21. Mensuration-II (Volumes and Surface Areas of a Cuboid and a Cube)

Exercise 21.1

1. Question

Find the volume of a cuboid whose

(i) length = 12 cm, breadth = 8 cm, height = 6 cm

(ii) length = 1.2 m, breadth = 30 cm, height = 15 cm

(iii) length = 15 cm, breadth = 2.5 dm, height = 8 cm.

Answer

(i) Length of cuboid = 12 cm

Breadth of cuboid = 8 cm

Height of cuboid = 6 cm

Hence,

Volume of cuboid = length × breadth × height = 12 × 8 ×6 = 576 cm^3

(ii) Length = 1.2 m = 120 cm

Breadth = 30 cm

Height = 15 cm

Hence,

Volume of cuboid = 120 × 30 × 15 = 54000 cm^3

(iii) Length = 15 cm

Breadth = 2.5 dm = 25 cm

Height = 8 cm

Hence,

Volume of cuboid = 15 × 25 × 8 = 3000 cm^3

2. Question

Find the volume of a cube whose side is

(i) 4 cm

(ii) 8 cm

(iii) 1.5 dm

(iv) 1.2 m

(v) 25 mm

Answer

(i) Given,

side of cube = 4 cm

volume of cube = $(side)^3 = 4^3 = 64\ cm^3$

(ii) side of cube = 8 cm

volume of cube =$(side)^3 = 8^3 = 512\ cm^3$

(iii) side of cube = 1.5 dm

volume of cube = $(side)^3 = 1.5^3 = 3.375\ dm^3 = 3375\ cm^3$

(iv) side of cube = 1.2 m

volume of cube = $1.2^3 = 1.728\ m^3$

(v) side of cube = 25 mm

volume of cube = $25^3 = 15625\ mm^3 = 15.625\ cm^3$

3. Question

Find the height of a cuboid of volume 100 cm^3, whose length and breadth are 5 cm and 4 cm respectively.

Answer

Given,

Volume of cuboid = 100 cm^3

Length = 5 cm

Breadth = 4 cm

Let height of cuboid = h cm

So,

$= l \times b \times h = 100\ cm$

$= h = \frac{100}{5 \times 4} = 5\ cm$

4. Question

A cuboidal vessel is 10 cm long and 5 cm wide. How high it must be made to hold 300 cm^3 of a liquid?

Answer

Given,

Length of cuboidal vessel = 10 cm

Width = 5 cm

Volume of liquid in it = 300 cm^3

Let height of vessel = h cm

So,

$= l \times b \times h = 300$

$= h = \frac{300}{10 \times 5} = 6\ cm$

5. Question

A milk container is 8 cm long and 50 cm wide. What should be its height so that it can hold 4 litres of milk?

Answer

Given,

Length of milk container = 8 cm

Width = 50 cm

Volume to hold = 4 litre = 4000 cm^3

Let height of container = h cm

So,

$= l \times b \times h = 4000$

$= h = \frac{4000}{50 \times 8} = 10\ cm$

6. Question

A cuboidal wooden block contains 36 cm^3 wood. If it be 4 cm long and 3 cm wide, find its height.

Answer

Given,

Volume of wood in cuboidal block = 36 cm^3

Length of block = 4 cm

Breadth of block = 3 cm

Let height of block = h cm

So,

$= l \times b \times h = 36$

$= h = \frac{36}{4 \times 3} = 3\ cm$

7. Question

What will happen to the volume of a cube, if its edge is

(i) halved (ii) trebled?

Answer

Given,

Let edge of cube = a

So volume of cube = a^3

Case (i)

Edge become = $\frac{a}{2}$

Volume become = $\left(\frac{a}{2}\right)^3 = \frac{a^3}{8} = \frac{1}{8} times$

Case (ii)

Edge becomes = 3a

Volume become = $(3a)^3 = 27a^3 = 27\ times$

8. Question

What will happen to the volume of a cuboid if its :

(i) Length is doubled, height is same and breadth is halved?

(ii) Length is doubled, height is doubled and breadth is same?

Answer

(i) Let ,

Length of cuboid = l

Breadth = b

Height = h

Volume of cuboid = lbh

Now,

Case(i)

Length become = 2l

Height = h

Breadth = $\frac{b}{2}$

Volume of cuboid = $2l \times \frac{b}{2} \times h = lbh = remain\ same$

(ii)

Case(ii)

Length become = 2l

Breadth = b

Height = 2h

Volume of cuboid = $2l \times b \times 2h = 4lbh = four\ times$

9. Question

Three cuboids of dimemsions 5 cm × 6cm × 7cm, 4cm × 7cm × 8cm and 2 cm×3 cm× 13 cm are melted and a cube is made. Find the side of cube.

Answer

Volume of First cuboids = 5 × 6 × 7 = 210 vm^3

Volume of second cuboids = 4 × 7 × 8 = 224 cm^3

Volume of third cuboids = 2 × 3 × 13 = 78 cm^3

Volume of cube = 210 + 224 + 78 = 512

Let side of cube = a

⇒ $a^3 = 512$

a = 8 cm

10. Question

Find the weight of solid rectangular iron piece of size 50 cm×40cm × 10 cm, if 1 cm^3 of iron weights 8 gm.

Answer

Given,

Dimension of rectangular iron piece = 50cm×40cm×10cm

Volume of solid rectangular = $50 \times 40 \times 10 = 20000\ cm^3$

Weight of 1 cm^3 iron = 8 gm

∴ weight of 20000 cm^3 iron = $8 \times 20000 = 160000gm = 160kg$

11. Question

How many wooden cubical blocks of side 25 cm can be cut from a log of wood of size 3 m by 75 cm by 50 cm, assuming that there is no wastage?

Answer

Given,

Dimensions of log of wood = 3m × 75 cm ×50cm

Side of cubical block = 25 cm

Hence,

No. of cubical block that can be made from wooden log = $\frac{volume\ of\ wooden\ block}{volume\ of\ cubical\ block}$

$= \frac{300\times75\times50}{25\times25\times25} = 72\ blocks$

12. Question

A cuboidal block of silver is 9 cm long, 4 cm broad and 3.5 cm in height. From it, beads of volume 1.5 cm^3 each are to be made. Find the number of beads that can be made from the block.

Answer

Given,

Dimensions of cuboidal block of silver = 9cm×4cm×3.5cm

Volume of beads made = 1.5 cm^3

So,

Number of beads can be made from cuboidal block = $\frac{9\times4\times3}{1.5} = 72\ beads$

13. Question

Find the number of cuboidal boxes measuring 2 cm by 3 cm by 10 cm which can be stored in a carton whose dimensions are 40 cm, 36 cm, and 24 cm.

Answer

Given,

Dimensions of cuboidal boxes = 2cm×3 cm×10 cm

Dimesions of carton = 40cm×36cm×24cm

So,

Number of boxes can be stored in carton = $\frac{volume\ of\ carton}{volume\ of\ one\ box} = \frac{40\times36\times24}{2\times3\times10} = 576\ boxes$

14. Question

A cuboidal block of solid iron has dimensions 50 cm, 45 cm, and 34 cm, How many cuboids of size 5 cm by 3 cm by 2 cm can be obtained from this block? Assume cutting causes no wastage.

Answer

Given,

Dimensions of cuboidal block of iron = 50cm×45cm×34cm

Size of small cuboids cutting from it = 5cm×3cm×2cm

So,

Number of small cuboids can be cut = $\frac{volume\ of\ large\ iron\ cuboid}{volume\ of\ small\ cuboids} = \frac{50\times45\times34}{5\times3\times2} = 2550\ blocks$

15. Question

A cube *A* has side thrice as long as that of cube *B*. What is the ratio of the volume of cube *A* to that of cube *B*?

Answer

Given,

Let side of cube B = X cm

Then, side of cube A = 3X cm

So,

$= \frac{volume\ of\ cube\ A}{volume\ of\ cube\ B} = \frac{(3x)^3}{(x)^3} = \frac{27x^3}{x^3} = \frac{27}{1}$

16. Question

An ice-cream brick measures 20 cm by 10 cm by 7 cm. How many such bricks can be stored in deep fridge whose inner dimensions are 100 cm by 50 cm by 42 cm?

Answer

Given,

Dimensions of ice cream brick = 20 cm× 10cm× 7cm

Dimensions of fridge = 100 cm×50cm×42 cm

So,

Number of bricks can be put in fridge = $\frac{volume\ of\ fridge}{volume\ of\ one\ ice\ brick} = \frac{100\times50\times42}{20\times10\times7} = 150\ ice\ cream$

17. Question

Suppose that there are two cubes, having edges 2 cm and 4 cm, respectively. Find the volumes V_1 and V_2 of the cubes and compare them.

Answer

Given,

Edge of one cube a_1 = 2 cm

Edge of second cube a_2 = 4 cm

Hence ,

$= v_1 = 2^3 = 8\ cm^3$

$= v_2 = 4^3 = 64\ cm^3$

$= v_2 = 8v_1$

18. Question

A tea-packet measures 10 cm×6 cm× 4 cm. How many such tea-packets can be placed in a cardboard box of dimensions 50 cm× 30cm× 0.2 m?

Answer

Given,

Dimensions of tea packet = 10 cm×6 cm×4cm

Dimension of cardboard box = 50cm×30cm×0.2 m

So,

Number of tea packets can be put in cardboard box = $\frac{volume\ of\ cardboard\ box}{volume\ of\ tea\ packet} = \frac{50\times30\times20}{10\times6\times4} = 125\ packets$

19. Question

The weight of a metal block of size 5 cm by 4 cm by 3 cm is 1 kg. Find the weight of a block of the same metal of size 15 cm by 8 cm by 3 cm.

Answer

Given,

Dimensions of metal block = 5cm×4cm×3cm

Weight of block = 1 kg

Volume of box = 5×4×3 = 60 cm^3

Dimension of new block = 15cm×8cm×3cm

Volume of new box = 15×8×3 = 360 cm^3

We know that,

$= 60\ cm^3 = 1\ kg$

$= 360\ cm^3 = 6 \times 60\ cm^3 = 6 \times 1 = 6\ kg$

20. Question

How many soap cakes can be placed in a box of size 56 cm×0.4 m× 0.25 m, if the size of a soap cake is 7 cm× 5cm× 2.5 cm?

Answer

Given,

Dimensions of box = 56cm×0.4m×0.25m

Dimensions of soap cake = 7cm×5cm×2.5cm

So,

Number of soap cakes can be placed in box $= \frac{volume\ of\ box}{volume\ of\ soap\ cake} = \frac{56 \times 40 \times 25}{7 \times 5 \times 2.5} = 640$

21. Question

The volume of acuboidal box is 48 cm^3. If its height and kength are 3 cm and 4 cm respectively, find its breadth.

Answer

Given,

Volume of cuboidal box = 48 cm^3

Height of box = 3 cm

Length of box = 4 cm

Let height of box = h cm

So,

$= l \times b \times h = 48$

$= h = \frac{48}{3 \times 4} = 4\ cm$

Exercise 21.2

1. Question

Find the volume in cubic metres (cu.m) of each of the cuboids whose dimensions are :

(i) length = 12 m, breadth = 10 m, height = 4.5 m

(ii) length = 14 m, breadth = 2.5m, height = 50 cm

(iii) length = 10m, breadth = 25 dm, height = 25 cm.

Answer

(i) Given,

Length of cuboid = 12 m

Breadth of cuboid = 10m

Height of cuboid = 4.5 m

So,

Volume of cuboid = $l \times b \times h = 12 \times 10 \times 4.5 = 540\ m^3$

(ii) Given,

length of cuboid = 14 m

breadth of cuboid = 2.5 m

height of cuboid = 50 cm = .50m

so.

Volume of cuboid = $l \times b \times h = 14 \times 2.5 \times .50 = 17.5\ m^3$

(iii) Given,

length of cuboid = 10m

breadth of cuboid = 25 dm = 2.5 m

height of cuboid = 25 cm = .25 m

so,

volume of cuboid = $l \times b \times h = 10 \times 2.5 \times .25 = 6.25\ m^3$

2. Question

Find the volume in cubic decimetre of each of the cubes whose side is

(i) 1.5 m

(ii) 2 dm 5 cm

Answer

(i) Given,

Side of cube = 1.5m = 15 dm

Volume of cube = 15^3 = 3375 dm^3

(ii) Given,

side of cube = 2dm 5 cm = 2.5 dm

volume of cube = 2.5^3 = 15.625 dm^3

3. Question

How much clay is dug out in digging a well measuring 3 m by 2 m by 5 m?

Answer

Given

Dimensions of well = 3m×2m×5m

So,

Volume of clay dug out from it = $l \times b \times h = 3 \times 2 \times 5 = 30\ m^3$

4. Question

What will be the height of a cuboid of volume 168 m^3, if the area of its base is 28 m^2?

Answer

Given,

Volume of cuboid = 168 m^3

Area of base = $l \times b = 28\, m^2$

Let height of cuboid = h m

So,

= $l \times b \times h = 168$

= $28 \times h = 168$

= $h = \frac{168}{28} = 6\, m$

So height of cuboid = 6 m

5. Question

A tank is 8 m long, 6 m broad and 2 m high. How much water can it contain?

Answer

Given,

Dimensions of a tank = 8m×6m×2m

So,

Capacity of tank = volume of tank = $l \times b \times h = 8 \times 6 \times 2 = 96\, m^3$ = 96000 litre

6. Question

The capacity of a certain cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its height and length are 10 m and 2.5 m respectively.

Answer

Given,

Capacity of cuboidal tank = 50000 litre = 50 m^3

Height of tank = 10 m

Length of tank = 2.5 m

Let breadth of tank = b m

So,

= $l \times b \times h = 50$

= $b = \frac{50}{10 \times 2.5} = 2$ m

Breadth of tank = 2 m

7. Question

A rectangular diesel tanker is 2 m long, 2 m wide and 40 cm deep. How many litres of diesel can it hold?

Answer

Given,

L = 2m

B = 2m

H = 40cm

Dimensions of rectangular diesel tank = 2m × 2m × 40 cm

So,

Amount of diesel it can hold = volume of tank = $2 \times 2 \times .40 = 1.6\, m^3$ = 1600 litre

8. Question

The length, breadth and height of a room are 5 m, 4.5 m and 3 m respectively. Find the volume of the air it contains.

Answer

Given,

L = 5m

B = 4.5m

H = 3m

Dimensions of a room = 5m × 4.5m × 3m

So,

Volume of air it contains = l × b × h = 5 × 4.5 × 3 = 67.5m^3

9. Question

A water tank is 3 m long, 2 m broad and 1 m deep. How many litres of water can it hold?

Answer

Given,

L = 2m

B = 2m

H = 40cm

Dimensions of water tank = 3m × 2m ×1m

So,

Capacity of water it can hold = $l \times b \times h = 3 \times 2 \times 1 = 6\, m^3 = 6000\, litre$

10. Question

How many planks each of which is 3 m long, 15 cm broad and 5 cm thick can be prepared from a wooden block 6 m long., 75 cm broad and 45 cm thick?

Answer

Given,

Dimensions of one plank = 3m × 15cm × 5cm = 300cm ×15cm×5cm

Dimensions of wooden block = 6m × 75cm ×45 cm = 600 cm×75 cm×45cm

So,

No. of planks can be made = $\frac{volume\ of\ wooden\ block}{volume\ of\ one\ plank} = \frac{600 \times 75 \times 45}{300 \times 15 \times 5}$ = 90 planks

11. Question

How many bricks each of size 25 cm× 10 cm× 8 cm will be required to build a wall 5 m long, 3 m high and

16 cm thick, assuming that the volume of sand and cement used in the construction is negligible?

Answer

Given,

Size of one brick = 25cm × 10cm × 8cm

Dimensions of wall = 5m × 3m × 16cm = 500 cm ×300 cm ×16cm

So,

Number of bricks needed to make the wall = $\frac{volume\ of\ wall}{volume\ of\ one\ brick} = \frac{500\times300\times16}{25\times10\times8} = 1200\ bricks$

12. Question

A village, having a population of 4000, required 150 litres water per head per day. It has a tank which is 20 m long, 15 m broad and 6 m high. For how many days will the water of this tank last?

Answer

Given,

Dimensions of water tank = 20m × 15m ×6m

Population of village = 4000

Water require per head per day = 150 litre

Total requirement of water per day = 150 ×4000 = 600000 litre

Volume of water tank = $20\times15\times6 = 1800\ m^3 = 1800000\ litre$

So,

$=no.\ of\ days\ till\ water\ in\ tank\ last = \frac{volume\ of\ tank}{(total\ requirement)} = \frac{1800000}{600000} = 3\ days$

13. Question

A rectangular field is 70 m long and 60 m broad. A well of dimensions 14 m × 8 m × 6 m is dug outside the field and the earth dug-out from this well is spread evenly on the field. How much will the earth level rise?

Answer

Given,

Dimension of rectangular field = 70m ×60m

Dimension of well = 14m × 8m ×6m

Amount of earth dug out from well = $14\times8\times6 = 672\ m^3$

So,

Rise in earth level of rectangular field = $\frac{70\times60}{672} = 0.16\ m = 16\ cm$

14. Question

A swimming pool is 250 m long and 130 m wide. 3250 cubic metres of water is pumped into it. Find the rise in the level of water.

Answer

Given,

Dimensions of swimming pool = 250 m ×130m

Volume of water pumped in it = 3250 m^3

So,

Rise in water level in pool = $\frac{volume\ of\ water\ pumped}{legth \times breadth} = \frac{3250}{250 \times 130} = 0.1\ m = 10\ cm$

15. Question

A beam 5 m long and 40 cm wide contains 0.6 cubic metre of wood. How thick is the beam?

Answer

Given,

Length of beam = 5 m

Width of beam = 40 cm = 0.4 m

Volume of wood in beam = 0.6 m^3

Let thickness of beam = h m

So,

= $l \times b \times h = 0.6$

= $h = \frac{0.6}{5 \times 0.4} = 0.3\ m$

16. Question

The rainfall on a certain day was 6 cm. How many litres of water fell on 3 hectares of field on that day?

Answer

Given,

Area of field = 3 hectare = 3×10000 m^2 = 30000 m^2

Depth of water on the field = 6 cm = $\frac{6}{100} = 0.06\ m$

∴ volume of water = area of field × depth of water

= 30000 × 0.06 = 1800 m^3

= $1\ m^3 = 1000\ litre$

∴ $1800\ m^3 = 1800 \times 1000 = 18 \times 10^5\ litre$

17. Question

An 8 m long cuboidal beam of wood when sliced produces four thousand 1 cm cubes and there is no wastage of wood in this process. If one edge of the beam is 0.5 m, find the third edge.

Answer

Given,

Length of cuboidal beam = 8 m

One edge of beam = 0.5 m

Let third edge of beam = h m

No. of cubes of side 1 cm (.01 m) produced = 4000

So,

= $volume\ of\ beam = no.of\ cubes \times volume\ of\ each\ cube$

= $8 \times 0.5 \times h = 4000 \times (.01)^3$

= $h = \frac{4000 \times 0.000001}{8 \times 0.5} = 0.001\ m$

Length of third edge = 0.001 m

18. Question

The dimensions of a metal block are 2.25 m by 1.5 m by 27 cm. It is melted and recast into cubes, each of the side 45 cm. How many cubes are formed?

Answer

Given,

Dimensions of metal block = 2.25m × 1.5m ×27 cm = 2.25m × 1.5m × .27m

Side of each cube formed = 45 cm = 0.45 m

So,

Number of cubes can formed = $\frac{volume\ of\ metal\ block}{volume\ of\ one\ cube} = \frac{2.25\times1.5\times.27}{0.45\times0.45\times0.45} = 10$ cubes

19. Question

A solid rectangular piece of iron measures 6 m by 6 cm by 2 cm. Find the weight of this piece, if 1 cm^3 of iron weighs 8 gm.

Answer

Given,

Dimensions of solid rectangular piece = 6m × 6cm ×2 cm

Volume of rectangular iron = $600\ cm\times6cm\times2cm = 7200\ cm^3$

Weight of 1 cm^3 iron = 8 gm

$\therefore weight\ of\ 7200\ cm^3 = 7200\times8 = 57600\ gm = 57.6\ kg$

20. Question

Fill in the blanks in each of the following so as to make the statement true :

(i) 1 m^3= ..... cm^3

(ii) 1 litre = ...cubic decimetre

(iii) 1 kl = ... m^3

(iv) The volume of a cube of side 8 cm is .....

(v) The volume of a wooden cuboid of length 10 cm and breadth 8 cm is 4000 cm^3. The height of the cuboid is ...50.... cm.

(vi) 1 cu. Dm = ... cu. Mm

(vii) 1 cu. Km = ... cu. M

(viii) 1 litre = ...cu. Cm

(ix) 1 ml = ... Cu. Cm

(x) 1 kl = ... cu. Dm = ... cu. Cm.

Answer

(i) $1\ m^3 = 1\times(100\times100\times100) = 10^6\ cm^3\ [1\ m = 100\ cm]$

(ii) $1\ litre = 1000\ cm^3 = 1000\times(0.1\times0.1\times0.1)\ dm^3 = 1\ dm^3\ [1cm = 0.1\ dm]$

(iii) $1\ kl = 1000\ litre\ = 1\ m^3\ [1m^3 = 1000\ litre]$

(iv) Side of cube = 8 cm

Volume of cube = $8^3 = 512\ cm^3$

(v) Volume of cuboid = 4000 cm^3

Length = 10 cm , breadth = 8cm

Then,

Height = $\frac{volume}{length \times breadth} = \frac{4000}{10 \times 8} = 50\ cm$

(vi) $1\ cu.dm = 1\ dm^3 = 1 \times 10 \times 10 \times 10 = 10^3\ cm^3\ [1dm = 10cm]$

$= 10^3 \times 10 \times 10 \times 10\ mm^3 = 10^6\ mm^3\ [1\ cm = 10\ mm\ ]$

(vii) $1\ km^3 = 1000 \times 1000 \times 1000 = 10^9\ m^3\ [1\ km = 1000m]$

(viii) $1\ litre = 1000\ cm^3 = 10^3 cm^3$

(ix) $1\ ml = \frac{1}{1000} litre = \frac{1}{1000} \times 1000 = 1cm^3\ \left[1ml = \frac{1}{1000} litre\right]$

(x) $1\ kl = 1 \times 1000\ litre = 1m^3 = 1 \times (10 \times 10 \times 10) dm^3\ [1m = 10dm]$

$1\ kl = 10^3\ dm^3\ = 1000 \times 1000 = 10^6 cm^3$

Exercise 21.3

1. Question

Find the surface area of a cuboid whose

(i) length = 10 cm, breadth = 12 cm, height = 14 cm

(ii) length = 6 dm, breadth = 8 dm, height = 10 dm

(iii) length = 2m, breadth = 4 m, height = 5 m

(iv) length = 3.2 m, breadth = 30 dm, height = 250 cm.

Answer

(i) Given,

Length = 10 cm

Breadth = 12 cm

Height = 14 cm

So,

Surface area of cuboid = $2(lb \times bh \times hl) = 2(10 \times 12 + 12 \times 14 + 14 \times 10)$

$= 2(120 + 168 + 140) = 2 \times 428 = 856\ cm^2$

(ii) Given,

Length = 6 dm

Breadth = 8 dm

Height = 10 dm

So,

Surface area of cuboid = $2(lb \times bh \times hl) = 2(6 \times 8 + 8 \times 10 + 10 \times 6)$

$= 2(48 + 80 + 60) = 2 \times 188 = 376\ dm^2$

(iii) Given,

Length = 2m

Breadth = 4m

Height = 5m

So,

Surface area of cuboid = $2(lb \times bh \times hl) = 2(2 \times 4 + 4 \times 5 + 5 \times 2)$

= $2(8 + 20 + 10) = 2 \times 38 = 76\, m^2$

(iv) Given,

length = 3.2 m= 32 dm

breadth = 30 dm

height = 250 cm= 25 dm

so,

surface area of cuboid = $2(lb \times bh \times hl) = 2(32 \times 30 + 30 \times 25 + 25 \times 32)$

= $2(960 + 750 + 800) = 2 \times 2510 = 5020\, dm^2$

2. Question

Find the surface area of a cube whose edge is

(i) 1.2 m

(ii) 27 cm

(iii) 3 cm

(iv) 6 m

(v) 2.1 m

Answer

(i) We have,

Edge of cube = 1.2 m

Surface area of cube = $6 \times side^2 = 6 \times 1.2^2 = 6 \times 1.44 = 8.64\, m^2$

(ii) We have,

Edge of cube = 27 cm

Surface area of cube = $6 \times side^2 = 6 \times 27^2 = 6 \times 729 = 4374\, cm^2$

(iii) We have,

Edge of cube = 3 cm

Surface area of cube = $6 \times side^2 = 6 \times 9 = 54\, cm^2$

(iv) We have,

Edge of cube = 6 m

Surface area of cube = $6 \times side^2 = 6 \times 6^2 = 216\, m^2$

(v) We have,

Edge of cube = 2.1 m

Surface area of cube = $6 \times side^2 = 6 \times 4.41 = 26.46\, m^2$

3. Question

A cuboidal box is 5 cm by 5 cm by 4 cm. Find its surface area.

Answer

Given,

Dimensions of cuboidal box = 5cm × 5cm × 4cm

Surface area of cuboid = $2(lb \times bh \times hl) = 2(5 \times 5 + 5 \times 4 + 5 \times 5) = 2 \times 65 = 130 cm^2$

4. Question

Find the surface area of a cube whose volume is

(i) 343 m^3

(ii) 216 dm^3

Answer

(i) Given,

Volume of cube = 343 m^3

Side of cube = a = $\sqrt[3]{343} = 7\ m$

So,

Surface area of cube = $6 \times side^2 = 6 \times 49 = 294\ m^2$

(ii) Given,

Volume of cube = 216 dm^3

Side of cube = $\sqrt[3]{216} = 6\ dm$

So,

Surface area of cube = $6 \times side^2 = 6 \times 36 = 216\ dm^2$

5. Question

Find the volume of a cube whose surface area is

(i) 96 cm^2

(ii) 150 m^2

Answer

(i) Given,

Surface area of cube = 96 cm^2

= $6 \times side^2 = 96$

= $side^2 = \frac{96}{6} = 16$

= $side = \sqrt{16} = 4\ cm$

So,

Volume of cube = $4^3 = 64\ cm^3$

(ii) Given,

Surface area of cube = 150 m^2

= $6 \times side^2 = 150$

= $side^2 = \frac{150}{6} = 25$

$=side = \sqrt{25} = 5\,m$

So,

Volume of cube = 5^3 = 125 m^3

6. Question

The dimensions of a cuboid are in the ratio 5 : 3 : 1 and its total surface area is 414 m^2. Find the dimensions.

Answer

Given,

Ratio of dimensions of cuboid = 5:3:1

Total surface area of cuboid = 414 m^2

Let dimensions are = $5x \times 3x \times x$

So,

$= 2(lb \times bh \times hl) = 414$

$= 2(15x^2 + 3x^2 + 5x^2) = 414$

$= 2 \times 23x^2 = 414$

$= x^2 = \frac{414}{46} = 9$

$= x = \sqrt{9} = 3$

So,

Dimensions are = $5x = 5 \times 3 = 15m$

$= 3x = 3 \times 3 = 9m$

$= x = 3\,m$

7. Question

Find the area of the cardboard required to make a closed box of length 25 cm, 0.5 m and height 15 cm.

Answer

Given,

Dimensions of closed box = 25 cm×0.5 m× 15 cm = 25 cm ×50 cm × 15 cm

So,

Area of cardboard required = $2(lb \times bh \times hl) = 2(25 \times 50 + 50 \times 15 + 15 \times 25)$

$= 2(1250 + 750 + 375) = 2 \times 2375 = 4750\,cm^2$

8. Question

Find the surface area of a wooden box whose shape is of a cube, and if the edge of the box is 12 cm.

Answer

Given,

Edge of a cubic wooden box = 12 cm

Surface area of cubic wooden box = $6 \times side^2 = 6 \times 12 \times 12 = 864\,cm^2$

9. Question

The dimensions of an oil tin are 26 cm× 26 cm× 45 cm. Find the area of the tin sheet required for making 20

such tins. If 1 square metre of the tin sheet costs Rs. 10, find the cost of tin sheet used for these 20 tins.

Answer

Given,

Dimensions of oil tin are = 26 cm × 26 cm × 45 cm

Then,

Area of tin sheet required for making one oil tin = total surface area of oil tin

$= 2(lb \times bh \times hl) = 2(26 \times 26 + 26 \times 45 + 45 \times 26) = 2(676 + 1170 + 1170)$

$= 2 \times 3016 = 6032\ cm^2$

Area of tin sheet required for 20 oil tins $= 20 \times 6032 = 120640\ cm^2 = 12.064\ m^2$

So,

Cost of 1 m^2 tin sheet = Rs. 10

∴cost of 12.064 m^2 tin sheet $= 10 \times 12.064 = Rs.\ 120.64$

10. Question

A classroom is 11 m long, 8 m wide and 5 m high. Find the sum of the areas of its floor and the four walls (including doors, windows etc.)

Answer

Given,

Dimensions of class room = 11m × 8 m ×5 m

Where,

Length = 11m , Breadth = 8m , Height = 5 m

Ten,

Area of floor $= length \times breadth = 11 \times 8 = 88 m^2$

Area of four walls (including doors & windows) $= 2(l \times h + b \times h) = 2(11 \times 5 + 8 \times 5)$

$= 2(55 + 40) = 190\ m^2$

∴ Sum of areas of floor and four walls = area of floor + area of four walls

= 88 + 190 = 278 m^2

11. Question

A swimming pool is 20 m long 15 m wide and 3 m deep. Find the cost of repairing the floor and wall at the rate of Rs. 25 per square metre.

Answer

Given,

Dimensions of swimming pool are = 20m × 15m ×3m

Where,

Length = 20m , Breadth = 15m , Height = 3m

Then,

Area of floor & walls of swimming pool $= l \times b + 2(l \times h + b \times h)$

$= 20 \times 15 + 2(20 \times 3 + 15 \times 3) = 300 + 2(60 + 45) = 300 + 210 = 510\ m^2$

So,

Cost of repairing 1 m^2 area = Rs.25

∴ Cost of repairing 510 m^2 = $510 \times 25 = Rs.\ 12750$

12. Question

The perimeter of a floor of a room is 30 m and its height is 3 m. Find the area of four walls of the room.

Answer

Given,

Perimeter of floor = 30 m

Height of floor = 3 m

So,

= $2(l + b) = 30$

= $l + b = \frac{30}{2} = 15\ m$

Area of four walls of room = $2(l \times h + b \times h) = 2h(l + b) = 2 \times 3 \times 15 = 90 m^2$

13. Question

Show that the product of the areas of the floor and two adjacent walls of a cuboid is the square of its volume.

Answer

Given,

Let length of cuboid = $l\ cm$

Let breadth of cuboid = b cm

Let height of cuboid = h cm

So,

Area of floor = $l \times b = lb\ cm^2$

Product of areas of two adjacent walls = $(l \times h) \times (b \times h) = lbh^2\ cm^4$

Product of areas of floor and two adjacent walls = $lb \times lbh^2\ cm^6$

= $l^2 b^2 h^2 = (lbh)^2\ cm^6$

= $(volume\ of\ cuboid)^2\ Proved\ ..$

14. Question

The walls and ceiling of a room are to be plastered. The length, breadth nad height of the room are 4.5 m, 3 m and 350 cm, respectively. Find the cost of plastering at the rate of Rs. 8 per square metre.

Answer

Given,

Length of room = 4.5 m

Breadth of wall = 3 m

Height of wall = 350 cm = $\frac{350}{100} = 3.5\ m$

So,

Area of ceiling + area of walls = $l \times b + 2(l \times h + b \times h)$

$= 4.5\times3 + 2(4.5\times3.5 + 3\times3.5) = 13.5 + 2(15.75 + 10.5)$

$=13.5 + 52.5 = 66\,m^2$

Cost of plastering 1 m^2 area = Rs.8

∴ Cost of plastering 66 m^2 area = 66 ×8 =Rs.528

15. Question

A cuboid has total surface area of 50 m^2 and lateral surface area os 30 m^2. Find the area of its base.

Answer

Given,

Total surface area of cuboid = 50 m^2

Lateral surface area of cuboid = 30 m^2

So,

$= 2(l\times h + b\times h) = 30$

And,

$= 2(l\times b) + 2(l\times h + b\times h) = 50$

$= 2(l\times b) + 30 = 50$

$=2(l\times b) = 50 - 30 = 20$

$= l\times b = \frac{20}{2} = 10$

So,

Area of base = $l\times b = 10\,m^2$

16. Question

A classroom is 7 m long, 6 m broad and 3.5 m high. Doors and windows occupy an area of 17 m^2. What is the cost of white washing the walls at the rate of Rs. 1.50 per m^2.

Answer

Given,

Dimensions of class room = 7m × 6m ×3.5m

Where,

Length = 7m, Breadth = 6m , Height = 3.5 m

Area of four walls (including doors & windows) = $2(l\times h + b\times h)$

$= 2(7\times3.5 + 6\times3.5) = 91\,m^2$

Then,

Area of walls without doors & windows =

= area including doors & windows – area occupied by doors & windows

Area of only walls = 91 – 17 = 74 m^2

So,

Cost of white washing 1 m^2 area of walls = Rs.1.50

∴ Total cost of white washing = 74 × 1.50 = Rs.111

17. Question

The central hall of a school is 80 m long and 8 m high. It has 10 doors each of size 3 m ×1.5 m and 10 windows each of size 1.5 m× 1 m. If the cost od white washing the walls of the hall at the rate of Rs. 1.20 per m^2 is Rs. 2385.60, find the breadth of the hall.

Answer

Given,

Dimensions of central hall of a school = Length = 80 m , height = 8m

Let breadth of hall = b metre

So,

Area of each door = 3m × 1.5 m = 4.5 m^2

∴ Area of 10 doors = 10 × 4.5 = 45 m^2

Area of each window = 1.5m × 1m = 1.5 m^2

∴ Area of 10 windows = 10 ×1.5 = 15 m^2

Area occupied by doors and windows = 45 + 15 = 60 m^2

Area of the walls of the hall including doors and windows = $2(l \times h + b \times h)$

= 2(80×8+b×8) = 2(640+8b) m^2

Then,

Area of only walls = (area of walls including doors & windows – area occupied by doors & windows)

= $[2(640 + 8b) - 60] = [1280 + 16b - 60\,] = (\,1220 + 16b)\,m^2$

Total cost of white washing = Rs.2385.60

Given, Rate of white washing = Rs.1.20 per m^2

So,

= $1.20 \times (area\ of\ walls\ only\,) = \ 2385.60$

= $1.20(1220 + 16b) = \ 2385.60$

=$1220 + 16\,b = \frac{2385.60}{1.20} = 1988$

= $16b = 1988 - 1220 = 768$

= $b = \frac{768}{16} = 48$

Hence,

Breadth of hall = 48 m

Exercise 21.4

1. Question

Find the length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high.

Answer

Given,

Length of room = 12 m

Breadth = 9m

Height = 8m

So,

Length of longest rod that can be placed in room = diagonal of room (cuboid) =
$\sqrt{l^2+b^2+h^2} = \sqrt{12^2+9^2+8^2} = \sqrt{144+81+64} = \sqrt{289} = 17\,m$

2. Question

If *V* is the volume of a cuboid of dimensions *a, b, c* and *S* is its surface area, then prove that

$\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Answer

Given,

V = volume of cuboid

S = surface area of cuboid

= a,b,c = dimensions of cuboid

So,

S =$2(ab+bc+ca)$

V = abc

$= \frac{S}{V} = \frac{[2(ab+bc+ca)]}{abc} = 2\left[\left(\frac{ab}{abc}\right)+\left(\frac{bc}{abc}\right)+\left(\frac{ca}{abc}\right)\right] = 2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$= \frac{1}{V} = \frac{2}{s}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\ Proved..$

3. Question

The areas of three adjacent faces of a cuboid are *x, y,* and *z*. If the volume is *V*, prove that $V_2 = xyz$.

Answer

Given,

Areas of three faces of cuboid =x, y, z

Let length of cuboid =$l\,, breadth = b\,, height = h$

So,

= $x = l{\times}b$

= $y = b{\times}h$

= $z = h{\times}a$

Or we can write ,

= $xyz = l^2b^2h^2 \ldots\ldots\ldots\ldots (i)$

If 'V' is volume of cuboid = V =lbh

= $V^2 = l^2b^2h^2 = xyz \ldots\ldots\ldots\ldots\ldots\ldots\ldots from\ (i)$

= $V^2 = xyz\ Proved.$

4. Question

A rectangular water reservoir contains 105 m^3 of water. Find the depth of the water in the reservoir if its base measures 12 m by 3.5 m.

Answer

Given,

Capacity of water reservoir = 105 m^3

Length of base of reservoir = 12 m

Width of base = 3.5 m

Let depth of reservoir = h m

So,

= $l \times b \times h = 105$

= $12 \times 3.5 \times h = 105$

= $h = \frac{105}{12 \times 3.5} = 2.5$ m

Depth of reservoir = 2.5 m

5. Question

Cubes *A, B, C* having edges 18 cm, 24 cm and 30 cm respectively are melted and moulded into a new cube *D*. Find the edge of the bigger cube *D.*

Answer

Given,

Edge length of cube A = 18 cm

Edge length of cube B = 24 cm

Edge length of cube C = 30 cm

So,

Volume of cube A = $v_1 = 18^3 = 5832\ cm^3$

Volume of cube B = $v_2 = 24^3 = 13824\ cm^3$

Volume of cube C = $v_3 = 30^3 = 27000\ cm^3$

Total volume of three cubes = 5832 + 13824 + 27000 = 46656 cm^3

Let ‘a’ be the length of edge of new cube formed .

= $a^3 = 46656$

= $a = \sqrt[3]{46656} = 36$

So,

Edge of bigger cube = 36 cm

6. Question

The breadth of a room is twice its height, one half of its length and the volume of the room is 512 cu. Dm. Find its dimensions.

Answer

Given,

Breadth of room is twice of its height = $b = 2h\ or\ h = \frac{b}{2} \ldots . . (i)$

Breadth is one half of length = $b = \frac{1}{2} l\ or\ l = 2b \ldots \ldots \ldots \ldots (ii)$

Volume of the room = $lbh = 512\ dm^3$

$= 2b \times b \times \frac{b}{2} = 512$

$= b^3 = 512$

$= b = \sqrt[3]{512} = 8$

Hence,

Breadth of cube = b = 8 dm

Length of cube = 2b = 2×8 = 16 dm

Height of cube $= \frac{b}{2} = \frac{8}{2} = 4\ dm$

7. Question

A closed iron tank 12 m long, 9 m wide and 4 m deep is to be made. Determine the cost of iron sheet used at the rate of Rs. 5 per metre sheet, sheet being 2 m wide.

Answer

Given,

Length of tank = 12 m

Width of tank = 9m

Depth of tank = 4m

So,

Area of sheet required = total surface area of tank

$= 2(lb \times bh \times hl) = 2(12 \times 9 + 9 \times 4 + 4 \times 12) = 2(108 + 36 + 48)$

$= 2 \times 192 = 384\ m^2$

Let l^1 *be the length and* b^1 *be the breadth of sheet.*

$= b^1 = 2\ m\ given$

$= l^1 \times b^1 = 384$

$= l^1 = \frac{384}{2} = 192\ m$

Then,

Cost of iron sheet at rate Rs.5 per metre = 5×192 = Rs. 960

8. Question

A tank open at the top is made of iron sheet 4 m wide. If the dimensions of the tank are 12m×8m×6m, find the cost of iron sheet at Rs. 17.50 per metre.

Answer

Given,

Dimensions of tank = 12m × 8m × 6m

Where length = 12m , breadth = 8m , height = 6m

Area of sheet required for making the tank = total surface area of tank with one top open

$= l \times b + 2(l \times h + b \times h) = 12 \times 8 + 2(12 \times 6 + 8 \times 6) = 96 + 240 = 336\ m^2$

Let l^1 *be the length of iron sheet and* b^1 *be the breadth of iron sheet.*

$= b^1 = 4m\ given,$

$= l^1 \times b^1 = 336$

$= l^1 = \frac{336}{4} = 84m$

So,

Cost of iron sheet at rate Rs.17.50 per metre = 17.50 ×84 = Rs.1470

9. Question

Three equal cubes are placedadjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

Answer

Given,

Let edge length of three equal cubes = a

Then,

Sum of surface area of 3 cubes = $3 \times 6a^2 = 18a^2$

When these cubes are placed in a row adjacently they form a cuboid.

Length of new cuboid formed = a+a+a = 3a

Breadth of cuboid = a

Height of cuboid = a

Total surface area of cuboid = $2(lb \times bh \times hl) = 2(3a \times a + a \times a + a \times 3a)$

$= 2(3a^2 + a^2 + 3a^2) = 2 \times 7a^2 = 14a^2$

Hence,

$= \frac{Total\ surface\ area\ of\ new\ cuboid}{sum\ of\ surface\ area\ of\ 3\ cuboids} = \frac{14}{18} = \frac{7}{9} = 7:9$

10. Question

The dimensions of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room; each door measures 2.5 m by 1.2 m and each window 1.5 m by 1 m. Find the cost of painting the walls at Rs. 3.50 per square metre.

Answer

Given,

Dimensions of room = 12.5m ×9m × 7m

Dimensions of each door = 2.5m ×1.2m

Dimensions of each window = 1.5m ×1m

Area of four walls including doors and windows = $2(l \times h + b \times h) = 2(12.5 \times 7 + 9 \times 7)$

$= 2(87.5 + 63) = 2 \times 150.5 = 301\ m^2$

Area of 2 doors and 4 windows = $2(2.5 \times 1.2) + 4((1.5 \times 1) = 6 + 6 = 12\ m^2$

Area of only walls = 301 – 12 = 289 m^2

Hence,

Cost of painting walls at rate Rs.3.50 per square metre = Rs.(3.50 ×289) = Rs.1011.50

11. Question

A field is 150 m long and 100 m wide. A plot (outside the field) 50 m long and 30 m wide is dug to a depth of 8 m and the earth taken out from the plot is spread evenly in the field. By how much is the level of field

raised?

Answer

Given,

Length of field = 150m

Width of field = 100m

Area of field = 150m × 100m = 15000 m^2

Length of plot = 50 m

Breadth = 30 m

Depth upto which it dug = 8 m

So volume of earth taken out from it = 50×30×8 = 12000 m^3

let raise in earth level of field on which it spread = h metre

so,

= $15000 \times h = 12000$

= $h = \frac{12000}{15000} = 0.8\, m$

The level of field raised by 0.8 metre

12. Question

Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Answer

Given,

Volume of each cube = 512 cm^3

Let length of edge of each cube = a cm

So,

= $a^3 = 512$

= a= $\sqrt[3]{512} = 8\, cm$

When these two cubes are joined end to end a cuboid is formed :

Length of cuboid = 8+8 = 16 cm

Breadth = 8 cm

Height = 8 cm

Surface area of resulting cuboid = $2(lb \times bh \times hl) = 2(16 \times 8 + 8 \times 8 + 8 \times 16)$

= $2(128 + 64 + 128) = 2 \times 320 = 640\, cm^2$

13. Question

Three cubes whose edges measure 3 cm, 4 cm, and 5 cm respectively are melted to form a new cube. Find the surface area of the new cube formed.

Answer

Given,

Edge of three cubes are respectively = 3cm , 4 cm , 5 cm

So,

Sum of volume of these cubes = $3^3 + 4^3 + 5^3 = 27 + 64 + 125 = 216\ cm^3$

After melted these cubes a new cube is formed.

Let edge length of this new cube = a cm

So,

= $a^3 = 216$

= $a = \sqrt[3]{216} = 6\ cm$

Edge of new cube = 6 cm

Surface area of new cube = $6 \times a^2 = 6 \times 36 = 216\ cm^2$

14. Question

The cost of preparing the walls of a room 12 m long at the rate of Rs. 1.35 per square metre is Rs. 340.20 and the cost of matting the floor at 85 paise per square metre is Rs. 91.80. Find the height of the room.

Answer

Given,

Length of room = 12 m

Let width of room = b metre

Let height of room = h metre

Now,

Area of floor = 12b m^2

Cost of matting floor @rate 85 paise per square metre = Rs.91.80

= $12b \times .85 = 91.80$

= $b = \frac{91.80}{12 \times .85} = 9\ m$

Breadth of room = 9 m

Area of 4 walls = $2(l \times h + b \times h) = 2(12h + 9h) = 42h\ m^2$

Cost of preparing walls at rate Rs.1.35 per square metre = Rs.340.20

= $42h \times 1.35 = 340.20$

= $h = \frac{340.20}{42 \times 1.35} = 6\ m$

Height of room = 6 m

15. Question

The length of a hall is 18 m and the width 12 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the wall.

Answer

Given,

Length of hall = 18 m

Width of hall = 12 m

Let height of hall = h metre

Then,

Sum of area of floor & flat roof = $l \times b + l \times b = 12 \times 18 + 12 \times 18 = 432\ m^2$

Sum of area of 4 walls = $2(l\times h + b\times h) = 2(18h + 12h) = \ 60h\ m^2$

Now,

= $60h = 432 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . given$

= $h = \frac{432}{60} = 7.2\ m$

Height of hall = 7.2 metre

16. Question

A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube.

Answer

Given,

Edge of metal cube = 12 cm

Edge of smaller two cubes = 6 cm , 8 cm

Let edge of third cube = a cm

So,

Volume of metal cube = sum of volume of three small cubes

= $12^3 = 6^3 + 8^3 + a^3$

= $a^3 = 1728 - (216 + 512) = \ 1728 - 728 = 1000$

= $a = \ \sqrt[3]{1000} = 10\ cm$

So,

Edge of third smaller cube would be = 10 cm

17. Question

The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the gall, if each person required 150 m^3 of air?

Answer

Given,

Dimensions of cinema hall are = 100m ×50m × 18m

Where , length = 100m , breadth = 50m , height = 18 m

Each person air requirement = 150 m^3

Now,

Volume of cinema hall = $lbh = 100\times 50\times 18 = 90000\ cm^3$

So,

Number of person can sit in cinema hall = $\frac{volume\ of\ hall}{volume\ of\ air\ required\ by\ one\ person} = \frac{90000}{150} = 600$

18. Question

The external dimensions of a closed wooden box are 48 cm, 36 cm, 30 cm. The box is made of 1.5 cm thick wood. How many bricks of size 6 cm× 3 cm× 0.75 cm can be put in this box?

Answer

Given,

External dimensions of wooden box = 48cm×36cm×30cm

Dimensions of bricks = 6cm×3cm×0.75cm

Thickness of wood = 1.5 cm

So,

Internal dimensions of box = $48-(2\times1.5)cm+36-(2\times1.5)cm+30-(2\times1.5)cm$

= 45cm ×33 cm× 27 cm

Hence,

Number of bricks can be put in box = $\frac{internal\ volume\ of\ box}{volume\ of\ one\ brick}=\frac{45\times33\times27}{6\times3\times0.75}=2970$ bricks

19. Question

The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

Answer

Given,

Ratio of dimensions of rectangular box = 2:3:4

Let length of box = 2x m

Let breadth = 3x m

Let height = 4x m

Area of sheet of paper required for covering it = total surface area of cuboid

= $2(lb\times bh\times hl)=2(6x^2+12x^2+8x^2)=\ 2\times26x^2=52x^2\ m^2$

Cost of covering it with sheet of paper at Rs.9.50 /m^2= $52x^2\times9.50=Rs.494\ x^2$

Cost of covering it with sheet of paper at rate Rs.8/m^2 = $52x^2\times8=Rs.416\ x^2$

= $494x^2-416x^2=1248\ Given$

= $78x^2=1248$

= $x^2=\frac{1248}{78}=16$

= $x=\sqrt{16}=4$

So,

Length of box = 2x = 2×4 = 8 m

Breadth of box = 3x = 3×4 = 12 m

Height of box = 4x = 4×4 = 16 m