Complex Numbers and Quadratic Equations

Introduction

- In Class10, We Studied about Quadratic Equations of the General form $aX^2 + bX + c = 0$.Also to find the Roots of this Quadratic Equation we used the."Quadratic Formula".

$$X = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Recall : If Discriminant $(D) = b^2 - 4ac > 0$: Real & Distinct Roots

$(D) = b^2 - 4ac = 0$: Real & Equal Roots

$(D) = b^2 - 4ac < 0$: No Real Roots or Imaginary Roots

- Toh Suno, the above third Case implies ki under root me negative value ajayegi i.e $\sqrt{-ve}$ for e.g -1 Jisko Solve karna abhi tak nhi seekha hai tumne......Right. Toh bas isi tarah ke questions or problems ko solve karne ke liye we needed the concept of "Complex No.s or Imaginary Numbers.

Complex Numbers

Let us denote -1 by the Symbol i (iota). Then $i^2 = -1$.

A number of the form a + ib, where a and b are real numbers, is called a Complex No.

Example:

3 + i5, (-2) + i 3 , (-9) + 19i , etc..... are examples of Complex Numbers.

*NOTE : Complex Number = Real Number + Imaginary Number

For the Complex number Z = a + ib, a is called the 'real part', denoted by Re(Z)

and b is called the 'imaginary part' denoted by Im(Z) of the complex number Z.

Example:

if Z = 5 + 7i, then Re(Z) = 5 and Im(Z) = 7.

Algebra of Complex Numbers

(i) Addition of two Complex numbers:

Let $Z_1 = a + ib$ and $Z_2 = c + id$ are two complex numbers.

Then $Z_1 + Z_2 = (a + c) + i(b + d)$

Example: (2 + 3i) + (3 + 5i) = (2 + 3) + (3 + 5)i = 5 + 8i

(ii) Subtraction of two Complex numbers:

For any two Complex numbers Z_1 and Z_2, the subtraction is done as follows: $Z_1 - Z_2 = Z_1 + (-Z_2)$

Example: $(2+3i) - (3+5i) = (2-3) + (3-5)i = -1 - 2i$

(iii) Multiplication of two Complex numbers:

Let $Z_1 = a + ib$ and $Z_2 = c + id$ are two complex numbens.
Then $Z_1 \times Z_2 = (ac - bd) + (ad + bc)i$

Example: $(2+3i) \times (3+5i) = (2\times3) + (2\times5i) + (3i\times3) + (3i\times5i)$

$= 6 + 10i + 9i + 15i^2$

$= 6 + 19i + 15(-1)$ [since $i^2 = -1$]

$= 6 + 19i - 15$

$= -9 + 19i$

(iv) Division of two Complex numbers:

For any two Complex numbers Z_1 and Z_2, the Division is done as follows:

Example: $Z_1 \div Z_2 = \frac{(2+3i)}{(3+5i)}$ USE RATIONALISATION method here

$$= \frac{(2+3i)}{(3+5i)} \times \frac{(3-5i)}{(3-5i)} = \frac{6-10i+9i-15i^2}{9-25i^2}$$

$$= \frac{6-i-15(-1)}{9-25(-1)} = \frac{6-i+15}{9+25}$$

$$= \frac{21-i}{34}$$

*NOTE: While Adding and Subtracting two Complex Numbers always remember that Real parts are to be added or subtracted together only, in the same way imaginary parts are solved with imaginary parts only.

Powers of i (iota)

Till now we know that $i = -1$ and $i^2 = -1$. Now some important powers of i are given below:

1. $i^3 = i^2 \times i = -1 \times i = -i$

2. $i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$

3. $i^5 = i^4 \times i = i^2 \times i^2 \times i = (-1) \times (-1) \times i = i$

4. $i^6 = (i^2)^3 = (-1)^3 = -1$

*NOTE: Agar badi powers aa Jaye toh hamesha power ko break karna i ki terms me..

5. $i^{39} = i^{38} \times i = (i^2)^{19} \times i = (-1)^{19} \times i = -1 \times i = -i$

6. $i^{61} = (i^2)^{30} \times i = (-1)^{30} \times i = 1 \times i = i$

Additive Inverse of complex numbers

For every Complex number $Z = a + ib$, there exists a Complex number $-a + i(-b)$ denoted as $-Z$, which is called the Additive Inverse or negative of Z.

Example:

The additive Inverse of the complex number $Z = 2 + 5i$ is given by

$$-Z = -2 - 5i$$

Multiplicative Inverse of complex numbers

For every non zero Complex number $Z = a + ib$, there exists a Complex number $1/Z$, which is called the multiplicative Inverse of Z such that $Z.1/Z = 1$.

Example:

If $Z = 2 + 3i$, then the multiplicative inverse of this complex number is given by

$$\frac{1}{Z} = \frac{1}{2 + 3i} \times \frac{2 - 3i}{2 - 3i}$$

$$\frac{1}{Z} = \frac{2 - 3i}{4 - 9i^2}$$

$$\frac{1}{Z} = \frac{2 - 3i}{4 - 9(-1)}$$

$$\frac{1}{Z} = \frac{2 - 3i}{13}$$

*Trick to remember easily - dekho additive inverse ka matlab hai Add ka ulta joki Subtract hota hai...theek hai aur multiplicative inverse ka matlab multiply ka ulta joki divide hota hai...aur yahi toh upar kiya abhi...OK

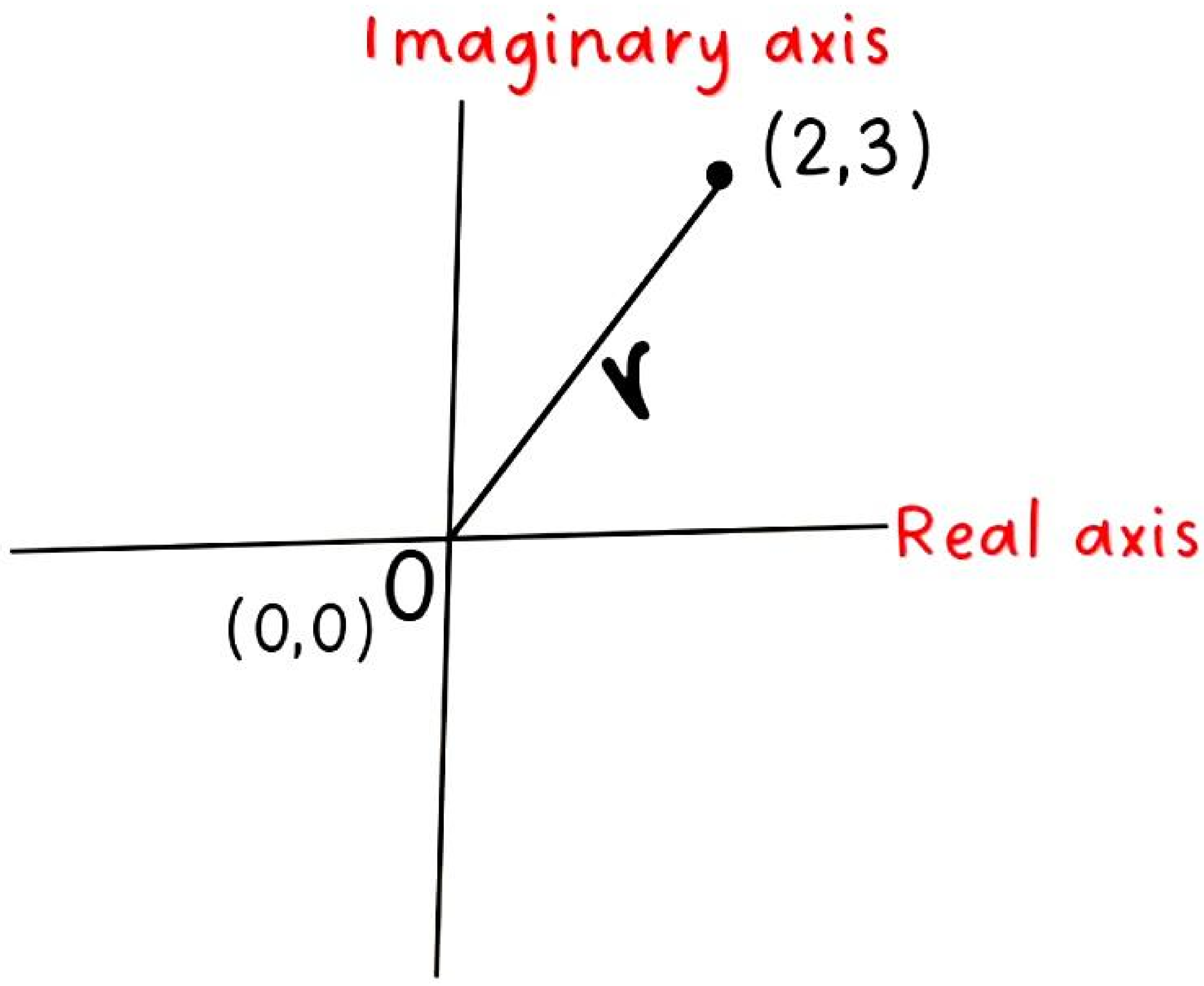
Argand Plane

In our earlier classes we have studied about the 'Cartesian Plane System' where we plotted an ordered pair of numbers (X,Y) in the XY- Plane.

Now for plotting the Complex numbers $Z = a + ib$, we use the Argand Plane.

- The plane having a Complex number assigned to each of its point is called the 'Complex Plane' or 'Argand Plane'.

Example: Plot $Z = 2 + 3i$ in the Argand Plane.
here $Re(Z) = 2$ & $Im(Z) = 3$.. So the point is (2,3).



*NOTE: The distance of any Point P(X,Y) from the Origin O(0,0) in the Argand Plane is r which is given by

$$r = \sqrt{X + Y^2}$$

- The X-axis and the Y-axis in the Argand Plane are called, respectively, the Real axis and the Imaginary axis.

Quadratic Equations

In Class10 we have studied about the 'Quadratic Equations' which were of the general
form $aX^2 + bX + c = 0$ and also to solve such equations we used the 'Quadratic formula'
Or Discriminant Method. $X = \frac{-b \pm D}{2a}$, where $D = b^2 - 4ac$

*NOTE: Here we will only have questions involving the value of discriminant(D) less than zero i.e. $D < 0$.

Example:

(i) Solve $2X^2 + X + 1 = 0$

Ans: Comparing the given quadratic equation with the general form $aX^2 + bX + c = 0$ we get

$a = 2$, $b = 1$, and $c = 1$. Now using the Quadratic formula

$$X = \frac{-b \pm D}{2a}$$, where $D = b^2 - 4ac$

$$D = (1)^2 - 4 \times (2) \times (1)$$

$$D = 1 - 8$$

$$D = -7$$

Therefore, $$X = \frac{-1 \mp -7}{2 \times 2}$$

$$X = \frac{-1 \pm 7i}{4}$$ (answer yaha par bhi chodd sakte ho)

Question:

If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then find the values of x and y.

Solution

We have $4x + i(3x - y) = 3 + i(-6)$ ... (1)

Equating the real and the imaginary parts of (1),

we get $4x = 3$, $3x - y = -6$, which, on solving simultaneously, give

$$X = \frac{3}{4} \qquad Y = \frac{33}{4}$$

Question:

Express $(5 - 3i)^3$ in the form a + ib.

Solution

We have, $(5 - 3i)^3 = 5^3 - 3 \times 5^2 \times (3i) + 3 \times 5 (3i)^2 - (3i)^3$

$= 125 - 225i - 135 + 27i$

$= - 10 - 198i.$

Question:

Find the multiplicative inverse of 2 − 3i.

Solution

Let z = 2 − 3i

Then $\bar{z} = 2 + 3i$ and $|z|^2 = 2^2 + (-3)^2 = 13$

Therefore, the multiplicative inverse of 2 - 3i is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{2 + 3i}{13} = \frac{2}{13} + \frac{3}{13}i$$

Question:

Find the conjugate of $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$

Solution

We have ,

$$\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$$

$$= \frac{6 + 9i - 4i + 6}{(1 + 2i)(2 - i)}$$

$$= \frac{12 + 5i}{4 + 3i} \times \frac{4 - 3i}{4 - 3i}$$

$$= \frac{48 - 36i + 20i + 15}{16 + 9}$$

$$= \frac{63 - 6i}{25}$$