Chapter 3

HCF AND LCM

KEY FACTS

1. The HCF of two or more numbers is the greatest number that divides each of them exactly.
2. **Methods of finding HCF**

   (*i*) **Prime Factorization :** Express each of the given numbers as the product of their prime factors. The HCF of the given numbers is the product of the least powers of common factors.

   $24 = 2^3 \times 3, 32 = 2^5 \Rightarrow$ HCF $(24, 32) = 2^3 = 8$.

   (*ii*) **Continued Division Method :** Divide the larger number by the smaller number. If the remainder is zero, the divisor is the HCF, otherwise divide the previous divisor by the remainder last obtained. Repeat this until the remainder becomes zero.

   To find the HCF of more than two numbers, first find the HCF of any two numbers and then find the HCF of the result and the third number and so on. The final HCF is the required HCF.

   Note : The HCF of two co-prime numbers is 1, as they have no factors in common.

3. The LCM of two or more numbers is the least number that is divisible by all these numbers.
4. **Methods of finding LCM**

   (*i*) **Prime Factorization Method :** Express each number as a product of prime factors. The LCM of the given numbers is the product of the greatest powers of these prime factors.

   $64 = 2^6, 56 = 2^3 \times 7 \Rightarrow$ LCM $(64, 56) = 2^6 \times 7 = 448$

   (*ii*) **Common Division Method** : Arrange the given numbers in a row in any order. Now divide by a prime number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the process till no numbers have a common factor other than 1. The product of the divisors and the remaining numbers is the LCM of the given numbers.

   Note: The LCM of two co-prime numbers is equal to their product.

5. The product of the HCF and LCM of two numbers is equal to the product of the numbers.
6. The HCF always completely divides the LCM for a given set of numbers.
7. HCF of given fractions $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

   LCM of given fractions $= \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

8. To find the HCF and LCM of decimal numbers, convert the given numbers to like decimals. Now find HCF and LCM of the numbers treating them as whole numbers (ignoring the decimal point). The number of decimal places in the answer are eaqual to the number of decimal places in the like decimals. Accordingly, put the decimal point in the answer.

Some Important Results

9. The greatest number that will divide x, y and z leaving remainders a, b and c respectively is given by the HCF of $(x - a)$, $(y - b)$, $(z - c)$.
10. The greatest number that will divide x, y and z leaving the same remainder in each case is given by HCF of $(x - y)$, $(y - z)$, $(z - x)$.
11. The least number which when divided by x, y and z leaves the same remainder R in each case is given by LCM of $(x, y, z) + R$.
12. The least number which when divided by x, y and z leaves the remainders a, b and c respectively is given by LCM of $(x, y, z) - p$ where $p = (x - a) = (y - b) = (z - c)$.

Solved Examples

Ex. 1. ***What is the least number which when divided by 15, 18 and 21 leaves remainders 2, 5 and 8 respectively ?***

Sol. Since the difference between the divisors and the respective remainders is same, i.e.,
$15 - 2 = 18 - 5 = 21 - 8 = 13$,
The required number = LCM of (15, 18, 21) – 13 = $(3 \times 5 \times 6 \times 7) - 13 = 630 - 13$ = **617.**

3	15, 18, 21
	5, 6, 7

Ex. 2. ***There are three numbers. The HCF of each pair is 15 and the LCM of all the three numbers is 1890. What is the product of the numbers ?***

Sol. Since the HCF of each pair of numbers is 15, the HCF of the three numbers is 15.
∴ Product of the numbers = HCF × LCM
= 15×1890 = **28350.**

Ex. 3. ***What is the least number which when divided by 4, 6, 8 and 9 leaves zero remainder in each case but when divided by 13 leaves a remainder of 7 ?***

Sol. LCM of 4, 6, 8 and 9 = 72
Dividing 72 by 13,

$$\begin{array}{r} 5 \\ 13\overline{)\ 72} \\ -65 \\ \hline 7 \end{array}$$

2	4, 6, 8, 9
2	2, 3, 4, 9
3	1, 3, 2, 9
	1, 1, 2, 3

∴ 72 is the required number.

Ex. 4. ***Find the least number which when divided by 12, 16, 18, 30 leaves remainder 4 in each case but it is completely divisible by 7 ?***

Sol. LCM of (12, 16, 18, 20) = 720

2	12, 16, 18, 30
2	6, 8, 9, 15
3	3, 4, 9, 15
	1, 4, 3, 5

∴ The required number is of the form $720k + 4$
Checking for values of k = 1, 2, 3,...... we see that the least value of k for which $720k + 4$ is divisible by 7 is $k = 4$,
∴ Required number = $720 \times 9 + 4$ = **2884.**

Ex. 5. ***The LCM and HCF of two positive numbers are 175 and 5 respectively. If the sum of the numbers is 60, what is the difference between them ?***

Sol. Let the two numbers be $5a$ and $5b$ as HCF of the two numbers = 5

$\Rightarrow$ Product of the two number = HCF × LCM

$\Rightarrow 5a \times 5b = 5 \times 175 \quad \Rightarrow \quad ab = \frac{175}{5} = 35$

$\therefore$ (a, b) can be (1, 35) or (5, 7)

Thus, the numbers can be (1 × 5 and 35 × 5) or (5 × 5 and 5 × 7), i.e., (5 and 175) or (25 and 35) The sum = 60 is satisfied by the pair (25, 35). Hence, the difference of the numbers is 10.

Question Bank–3

1. Find the number of pairs of natural numbers with LCM as 56.
(a) 3 (b) 4
(c) 10 (d) Can't be determined

2. A General can draw up his soldiers in the rows of 10, 15 and 18 soldiers and he can also draw them up in the form of a solid square. Find the least number of soldiers with the general.
(a) 100 (b) 3600
(c) 900 (d) 90

3. The circumferences of the fore and hind wheels of a carriage are $6\frac{3}{14}$ m and $8\frac{1}{18}$ m respectively. At any given moment, a chalk mark is put on the point of contact of each wheel with the ground. Find the distance travelled by the carriage so that both the chalk marks are again on the ground at the same time.
(a) 218 m (b) 217.5 m
(c) 218.25 m (d) 217 m

4. The LCM of two numbers is 28 times of their HCF. The sum of their LCM and HCF is 1740. If one of the numbers is 240, find the other number.
(a) 240 (b) 620
(c) 540 (d) 420

5. Find the two largest numbers of four digits having 531 as their HCF.
(a) 9231, 9762 (b) 9027, 9558
(c) 9037, 9568 (d) 9127, 9658

6. Find the greatest number of five digits which become exactly divisible by 10, 12, 15 and 18 when 3769 is added to it.
(a) 99819 (b) 99911
(c) 99900 (d) 99111

7. Two numbers both greater than 29 have HCF = 29 and LCM = 4147. The sum of the numbers is
(a) 666 (b) 669
(c) 696 (d) 966

8. The HCF of two numbers each consisting of 4 digits is 103 and their LCM is 19261. The numbers are
(a) 1133, 1751 (b) 1053, 1657
(c) 1061, 1111 (d) 1591, 1377

9. Four prime numbers are written in ascending order of their magnitudes. The product of the first three is 715 and that of the last three is 2431. What is the largest given prime number ?
(a) 5 (b) 19
(c) 17 (d) 23

10. A number lying between 1000 and 2000 is such that on division by 2, 3, 4, 5, 6, 7 and 8 leaves remainders 1, 2, 3, 4, 5, 6 and 7 respectively. The number is
(a) 518 (b) 416
(c) 364 (d) 1679

11. Find the greatest number of five digits which when divided by 4, 6, 14 and 20 leaves respectively 1, 3, 11 and 17 as remainders.
(a) 99930 (b) 99960
(c) 99997 (d) 99957

12. Find the least number which when divided by 12, 24, 36 and 40 leaves a remainder 1, but when divided by 7 leaves no remainder.
(a) 361 (b) 1080
(c) 721 (d) 371

13. What is the least number which when divided by the numbers 3, 5, 6, 8, 10 and 12 leaves in each case a remainder 2, but when divided by 13 leaves no remainder.

(a) 312 (b) 962
(c) 1562 (d) 1586

14. A heap of stones can be made up into groups of 21. When made up into groups of 16, 20, 25 and 45, there are 3 stones left in each case. How many stones at least can there be in the heap ?
(a) 7203 (b) 2403
(c) 3603 (d) 4803

15. Find the least number which when divided by 2, 3, 4, 5 and 6 leaves 1, 2, 3, 4 and 5 as remainders respectively, but when divided by 7 leaves no remainder.
(a) 210 (b) 119
(c) 126 (d) 154

16. The HCF and LCM of two numbers are 12 and 72 respectively. If the sum of the two numbers is 60, then one of the numbers will be
(a) 12 (b) 24
(c) 60 (d) 72

17. The difference of two numbers is 20 and their product is 56.25 times their difference. Find the LCM of the numbers.
(a) 70 (b) 1125
(c) 225 (d) 5

18. There are 4 numbers, The HCF of each pair is 7 and the LCM of all the numbers is 1470. What is the product of the 4 numbers ?
(a) 504210 (b) 502410
(c) 504120 (d) 501420

19. Two persons *A* and *B* walk round a circle whose diameter is 1.4 km. *A* walks at a speed of 165 metres per munute while *B* walks at a speed of 110 metres per minute. If they both start at the same time from the same point and walk in the same direction at what interval of time would they both be at the same starting point again.

(a) 1 h (b) $1\frac{1}{3}$ h
(c) $1\frac{2}{3}$ h (d) $1\frac{1}{2}$ h

20. The LCM of two numbers is 495 and their HCF is 5. If the sum of the numbers is 100, then their difference is
(a) 10 (b) 46
(c) 70 (d) 90

Answers

1. (c)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (d)
11. (d)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)

Hints and Solutions

1. (c) Let *a*, *b* be the two numbers with LCM 56. Since 56 is the LCM *a* and *b* will be the divisors of 56. The divisors of 56 are 1, 2, 4, 7, 8, 14, 28 and 56. Combining two at a time, the possible pairs with LCM as 56 are (1, 56), (2, 56), (4, 56), (7, 56), (8, 56), (14, 56), (28, 56), (8, 14), (8, 28) and (7, 8). Thus, in all there are 10 pairs.

2. (c) LCM of (10, 15, 18) = 2 × 3 × 5 × 3 = 90

2	10,	15,	18
3	5,	15,	9
5	5,	5,	3
	1,	1,	3

∴ To draw the soldier in the form of a solid square the number of soldiers = 90*k*, where *k* is a natural number. The least value of *k* for which 90 *k* is a square number is *k* = 10, i.e, the required number of soldiers = 90 × 10 = **900**.

3. (b) Required distance travelled

$$= \text{LCM of } \left(6\frac{3}{14}\text{ m}, 8\frac{1}{18}\text{ m}\right)$$

$$\boxed{\begin{array}{l}87 = 3\times 29\\ 145 = 5\times 29\end{array}} = \text{LCM of } \left(\frac{87}{14}, \frac{145}{18}\right)$$

$$\boxed{\begin{array}{l}14 = 2\times 7\\ 18 = 2\times 9\end{array}} \quad = \frac{\text{LCM of }(87,145)}{\text{HCF of }(14,18)}$$

$$= \frac{3\times 5\times 29}{2} = \frac{435}{2} = \mathbf{217.5\ m.}$$

4. (d) LCM = 28 HCF
Also, LCM + HCF = 1740
$\Rightarrow$ 28 HCF + HCF = 1740
$\Rightarrow$ 29 HCF = 1740 $\Rightarrow$ HCF = $\frac{1740}{29} = 60$
$\Rightarrow$ LCM = 28 × 60 = 1680
Since, one number = 240

$$\therefore \text{ Other number} = \frac{\text{HCF}\times\text{LCM}}{\text{One number}}$$

$$= \frac{60\times 1680}{240} = \mathbf{420}$$

5. (b) The largest number of four digits = 9999

Dividing 9999 by 531, we get

$$\begin{array}{r} 18 \\ 531\overline{)9999} \\ -531 \\ \hline 4689 \\ -4248 \\ \hline 441 \end{array}$$

∴ Greatest number of four digits divisible by 531

= 9999 – 441 = **9558**

The other number = 9558 – 531 = **9027.**

6. (b) LCM of (10, 12, 15 and 18)

= 2 × 3 × 5 × 2 ×3 = 180

2	10, 12, 15, 18
3	5, 6, 15, 9
5	5, 2, 5, 3
	1, 2, 1, 3

The greatest five digit number = 99999

Dividing (99999 + 3769) = 103768 by 180 , we get

$$\begin{array}{r} 180)\overline{103768}(576 \\ -900 \\ \hline 1376 \\ -1260 \\ \hline 1168 \\ -1080 \\ \hline 88 \end{array}$$

Remainder = 88

∴ Required number = 99999 – 88 = **99911.**

7. (c) Since the HCF of both the numbers is 29, let the numbers be 29*a* and 29*b*, where *a* and *b* are co-prime to each other.

Given, 29*a* × 29*b* = 29 × 4147 ⇒ *ab* = 143

The co-prime pairs (*a*, *b*) whose product = 143 are (1, 143) and (11, 13)

Since both the required numbers are greater than 29, the co-prime pair satisfying the condition is (11, 13).

Therefore the numbers are 29 × 11 and 29 × 13, i.e., 319 and 377.

∴ Sum of the numbers = 319 + 377 = **696.**

8. (a) Similar to Q. No. 7.

9. (c) Let the four prime numbers in the ascending order of their magnitudes be *a*, *b*, *c* and *d*.

Given, *a* × *b* × *c* = 715 and *b* × *c* × *d* = 2431

HCF of (*abc* and *bcd*) = *bc*

∴ Now, 715 = 11 × 13 × 5 and

2431 = 11 × 13 × 17

⇒ HCF of 715 and 2431 = 11 × 13 = 143

∴ Largest number (*d*) = $\frac{bcd}{bc} = \frac{2431}{143} = \mathbf{17}$.

10. (d) The common difference between the divisors and the respective remainders = (2 – 1) (3 – 2) = (4 – 3) = ... = (8 – 7) = 1

LCM of 2, 3, 4, 5, 6, 7 and 8 = 840

∴ Required number = 840 *k* – 1

Since the number lies between 1000 and 2000, *k* = 2.

∴ Required number = 840 × 2 – 1

= 1680 – 1 = **1679.**

11. (d) Common difference between divisors and respective remainders

= (4 – 1) = (6 – 3) = (14 – 11) = (20 – 17) = 3

LCM of (4, 6, 14, 20) = 2 × 2 × 3 × 7 × 5

= 420.

2	4, 6, 14, 20
2	2, 3, 7, 10
	1, 3, 7, 5

Greatest number of five digits = 99999

Dividing 99999 by 420 and subtracting the remainder 39 from 99999, we get 99999 – 39 = 99960.

$$\begin{array}{r} 420)\overline{99999}(238 \\ -840 \\ \hline 1599 \\ -1260 \\ \hline 3399 \\ -3360 \\ \hline 39 \end{array}$$

∴ The required number = 99960 – 3

= **99957.**

12. (c) LCM of 12, 24, 36 and 40 = 360

Any number which when divided by 12, 24, 36 and 40 leaving a remainder 1 is of the form 360*k* + 1. Now, we have to find the least value of *k* for which 360*k* + 1 is divisible by 7.

$$\begin{array}{r} 51k \\ 7\overline{)360k+1} \\ -357k \\ \hline 3k+1 \end{array}$$

By inspection we find that for *k* = 2, 3 × 2 + 1 = 7

∴ Required number = 360 × 2 + 1 = 721.

13. (b) Similar to Q. No. 12.

14. (a) Number of stones
= LCM of (16, 20, 25, 45) × k + 3
= 3600 k + 3
Since the stones can be made up into groups of 21.

$$21\ )\ \overline{3600k + 3}\ (171k \quad -3591k \quad \overline{9k+3}$$

∴ $9k + 3$ is divisible by 21 when $k = 2$
∴ Least number of stones = 3600 × 2 + 3 = **7203.**

15. (b) Common difference between divisors and respective remainders
= (2 – 1) = (3 – 2) = (4 – 3)
= (5 – 4) = (6 – 5) = 1
LCM of (2, 3, 4, 5, 6) = 60
∴ Required number = 60 k – 1
Now we have to find the least value of k for which $60k – 1$ is divisible by 7.
By inspection, we find that for $k = 2$, 4 × 2 – 1 = 7

$$7)\ 60k-1\ (8k \quad -56k \quad \overline{4k-1}$$

∴ Required number = 60 × 2 – 1 = 120 – 1 = 119.

16. (b) Given, HCF = 12, LCM = 72
One number = x, othe number = 60 – x
∴ Product of the two numbers = HCF × LCM
⇒ $x(60 – x) = 12 × 72$
⇒ $x^2 – 60x + 864 = 0$
⇒ $x^2 – 36x – 24x + 864 = 0$
⇒ $x(x – 36) – 24(x – 36) = 0$
⇒ $(x – 36)(x – 24) = 0$ ⇒ x = **36** or **24.**
∴ One of the number is 24.

17. (c) Let the numbers be a and b. Then,
$a – b = 20$ .... (i)
$ab = 56.25 × 20 = 1125$

$$\Rightarrow b = \frac{1125}{a}$$

∴ Putting the value of b in (i) we get

$$a - \frac{1125}{a} = 20 \Rightarrow a^2 - 1125 = 20a$$

⇒ $a^2 – 20a – 1125 = 0$
⇒ $a^2 – 45a + 25a – 1125 = 0$
⇒ $a(a – 45) + 25(a – 45) = 0$
⇒ $(a – 45)(a + 25) = 0$
⇒ $a = 45$ or $–25$

Neglecting the negative value of a, $b = 45 – 20 = 25$
Now, $a = 45 = 3^2 × 5$, $b = 25 = 5^2$
∴ LCM of (45, 25) = $3^2 × 5^2$ = 225

18. (a) Since the HCF of each pair = 7, let the four numbers be $7a$, $7b$, $7c$, $7d$.
Also, LCM = $abcd$ × HCF

$$\Rightarrow abcd = \frac{1470}{7} = 210$$

∴ Product of the numbers
= $7a × 7b × 7c × 7d$
= $7^4 × abcd = 7^4 × 210$ = **504210.**

19. (b) Diameter of the circle = 1.4 cm
⇒ Circumference = $2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{1.4}{2} \text{km} = 4.4 \text{ km}$$

A's speed = 165 m/min
∴ Time taken by A to travel 4.4 km (4400 m)

$$= \frac{4400}{165} \text{min} = \frac{80}{3} \text{min}$$

B's speed = 110 m/ min
∴ Time taken by B's to travel 4.4 km (4400 m)

$$= \frac{4400}{110} \text{min} = 40 \text{ min}$$

Required interval of time = LCM of $\left(\frac{80}{3}, 40\right)$

$$= \frac{\text{LCM}(80, 40)}{\text{HCF}(3, 1)} = \frac{80}{1} \text{min}$$

$$= \frac{80}{60} \text{hrs} = \mathbf{1\frac{1}{3}\ hrs}$$

20. (a) Let one number = x. Then,
Other number = 100 – x
LCM = 495, HCF = 5
∴ $x(100 – x) = 495 × 5$
⇒ $100x – x^2 = 2475$
⇒ $x^2 – 100x + 2475 = 0$
⇒ $x^2 – 45x – 55x + 2475 = 0$
⇒ $x(x – 45) – 55(x – 45) = 0$
⇒ $(x – 45)(x – 55) = 0$
⇒ $x = 45$ or 55.
∴ The numbers and 45, 100 – 45 = 55 or 55, 100 – 55 = 45
Required difference = 55 – 45 = **10.**

Self Assessment Sheet–3

1. What is the greatest number of 4-digits that, which when divided by any of the numbers 6, 9, 12 and 17 leaves a remainder of 1?
   (a) 9997 (b) 9793
   (c) 9895 (d) 9487
2. Find the greatest number that will divide 55, 127 and 175, so as to leave the same remainder in each case.
   (a) 1 (b) 16
   (c) 24 (d) 15
3. The sum of two numbers is 1215 and their HCF is 81. How many pairs of such numbers can be formed?
   (a) 2 (b) 6
   (c) 4 (d) None
4. How many numbers between 200 and 600 are exactly divisible by 4, 5 and 6?
   (a) 5 (b) 16
   (c) 10 (d) 6
5. The HCF of two numbers of same number of digits is 45 and their LCM is 540. The numbers are
   (a) 270, 540 (b) 135, 270
   (c) 180, 270 (d) 135, 180
6. Find the least number which on being divided by 5, 6, 8, 9, 12 leaves in each case a remainder 1 but when divided by 13 leaves no remainder?
   (a) 3601 (b) 1469
   (c) 2091 (d) 4879
7. If $x = 103$, then the LCM of $x^2 - 4$ and $x^2 - 5x + 6$ is
   (a) 105105 (b) 1051050
   (c) 106050 (d) 1060500
8. The LCM of two numbers is 12 times their HCF. The sum of HCF and LCM is 403. If one of the numbers is 93, then the other number is
   (a) 124 (b) 128
   (c) 134 (d) 138
9. The HCF and LCM of two numbers x and y is 6 and 210 respectively. If $x + y = 72$, which of the following relation is correct?
   (a) $\frac{1}{x} + \frac{1}{y} = \frac{3}{35}$ (b) $\frac{1}{x} + \frac{1}{y} = \frac{2}{35}$
   (c) $\frac{1}{x} + \frac{1}{y} = \frac{35}{2}$ (d) not sufficient
10. If the HCF of $x^2 - x - 6$ and $x^2 + 9x + 14$ is $(x + m)$, then the value of m is:
    (a) 1 (b) 2
    (c) – 2 (d) – 1

Answers

1. (b)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (b)