GUIDED REVISION

SECTION-I

Single Correct Answer Type **3 Q. [3 M (–1)]**

1. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b/2?

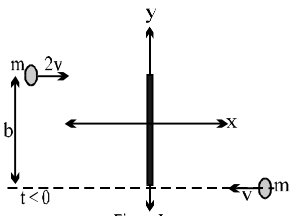


Figure I

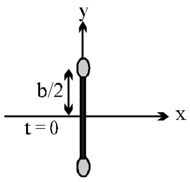


Figure II

(A) $x = 2vt, y = b/2$

(B) $x = vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(C) $x = 0.5vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(D) $x = 0.5vt + 0.5b\sin(6vt/b), y = 0.5b\cos(6vt/b)$

2. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then :- **[IIT-JEE 2009]**

(A) at θ =30°, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

3. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h (<ℓ) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are: (g = 10 ms^{-2}) **[JEE Advanced-2016]**

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}$N

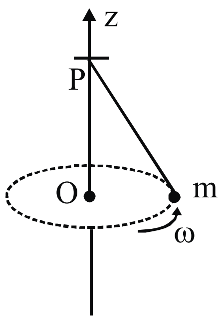
(B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}$N

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}$N

(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}$N

Multiple Correct Answer Type 4 Q. [4 M (–1)]

4. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then **[IIT-JEE 2012]**



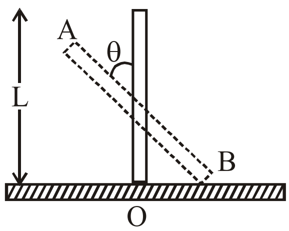
(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

(D) $|\vec{L}_O|$ and $|\vec{L}_P|$ both do not vary with time

5. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct ? **[JEE Advanced-2017]**



(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cosθ)

(B) The midpoint of the bar will fall vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to sinθ

(D) The trajectory of the point A is a parabola

6. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0\ Ns^{-1}$ and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true? **[JEE Advanced-2018]**

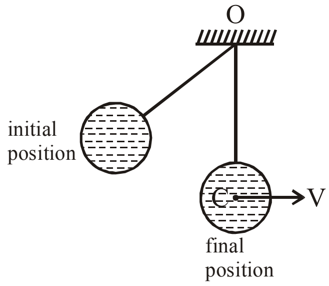
(A) $|\vec{\tau}| = \frac{1}{3}$Nm

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})ms^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}$m

7. A massles rod has a massless hollow sphere attached to it. This sphere can be fully filled either with a liquid (non viscous) or with a solid (rigidly fitted into sphere) of same mass. System is released from rest from initial position (as shown). When it reaches final position which of the following is/are true for the system.

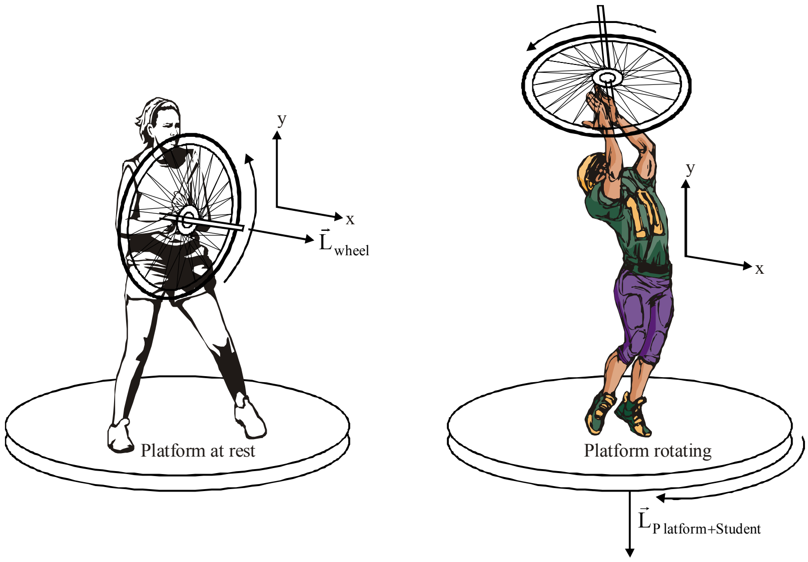


(A) Kinetic energy in case of liquid will be more than in case of solid.
(B) Velocity of centre (V) in case of liquid will be more than in case of solid.
(C) About C angular momentum in case of liquid will be more than in case of solid.
(D) About C angular momentum in case of liquid will be less than in case of solid.

Linked Comprehension Type (2 Para × 3Q.) [3 M (-1)]
(Single Correct Answer Type)

Paragraph for Question no. 8 to 10

A demonstration often done in physics classes is for a student to hold a spinning bicycle wheel while standing on a platform that is free to rotate.The wheel's rotation axis is initially horizontal. Then the student repositions the wheel so that its axis of rotation is vertical. As he repositions the wheel, the platform begins to rotate opposite to the wheel's rotation. If we assume no friction acts to resist rotation of the platform, then the platform continues to rotate as long as the wheel is held with its axis vertical. If the student returns the wheel to its original orientation, the rotation of the platform stops.



The platform is free to rotate about a vertical axis. As a result, once the student steps onto the platform, the vertical component L_y of the angular momentum of the system (student + platform + wheel) is conserved. The horizontal components of $\vec{L}$ are not conserved. The platform is not free to rotate about any horizontal axis since the floor can exert external torques to keep it from doing so. In vector language, we would say that only the vertical component of the external torque is zero, so only the vertical component of angular momentum is conserved.

Initially $L_y = 0$ since the student and the platform have zero angular momentum and the wheel's angular momentum is horizontal. When the wheel is repositioned so that it spins with an upward angular momentum ($L_y > 0$), the rest of the system (the student and the platform) must acquire an equal magnitude of downward angular momentum ($L_y < 0$) so that the vertical component of the total angular momentum is still zero.

The student and the wheel apply torques to each other to transfer angular momentum from one part of the system to the other. As the student lifts the wheel, he feels a strange twisting force that tends to rotate him about a horizontal axis. The platform prevents the horizontal rotation by exerting unequal normal forces on the student's feet. The horizontal component of the torque is so counterintuitive that, if the student is not expecting it, he can easily be thrown from the platform.

8. The torque exerted by the wheel on the student is :
(A) having horizontal component only
(B) having vertical component only
(C) having horizontal as well as vertical component
(D) there is no torque at any time on the student

9. As indicated in paragraph :
(A) Ground exerts only a torque in horizontal direction
(B) Ground exerts only a torque in vertical direction
(C) Ground exerts a torque in both horizontal and vertical direction
(D) Ground exerts no torque

10. Accroding to the paragraph
(A) there is no friction between the student and the platform but there is friction between platform and the ground.
(B) there is friction between the student and the platform as well as between platform and the ground.
(C) there is no friction between the student and the platform as well as between platform and the ground.
(D) there is friction between the student and the platform but there is no friction between platform and the ground.

Paragraph for Questions no. 11 to 13

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2 ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. **[JEE 2007]**

11. The ratio x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

12. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{3t}$ (C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

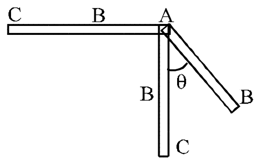
13. The loss of kinetic energy the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$ (C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

SECTION-II

Numerical Answer Type Question 1 Q. [3(0)]
(upto second decimal place)

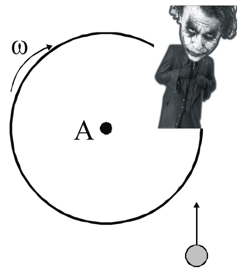
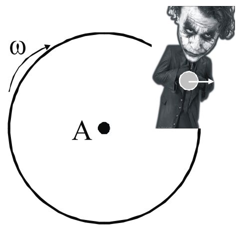
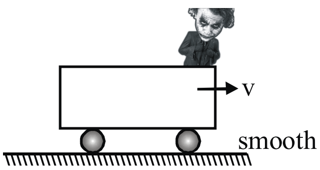
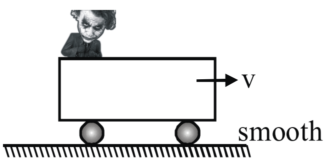
1. A rod hinged at one end is released from the horizontal position as shown in the figure. When it becomes vertical its lower half separates without exerting any reaction at the breaking point. Then find the maximum angle 'θ' made by the hinged upper half with the vertical.



SECTION-IV

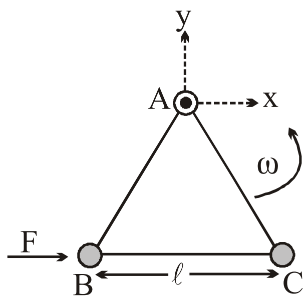
Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. Column-I depicts various situations where some sudden events are taking place. Column-II describes changes in various parameters of systems immediately after the events taking place in column-I.

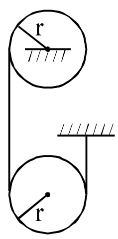
Column-I	**Column-II**
(A) Joker is standing on revolving platform and batman throws the ball and joker catches the ball while it was moving horizontally.  Joker, ball and platform is system.	(P) Linear momentum remains conserved.
(B) Joker throws the ball horizontally and perpendicular to his motion while standing on the revolving platform.  Joker, ball and platform is system.	(Q) Mechanical energy is conserved
(C) Joker jumps horizontally towards right from the cart which is moving at speed v on smooth horizontal floor.  Joker and cart is the system	(R) Mechanical energy increases.
(D) Joker drops himself vertically from the moving cart with no horizontal velocity relative to cart.  Joker and cart is the system	(S) Mechanical energy decreases.
	(T) v or ω changes

Subjective Type 9 Q. [4 M (0)]

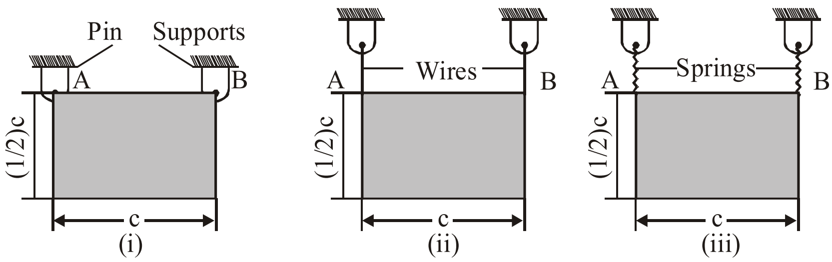
1. Three particles A,B, and C each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.
(a) Find the magnitude of the horizontal force exerted by the hinge on the body.
(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown).Obtain the x–component and the y–component of the force exerted by the hinge on the body, immediately after time T. **[IIT-JEE' 2001]**



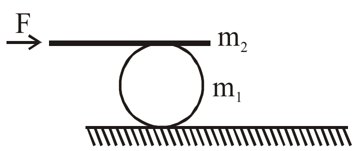
2. Two uniform cylinders, each of mass m = 10 kg and radius r = 150 mm, are connected by a rough belt as shown. If the system is released from rest, determine
(a) the velocity of the centre of cylinder A after it has moved through 1.2 m &
(b) the tension in the portion of the belt connecting the two cylinders.



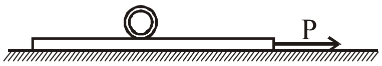
3. A uniform plate of mass m is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released ;
(a) the angular acceleration of the plate .
(b) the acceleration of its mass center .



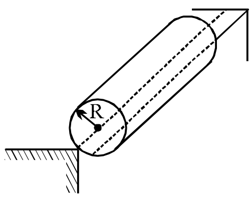
4. A man pushes a cylinder of mass m_1 with help of a plank of mass m_2 as shown. There is no slipping at any contact. the horizontal component of the force applied by the man is F. Find:
(a) the accelerations of the plank and the centre of mass of the cylinder, and
(b) the magnitudes and directions of frictional forces at contact points. **[JEE '99, 6 + 4]**



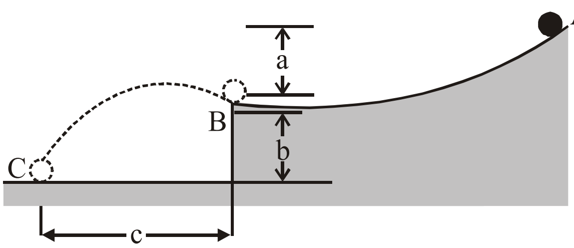
5. A 160 mm diameter pipe of mass 6 kg rests on a 1.5 kg plate. The pipe and plate are initially at rest when a force P of magnitude 25 N is applied for 0.75s. Knowing that $\mu_s = 0.25$ & $\mu_k = 0.20$ between the plate and both the pipe and the floor, determine;
(a) whether the pipe slides with respect to the plate .
(b) the resulting velocities of the pipe and of the plate .



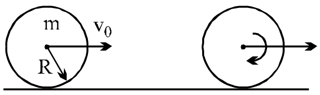
6. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in figure. there is sufficient friction present at the edge so that a very small displacement cause the cylinder to roll of the edge without slipping. Determine
(a) the angle θ_C through which the cylinder rotates before it leaves contact with the edge,
(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and
(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.



7. A small sphere of mass m and radius r is released from rest at A and rolls without sliding on the curved surface to point B where it leaves the surface with a horizontal velocity. Knowing that a = 1.5 m & b = 1.2 m, determine;
(a) the speed of the sphere as it strikes the ground at C .
(b) the corresponding distance c .



8. A uniform disk of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds it acquires a purely rolling motion as shown in figure.
(i) Calculate the velocity of the centre of mass of the disk at t_0.
(ii) Assuming the coefficient of friction to be μ calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.



9. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.
(a) Find the ratio m/M . **[IIT-JEE' 2000]**
(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.
(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

SECTION-I

Single Correct Answer Type **3 Q. [3 M (–1)]**

1. Ans. (C) **2. Ans. (B)** **3. Ans. (D)**

Multiple Correct Answer Type **4 Q. [4 M (–1)]**

4. Ans. (C,D) **5. Ans. (A), (B), (C)** **6. Ans. (A,C)** **7. Ans. (B,D)**

Linked Comprehension Type **(2 Para × 3Q.)** **[3 M (-1)]**

(Single Correct Answer Type)

8. Ans. (C) **9. Ans. (A)** **10. Ans. (D)** **11. Ans. (C)** **12. Ans. (A)** **13. Ans. (B)**

SECTION-II

Numerical Answer Type Question **1 Q. [3(0)]**

(upto second decimal place)

1. Ans. 60°

SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. Ans. (A) ST (B) R (C) PRT (D) PQ

Subjective Type **9 Q. [4 M (0)]**

1. Ans. (a) $\sqrt{3}\ ml\omega^2$ (b) $(F_{net})_x = -\frac{F}{4}$, $(F_{net})_y = \sqrt{3}\ ml\omega^2$ **2. Ans.** (a) $4\sqrt{\frac{3}{7}}$ m/s, (b) $\frac{200}{7}$ N

3. Ans. **(i)** **(a)** $\frac{1.2g}{c}$ (cw) **(b)** $-03.(\hat{i} + 2\hat{j})g$ **(ii)** **(a)** $24g/17c$ (cw) **(b)** $12g/17\downarrow$

(iii) **(a)** $2.4g/c$ (cw) **(b)** $0.5g\downarrow$

4. Ans. **(a)** $a_c = \frac{4F}{3m_1 + 8m_2}$, $a_p = 2a_c$

(b) friction at the top of the cylinder = $3m_1F(3m_1+8m_2)$ towards right; friction at the bottom = $m_1F/(3m_1+8m_2)$ towards right.

5. Ans. **(a)** pipe rolls without sliding **(b)** pipe : 5/6 m/s →, 125/12 rad/s (anticlockwise); plate : 5/3 m/s →

6. Ans. (a) $\theta_C = \cos^{-1}(4/7)$, (b) $v = \sqrt{4/7\ gR}$, (c) $K_T/K_R = 6$

7. Ans. **(a)** 6.68 m/s **(b)** 2.27 m

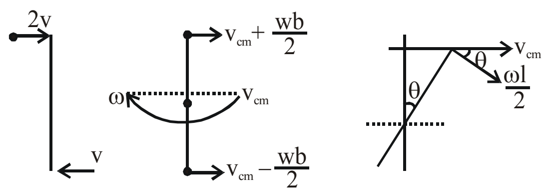
8. Ans. (i) $2v_0/3$, (ii) $t = v_0/3\mu g$, $W = \frac{1}{2}[3\mu^2 mg^2t^2 - 2\mu\, m\, g\, t\, v_0]$ $(t < t_0)$, $W = -\frac{1}{6}mv_0^2$ $(t > t_0)$

9. Ans. (a) $\frac{m}{M} = \frac{1}{4}$; (b) $x = \frac{2L}{3}$; (c) $\frac{v_0}{2\sqrt{2}}$

SOLUTIONS
SECTION-I

Single Correct Answer Type **3 Q. [3 M (–1)]**

1. Ans. (C)

Sol. 

$2mv_{cm} = m2v - mv \qquad \therefore\ v_{cm} = \frac{v}{2}$

$$2vm\frac{b}{2}+mv\frac{b}{2}=m\left(v_{cm}+\frac{\omega b}{2}\right)\frac{b}{2}-m\left(v_{cm}-\frac{\omega b}{2}\right)$$

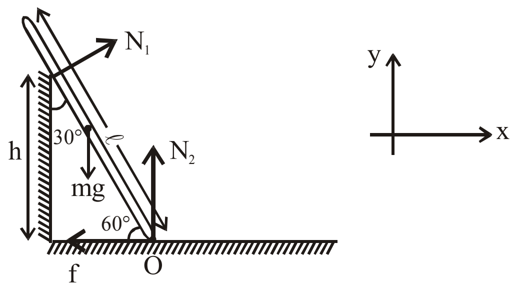
$\therefore\ 3m\frac{vb}{2}=m\omega b\frac{b}{2} \qquad \therefore\ \omega = \frac{3v}{b}$

2. Ans. (B)

Sol. Block will slide at 60° ($\tan^{-1}\mu$) for toppling

$f\frac{15}{2}=mg\cos\theta\frac{10}{2} \qquad \therefore\ mg\sin\theta\ 15 = mg\cos\theta\ 10 \qquad \therefore\ \tan\theta = \frac{2}{3}$ for toppling

3. Ans. (D)

Sol. 

Force equation in x-direction,

$N_1 \cos 30° - f = 0$ ... (i)

Force equation in y-direction,

$N_1 \sin 30° + N_2 - mg = 0$ ... (ii)

Torque equation about O,

$mg\frac{\ell}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0$ ... (iii)

Also, given $N_1 = N_2$ ... (iv)

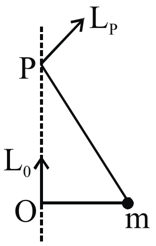
[Note taking reaction from floor as normal reaction only]

solving (i), (ii), (iii) & (iv) we have

$\frac{h}{\ell}=\frac{3\sqrt{3}}{16}\ \&\ f=\frac{16\sqrt{3}}{3}$

Multiple Correct Answer Type 4 Q. [4 M (–1)]

4. **Ans. (C,D)**

Sol. 

5. **Ans. (A), (B), (C)**

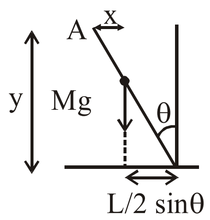
Sol. When the bar makes an angle θ; the height of its COM (mid point) is $\frac{L}{2}\cos\theta$

$\therefore$ displacement $= L - \frac{L}{2}\cos\theta = \frac{L}{2}(1-\cos\theta)$

Since force on COM is only along the vertical direction, hence COM is falling vertically downward.
Instantaneous torque about point of contact is

$Mg \times \frac{L}{2}\sin\theta$

Now; $x = \frac{L}{2}\sin\theta$



$y = L\cos\theta$

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$$

Path of A is an ellipse.

6. **Ans. (A,C)**

Sol. $\vec{F} = (\alpha t)\hat{i} + \beta\hat{j}$ [At t = 0, v = 0, $\vec{r} = \vec{0}$]

$\alpha = 1$, $\beta = 1$

$\vec{F} = t\hat{i} + \hat{j}$

$m\frac{d\vec{v}}{dt} = t\hat{i} + \hat{j}$

On integrating

$m\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$ [m = 1kg]

$\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$ [$\vec{r} = \vec{0}$ at t = 0]

On integrating

$\vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

At t = 1 sec, $\vec{\tau} = (\vec{r}\times\vec{F}) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right)\times\left(\hat{i}+\hat{j}\right)$

$\vec{\tau} = -\frac{1}{3}\hat{k}$

$$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$$

At t = 1 $\vec{v} = \left(\frac{1}{2}\hat{i} + \hat{j}\right) = \frac{1}{2}\left(\hat{i} + 2\hat{j}\right) m/sec$

At t = 1 $\vec{s} = \vec{r}_1 - \vec{r}_0$

$$= \left[\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right] - [\vec{0}]$$

$$\vec{s} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$$

$$|\vec{s}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2} \Rightarrow \frac{\sqrt{10}}{6} m$$

7. Ans. (B,D)

Sol. In case of liquid motion is translational

$\Rightarrow I = m\ell^2$

In case of solid motion is CRTM $\Rightarrow I = \frac{2}{5}mR^2 + m\ell^2$

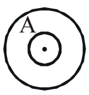
Linked Comprehension Type (2 Para × 3Q.) [3 M (-1)]
(Single Correct Answer Type)

8. Ans. (C)

9. Ans. (A)

10. Ans. (D)

11. Ans. (C)

Sol. 

$$\frac{1}{2}kx_1^2 = \frac{1}{2}I4\omega^2$$

$$\frac{1}{2}kx_2^2 = \frac{1}{2}2I\omega^2$$

$\therefore \frac{x_1^2}{x_2^2} = 2$ $\qquad \therefore \frac{x_1}{x_2} = \sqrt{2}$

12. Ans. (A)

Sol. $I2\omega + 2I\omega = 3I\omega'$ $\qquad \therefore \omega' = \frac{4}{3}\omega$

$$\therefore \tau_{avg} = \frac{\Delta L}{\Delta t} = \left|\frac{I\frac{4}{3}\omega - 2I\omega}{t}\right| = \frac{2I\omega}{3t}$$

13. Ans. (B)

Sol. $KE_f - KE_i = \left|\frac{1}{2}3I\omega'^2 - \frac{1}{2}2I\omega^2 - \frac{1}{2}I4\omega^2\right|$

$$= \frac{1}{2}I\omega^2\left(3\times\frac{16}{9}-2-4\right) = \frac{I\omega^2}{3}$$

SECTION-II

Numerical Answer Type Question **1 Q. [3(0)]**

(upto second decimal place)

1. **Ans.** 60°

Sol. $mg\frac{\ell}{2} = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$ $\quad\therefore\ \omega^2 = \frac{3g}{\ell}$ at bottom

$W_g = \Delta KE$

$$\Rightarrow -\frac{m}{2}g\frac{\ell}{4}(1-\cos\theta) = 0-\frac{1}{2}\left(\frac{m}{2}\right)\frac{\ell^2}{4}\times\frac{1}{3}\omega^2$$

$$\therefore\ \frac{mg\ell}{8}(1-\cos\theta) = \frac{mg\ell}{16}$$

$$\therefore\ \cos\theta = \frac{1}{2}\ \therefore\ \theta = 60°$$

SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. **Ans. (A) ST (B) R (C) PRT (D) PQ**

Sol. For (A) : Hinge will apply external impulse hence momentum will not be conserved.
So (P) is not correct.
Mechanical energy reduce hence (S)
ω decreases by conservation of angular momentum hence (T)
For (B) **:** Hinge will apply external impulse hence momentum will not be conserved.
So (P) is not correct.
Mechanical energy will increase hence (R)
ω does not change by conservation of angular momentum hence (No 'T')
For (C) : No exernal horizontal impulse so momentum is conserved. (P)
Mechanical energy will increase hence (R)
V will change so (T)
For (D) : There is no external impulse and no change in momentum as velocity of joker w.r.t. car is still same so P Not 'T'
Mechanical energy remains same immediately after joker drops as he has no vertical velocity so (Q)

Subjective Type **9 Q. [4 M (0)]**

1. **Ans.** (a) $\sqrt{3}\ ml\omega^2$ (b) $(F_{net})_x = -\frac{F}{4}$, $(F_{net})_y = \sqrt{3}\ ml\omega^2$

Sol. (a) $3mw^2\left(\frac{\sqrt{3}}{2}\ell\frac{2}{3}\right) = \sqrt{3}\,mw^2\ell$

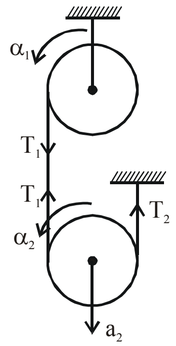
(b) $F_y = \sqrt{3}m\ell\omega^2$ $\quad$ but for Fx

$$F\frac{\sqrt{3}}{2}\ell = 2m\ell^2\alpha$$

$$\& \ F - F_x = \frac{F}{4m} 3m$$

2. **Ans.** (a) $4\sqrt{\frac{3}{7}}$ m/s, (b) $\frac{200}{7}$ N

Sol. $T_1 r = \frac{mr^2}{2} \alpha_1$



$$T_2 r - T_1 r = = \frac{mr^2}{2} \alpha_2 = \frac{ma_2}{2}$$

$\alpha_1 r = a_2 + \alpha_2 r = 2a_2$

$mg - T_1 - T_2 = ma_2$

$a_2 = \alpha_2 r$

$\therefore \ T_1 = ma$

$$T_2 - T_1 = \frac{ma}{2}$$

$$T_2 = \frac{ma}{2} + T_1 = \frac{3ma}{2}$$

$$mg - T_1 - T_2 = ma \qquad mg - T_1 = ma + T_2 = \frac{5}{2} ma$$

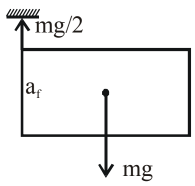
3. **Ans. (i) (a)** $\frac{1.2g}{c}$ (cw) **(b)** $-03.(\hat{i} + 2\hat{j})g$ **(ii) (a)** $24g/17c$ (cw) **(b)** $12g/17\downarrow$ **(iii) (a)** $2.4g/c$ (cw) **(b)** $0.5g\downarrow$

Sol. $mg\frac{C}{2} = I\alpha$

$$= \left(m\frac{C^2}{3} + \frac{m\left(\frac{C}{2}\right)^2}{3} \right) \alpha$$

$$\Rightarrow \alpha = 1.2\frac{g}{c}$$

$$-\alpha\frac{C}{2}\hat{j} - \alpha\frac{C}{4}\hat{i} = .6g\hat{j} - .3g\hat{i}$$

(ii) 

$$\frac{TC}{2} = \left(\frac{mC^2}{12} + \frac{m(C/2)^2}{12} \right) \alpha$$

$$mg - T = ma_y = \frac{m\alpha C}{2}$$

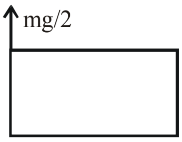
$$\frac{\alpha C}{2} = a_y$$

$$\therefore\ T\frac{C}{2} = \frac{5}{4}\frac{mL^2}{12}\alpha$$

$$\therefore\ mg = \left(\frac{5}{24} + \frac{1}{2}\right)mC\alpha$$

$$\therefore\ \alpha = \frac{24}{17}gC$$

(b) $a_y = \dfrac{\alpha C}{2} = \dfrac{12g}{17}$

(iii) 

$$\frac{mg}{2}\frac{C}{2} = \frac{5}{4}\frac{mC^2}{12}\alpha$$

$$\therefore\ \alpha = \frac{2.4g}{C}$$

$$mg - \frac{mg}{2} = ma_{Cm} \Rightarrow \frac{g}{2} = a_{cm}$$

4. **Ans. (a)** $a_c = \dfrac{4F}{3m_1 + 8m_2}$, $a_p = 2a_c$

(b) friction at the top of the cylinder = $3m_1F(3m_1+8m_2)$ towards right;
friction at the bottom = $m_1F/(3m_1+8m_2)$ towards right.

Sol. $F - f_1 = M_2(a + \alpha r) = m_2(2a)$

$f_1 - f_2 = m_1a$

$$(f_1 + f_2)\,r = \frac{m_1r^2}{2}\alpha = \frac{m_1a}{2}$$

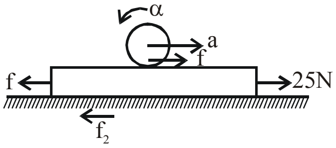
$$f_1 = \frac{3}{4}m_1a$$

$$\therefore\ F = 2m_2a + \frac{3}{4}m_1a$$

$$\therefore\ a = \frac{4F}{8m_2 + 3m_1}$$

5. **Ans. (a)** pipe rolls without sliding

(b) pipe : 5/6 m/s →, 125/12 rad/s (anticlockwise); plate : 5/3 m/s →

Sol. 

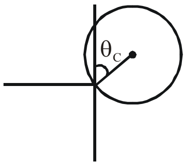
$25 - f - f_2 = 1.5(a + \alpha R)$

$f = 6a$

$fR = 6R^2\,\alpha$

6. **Ans.** (a) $\theta_C = \cos^{-1}(4/7)$, (b) $v = \sqrt{4/7\ gR}$, (c) $K_T/K_R = 6$

Sol. $mg\cos\theta_C = \frac{mv^2}{R}$

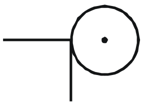


$mg\,R(1-\cos\theta_C) = \frac{1}{2}mV_C^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$

$mgR(1-\cos\theta) = \frac{3}{4}mV_1^2$

$\therefore\ \frac{1-\cos\theta}{\cos\theta} = \frac{3}{4}\ \ \therefore\ \frac{4}{7} = \cos\theta_C$

(b) $V_c = \sqrt{\frac{4}{7}gR}$ $\qquad \therefore \omega_C = \sqrt{\frac{4}{7}\frac{g}{R}}$

(c)  $\qquad mgR = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2\omega^2}{2}$

7. **Ans. (a)** 6.68 m/s **(b)** 2.27 m

Sol. $mg\,(1.5) = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}m\omega^2r^2$

$\therefore\ v_A^2 = \frac{10}{7}\times 10\times 1.5$

Now $mg\,(1.2) = \frac{1}{2}mv_c^2 + \frac{1}{2}mr^2\omega^2 - \left(\frac{1}{2}mv_A^2 + \frac{1}{2}mr^2\omega^2\right)$

$\therefore\ 24 = v_c^2 - v_A^2$

$\Rightarrow\ v_c^2 = \left(24 + \frac{150}{7}\right)(.98)$

$V_C = (6.67)\text{m/s}$

(b) $\sqrt{\frac{2b}{g}}v_A = \sqrt{\frac{2\times1.2}{9.8}}\times\sqrt{\frac{10\times1.5\times9.8}{7}} = 2.27\text{m}$

8. **Ans.** (i) $2v_0/3$, (ii) $t = v_0/3\mu g$, $W = \frac{1}{2}[3\mu^2 mg^2t^2 - 2\mu\, m\, g\, t\, v_0]$ $(t < t_0)$, $W = -\frac{1}{6}mv_0^2$ $(t > t_0)$

Sol. $mv_0R = mv'R + \frac{mR^2}{2}\omega = \frac{3}{2}mv'$

$\therefore\ v' = \frac{2v_0}{3}$

$a = \mu g$ $\qquad \therefore\ \frac{v_0 - v'}{\mu g} = t = \frac{v_0}{3\mu g}$

$v = v_0 - \mu gt$

$\mu_0 mgR = \frac{mR^2}{2}\alpha$

$\therefore \ \alpha = \dfrac{2\mu_0 g}{R}$ $\qquad$ $\therefore \ \omega = \dfrac{2\mu_0 g}{R} t$

$KE_f - KE_i = W_{fr}$

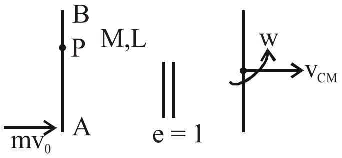
$$\frac{1}{2}m\left(v_0 - \mu g t\right)^2 + \frac{1}{2}\frac{mR^2}{2}w^2$$

$$= \frac{1}{2}m\left(v_0^2 + \mu^2 g^2 t^2 - 2\mu v_0 g t\right) + \frac{1}{2}\frac{mR^2}{2}\frac{4\mu_0^2 g^2 t^2}{R^2}$$

after $t = t_0$, $w_{fr} = \text{constant} = -\dfrac{1}{6}mv_0^2$

9. **Ans.** (a) $\dfrac{m}{M} = \dfrac{1}{4}$; (b) $x = \dfrac{2L}{3}$; (c) $\dfrac{v_0}{2\sqrt{2}}$

Sol.



$$e = \frac{v'_2 - v'_1}{v_1 - v_2} = \frac{\frac{\omega L}{2} + v_{cm}}{v_0}$$

$$v_{CM} = \frac{mv_0}{M} \ \& \ mv_0\frac{L}{2} = \frac{m_1 L^2}{12}\omega$$

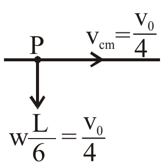
$$\omega = \frac{6mv_0}{ML}$$

$$\frac{\frac{3mv_0}{M} + \frac{mv_0}{M}}{v_0} = 1 \Rightarrow \frac{m}{M} = \frac{1}{4}$$

(b) $v_{cm} - \omega x = 0$

$\therefore \ x = \dfrac{L}{6} \ \therefore \ \dfrac{L}{2} + \dfrac{L}{6} = \dfrac{2L}{3}$

(c) $\omega\ell = \dfrac{3}{2}v_0$



$$\omega t = \frac{\omega\pi\ell}{3v_0}$$

$$\frac{\pi\ell}{3v_0} \times \frac{3v_0}{2\ell} = \frac{\pi}{2}$$

$\therefore$ after $\dfrac{\pi}{2}$