1. **(d)** $\frac{3\pi}{2} < 5 < \frac{5\pi}{2}$

$\Rightarrow \sin^{-1}(\sin 5) = 5 - 2\pi$

Given $\sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow x^2 - 4x + 4 < 9 - 2\pi$

$\Rightarrow (x - 2)^2 < 9 - 2\pi$

$\Rightarrow -\sqrt{9-2\pi} < x - 2 < \sqrt{9-2\pi}$

$\Rightarrow 2-\sqrt{9-2\pi} < x < 2+\sqrt{9-2\pi}$

2. **(c)** We know that $\cot A > 1$ if $0 < A < \frac{\pi}{4}$

and $\cot A < 1$ if $\frac{\pi}{4} < A < \frac{\pi}{2}$

$$\tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A) = \pi + \tan^{-1}\frac{\cot A + \cot^3 A}{1 - \cot^4 A},$$

If $0 < A < \frac{\pi}{4}$

and $\tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

$$= \tan^{-1} \frac{\cot A + \cot^3 A}{1 - \cot^4 A} \text{ if } \frac{\pi}{4} < A < \frac{\pi}{2}$$

Also, $\frac{\cot A + \cot^3 A}{1 - \cot^4 A} = \frac{\cot A \operatorname{cosec}^2 A . \sin^4 A}{\sin^4 A - \cos^4 A}$

$$= \frac{\sin A \cos A}{(\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)}$$

$$= -\frac{\sin 2A}{2 \cos 2A} = -\frac{1}{2} \tan 2A$$

Hence,

$$\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A),$$

$$= \begin{cases} \pi & \text{if } 0 < A < \frac{\pi}{4} \\ 0 & \text{if } \frac{\pi}{4} < A < \frac{\pi}{2} \end{cases}$$

[Since, $\tan^{-1}(-x) = -\tan^{-1} x$]

3. **(b)** We have, $\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right| < \frac{\pi}{3}$

$$\Rightarrow -\frac{\pi}{3} < \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) < \frac{\pi}{3}$$

$$\Rightarrow 0 \le \cos^{-1} \frac{1-x^2}{1+x^2} < \frac{\pi}{3} \Rightarrow \frac{1}{2} < \frac{1-x^2}{1+x^2} \le 1$$

$$\Rightarrow 1 + x^2 < 2(1 - x^2) \le 2(1 + x^2)$$

$$\Rightarrow 0 \le x^2 < \frac{1}{3} \Rightarrow -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

4. **(d)** We have

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = \frac{\pi}{2}$$

Put x = tan θ and y = tan φ, we get

$$\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+\cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)=\frac{\pi}{2}.$$

$$\Rightarrow \cos^{-1}(\cos 2\theta) + \cos^{-1}(\cos 2\varphi) = \frac{\pi}{2}$$

$$\Rightarrow 2(\theta+\phi)=\frac{\pi}{2} \Rightarrow \theta+\phi=\frac{\pi}{4}$$

So, $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right)=\tan^{-1}1$$

$$\Rightarrow x + y = 1 - xy \Rightarrow x + y + xy = 1$$

5. **(c)** $\sin^{-1}(\log[x])$ is defined if $-1\le \log[x]\le 1$ and $[x]>0$

$$\frac{1}{e}\le[x]\le e \qquad [x] = 1, 2 \qquad x\in[1,3)$$

Again, $\log(\sin^{-1}[x])$ is defined if

$\sin^{-1}[x]>0$ and $-1\le[x]\le 1$

$\Rightarrow [x]>0$ and $-1\le[x]\le 1 \Rightarrow 0<[x]\le 1$

$x\in[1,2)$

Domain of $f(x)=[1,2)$

For $1\le x<2$, $[x]=1$

$$f(x)=\sin^{-1}0+\log\frac{\pi}{2}=\log\frac{\pi}{2}, \forall x\in[1,2)$$

Range of $f(x)=\left\{\log\frac{\pi}{2}\right\}$

6. **(b)** We have, $\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots$

$$+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)$$

$$= \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2 a_3}\right) + \ldots$$

$$+ \tan^{-1}\left(\frac{a_n - a_{n-1}}{1 + a_{n-1} a_n}\right)$$

$$= \left(\tan^{-1} a_2 - \tan^{-1} a_1\right) + \left(\tan^{-1} a_3 - \tan^{-1} a_2\right) + \ldots.$$
$$+ \left(\tan^{-1} a_n - \tan^{-1} a_{n-1}\right)$$

$$= \tan^{-1} a_n - \tan^{-1} a_1 = \tan^{-1}\left(\frac{a_n - a_1}{1 + a_n a_1}\right)$$

$$= \tan^{-1}\left(\frac{(n-1)d}{1 + a_1 a_n}\right)$$

$$\therefore \tan\left[\tan^{-1}\left(\frac{d}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{d}{1 + a_2 a_3}\right) + \ldots..\right.$$

$$\left.\ldots + \tan^{-1}\left(\frac{d}{1 + a_{n-1} a_n}\right)\right] = \frac{(n-1)d}{1 + a_1 a_n}$$

7. **(c)** Let $S_\infty = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots.$

$\therefore T_n = \cot^{-1} 2n^2$

$$= \tan^{-1} \frac{1}{2n^2}$$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}\right)$$

$= \tan^{-1}(2n + 1) - \tan^{-1}(2n - 1)$

$$\therefore S_n = \sum_{n=1}^{\infty} \{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\} \qquad = \tan^{-1} \infty - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

8. **(d)** Let $\cos^{-1} x + \cos^{-1} y = \frac{2\pi}{7}$

$$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1} x\right) + \left(\frac{\pi}{2} - \sin^{-1} y\right) = \frac{2\pi}{7}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \pi - \frac{2\pi}{7} = \frac{5\pi}{7}.$$

9. **(a)** Since, $1 \pm \sin x = \left(\cos\frac{x}{2} \pm \sin\frac{x}{2}\right)^2$

$$\therefore \cot^{-1}\left\{\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right\}$$

$$= \cot^{-1}\left[\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) + \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) - \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}\right]$$

$$= \cot^{-1}\left\{-\cot\frac{x}{2}\right\} = \cot^{-1}\left\{\cot\left(\pi - \frac{x}{2}\right)\right\} = \pi - \frac{x}{2}$$

10. **(d)** Let $\sin^{-1} a = x$ $\therefore a = \sin x$

$\sin^{-1} b = y$ $\therefore b = \sin y$

$\sin^{-1} c = z$ $\therefore c = \sin z$

$\therefore$ $a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2}$

$= \sin x \cos x + \sin y \cos y + \sin z \cos z$

$= (1/2)(\sin 2x + \sin 2y + \sin 2z) = (1/2)(4 \sin x \sin y \sin z)$

$= 2 \sin x \sin y \sin z = = 2abc$

11. **(a)** We have

$$\alpha + \beta = \left(\sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2}\right) + \left(\sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Since $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all x

Also, $\alpha=\frac{\pi}{3}+\sin^{-1}\frac{1}{3}<\frac{\pi}{3}+\sin^{-1}\frac{1}{2}$

as $\sin\theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$$\therefore\ \alpha<\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}\quad\Rightarrow\beta>\frac{\pi}{2}>\alpha\Rightarrow\alpha<\beta$$

12. (b) $\frac{1}{1+r+r^2}=\frac{1}{1+r(r+1)}=\frac{\frac{1}{r(r+1)}}{1+\frac{1}{r(r+1)}}=\frac{\frac{1}{r}-\frac{1}{r+1}}{1+\frac{1}{r}.\left(\frac{1}{r+1}\right)}$

$$\therefore\ \tan^{-1}\left(\frac{1}{1+r+r^2}\right)=\tan^{-1}\frac{1}{r}-\tan^{-1}\frac{1}{r+1}$$

$$\therefore\ \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)=\tan^{-1}1=\frac{\pi}{4}$$

13. (c) $\because a-b<0$, so $\cot^{-1}\frac{ab+1}{a-b}=\cot^{-1}b-\cot^{-1}a+\pi$

$b-c<0$, so $\cot^{-1}\frac{bc+1}{b-c}=\cot^{-1}c-\cot^{-1}b+\pi$

$c-a>0$, so $\cot^{-1}\frac{ca+1}{c-a}=\cot^{-1}a-\cot^{-1}c$

Adding we get

$$\cot^{-1}\frac{ab+1}{a-b}+\cot^{-1}\frac{bc+1}{b-c}+\cot^{-1}\frac{ca+1}{c-a}=2\pi$$

14. (b) We have, $A=\tan^{-1}2\Rightarrow\tan A=2$

and $B=\tan^{-1}3\Rightarrow\tan B=3$

since A, B, C are angles of a triangle, $A+B+C=\pi$

$\Rightarrow\ C=\pi-(A+B)$ ... (1)

Now, $A+B=\tan^{-1}2+\tan^{-1}3\ =\pi+\tan^{-1}\left[\frac{2+3}{1-2.3}\right]$

$$\left[\begin{array}{l}\therefore \tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left[\dfrac{x+y}{1-xy}\right] \\ \text{for } x > 0,\ y > 0 \text{ and } xy > 1\end{array}\right]$$

$$= \pi + \tan^{-1}(-1) = \pi - \tan^{-1} 1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$\therefore$ From (1), $C = \pi - \dfrac{3\pi}{4} = \dfrac{\pi}{4}$

15. (b) We have, $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

or $x\sqrt{(1-y^2)} + y\sqrt{(1-x^2)} = z$

or $x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$

or $(x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$

or $x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2$

$+4x^2y^2z^2 - 4y^2z^2 = 0$

or $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

$\therefore \quad k = 2$

16. (c) ✌ $T_r = \sin^{-1}\left(\dfrac{\sqrt{r} - \sqrt{(r-1)}}{\sqrt{r(r+1)}}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{r} - \sqrt{(r-1)}}{1 + \sqrt{r}\sqrt{(r-1)}}\right)$$

$$S_n = \sum_{r=1}^{n} \tan^{-1}\left(\frac{\sqrt{r} - \sqrt{(r-1)}}{1 + \sqrt{r}\sqrt{(r-1)}}\right)$$

$$= \sum_{r=1}^{n} \{\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{(r-1)}\}$$

$$= \tan^{-1}\sqrt{n} - \tan^{-1}\sqrt{0}$$

$$= \tan^{-1}\sqrt{n} - 0$$

$\therefore \quad S_\infty = \tan^{-1}\infty = \frac{\pi}{2}$

17. **(c)** $\sin^{-1}(x-1) \Rightarrow -1 \le x-1 \le 1 \Rightarrow 0 \le x \le 2$

$\cos^{-1}(x-3) \Rightarrow -1 \le x-3 \le 1 \Rightarrow 2 \le x \le 4$

$\tan^{-1}\left(\frac{x}{2-x^2}\right) \Rightarrow x \in R, \ x \ne \sqrt{2}, -\sqrt{2}$

$\sin^{-1}(2-1) + \cos^{-1}(2-3) + \tan^{-1}\frac{2}{2-4} = \cos^{-1}k + \pi$

$\Rightarrow \sin^{-1}1 + \cos^{-1}(-1) + \tan^{-1}(-1) = \cos^{-1}k + \pi$

$\frac{\pi}{2} + \pi - \frac{\pi}{4} = \cos^{-1}k + \pi$

$\Rightarrow \cos^{-1}k = \frac{\pi}{4} \Rightarrow k = \frac{1}{\sqrt{2}}$

18. **(c)** $8x^2 + 22x + 5 = 0 \Rightarrow x = -\frac{1}{4}, \ -\frac{5}{2}$

$\because \quad -1 < -\frac{1}{4} < 1$ and $-\frac{5}{2} < -1$

$\therefore \quad \sin^{-1}\left(-\frac{1}{4}\right)$ exists but $\sin^{-1}\left(-\frac{5}{2}\right)$ does not exist

$\sec^{-1}\left(-\frac{5}{2}\right)$ exists but $\sec^{-1}\left(-\frac{1}{4}\right)$ does not exist,

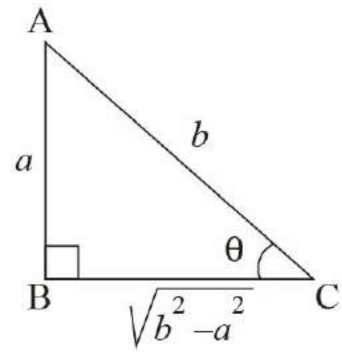
$\tan^{-1}\left(-\frac{1}{4}\right)$ and $\tan^{-1}\left(-\frac{5}{2}\right)$ both exist

19. **(a)** $\cot(\cos^{-1}x) = \sec\left(\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right)$ …(i)

Let $\theta = \tan^{-1}\frac{a}{\sqrt{b^2-a^2}}$

$\therefore \ \tan\theta = \frac{a}{\sqrt{b^2-a^2}}$

In ΔABC,



$$AC^2 = a^2 + (\sqrt{b^2 - a^2})^2$$

$$AC = b$$

$$\therefore \sec\theta = \frac{b}{\sqrt{b^2 - a^2}} \quad \dots \text{(ii)}$$

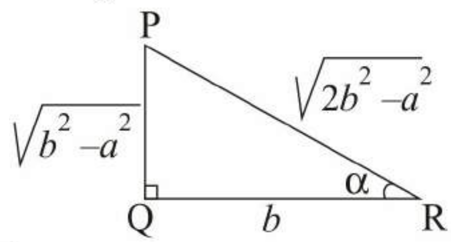
From (i), $\cot(\cos^{-1} x) = \sec\theta$

$$\cot(\cos^{-1} x) = \frac{b}{\sqrt{b^2 - a^2}} \quad \text{[using (ii)]}$$

$$\cos^{-1} x = \cot^{-1}\left(\frac{b}{\sqrt{b^2 - a^2}}\right) \quad \dots \text{(iii)}$$

Again let $\alpha = \cot^{-1}\left(\frac{b}{\sqrt{b^2 - a^2}}\right)$

$$\cot\alpha = \frac{b}{\sqrt{b^2 - a^2}}$$



$$\therefore PR^2 = (\sqrt{b^2 - a^2})^2 + b^2$$

$$PR = \sqrt{2b^2 - a^2}$$

$$\therefore \cos\alpha = \frac{b}{\sqrt{2b^2 - a^2}} \quad \dots \text{(iv)}$$

From (iii),

$$\cos^{-1} x = \alpha \text{ or } x = \cos\alpha$$

$$x = \frac{b}{\sqrt{2b^2 - a^2}} \quad \text{[using (iv)]}$$

20. **(a)** $\tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z)$
$- \cot(\cot^{-1} x + \cot^{-1} y + \cot^{-1} z)$

$= \tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z)$

$- \cot\left(\frac{\pi}{2} - \tan^{-1} x + \frac{\pi}{2} - \tan^{-1} y + \frac{\pi}{2} - \tan^{-1} z\right)$

$\left(\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right)$

$= \tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z)$

$- \cot\{3\pi/2 - (\tan^{-1} x + \tan^{-1} y + \tan^{-1} z)$

$= \tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z)$

$- \tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z) = 0$

21. **(30)** $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{\sqrt{3}x}{2K - x} - \frac{2x - K}{\sqrt{3}K}}{1 + \frac{\sqrt{3}x}{2K - x}\frac{2x - K}{\sqrt{3}K}}$$

$$= \frac{3Kx - (2x - K)(2K - x)}{(2K - x)\sqrt{3}K + \sqrt{3}x(2x - K)}$$

$$= \frac{3Kx - (4Kx - 2x^2 - 2K^2 + Kx)}{2\sqrt{3}K^2 - \sqrt{3}Kx + 2\sqrt{3}x^2 - \sqrt{3}Kx}$$

$$= \frac{2x^2 - 2Kx + 2K^2}{2\sqrt{3}x^2 - 2\sqrt{3}Kx + 2\sqrt{3}k^2} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$\therefore A - B = 30^\circ$

22. **(5)** $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6} \Rightarrow \frac{\pi}{6} < \cot^{-1}\frac{n}{\pi} < \pi \Rightarrow -\infty < \frac{n}{\pi} < \sqrt{3}$

$\Rightarrow n < \pi\sqrt{3} \ (\because n > 0) \Rightarrow n \le 5 \ (\because 5 < \pi\sqrt{3} < 6)$

23. **(1)** $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right) +$

$\cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$

This is true only when $x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots = x^2 - \frac{x^4}{2} + \frac{x^6}{4} \ldots\ldots \Rightarrow \frac{x}{1+\frac{x}{2}} = \frac{x^2}{1+\frac{x^2}{2}}$

(Common ratios are $-\frac{x}{2}$ & $-\frac{x^2}{2}$ & |common ratios | < 1, in the given interval)

$\frac{2x}{2+x} = \frac{2x^2}{2+x^2} \Rightarrow x = 0$ or $x = 1 \Rightarrow x = 1$,

{x cannot be zero as $0 < |x| < \sqrt{2}$ }.

24. **(2)** The given question can be written as

$$\sin^{-1}\left(\frac{1}{5}\right) + \sec^{-1}(5) + \sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right)$$

$$+2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + 2\tan^{-1}(\sqrt{3}) = k\pi$$

or $$\left\{\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}\left(\frac{1}{5}\right)\right\} + \left\{\sec^{-1}(2) + \operatorname{cosec}^{-1}(2)\right\}$$

$$+\left\{2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + 2\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)\right\} = k\pi$$

or $$\frac{\pi}{2} + \frac{\pi}{2} + 2\left\{\tan^{-1}\frac{1}{\sqrt{3}} + \cot^{-1}\frac{1}{\sqrt{3}}\right\} = k\pi$$

or $$\pi + 2\left(\frac{\pi}{2}\right) = k\pi$$

or $\pi + \pi = k\pi$

or $2\pi = k\pi$

or $k = 2$

25. **(11.5)** $\tan\left[\cos^{-1}\left(\frac{1}{\sqrt{82}}\right)-\sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$

$= \tan(\tan^{-1}9 - \tan^{-1}5)$

$= \tan\left\{\tan^{-1}\left(\frac{9-5}{1+9\times 5}\right)\right\} = \frac{2}{23}$