is true for i_1 and i_2.

According to Snell's law $\sin i_1 = \mu \sin r_1$ or $i_1 = \mu r_1$

$\sin i_2 = \mu \sin r_2$ oqr $i_2 = \mu r_2$

Therefore, deviation $\delta = (i_1 - r_1) + (i_2 - r_2)$

$\delta = (r_1 + r_2) + (i_2 - r_2)$

$\delta = A(\mu - 1)$

Note : That the deviation for a small angled prism is independent of the angle of incidence.

Illustration :

A thin prism of angle A = 6° produces a deviation δ = 3°. Find the refractive index of the material of prism.

Sol. *We know that* $\delta = A(\mu - 1)$

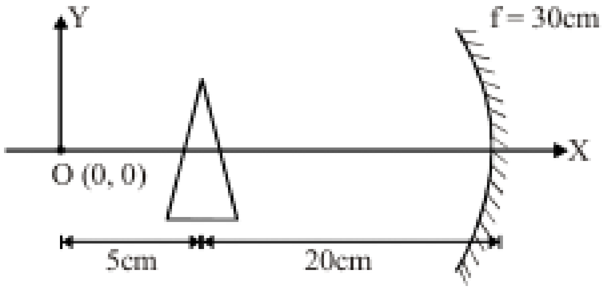
or $\mu = 1 + \frac{\delta}{A}$

Here $A = 6°,\ \delta = 3°$, *therefore*

$$\mu = 1 + \frac{3}{6} = 1.5$$

Illustration :

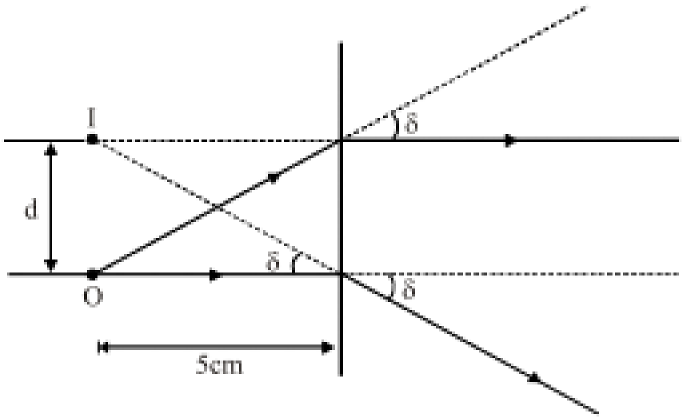
Find the co-ordinates of image of the point object 'O' formed after reflection from concave mirror as shown in figure assumming prism to be thin and small in size of prism angle 2°. Refractive index of prism materal is 3/2.



Sol. *Consider image formation through prism. All incident rays will be deviated by*

$$\delta = (\mu - 1)A = \left(\frac{3}{2} - 1\right)2° = 1° = \frac{\pi}{180} \text{ rad}$$

Now as prism is thin so object and imae will be in same plane as shown in figure.



It is clear $\frac{d}{5} = \tan\delta \approx \delta$ *($\therefore$ δ is very small)*

or $d = \frac{\pi}{36}$ *cm*

Now this image will act as an object for concave mirror.

$u = -25$ *cm,* $f = -30$ *cm*

$\therefore \quad v = \frac{uf}{u - f} = 150$ *cm*

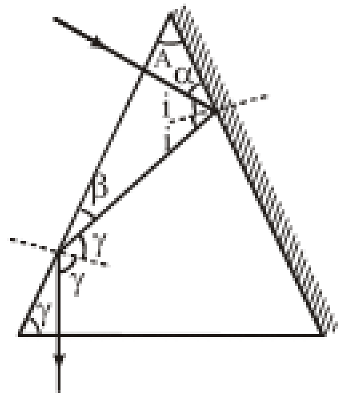
Also, $m = \frac{-v}{u} = +6$

$\therefore$ *Distance of image from principal axis* $= \frac{\pi}{36} \times 6 = \frac{\pi}{6}$ *cm*

Hence, co-ordinates of image formed after reflection from concave mirror are $\left(175\text{cm}, \frac{\pi}{6}\text{cm}\right)$

Illustration :

The cross-section of the glass prism has the form of an isosceles triangle. One of the equal faces is silvered. A ray of light incident normally on the other equal face and after being reflected twice, emerges through the base of prism along the normal. Find the angle of the prism.



Sol. *From the figure,*

$$\alpha = 90^\circ - A$$

$$i = 90^\circ - \alpha = A \qquad \ldots(i)$$

Also $\beta = 90^\circ - 2i = 90^\circ - 2A$

and $\gamma = 90^\circ - \beta = 2A$

Thus, $\gamma = r = 2A$

From geometry,

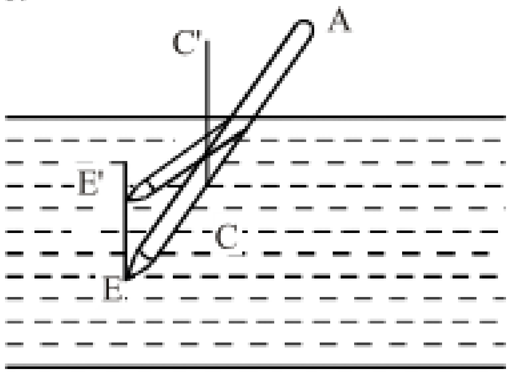
$$A + \gamma + \gamma = 180^\circ.$$

or $A = \frac{180}{5} = 36^\circ$.

Some interesting Facts related to refraction and total internal reflection

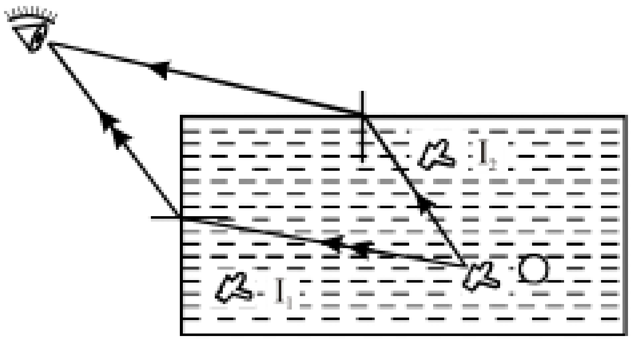
Bending of an object

When a point object in a denser medium is seen from a rarer medium it appears to be at a depth (d/μ). so if a linear object is dipped inclined to the surface of a liquid, (say water) actual depth will be different for its different points and so apparent depth. Due to this the object appears to be inclined from its actual position or BE as shown in figure.



Visibility of two images of an object

When an object is in a glass container and is seen from a level higher than that of liquied in the container as shown in figure , two images I_1 and I_2 of object O can be seen simultaneously-one due to refraction at the upper surface while the other at the side surface.



The sun is oval shaped at the time of rising and setting.

In the moring or eveing, the sunin at the horzon and refactive index in the atmosphere of the earth decreases with height. Due to this, light reaching earth's atmophere from different parts of vertical diameter of the sun enters at different heights in earth's atmophere and so travels in media of different refractive indices at the same instant and hence bends unequally.

Due to this unequal bending of light from vertical diameter, the image of the sun gest distorted and it apperars oval and larger. Howver at noon when the sun is overhead, then due to normal incidence there will be no bending and the sun will appear circular.

Similarly you can explain **Sun rises before it actually rises and sets after it actually sets.**

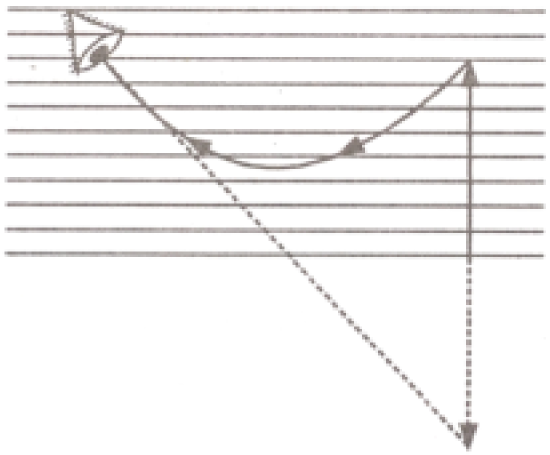
Stars twinkle.

Stars are self - luminous distant object, so only a few rays of light reach the eye through the atmosphere. However, due to fluctuations in refractive index of atmosphere the refraction becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars. If from moon or free space we look at a star this effect will not take place and star light will reach the eye continuously.

Tree appear inverted in deserts (mirage)

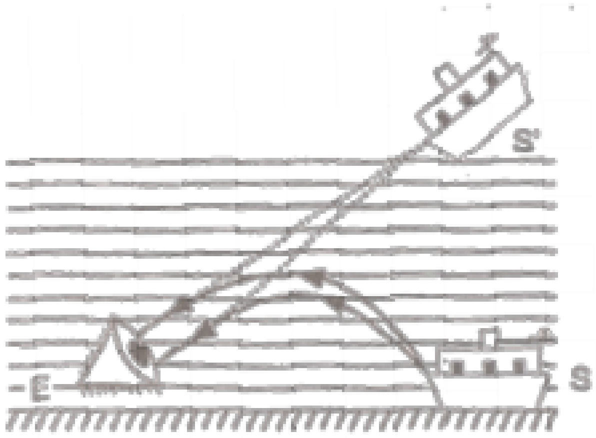
It is an optical illusion created due to the phenomenon of total internal reflection. This is seen in hot region. In hot areas like deserts surface of earth is very hot. So, air in the lower regions of atmosphere is hot as compared to that in higher regions. This result in variation of density with height and it increases as we go up. In this situation atmosphere can be assumed to be made of large number of thin layers of air.

A beam of light starting from an object say a tree and travelling downward finds itself going from denser to rater medium. Therefore, its angle of incidence at consecutive layers goes on increasing gradually till it surpasses the critical value and is reflected back due to total-internal reflection. A virtual image of the object is seen by eye at E. Due to the disturbance of air, the mirage is wavy in nature, thus giving an illusion for the presence of water which is actually not there. This effect is also called inferior mirage.



Ships appear above in the air in cold countries (looming)

This effect occurs when the density of air decreases much more rapidly with increasing height than it does under normal conditions. This situation sometimes happens in cold region particularly in the vicinity of the cold surface of sea or of a lake. Light rays starting from an object S (say a ship) are curved downward and on entering the eye the rays appear to come from S′, thus giving an impression that the ship is floating in air. This effect is also called superior mirage.

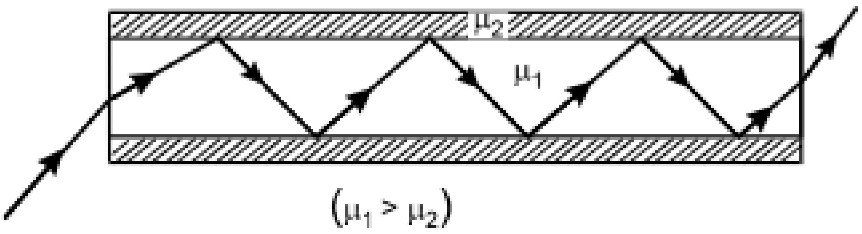


An eye placed inside water sees the external world within a cone and rest surface of water appears as a vast sheet of mirror. Radius is $r = \frac{h}{\sqrt{\mu^2 - 1}}$

Diamond and glass both shine if cut to the special shape, but diamond shine more than glass piece cut to same shape.

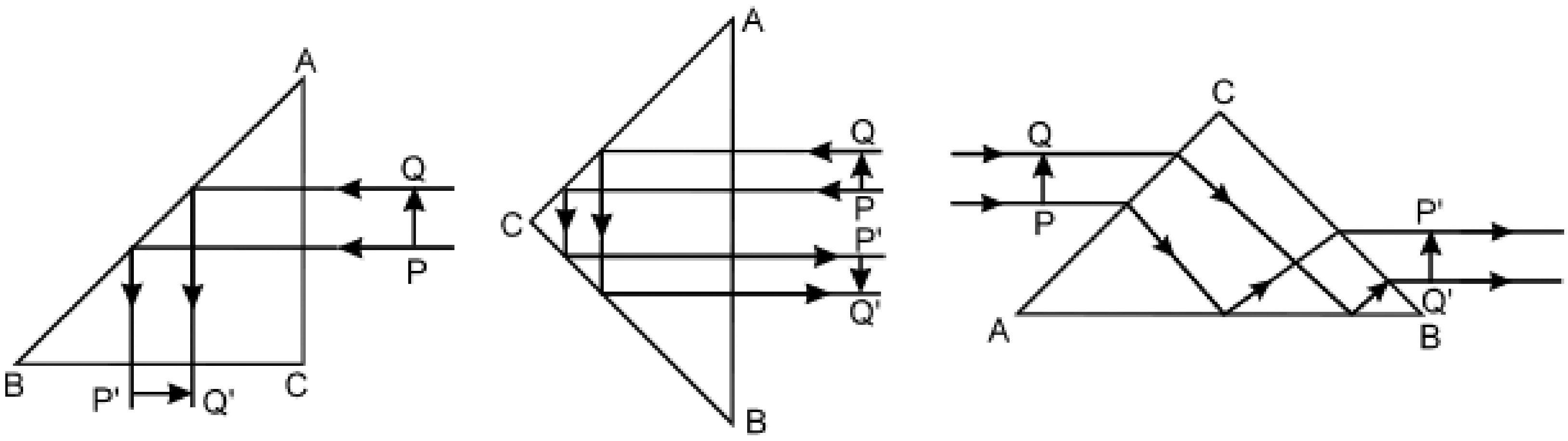
Optical fibre

In optical fibre a fine material of high refractive index is coated with a material of relatively low refractive index. When a light is incident in it, it suffers a no. of total internal reflection and comes out of it. It is used as light pipe.



$(\mu_1 > \mu_2)$

reflecting prisms.



Dispersion of Light

When a beam of light (containing several wavelengths) falls on one face of a prism, it splits into its constituent colours. This phenomenon of splitting of light into its constituent colours is called dispersion of light and the band of colours obtained on a screen is called spectrum. The cause of dispersion is variation of refractive index with wavelength of light. An approximate empirical relation as proposed by Cauchy is given by

$$\mu(\lambda) = A + \frac{B}{\lambda^2}$$

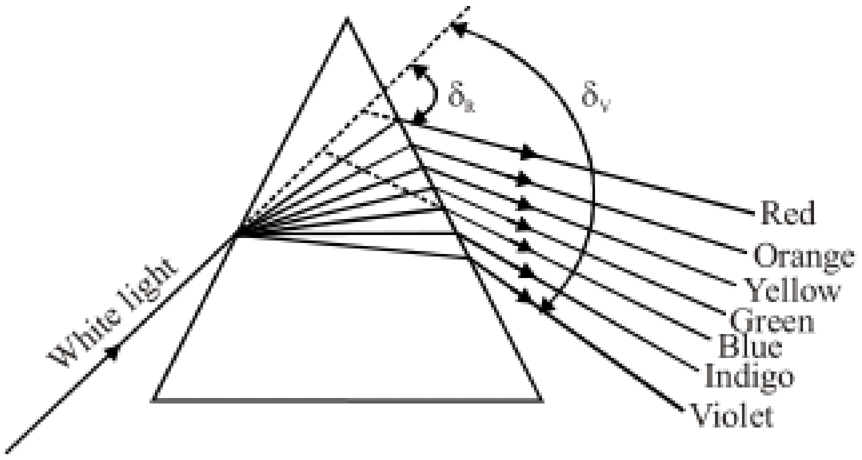
where A and B are known as Cauchy's constant. The value of A and B depends on material of prism.

We know that $\lambda_{Red} > \lambda_{Violet}$

$\therefore$ $\mu_{Red} < \mu_{Violet}$

Hence, $\delta_{Red} < \delta_{Violet}$

The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for these colours. The angle between the violet and red colours is known as angular dispersion. We know that for small angle of prism, deviation is given by



$\delta = (\mu - 1)A$

$\therefore$ δ_V = Deviation in violet colour $= (\mu_V - 1)A$

δ_R = Deviation in red colour $= (\mu_R - 1)A$

Hence, Angular Dispersion (A.D) $= \delta_V - \delta_R$

$= (\mu_V - \mu_R)A$

It is clear from above relation that angular dispersion depends upon (i) the nature of material of the prism and (ii) the angle of the prism. This is also defined as the rate of change of angle of deviation with wavelngth i.e., A.D. $= \frac{d\delta}{d\lambda}$.

Dispersive power of a prism is defined as the ratio between angular dispersion to mean deviation produced by the prism.

ω = Dispersive power

$$= \frac{\delta_V - \delta_R}{\delta_Y} = \frac{\mu_V - \mu_R}{\delta_Y - 1} = \frac{d\mu}{\mu_Y - 1}$$

Where $d\mu$ denotes the difference between the refractive indices of material of prism for violet and red light. It is also defined as dispersion per unit deviation. Yellow colour is taken as mean colour.

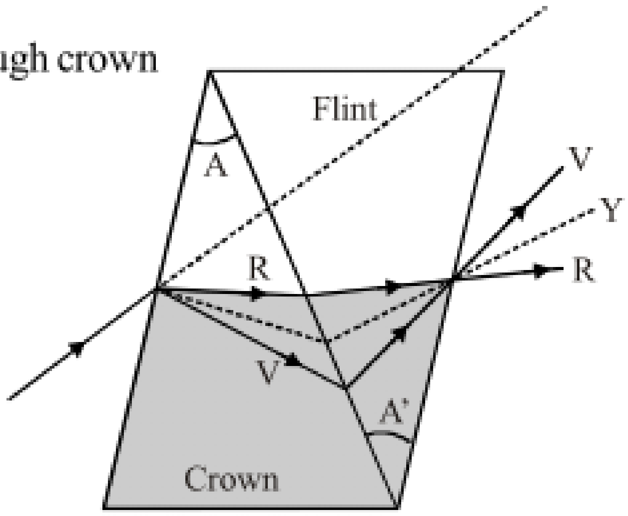
Also, $\mu_Y = \frac{\mu_V + \mu_R}{2}$ or $\frac{\mu_B + \mu_R}{2}$

Dispersion without Deviation

Let us consider a crown glass prism combined with a flint glass prism in position as shown in figure. Let A and A' be the angles of crown glass prism and flint glass prism respectively. Let μ_0, μ and μ_r be the refractive indices of the crown glass for violet, yellow and red colours respectively.

Let μ_0', μ' and μ_r' be the corresponding values for the flint glass prism.

Let δ and δ' be the deviations suffered by yellow light through crown glass prism and flint glass prism respectively.



If the combination does not produce any deviation, then

$$\delta + \delta' = 0$$

or $$(\mu - 1)A + (\mu' - 1)A' = 0$$

or $$(\mu' - 1)A' = -(\mu - 1)A$$

or $$A' = -\left(\frac{\mu - 1}{\mu' - 1}\right)A$$

This is the condition for no devition. The negative sign indicates that the two prisms are to be placed in position as shown in figure.

Net angular dispersion $= [(d_v - d_r) + (d'_v - d'_r)] = (\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A'$

$$= A\left[(\mu_v - \mu_r) + (\mu_v' - \mu_r')\frac{A'}{A}\right]$$

$$= A\left[(\mu_v - \mu_r) + \left(\frac{\mu - 1}{\mu' - 1}\right)(\mu_v' - \mu_r')\right]$$

$$= (\mu - 1)A\left[\frac{\mu_v - \mu_r}{\mu - 1} - \frac{\mu_v' - \mu_r'}{\mu' - 1}\right] = \delta(\omega - \omega')$$

Here ω and ω' are the dispersive powers of crown glass and flint glass respectively. As the dispersive powers ω and ω' are not equal, in such a combination there will be resultant dispersion and the final dispersed beam is parallel to the incident beam.

Since $\omega' > \omega$ therefore the net angular dispersion is negative. This explains why the order of colours in the spectrum due to combination is opposite to that in the crown glass prism.

Deviation without dispersion :

For the combination of prism shown in figure, if there is to be no angular dispersion, then

$$(\delta_v - \delta_r) + (\delta_v' - \delta_r') = 0$$

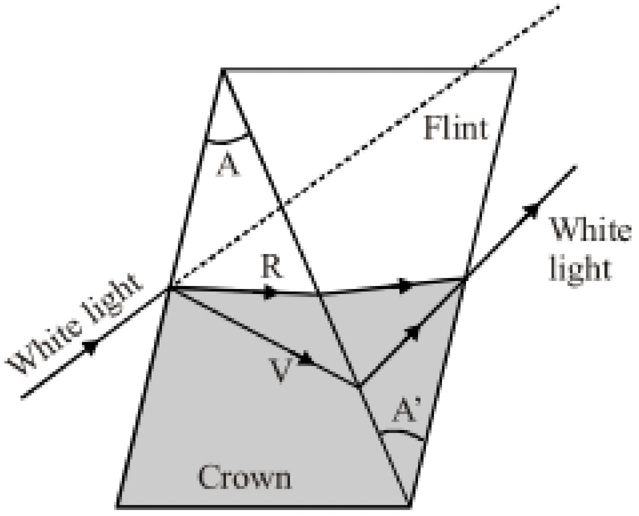
or $$(\mu_v - \mu_r)A + (\mu_v' - \mu_r')A' = 0$$

or $$(\mu_v' - \mu_r')A' = -(\mu_v - \mu_r)A$$

or $$A' = -\left(\frac{\mu_v - \mu_r}{\mu_v' - \mu_r'}\right)A$$

This is the condition for achromatism i.e., the condition for no dispersion. This condition can be written in another form as given below.

From equation (1),



$$(\mu - 1)A\,\frac{\mu_v - \mu_r}{\mu - 1} + (\mu' - 1)A\,\frac{\mu_v' - \mu_r'}{\mu' - 1} = 0$$

or $$\delta\omega + \delta'\omega' = 0$$

or $$\frac{\omega}{\omega'} = -\frac{\delta'}{\delta}$$

Since, $\omega' > \omega$, $\quad \therefore \delta > \delta'$

or $(\mu - 1)A > (\mu' - 1)A'$

But, $(\mu - 1) < (\mu' - 1)$ $\quad \therefore A > A'$

So, the crown glass prism should have a larger angle than the flint glass prism.

Net deviation $= \delta + \delta'$

$$= (\mu - 1)A + (\mu' - 1)A' = (\mu - 1)A\left[1 + \frac{(\mu'-1)A'}{(\mu-1)A}\right]$$

$$= (\mu - 1)A\left[1 - \frac{\mu'-1}{\mu-1}\,\frac{\mu_v - \mu_r}{\mu_v' - \mu_r'}\right] \qquad \text{(Using equation (2))}$$

$$= (\mu - 1)A\left[1 - \frac{\mu_v - \mu_r}{\mu - 1} \times \frac{\mu_v - \mu_r}{\mu - 1} \times \frac{\mu'-1}{\mu_v' - \mu_r'}\right] = \delta\left(1 - \frac{\omega}{\omega'}\right)$$

Since $\omega' > \omega$, therefore the net deviation is in the direction of the deviation produced by crown glass prism.

Remark :

A parallel sided glass slab can be looked upon as the combination of two prisms producing no deviation and no dispersion.

Practice Exercise

Q.1 A ray of light undergoes deviation of 30^o when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism?

Q.2 A thin prism P_1 with angle 4^o and made from glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. What is the angle of the prism P_2?

Q.3 A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism.

Q.4 A glass prism of angle 72° and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. Find the angle of minimum deviation for a parallel beam of incident light passing through the prism.

Answers

Q.1 0 Q.2 3° Q.3 $\sqrt{3}$ Q.4 22.37°