[SINGLE CORRECT CHOICE TYPE]

1. For all pairs of angles (A, B), measured in degrees such that $\sin A + \sin B = \sqrt{2}$ and $\cos A + \cos B = \sqrt{\sqrt{2}}$, both hold simultaneously. The smallest possible value of $|A - B|$ in degrees is

(A) 15 (B) 30 (C) 45 (D) 60

2. Which one of the following trigonometric statement does not hold good ?

(A) $\tan\left(\frac{\pi}{4}+x\right) = \cot\left(\frac{\pi}{4}-x\right)$ (B) $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$

(C) $\tan\left(\frac{\pi}{4}+x\right) = \sec 2x + \tan 2x$ (D) $\tan\left(\frac{\pi}{4}+x\right) = \frac{\cos 2x}{1+\sin 2x}$

3. $\tan(x + y) = a$ $(a \neq 0)$ & $\tan(x - y) = b$ $(b \neq 0)$ such that $\tan(x + y) + \tan(x - y) = \tan 2x$ then

(A) $a^2 + b = 0$ (B) $a + b = 0$ (C) $ab = 1$ (D) $a - b = 0$

4. Suppose $\sin\theta - \cos\theta = 1$ then the value of $\sin^3\theta - \cos^3\theta$ is $(\theta \in R)$

(A) 1 (B) – 2 (C) – 1 (D) 0

5. If $90^0 < \alpha < 180^0$ and $0 < \beta < 90^0$ such that $\sin\alpha = \frac{4}{5}$, $\cos\beta = \frac{5}{13}$ and $\tan\left(\frac{\alpha+\beta}{2}\right) = p$, $\tan\left(\frac{\alpha-\beta}{2}\right) = q$, then $\frac{p}{q}$ is equal to

(A) 14 (B) –14 (C) 16 (D) –16

6. If $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -\frac{3}{2}$, $\cos\alpha + \cos\beta + \cos\gamma = p$ & $\sin\alpha + \sin\beta + \sin\gamma = q$, then

(A) $p = q = 0$ (B) $p + q = 1$ (C) $p = q = 1$ (D) $p - q = 1$

[MULTIPLE CORRECT CHOICE TYPE]

7. If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then

(A) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{(a^2+b^2)}$ (B) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{(a^2-b^2)}$

(C) $\tan\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$ (D) $\cos(\theta - \phi) = \frac{a^2+b^2-2}{2}$

8. If $\cos\alpha = \frac{3}{5}$ and $\cos\beta = \frac{5}{13}$, (where $\alpha, \beta, \in$ 1st quadrant)

(A) $\cos(\alpha + \beta) = \frac{33}{65}$ (B) $\sin(\alpha + \beta) = \frac{56}{65}$

(C) $\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{65}$ (D) $\cos(\alpha - \beta) = \frac{63}{65}$

[MATRIX TYPE]

Q.9 & Q. 10 has **four** statements (A,B,C and D) given in **Column-I** and **four** statements (P, Q, R and S) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II.**

9.

Column-I		Column-II	
(A)	Number of real solutions of the equation $\lvert x-1 \rvert + \lvert x-3 \rvert = \frac{3}{2}$ is	(P)	-1
(B)	If $\sin x + \sin^2 x = 1$ then the value of $\cos^2 x + \cos^4 x$ equals	(Q)	0
(C)	If $\log_{10}(x^2 + x) = \log_{10}(x^3 - x)$ then the product of all solutions of the equation is	(R)	1
(D)	If $1 + x + x^2 + x^3 = 0$ where $x \in R$ then the value of $1 + x + x^2 + x^3 + x^4 + \ldots\ldots + x^{2018} + x^{2019}$ equals	(S)	2

10.

Column-I		Column-II	
(A)	If x, y, z be positive real numbers such that $\log_{2x}(z) = 3$, $\log_{5y}(z) = 6$ and $\log_{xy}(z) = 2/3$ then the value of z is in the form of m/n in lowest form then $(n - m)$ is equal to	(P)	4
(B)	Let $0 \le a, b, c, d \le \pi$ where b and c are not complementary such that $2\cos a + 6\cos b + 7\cos c + 9\cos d = 0$ and $2\sin a - 6\sin b + 7\sin c - 9\sin d = 0$. If $\frac{\cos(a+d)}{\cos(b+c)} = \frac{m}{n}$ where *m* and *n* are relatively prime positive numbers, then the value of $(m + n)$ is equal to	(Q)	8
(C)	Suppose A and B are two angles such that $A, B \in (0, \pi)$ and satisfy $\sin A + \sin B = 1$ and $\cos A + \cos B = 0$. The value of $12\cos 2A + 4\cos 2B$, is equal to	(R)	9
(D)	α and β are the positive acute angles and satisfying simultaneously the equation $5\sin 2\beta = 3\sin 2\alpha$ and $\tan\beta = 3\tan\alpha$. The value of $\tan\alpha + \tan\beta$ is	(S)	10

[SUBJECTIVE]

11. If $5\sin x = \sin(x + 2y)$, then prove that $2\tan(x + y) = 3\tan y$

12. If $\sin(A + 2B) = \cos(2A + B)$ & $B - A = \frac{\pi}{3}$ where $A,B \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then $\left|\frac{B}{A}\right|$ is

13. Prove that $\tan\left(\frac{\pi}{4}+\theta\right)\times\tan\left(\frac{3\pi}{4}+\theta\right) = -1$

14. If $\tan\theta = \frac{k}{k+1}$ and $\tan\phi = \frac{k}{k-1}$, find $\tan(\theta + \phi)$

15. If 2 $\sin\alpha\ \cos\beta\sin\gamma = \sin\beta\sin(\alpha + \gamma)$. Then show $\tan\alpha$, $\tan\beta$ and $\tan\gamma$ are in Harmonic Progression

16. If $3\tan\theta\tan\phi = 1$, then prove that $2\cos(\theta + \phi) = \cos(\theta - \phi)$

17. Show that $2\cos\theta = -\sqrt{2+\sqrt{2+2\cos 4\theta}}$ where $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}+$

18. Prove that $2\cos\left(\frac{A+3B}{2}\right)\cos\left(\frac{3A-B}{2}\right) = \cos(2A+B)+\cos(A-2B)$

19. Prove that $\sin(A+B).\sin(A-B) = \frac{1}{2}[2\sin(A+B).\sin(A-B)] = \frac{1}{2}(\cos 2B - \cos 2A)$

20. Prove the following:

(A) $\dfrac{\sin(x+y)-2\sin x+\sin(x-y)}{\cos(x+y)-2\cos x+\cos(x-y)} = \tan x$

(B) $\dfrac{\sin(a-c)+2\sin a+\sin(a+c)}{\sin(b-c)+2\sin b+\sin(b+c)} = \dfrac{\sin a}{\sin b}$

(C) $\dfrac{\sin A+\sin B}{\sin A-\sin B} = \tan\dfrac{A+B}{2}\cot\dfrac{A-B}{2}$

(D) $\dfrac{\cos A+\cos B}{\cos A-\cos B} = -\cot\dfrac{A+B}{2}\cot\dfrac{A-B}{2}$

(E) $\cos(-B + C + A) + \cos(-A + B + C) + \cos(A + B - C) + \cos(A + B + C = 4\cos A\cos B\cos C$

Answers

RACE # 20

1. (C) **2.** (D) **3.** (B) **4.** (A) **5.** (A) **6.** (A) **7.** (ACD) **8.** (BCD)

9. (A) Q; (B) R; (C) S; (D) Q **10.** (A) R; (B) S; (C) Q; (D) P **12.** (3) **14.** $-2k^2$