Determinants

Using the properties of determinants in Exercises 1 to 6, evaluate:

1. $\begin{vmatrix} x^2 \;\; x \;\; 1 & x \;\; 1 \\ x \;\; 1 & x \;\; 1 \end{vmatrix}$

2. $\begin{vmatrix} a \;\; x & y & z \\ x & a \;\; y & z \\ x & y & a \;\; z \end{vmatrix}$

3. $\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$

4. $\begin{vmatrix} 3x & x \;\; y & x \;\; z \\ x \;\; y & 3y & z \;\; y \\ x \;\; z & y \;\; z & 3z \end{vmatrix}$

5. $\begin{vmatrix} x \;\; 4 & x & x \\ x & x \;\; 4 & x \\ x & x & x \;\; 4 \end{vmatrix}$

6. $\begin{vmatrix} a \;\; b \;\; c & 2a & 2a \\ 2b & b \;\; c \;\; a & 2b \\ 2c & 2c & c \;\; a \;\; b \end{vmatrix}$

Using the proprties of determinants in Exercises 7 to 9, prove that:

7. $\begin{vmatrix} y^2z^2 & yz & y \;\; z \\ z^2x^2 & zx & z \;\; x \\ x^2y^2 & xy & x \;\; y \end{vmatrix} = 0$

8. $\begin{vmatrix} y \;\; z & z & y \\ z & z \;\; x & x \\ y & x & x \;\; y \end{vmatrix} = 4xyz$

9. $\begin{vmatrix} a^2 \;\; 2a & 2a \;\; 1 & 1 \\ 2a \;\; 1 & a \;\; 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a \;\; 1)^3$

10. If A + B + C = 0, then prove that $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 0$

11. If the co-ordinates of the vertices of an equilateral triangle with sides of length 'a' are (x_1, y_1), (x_2, y_2), (x_3, y_3), then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}$.

12. Find the value of θ satisfying $\begin{bmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{bmatrix} = 0$.

13. If $\begin{matrix} 4 & x & 4 & x & 4 & x \\ 4 & x & 4 & x & 4 & x \\ 4 & x & 4 & x & 4 & x \end{matrix} = 0$, then find values of x.

14. If $a_1, a_2, a_3, \ldots, a_r$ are in G.P., then prove that the determinant $\begin{vmatrix} a_{r\ 1} & a_{r\ 5} & a_{r\ 9} \\ a_{r\ 7} & a_{r\ 11} & a_{r\ 15} \\ a_{r\ 11} & a_{r\ 17} & a_{r\ 21} \end{vmatrix}$ is independent of r.

15. Show that the points $(a + 5, a - 4)$, $(a - 2, a + 3)$ and (a, a) do not lie on a straight line for any value of a.

16. Show that the ΔABC is an isosceles triangle if the determinant

$$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{bmatrix} = 0.$$

17. Find A^{-1} if $A \begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$ and show that $A^{-1} \; \frac{A^2 \quad 3I}{2}$.

18. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the system of linear equations $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$.

19. Using matrix method, solve the system of equations $3x + 2y - 2z = 3$, $x + 2y + 3z = 6$, $2x - y + z = 2$.

20. Given $A \begin{matrix} 2 & 2 & 4 \\ 4 & 2 & 4 \\ 2 & 1 & 5 \end{matrix}$, $B \begin{matrix} 1 & 1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{matrix}$, find BA and use this to solve the

system of equations $y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$.

21. If $a + b + c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then prove that $a = b = c$.

22. Prove that $\begin{vmatrix} bc & a^2 & ca & b^2 & ab & c^2 \\ ca & b^2 & ab & c^2 & bc & a^2 \\ ab & c^2 & bc & a^2 & ca & b^2 \end{vmatrix}$ is divisible by $a + b + c$ and find the quotient.

23. If $x + y + z = 0$, prove that $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Multiple Choice Questions

Choose the correct answer from given four options in each of the Exercises from 24 to 37.

24. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} \quad \begin{vmatrix} 6 & 2 \\ 7 & 3 \end{vmatrix}$, then value of x is

(A) 3 (B) ± 3

(C) ± 6 (D) 6

25. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$

(A) $a^3 + b^3 + c^3$ (B) $3\ bc$

(C) $a^3 + b^3 + c^3 - 3abc$ (D) none of these

26. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be

(A) 9 (B) 3

(C) – 9 (D) 6

27. The determinant $\begin{vmatrix} b^2 & ab & b & c & bc & ac \\ ab & a^2 & a & b & b^2 & ab \\ bc & ac & c & a & ab & a^2 \end{vmatrix}$ equals

(A) $abc\ (b-c)\ (c-a)\ (a-b)$ (B) $(b-c)\ (c-a)\ (a-b)$

(C) $(a + b + c)\ (b - c)\ (c - a)\ (a - b)$ (D) None of these

28. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$ 0 in the interval

$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is

(A) 0 (B) 2

(C) 1 (D) 3

29. If A, B and C are angles of a triangle, then the determinant

$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} \text{ is equal to}$$

(A) 0 (B) -1

(C) 1 (D) None of these

30. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then $\lim_{t \to 0} \frac{f(t)}{t^2}$ is equal to

(A) 0 (B) -1

(C) 2 (D) 3

31. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1\ \sin & 1 \\ 1\ \cos & 1 & 1 \end{vmatrix}$ is (θ is real number)

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{2}$

(D) $\frac{2\sqrt{3}}{4}$

32. If $f(x) = \begin{vmatrix} 0 & x\ a & x\ b \\ x\ a & 0 & x\ c \\ x\ b & x\ c & 0 \end{vmatrix}$, then

(A) $f(a) = 0$

(B) $f(b) = 0$

(C) $f(0) = 0$

(D) $f(1) = 0$

33. If A = $\begin{matrix} 2 & & 3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{matrix}$, then A^{-1} exists if

(A) $\lambda = 2$

(B) $\lambda \neq 2$

(C) $\lambda \neq -2$

(D) None of these

34. If A and B are invertible matrices, then which of the following is not correct?

(A) $adj\ A = |A|.\ A^{-1}$

(B) $\det(A)^{-1} = [\det(A)]^{-1}$

(C) $(AB)^{-1} = B^{-1}A^{-1}$

(D) $(A + B)^{-1} = B^{-1} + A^{-1}$

35. If x, y, z are all different from zero and $\begin{vmatrix} 1\ x & 1 & 1 \\ 1 & 1\ y & 1 \\ 1 & 1 & 1\ z \end{vmatrix}$ 0, then value of $x^{-1} + y^{-1} + z^{-1}$ is

(A) $x\,y\,z$ (B) $x^{-1}\,y^{-1}\,z^{-1}$

(C) $-x\;-y\;-z$ (D) -1

36. The value of the determinant $\begin{vmatrix} x & x\ \ y & x\ \ 2y \\ x\ \ 2y & x & x\ \ y \\ x\ \ y & x\ \ 2y & x \end{vmatrix}$ is

(A) $9x^2\,(x+y)$ (B) $9y^2\,(x+y)$

(C) $3y^2\,(x+y)$ (D) $7x^2\,(x+y)$

37. There are two values of a which makes determinant, $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & 1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then

sum of these number is

(A) 4 (B) 5

(C) -4 (D) 9

Fill In Blanks Type Questions

38. If A is a matrix of order 3×3, then $|3A| =$ ________ .

39. If A is invertible matrix of order 3×3, then $|A^{-1}|$ ________ .

40. If $x, y, z \in$ R, then the value of determinant $\begin{vmatrix} 2^x\ \ 2^{-x}\ {}^2 & 2^x\ \ 2^{-x}\ {}^2 & 1 \\ 3^x\ \ 3^{-x}\ {}^2 & 3^x\ \ 3^{-x}\ {}^2 & 1 \\ 4^x\ \ 4^{-x}\ {}^2 & 4^x\ \ 4^{-x}\ {}^2 & 1 \end{vmatrix}$ is

equal to ______.

41. If $\cos 2\theta = 0$, then $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2 =$ ________.

42. If A is a matrix of order 3 × 3, then $(A^2)^{-1} =$ _______.

43. If A is a matrix of order 3 × 3, then number of minors in determinant of A are ________.

44. The sum of the products of elements of any row with the co-factors of corresponding elements is equal to ________.

45. If $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then other two roots are _________.

46. $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix} =$ _________.

47. If $f(x) = \begin{vmatrix} (1+x)^{17} & (1+x)^{19} & (1+x)^{23} \\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34} \\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47} \end{vmatrix} = A + Bx + Cx^2 + \ldots$, then

A = _______.