GP_3480

15 Matrices and Determinants

Section-A : JEE Advanced/ IIT-JEE

A 1. $t=0$ 2. $x=-1, 2$ 3. 3/16 4. 2, 7 5. $\lambda = 0$ 6. 0 7. 0

B 1. F 2. F

C 1. (b) 2. (a) 3. (d) 4. (a) 5. (b) 6. (a) 7. (d)
8. (b) 9. (b) 10. (d) 11. (a) 12. (b) 13. (c) 14. (c)
15. (a) 16. (d) 17. (a) 18. (a) 19. (d) 20. (d) 21. (b)

D 1. (b, e) 2. (d) 3. (c) 4. (a, d) 5. (c, d) 6. (b, c, d) 7. (c, d)
8. (a, b) 9. (b, c) 10. (c, d) 11. (b, c) 12. (b, c, d)

E 1. $x = b,\ y = \frac{-2b}{15},\ z = \frac{2b}{5},\ b \in R$ 5. $n\pi$ or $n\pi + (-1)^n \pi/6,\ n \in Z$

8. 2 10. $n\pi$ or $n\pi + \pi/4$

12. $\dfrac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$ 14. 4

F 1. (A) $\to$ s; (B) $\to$ p, q; (C) $\to$ r; (D) $\to$ p, q, s 2. (A) $\to$ r; (B) $\to$ q, s; (C) $\to$ r, s; (D) $\to$ p, r

G 1. (d) 2. (b) 3. (a) 4. (a) 5. (b) 6. (b) 7. (d) 8. (c) 9. (d) 10. (d) 11. (a)
12. (b)

H 1. (a)

I 1. 0 2. 4 3. 9 4. 2 5. 1

Section-B : JEE Main/ AIEEE

1. (c) 2. (d) 3. (b) 4. (c) 5. (a) 6. (a) 7. (d)
8. (d) 9. (a) 10. (d) 11. (b) 12. (b) 13. (d) 14. (d)
15. (a) 16. (d) 17. (d) 18. (c) 19. (d) 20. (b) 21. (c)
22. (b) 23. (c) 24. (a) 25. (a) 26. (d) 27. (c) 28. (b)
29. (a) 30. (d) 31. (a) 32. (b) 33. (b) 34. (d)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. As given equation is an identity in λ, it must be true for all values of λ.

$\therefore$ For $\lambda = 0$ also. Putting $\lambda = 0$ we get $t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = 0$

2. Given equation is, $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

Clearly on expanding the det. we will get a quadratic equation in x.

$\therefore$ It has 2 roots. We observe that R_3 becomes identical to R_1 if $x = 2$. thus at $x = 2 \Rightarrow \Delta = 0$

$\therefore$ $x = 2$ is a root of given eq.

Similarly R_3 becomes identical to R_2 if $x = -1$.

thus at $x = -1 \Rightarrow \Delta = 0$

$\therefore$ $x = -1$ is a root of given eq.

Hence equation has roots as –1 and 2.

3. With 0 and 1 as elements there are $2 \times 2 \times 2 \times 2 = 16$ determinants of order 2×2 out of which only

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$ are the three det whose value is +ve.

$\therefore$ Req. prob. = 3/16

4. $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Operating $R_1 \to R_1 + R_2 + R_3$ we get

$$\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Operating $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 0 & 0 \\ 2 & x-2 & 0 \\ 7 & -1 & x-7 \end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow (x+9)(x-2)(x-7) = 0$

$\Rightarrow x = -9, 2, 7$

$\therefore$ Other roots are 2 and 7.

5. The given homogeneous system of equations will have non zero solution if $D = 0$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 1 \\ -1 & \lambda & 1 \\ -1 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda(\lambda^2+1) - 1(-\lambda+1) + 1(1+\lambda) = 0 \Rightarrow \lambda^3 + 3\lambda = 0$

$\Rightarrow \lambda(\lambda^2+3) = 0$, but $\lambda^2 + 3 \neq 0$ for real $\lambda \Rightarrow \lambda = 0$

6. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$

Operating $R_1 \to R_1 - R_2;\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= (a-b(b-c)\begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2-ab \end{vmatrix} = 0$$

7. Given x, y, z and + ve numbers, then value of

$$D = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix} \quad \left(\because \log_b a = \frac{\log a}{\log b}\right)$$

Taking $\frac{1}{\log x}, \frac{1}{\log y}$, and $\frac{1}{\log z}$ common from R_1, R_2 and R_3 respectively

$$D = \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

B. True/False

1. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = (-1)^2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$[C_1 \Leftrightarrow C_3$ and then $C_2 \Leftrightarrow C_3]$

$\therefore$ Equal. Hence statement is *F*.

2. (i) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

$Ar(\Delta_1) = Ar(\Delta_2)$

Where Δ_1 is the area of triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3); and Δ_2 is the area of triangle with ; vertices $(a_1, b_1), (a_2, b_2)$ and (a_3, b_3). But two Δ's of same area may not be congruent.

$\therefore$ Given statement is false.

C. MCQs with ONE Correct Answer

1. **(b)** For every 'det, with value 1' ($\in B$) we can find a det. with value – 1 by changing the sign of one entry of '1'. Hence there are equal number of elements in B and C. $\therefore$ (b) is the correct option

2. **(a)** $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$

Operating $R_1 \to R_1 - R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega+1 & -1 \end{vmatrix} \quad (\because 1+\omega+\omega^2=0)$$
$= 0$

3. (d) Let $\frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y, \frac{z^2}{c^2} = Z$

Then the given system of equations becomes
$X + Y - Z = 1$
$X - Y + Z = 1, \quad -X + Y + Z = 1$
This is the new system of equations
For new system, we have

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1(-1-1) - 1(1+1) - 1(1-1)$$
$= -4 \neq 0$
$\therefore$ New system of equations has unique solution.

$$D_1 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1(-1-1) - 1(1-1) - 1(1+1) = -4$$

$$D_2 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1(1-1) - 1(1+1) - 1(1+1) = -4$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1(-1-1) - 1(1+1) + 1(1-1) = -4$$

Now, $X = \frac{D_1}{D} = \frac{-4}{-4} = 1, \; Y = \frac{D_2}{D} = \frac{-4}{-4} = 1$

$Z = \frac{D_3}{D} = \frac{-4}{-4} = 1 \Rightarrow x = \pm a, \; y = \pm b, \; z = \pm c$

4. (a) If A and B are square matrices of same degree then matrices A and B can be added or subtracted or multiplied. By algebra of matrices the only correct option is $A + B = B + A$

5. (b) Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2 & a & a^2 \\ 2\cos px \cos dx & \cos px & \cos(p+d)x \\ 2\sin px \cos dx & \sin px & \sin(p+d)x \end{vmatrix}$$

$C_1 \to C_1 - (2\cos dx)C_2$

$$\Delta = \begin{vmatrix} 1+a^2-2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

Expanding along C_1, we get
$\Delta = (1 + a^2 - 2a \cos dx)\,[\sin (p + d)x \cos px - \sin px \cos (p + d)x]$
$\Rightarrow \Delta = (1 + a^2 - 2a \cos dx)\,[\sin \{(p + d)x - px\}]$
$\Rightarrow \Delta = (1 + a^2 - 2a \cos dx)\,[\sin dx]$
which is independent of p.

6. (a)

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

$$= \begin{vmatrix} x+1 & x & x+1 \\ (x+1)x & x(x-1) & (x+1)x \\ (x+1)x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$
$[C_1 \to C_1 + C_2]$

$= 0 \quad [\because C_1$ and C_2 are identical$]$
which is free of x, so the function is true for all values of x. Therefore, at $x = 100, f(x) = 0$, i.e., $f(100) = 0$

7. (d) For the given homogeneous system to have non zero solution determinant of coefficient matrix should be zero; i.e.,

$$= \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 1(1+1) + k(-k+1) - 1(k+1) = 0$$
$\Rightarrow 2 - k^2 + k - k - 1 = 0 \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

8. (b) Given that $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \; \omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$
Also $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$
Now given det. is

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$
[Using $\omega = -1 - \omega^2$ and $\omega^3 = 1$]

Operating $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix} \quad (\text{as } 1+\omega+\omega^2=0)$$

Expanding along C_1, we get
$3(\omega^2 - \omega^4) = 3(\omega^2 - \omega) = 3\omega(\omega - 1)$

9. (b) For infinitely many solutions the two equations become identical

$\Rightarrow \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1} \Rightarrow k = 1$

10. (d) Given that $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$
and $A^2 = B$

$\Rightarrow \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 1$ and $\alpha + 1 = 5 \Rightarrow \alpha = \pm 1$ and $\alpha = 4$
$\because$ There is no common value
$\therefore$ There is no real value of α for which $A^2 = B$

11. (a) The given system is, $x + ay = 0$

$az + y = 0$

$ax + z = 0$

It is system of homogeneous equations therefore, it will have infinite many solutions if determinant of coefficient matrix is zero. i.e.,

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(1-0) - a(0-a^2) = 0 \Rightarrow 1 + a^3 = 0$

$\Rightarrow a^3 = -1 \Rightarrow a = -1$

12. (b) Since the system has no solution, $\Delta = 0$ and any one amongst $\Delta_x, \Delta_y, \Delta_z$ is non-zero.

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1$$

$$\text{Also, } \Delta_z = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & 4 \end{vmatrix} = 6 \neq 0$$

13. (c) $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125 \Rightarrow |A|^3 = 125$

Now, $|A| = \alpha^2 - 4$

$\Rightarrow (\alpha^2 - 4)^3 = 125 = 5^3 \Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$

14. (c) Given $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$

$\therefore$ Characteristic eqn of above matrix A is given by

$$|A - \lambda I| = 0 \Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -2 & 4-\lambda \end{bmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)(4 - 5\lambda + \lambda^2 + 2) = 0 \Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

Also by Cayley Hamilton thm (every square matrix satisfies its characteristic equation) we obtain

$A^3 - 6A^2 + 11A - 6I = 0$

Multiplying by A^{-1}, we get

$$A^2 - 6A + 11I - 6A^{-1} = 0 \Rightarrow A^{-1} = \frac{1}{6}(A^2 - 6A + 11I)$$

Comparing it with given relation,

$$A^{-1} = \frac{1}{6}(A^2 - cA + dI)$$

we get $c = -6$ and $d = 11$

15. (a) Given that, $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $X = P^T Q^{2005} P$

We observe that $Q = PAP^T$

$\Rightarrow \quad Q^2 = (PAP^T)(PAP^T)$

$= PA(P^T P)AP^T = PA(IA)P^T$

$\therefore PA^2 P^T$

Proceeding in the same way, we get

$Q^{2005} = PA^{2005} P^T$

Also $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

and proceeding in the same way $A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

Now, $X = P^T Q^{2005} P$

$= P^T (PA^{2005} P^T) P = (P^T P) A^{2005} (P^T P)$

$= IA^{2005} I = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

16. (d) The given points are $P(-\sin(\beta - \alpha), -\cos\beta)$,

$Q(\cos(\beta - \alpha), \sin\beta)$

$R(\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$

Where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$

$$\therefore \Delta = \begin{vmatrix} 1 & 1 & 1 \\ -\sin(\beta - \alpha) & \cos(\beta - \alpha) & \cos(\beta - \alpha + \theta) \\ -\cos\beta & \sin\beta & \sin(\beta - \theta) \end{vmatrix}$$

Operating $C_3 - C_1 \sin\theta - C_2 \cos\theta$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 1 - \sin\theta - \cos\theta \\ -\sin(\beta - \alpha) & \cos(\beta - \alpha) & 0 \\ -\cos\beta & \sin\beta & 0 \end{vmatrix}$$

$= (1 - \sin\theta - \cos\theta)[\cos\beta\cos(\beta - \alpha) - \sin\beta\sin(\beta - \alpha)]$

$\Rightarrow \Delta = [1 - (\sin\theta + \cos\theta)]\cos(2\beta - \alpha)$

$\because 0 < \alpha, \beta, \theta < \frac{\pi}{4} \qquad \therefore \sin\theta + \cos\theta \neq 1$

Also $2\beta - \alpha < \frac{\pi}{4} \Rightarrow \cos(2\beta - \alpha) \neq 0$

$\therefore \Delta \neq 0 \Rightarrow$ the three points are non collinear.

17. (a) Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ where a_i, b_i, c_i have values 0 or 1 for $i = 1, 2, 3$.

Then the given system is equivalent to

$a_1 x + b_1 y + c_1 z = 1,$

$a_2 x + b_2 y + c_2 z = 0,$

$a_3 x + b_3 y + c_3 z = 0$

Which represents three distinct planes. But three planes can not intersect at two distinct points, therefore no such system exists.

18. (a) For the given matrix to be non-singular

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$\Rightarrow 1-(a+c)\,\omega + a\,c\omega^2 \neq 0 \Rightarrow (1-a\omega)\,(1-c\omega) \neq 0$

$\Rightarrow a \neq \omega^2$ and $c \neq \omega^2$ where ω is complex cube root of unity.

As a, b and c are complex cube roots of unity

$\therefore$ a and c can take only one value i.e. ω while b can take 2 values i.e. ω and ω^2.

$\therefore$ Total number of distinct matrices $= 1 \times 1 \times 2 = 2$

19. (d) We have

$$|Q| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$= 2^2.2^3.2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2 a_{31} & 2^2 a_{32} & 2^2 a_{33} \end{vmatrix}$$

$$= 2^9.2.2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 2^{12} \times |P| = 2^{12} \times 2 = 2^{13}$$

20. (d) We have $P^T = 2P + I$

$\Rightarrow P = 2P^T + I \Rightarrow P = 2(2P+I) + I$

$\Rightarrow P = 4P + 3I \Rightarrow P + I = 0$

$\Rightarrow PX + X = 0 \Rightarrow PX = -X$

21. (b) $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = I + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} = I + A$

$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ $A^n = O, \forall\, n \geq 3$

Now $P^{50} = (I+A)^{50} = {}^{50}C_0\, I^{50} + {}^{50}C_1\, I^{49} A + {}^{50}C_2\, I^{48} A^2 + O$

$= I + 50A + 25 \times 49 A^2.$

$\therefore$ $Q = P^{50} - I = 50A + 25 \times 49 A^2.$

$\Rightarrow$ $q_{21} = 50 \times 4 = 200 \Rightarrow q_{31} = 50 \times 16 + 25 \times 49 \times 16 = 20400$

$\Rightarrow$ $q_{32} = 50 \times 4 = 200$

$\therefore$ $\dfrac{q_{31} + q_{32}}{q_{21}} = \dfrac{20600}{200} = 103$

D. MCQs with ONE or MORE THAN ONE Correct

1. (b, e) ATQ $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$

Operating $C_3 \to C_3 - C_1\,\alpha - C_2$, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & -(a\alpha^2+b\alpha+b\alpha+c) \end{vmatrix} = 0$$

$\Rightarrow$ $(a\alpha^2 + 2b\,\alpha + c) \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & 1 \end{vmatrix} = 0$

$\Rightarrow$ $(ac - b^2)\,(a\alpha^2 + 2b\alpha + c) = 0$

$\Rightarrow$ either $ac - b^2 = 0$ or $a\alpha^2 + 2b\,\alpha + c = 0$

$\Rightarrow$ either a, b, c are in G.P. or $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

$\Rightarrow$ (b) and (e) are the correct answers.

2. (d) $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ (given)

$\Rightarrow$ $-3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0$

[$\because$ C_2 and C_3 are identical]

$\Rightarrow$ $x + iy = 0 \Rightarrow x = 0, y = 0$

3. (c) [As a skew symmetric matrix of order 3 cannot be non singular, therefore the data given in the question is inconsistent.]

We have

$M^2N^2\,(M^T N)^{-1}\,(MN^{-1})^T = M^2N^2N^{-1}\,(M^T)^{-1}\,(N^{-1})^T M^T$

$= M^2 N\,(M^T)^{-1}\,(N^{-1})^T M^T = -M^2NM^{-1}N^{-1}M$

$(\because M^T = -M, N^T = -N$ and $(N^{-1})^T = (N^T)^{-1}$

$= -M\,(NM)\,(NM)^{-1}\,M$ $\quad(\because MN = NM)$

$= -MM = -M^2$

4. (a, d)

We know for a third order matrix P, $|\text{Adj}\, P| = |P|^2$

Where $|\text{Adj}\, P| = 1\,(3-7) - 4(6-7) + 4\,(2-1) = 4$

$\therefore$ $|P|^2 = 4 \Rightarrow |P| = 2$ or -2

5. (c, d)

(a) $(N'\,MN)' = (MN)'N = N'M'N = N'MN$ or $-N'MN$
According as M is symm. or skew symm. $\therefore$ correct

(b) $(MN - NM)' = (MN)' - (NM)' = N'M' - M'N'$
$= NM - MN = -(MN - NM)$
$\therefore$ It is skew symm. Statement B is also correct.

(c) $(MN)' = N'M' = NM \neq MN$
$\therefore$ Statement C is incorrect

(d) $(adj\,M)\,(adj\,N) = adj\,(MN)$ is incorrect.

6. (b, c, d)

For $n = 3$, $P = \begin{bmatrix} w^2 & w^3 & w^4 \\ w^3 & w^4 & w^5 \\ w^4 & w^5 & w^6 \end{bmatrix}$ and $P^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

It shows $P^2 = 0$ if n is a multiple of 3.

So for $P^2 \neq 0$, n should not be a multiple of 3 i.e. n can take values 55, 58, 56

7. **(c, d)** Let $M=\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ where a, b, c are integers.

M is invertible if $\begin{vmatrix} a & b \\ b & c \end{vmatrix} \neq 0 \Rightarrow ac \neq b^2$

Then $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix} \Rightarrow a=b=c \Rightarrow ac=b^2$.

$\therefore$ (a) is not correct.

If $\begin{bmatrix} b & c \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix} \Rightarrow b=a=c \Rightarrow ac=b^2$

$\therefore$ (b) is not correct.

If $M=\begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$, then $|M| = ac \neq 0$

$\therefore$ M is invertible.

(c) is correct

As $ac \neq (\text{integer})^2 \Rightarrow ac \neq b^2$

$\therefore$ (d) is correct.

8. **(a, b)** Given $MN = NM$, $M \neq N^2$ and $M^2 = N^4$.

Then $M^2 = N^4 \Rightarrow (M+N^2)(M-N^2) = 0$

$\Rightarrow$ (i) $M+N^2=0$ and $M-N^2 \neq 0$

(ii) $|M+N^2| = 0$ and $|M-N^2| = 0$

In each case $|M+N^2| = 0$

$\therefore |M^2+MN^2| = |M|\,|M+N^2| = 0$

$\therefore$ (a) is correct and (c) is not correct.

Also we know if $|A| = 0$, then there can be many matrices U, such that $AU = 0$

$\therefore (M^2+MN^2)U = 0$ will be true for many values of U.

Hence (b) is correct.

Again if $AX = 0$ and $|A| = 0$, then X can be non-zero.

$\therefore$ (d) is not correct.

9. **(b, c)** $R_2 - R_1, \quad R_3 - R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 2\alpha+5 & 4\alpha+5 & 6\alpha+5 \end{vmatrix} = -648\alpha$$

$R_3 - R_2$

$$2\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 1 & 1 & 1 \end{vmatrix} = -648\alpha$$

$C_2 - C_1, \quad C_3 - C_2$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(3\alpha+2) & \alpha(5\alpha+2) \\ 2\alpha+3 & 2\alpha & 2\alpha \\ 1 & 0 & 0 \end{vmatrix} = -324\alpha$$

$\Rightarrow 2\alpha^2(-2\alpha) = -324\alpha \Rightarrow \alpha^3 - 81\alpha = 0 \Rightarrow \alpha = 0, 9, -9$

10. **(c, d)**

$X' = -X,\ Y' = -Y,\ Z' = Z$

$(Y^3Z^4 - Z^4Y^3)' = (Z^4)'(Y^3)' - (Y^3)'(Z^4)'$

$= (Z')^4(Y')^3 - (Y')^3(Z')^4$

$= -Z^4Y^3 + Y^3Z^4 = Y^3Z^4 - Z^4Y^3$

$\therefore$ Symmetric matrix.

Similarly $X^{44} + Y^{44}$ is symmetric matrix and $X^4Z^3 - Z^3X^4$ and $X^{23} + Y^{23}$ are skew symmetric matrices.

11. **(b, c)** $PQ = kI \Rightarrow \frac{P.Q}{k} = I \Rightarrow P^{-1} = \frac{Q}{k}$

Also $|P| = 12\alpha + 20$

Comparing the third elements of 2nd row on both sides, we get

$-\left(\frac{3\alpha+4}{12\alpha+20}\right) = \frac{1}{k} \times \frac{-k}{8} \Rightarrow 24\alpha + 32 = 12\alpha + 20 \Rightarrow \alpha = -1$

$\therefore |P| = 8$

Also $PQ = kI \Rightarrow |P|\,|Q| = k^3$

$\Rightarrow 8 \times \frac{k^2}{2} = k^3 \Rightarrow k = 4 \Rightarrow |Q| = \frac{k^2}{2} = 8$

(b) $4\alpha - k + 8 = 4 \times (-1) - 4 + 8 = 0$

(c) Now det (P adj Q) = |P| adj Q|

$= |P|\,|Q|^2 = 8 \times 8^2 = 2^9$

(d) $|Q \text{ adj } P| = |Q|\,|P|^2 = 2^9$

12. **(b, c, d)**

$ax + 2y = \lambda$

$3x - 2y = \mu$

For unique solution, $\frac{a}{3} \neq \frac{2}{-2} \Rightarrow a \neq -3$

$\therefore$ (b) is the correct option.

For infinite many solutions and $a = -3$

$\frac{-3}{3} = \frac{2}{-2} = \frac{\lambda}{\mu} \Rightarrow \frac{\lambda}{\mu} = -1$ or $\lambda + \mu = 0$

$\therefore$ (c) is the correct option.

Also if $\lambda + \mu \neq 0$, then $\frac{-3}{3} = \frac{2}{-2} \neq \frac{\lambda}{\mu}$

$\Rightarrow$ system has no solution.

$\therefore$ (d) is the correct option.

E. Subjective Problems

1. We should have,

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow 1(-4k+6) - k(-12+4) + 3(9-2k) = 0$

$\Rightarrow -2k + 33 = 0 \Rightarrow k = \frac{33}{2}$

Substituting $k = \frac{33}{2}$ and putting $x = b$, where $b \in Q$, we get the system as

$33y + 6z = -2b$ ...(1)

$33y - 4z = -6b$ ...(2)

$3y - 4z = -2b$ ...(3)

$(1) - (2) \Rightarrow 10z = 4b \Rightarrow z = \frac{2}{5}b$

$(1) \Rightarrow 33y = -2b - \frac{12b}{5} = -\frac{22b}{5} \Rightarrow y = \frac{-2b}{15}$

$\therefore$ The solution is $x = b,\ y = \frac{-2b}{15},\ z = \frac{2b}{5}$

2. The given det, on expanding along R_1, we get

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc-a^2) - b(b^2-ac) + c(ab-c^2)$$

$= 3abc - a^3 - b^3 - c^3 = -(a^3+b^3+c^3-3abc)$
$= -(a+b+c)[a^2+b^2+c^2-ab-bc-ca]$

$$= -\frac{1}{2}(a+b+c)[2a^2+2b^2+2c^2-2ab-2bc-2ca]$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

As $a, b, c > 0$
$\therefore \quad a+b+c>0$
Also $a \neq b \neq c$
$\therefore \quad (a-b)^2+(b-c)^2+(c-a)^2>0$
Hence the given determinant is – ve.

3. $$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA+B$$

$$\text{L.H.S.} = \begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$$

Operation $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ x-1 & x-2 & x+1 \\ x+3 & x-2 & x+1 \end{vmatrix}$$

$$= \begin{vmatrix} x^2 & x+1 & x-2 \\ 0 & x-2 & x+1 \\ 0 & x-2 & x+1 \end{vmatrix} + \begin{vmatrix} x & x+1 & x-2 \\ x-1 & x-2 & x+1 \\ x+3 & x-2 & x+1 \end{vmatrix}$$

$$= 0 + \begin{vmatrix} x & x+1 & x-2 \\ x-1 & x-2 & x+1 \\ x+3 & x-2 & x+1 \end{vmatrix}$$

Operating $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} x & x+1 & x-2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} x & x & x \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix}$$

$$= x\begin{vmatrix} 1 & 1 & 1 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -1 & -3 & 3 \\ 4 & 0 & 0 \end{vmatrix}$$

$= xA + B =$ R.H. S
Hence Proved.

4. On L.H.S.$= D$, applying operations $C_2 \to C_2 + C_1$ and $C_3 \to C_3 + C_2$ and using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$, we get

$$D = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$$

Operating $C_3 + C_2$ and using the same result, we get

$$D = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix} = \text{RHS}$$

Hence proved

5. The system will have a non-trivial solution if

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

Expanding along C_1, we get
$\Rightarrow \quad (28-21)\sin 3\theta - (-7-7)\cos 2\theta + 2(-3-4) = 0$
$\Rightarrow \quad 7\sin 3\theta + 14\cos 2\theta - 14 = 0$
$\Rightarrow \quad \sin 3\theta + 2\cos 2\theta - 2 = 0$
$\Rightarrow \quad 3\sin - 4\sin^3\theta + 2(1 - 2\sin^2\theta) - 2 = 0$
$\Rightarrow \quad 4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$
$\Rightarrow \quad \sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$
$\sin\theta = 0$ or $\sin\theta = 1/2$ ($\sin\theta = -3/2$ not possible)
$\Rightarrow \quad \theta = n\pi$ or $\theta = n\pi + (-1)^n \pi/6, n \in Z$.

6. We have

$$\Delta a = \begin{vmatrix} (a-1) & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$\text{Then } \sum_{a=1}^{n} \Delta a = \begin{vmatrix} (1-1) & n & 6 \\ (1-1)^2 & 2n^2 & 4n-2 \\ (1-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$+ \begin{vmatrix} (2-1) & n & 6 \\ (2-1)^2 & 2n^2 & 4n-2 \\ (2-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix} + \ldots..$$

$$+ \begin{vmatrix} (n-1) & n & 6 \\ (n-1)^2 & 2n^2 & 4n-2 \\ (n-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \begin{vmatrix} 1+2+3+\ldots.+(n-1) & n & 6 \\ 1^2+2^2+3^2+\ldots\ldots+(n-1)^2 & 2n^2 & 4n-2 \\ 1^3+2^3+3^3+\ldots.+(n-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{n(n-1)(2n-1)}{6} & 2n^2 & 4n-2 \\ \left(\frac{n(n-1)}{2}\right)^2 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \frac{n^2(n-1)}{12}\begin{vmatrix} 6 & 1 & 6 \\ 2(2n-1) & 2n & 2(2n-1) \\ 3n(n-1) & 3n^2 & 3n(n-1) \end{vmatrix}$$

(Taking $\frac{n(n-1)}{12}$ common from C_1 and n from C_2)

$= 0$ (as C_1 and C_3 are identical)

Thus, $\sum_{a=1}^{n} \Delta a = 0 \Rightarrow \sum_{a=1}^{n} \Delta a = c$ (a constant) where $c = 0$

7. Given that A, B, C are integers between 0 and 9 and the three digit numbers $A28$, $3B9$ and $62C$ are divisible by a fixed integer k.

Now, $D = \begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$

On operating $R_2 \to R_2 + 10R_3 + 100R_1$, we get

$$= \begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} = \begin{vmatrix} A & 3 & 6 \\ kn_1 & kn_2 & kn_3 \\ 2 & B & 2 \end{vmatrix}$$

[As per question $A28$, $3B9$ and $62C$ are divisible by k, therefore,

$A28 = kn_1$
$3B9 = kn_2$
$62C = kn_3$]

$$= k\begin{vmatrix} A & 3 & 6 \\ n_1 & n_2 & n_3 \\ 2 & B & 2 \end{vmatrix} = k \times \text{some integral value.}$$

$\Rightarrow D$ is divisible by k.

8. Consider $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

Operating $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ we get

$$\begin{vmatrix} p-a & -(q-b) & c \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0$$

Taking $(p-q)$, $(q-b)$ and $(r-c)$ common from C_1, C_2 and C_3 resp, we get

$$\Rightarrow (p-a)(q-b)(r-c)\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \frac{a}{p-a} & \frac{b}{q-b} & \frac{r}{r-c} \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow (p-a)(q-b)(r-c)\left[1\left(\frac{r}{r-c}+\frac{b}{q-b}\right)+\frac{a}{p-a}\right]=0$$

As given that $p \neq a, q \neq b, r \neq c$

$$\therefore \frac{r}{r-c}+\frac{b}{q-b}+\frac{a}{p-a}=0$$

$$\Rightarrow \frac{r}{r-c}+\frac{q-(q-b)}{q-b}+\frac{p-(p-a)}{p-a}=0$$

$$\Rightarrow \frac{r}{r-c}+\frac{q}{q-b}-1+\frac{p}{p-a}-1=0$$

$$\Rightarrow \frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$$

9. $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$

$$= n!\,(n+1)!\,(n+2)!\begin{vmatrix} 1 & n+1 & (n+2)(n+1) \\ 1 & n+2 & (n+3)(n+2) \\ 1 & n+3 & (n+4)(n+3) \end{vmatrix}$$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$, we get

$$D = (n!)^3(n+1)^2(n+2)\begin{vmatrix} 1 & n+1 & n^2+3n+2 \\ 0 & 1 & 2n+4 \\ 0 & 1 & 2n+6 \end{vmatrix}$$

Operating $R_3 \to R_3 - R_2$

$$D = (n!)^3(n+1)^2(n+2)\begin{vmatrix} 1 & n+1 & n^2+3n+2 \\ 0 & 1 & 2n+4 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= (n!)^3(n+1)^2(n+2)\,1\,[2]$$

$$\Rightarrow \frac{D}{(n!)^3} = 2\,(n+1)^2(n+2)$$

$$\Rightarrow \frac{D}{(n!)^3} - 4 = 2(n^3+4n^2+5n+2)-4$$

$$= 2(n^3+4n^2+5n) = 2n(n^2+4n+5)$$

$\Rightarrow \frac{D}{(n!)^3} - 4$ is divisible by n.

10. Given that $\lambda, \alpha \in R$ and system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$
$x + (\cos\alpha)y + (\sin\alpha)z = 0$
$-x(\sin\alpha)y - (\cos\alpha)z = 0$

has a non trivial solution, therefore $D = 0$

$$\Rightarrow \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - \sin\alpha(-\cos\alpha + \sin\alpha) + \cos\alpha(\sin\alpha + \cos\alpha) = 0$$

$\Rightarrow -\lambda + \sin\alpha\cos\alpha - \sin^2\alpha + \sin\alpha\cos\alpha + \cos^2\alpha = 0$
$\Rightarrow \lambda = \cos^2\alpha - \sin^2\alpha + 2\sin\alpha\cos\alpha$
$\Rightarrow \lambda = \cos 2\alpha + \sin 2\alpha$
For $\lambda = 1$, $\cos 2\alpha + \sin 2\alpha = 1$

$$\frac{1}{\sqrt{2}}\cos 2\alpha + \frac{1}{\sqrt{2}}\sin 2\alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos 2\alpha\cos\pi/4 + \sin 2\alpha\sin\pi/4 = \frac{1}{\sqrt{2}}$$

$\Rightarrow \cos(2\alpha - \pi/4) = \cos\pi/4 \Rightarrow 2\alpha - \pi/4 = 2n\pi \pm \pi/4$
$\Rightarrow 2\alpha = 2\pi r + \pi/4 + \pi/4;\ 2n\pi - \pi/4 + \pi/4$
$\Rightarrow \alpha = n\pi + \pi/4$ or $n\pi$

11. L.H.S.

$$= \begin{vmatrix} \cos A\cos P + \sin A\sin P & \cos(A-Q) & \cos(A-R) \\ \cos B\cos P + \sin B\sin P & \cos(B-Q) & \cos(B-R) \\ \cos C\cos P + \sin C\sin P & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

$$= \cos P\begin{vmatrix} \cos A & \cos(A-Q) & \cos(A-R) \\ \cos B & \cos(B-Q) & \cos(B-R) \\ \cos C & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

$$+ \sin P\begin{vmatrix} \sin A & \cos(A-Q) & \cos(A-R) \\ \sin B & \cos(B-Q) & \cos(B-R) \\ \sin C & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

Operating; $C_2 \to C_2 - C_1(\cos Q)$; $C_3 \to C_3 - C_1(\cos R)$ on first determinant and $C_2 \to C_2 - (\sin Q)C_1$ and $C_3 \to C_3 - (\sin R)C_1$ on second determinant, we get

$$= \cos P\begin{vmatrix} \cos A & \sin A\sin Q & \sin A\sin R \\ \cos B & \sin B\sin Q & \sin B\sin R \\ \cos C & \sin C\sin Q & \sin C\sin R \end{vmatrix}$$

$$+ \sin P\begin{vmatrix} \sin A & \cos A\cos Q & \cos A\cos R \\ \sin B & \cos B\cos Q & \cos B\cos R \\ \sin C & \cos C\cos Q & \cos C\cos R \end{vmatrix}$$

$$= \cos P\sin Q\sin R\begin{vmatrix} \cos A & \sin A & \sin A \\ \cos B & \sin B & \sin B \\ \cos C & \sin C & \sin C \end{vmatrix}$$

$$+ \sin P\cos Q\cos R\begin{vmatrix} \sin A & \cos A & \cos A \\ \sin B & \cos B & \cos B \\ \sin C & \cos C & \cos C \end{vmatrix}$$

$= 0 + 0$ [Both determinants become zero as $C_2 \equiv C_3$]
$= 0 =$ R.H.S.

12. Let us denote the given determinant by Δ. Taking

$\dfrac{1}{a(a+d)(a+2d)}$ as common from

R_1, $\dfrac{1}{(a+d)(a+2d)(a+3d)}$ from R_2 and

$\dfrac{1}{(a+2d)(a+3d)(a+4d)}$ from R_3, we get

$$\Delta = \frac{1}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}\Delta_1$$

where

$$\Delta_1 = \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(a+3d) & a+3d & a+d \\ (a+3d)(a+4d) & a+4d & a+2d \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$, we get

$$\Delta_1 = \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(2d) & d & d \\ (a+3d)(2d) & d & d \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\Delta_1 = \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(2d) & d & d \\ 2d^2 & 0 & 0 \end{vmatrix}$$

Expanding along R_3, we get

$$\Delta_1 = (2d^2)\begin{vmatrix} a+2d & a \\ d & d \end{vmatrix} = (2d)^2(d)(a+2d-a) = 4d^4$$

$$\text{Thus, } \Delta = \frac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$$

13. $R_2 \to R_2 + R_3$,

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ 2\sin\theta\cos\frac{2\pi}{2} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin 2\theta\cos\frac{4\pi}{3} \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

$$= \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$$

14. Given that $A^T A = I$
$\Rightarrow |A^T A| = |A^T||A| = |A||A| = 1 \qquad [\because |I| = I]$
$\Rightarrow |A|^2 = 1$ ....(1)

$$\text{From given matrix } A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc \qquad ....(2)$$

$\therefore (a^3 + b^3 + c^3 - 3abc)^2 = 1$ (From (1) and (2)]
$\Rightarrow a^3 + b^3 + c^3 - 3abc = 1$ or -1
But for a^3, b^3, c^3 using $AM \geq GM$

We get $\dfrac{a^3 + b^3 + c^3}{3} \geq \sqrt[3]{a^3b^3c^3} \Rightarrow a^3 + b^3 + c^3 - 3abc \geq 0$

$\therefore$ We must have $a^3 + b^3 + c^3 - 3abc = 1$
$\Rightarrow a^3 + b^3 + c^3 = 1 + 3 \times 1 = 4$ [Using $abc = 1$]

15. We are given that $MM^T = I$ where M is a square matrix of order 3 and det. $M = 1$.

Consider det $(M - I) = \det(M - MM^T)$ [Given $MM^T = I$]

$= \det[M(I - M^T)]$

$= (\det M)(\det(I - M^T))$

$[\because |AB| = |A||B|]$

$= -(\det M)(\det(M^T - I))$

$= -1[\det(M^T - I)]$ $[\because \det(M) = 1]$

$= -\det(M - I)$

$[\because \det(M^T - I) = \det[(M - I)^T] = \det(M - I)]$

$\Rightarrow 2\det(M - I) = 0 \Rightarrow \det(M - I) = 0$

Hence Proved

16. Given that,

$$A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

and $AX = U$ has infinite many solutions.

$\Rightarrow |A| = 0 = |A_1| = |A_2| = |A_3|$

Now, $|A| = 0$

$$\Rightarrow \begin{vmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{vmatrix} = a(bc - bd) - 1(c - d) = 0$$

$\Rightarrow (ab - 1)(c - d) = 0$

$\Rightarrow ab = 1$ or $c = d$ ....(1)

$$\text{And } |A_1| = \begin{vmatrix} f & 1 & 0 \\ g & b & d \\ h & b & c \end{vmatrix} = 0$$

$\Rightarrow f(bc - bd) - 1(gc - hd) = 0$

$\Rightarrow fb(c - d) = gc - hd$ ....(2)

$$|A_2| = \begin{vmatrix} a & 1 & f \\ 1 & b & g \\ 1 & b & h \end{vmatrix} = 0$$

$\Rightarrow a(gc - hd) - f(c - d) = 0 \Rightarrow a(gc - hd) = f(c - d)$

$$|A_3| = \begin{vmatrix} a & 1 & f \\ 1 & b & g \\ 1 & b & h \end{vmatrix} = 0$$

$\Rightarrow a(bh - bg) - 1(h - g) + f(b - b) = 0$

$\Rightarrow ab(h - g) - (h - g) = 0$

$\Rightarrow ab = 1$ or $h = g$ ....(3)

Taking $c = d \Rightarrow h = g$ and $ab \neq 1$ (from (1), (2) and (3))

Now taking $BX = V$

$$\text{where } B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Then } |B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix} = 0$$

[$\because$ In vew of $c = d$ and $g = h$, C_2 and C_3 are identical]

$\Rightarrow BX = V$ has no unique solution

$$\text{And } |B_1| = \begin{vmatrix} a^2 & 1 & 1 \\ 0 & d & c \\ 0 & g & h \end{vmatrix} = 0 \quad (\because c = d, g = h)$$

$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2cf = a^2df \quad (\because c = d)$$

$$|B_3| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = a^2df$$

$\Rightarrow$ If $adf \neq 0$ then $|B_2| = |B_3| \neq 0$

Hence no solution exist.

F. Match the Following

1. The given lines are

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

(A) Three lines L_1, L_2, L_3 are concurrent if

$$\begin{vmatrix} 1 & 3 & 5 \\ 3 & -k & 1 \\ 5 & 2 & 12 \end{vmatrix} = 0 \Rightarrow 13k - 65 = 0 \Rightarrow k = 5$$

$\therefore$ (A) $\rightarrow$ (s)

(B) For $L_1 \parallel L_2 \Rightarrow \frac{1}{3} = \frac{-3}{k} \Rightarrow k = -9$

and $L_2 \parallel L_3 \Rightarrow \frac{3}{5} = \frac{-k}{2} \Rightarrow k = -\frac{6}{5}$

$\therefore$ (B) $\rightarrow$ (p), (q)

(C) Three lines L_1, L_2, L_3 will form a triangle if no two of them are parallel and no three are concurrent

$\therefore k \neq 5, -9, -6/5$ $\therefore$ (C) $\rightarrow$ r

(D) L_1, L_2, L_3 do not form a triangle if either any two of these are parallel or the three are concurrent i.e.

$k = 5, -9, -6/5$

$\therefore$ (D) $\rightarrow$ (p), (q), (s)

2. (A) Let $y = \frac{x^2 + 2x + 4}{x + 2} \Rightarrow \frac{dy}{dx} = \frac{x^2 + 4x}{(x+2)^2} = 0$

$\Rightarrow$ $x = 0, -4$

$$\frac{d^2y}{dx^2} = \frac{8}{(x+2)^3}$$

At $x = 0$, $\frac{d^2y}{dx^2}$ is true

$\therefore$ y is min when $x = 0$, $\therefore$ y min $= 2$

(B) As A is symmetric and B is skew symmetric matrix, we should have

$A^t = A$ and $B^t = -B$ ...(1)

Also given that

$(A+B)(A-B) = (A-B)(A+B)$

$\Rightarrow A^2 - AB + BA - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow 2BA = 2AB$ or $AB = BA$ ...(2)

Now given that

$(AB)^t = (-1)^k AB$

$\Rightarrow (BA)^t = (-1)^k AB$ (using equation (2))

$\Rightarrow A^t B^t = (-1)^k AB$

$\Rightarrow -AB = (-1)^k AB$ [using equation(1)]

$\Rightarrow k$ should be an odd number

$\therefore$ (B) $\rightarrow$ (q), (s)

(C) Given that $a = \log_3 \log_3 2$

$\Rightarrow \log_3 2 = 3^a \Rightarrow \dfrac{1}{\log_2 3} = 3^a$ or $\log_2 3 = 3^{-a}$

$\Rightarrow \quad 3 = 2^{(3^{-a})}$ ...(1)

Now $1 < 2^{(-k+3^{-a})} < 2 \Rightarrow 1 < 2^{-k}.2^{3^{-a}} < 2$

$\Rightarrow 1 < 2^{-k}.3 < 2$ (using eq (1))

$\Rightarrow \dfrac{1}{3} < 2^{-k} < \dfrac{2}{3} \Rightarrow \dfrac{3}{2} < 2^k < 3 \Rightarrow k = 1$

$\therefore k$ is less than 2 and 3

$\therefore$ (C) $\rightarrow$ (r), (s).

(D) Given that $\sin\theta = \cos\phi \Rightarrow \cos\left(\dfrac{\pi}{2} - \theta\right) = \cos\phi$

$\Rightarrow \dfrac{\pi}{2} - \theta = 2n\pi \pm \phi,\ n \in Z \Rightarrow \theta \pm \phi - \dfrac{\pi}{2} = -2n\pi$

$\Rightarrow \dfrac{1}{\pi}\left(\theta \pm \phi - \dfrac{\pi}{2}\right) = -2n$

$\therefore$ Here possible values of $\dfrac{1}{\pi}\left(\theta \pm \phi - \dfrac{\pi}{2}\right)$ are 0 and 2 for $n = 0, -1$.

$\therefore$ D $\rightarrow$ (p),(r).

G. Comprehension Based Questions

1. **(d)** Let $U_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ then $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow a = 1, b = -2, c = 1$

$\therefore U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ Similarly, $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

$\therefore U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \Rightarrow |U| = 3$

2. **(b)** $U^{-1} = \dfrac{1}{3}\begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$

$\Rightarrow$ Sum of elements of $U^{-1} = \dfrac{1}{3}(0) = 0$

3. **(a)** $[3\ 2\ 0]\begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = [3\ 2\ 0]\begin{bmatrix} 7 \\ -8 \\ -5 \end{bmatrix} = 5$

4. **(a)** Each element of set A is 3×3 symmetric matrix with five of its entries as 1 and four of its entries as 0, we can keep in diagonal either 2 zero and one 1 or no zero and three 1 so that the left over zeros and one's are even in number.

Hence taking 2 zeros and one 1 in diagonal the possible cases are $\dfrac{3!}{2!} \times \dfrac{3!}{2!} = 9$

and taking 3 ones in diagonal the possible cases are

$1 \times \dfrac{3!}{2!} = 3$

$\therefore$ Total elements A can have $= 9 + 3 = 12$

5. **(b)** The given system will have unique solution if $|A| \neq 0$ which is so for the matrices.

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$;

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$;

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

which are 6 in number.

6. **(b)** For the given system to be inconsistent $|A| = 0$. The matrices for which $|A| = 0$ are

$\underset{(i)}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}}, \underset{(ii)}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}, \underset{(iii)}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

and $\underset{(iv)}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}, \underset{(v)}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}, \underset{(vi)}{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}}$

On solving $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

We find for A = (i)

By Cramer's rule $D_1 = 0 = D_2 = D_3$

$\therefore$ infinite many solution

For A = (ii)

By Cramer's rule $D_1 \neq 0$

$\Rightarrow$ no solution i.e. inconsistent.

Similarly we find the system as inconsistent in cases (iii), (v) and (vi).

Hence for four cases system is inconsistent.

7. **(d)** If A is symmetrie then b = c

$\Rightarrow |A| = a^2 - b^2 = (a+b)(a-b)$

Which is divisible by p if $(a + b)$ is divisible by p or $(a - b)$ is divisible by p.

Now $(a + b)$ is divisible by p if (a, b) can take values $(1, p-1), (2, p-2), (3, p-3), ...(p-1, 1)$

$\therefore (p-1)$ ways.

Also $(a - b)$ is divisible by p only when $a - b = 0$

i.e. $a = b$, then (a, b) can take values (0,0) (1,1), (2,2)..... $(pÝ, p-2)$

$\therefore p$ ways.

If A is skew symmetric, then $a = 0$ and $b = -c$ or $b + c = 0$ which gives $|A| = 0$ when $b^2 \Rightarrow b = 0, c = 0$

But this possibility is already included when A is symmetric and $(a, b) = (0, 0)$.

Again if A is both symmetric and skew symmetric, then clearly A is null matrix which case is already included.

Hence total number of ways $= p + (p - 1) = 2p - 1$

8. **(c)** Trace $A = a + a = 2a$ is not divisible by p

$\Rightarrow a$ is not divisible by $p \Rightarrow a \neq 0$

But $|A|$ is divisible by $p \Rightarrow a^2 - bc$ is divisible by p

It will be so if on dividing a^2 by p suppose we get $m\frac{l}{p}$

then on dividing bc by p we should get $n\frac{l}{p}$ for some integeral values of m, n and l.

i.e. the remainder should be same in each case, so that

$$\frac{a^2 - bc}{p} = \left(m + \frac{l}{p}\right) - \left(n + \frac{l}{p}\right) = (m - n) = \text{an integer}$$

For this to happen a can take any value from 1 to $p-1$, also if b takes any value from 1 to $p-1$ then c should take only that value corresponding to which the remainder is same.

$\therefore$ No. of ways $= (p - 1) \times (p - 1) \times 1 = (p - 1)^2$.

9. **(d)** Total number of matrices

= total number of ways a, b, c can be selected

$= p \times p \times p = p^3$.

Number of ways when det (A) is divisible by p and trace (A) $\neq$ 0 are equal to number of ways det (A) is divisible by p and trace (A) is not divisible by $p = (p - 1)^2$

Also number of ways when det (A) is divisible by p and trace A = 0 are the ways when bc is multiple of p

$\Rightarrow b = 0$ or $c = 0$

for $b = 0$, c can take values $0, 1, 2, ..., p - 1$

For $c = 0$, b can take values $0, 1, 2, ..., p - 1$

Here $(b, c) = (0, 0)$ is coming twice.

$\therefore$ Total ways of selecting b and $c = p + p - 1$

$= 2p - 1$

$\therefore$ Total number of ways when det (A) is divisible by $p = (p - 1)^2 + 2p - 1 = p^2$

Hence the number of ways when det (A) is not divisible by $p = p^3 - p^2$.

10. **(d)** From equation (E), we get

$$a + 8b + 7c = 0$$
$$9a + 2b + 3c = 0$$
$$a + b + c = 0$$

Here $\begin{vmatrix} 1 & 8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$,

Therefore system has infinite many solutions.

Solving these, we get $b = 6a$ and $c = -7a$

Now (a, b, c) lies on $2x + y + z = 1 \Rightarrow b = 6, c = -7$

$\therefore 2a + 6a - 7a = 1 \Rightarrow a = 1$

$\therefore 7a + b + c = 7 + 6 - 7 = 6 \Rightarrow b = 6, c = -7$

11. **(a)** If $a = 2$ then $b = 12, c = -14$

$$\therefore \frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}}$$

$= 3\omega + 1 + 3\omega^2 = 1 - 3 = -2$

12. **(b)** If $b = 6$ then $a = 1, c = -7$

$\therefore$ Equation becomes $x^2 + 6x - 7 = 0$ or $(x + 7)(x - 1) = 0$ whose roots are 1 and -7.

Let $\alpha = 1$ and $\beta = -7$

$$\therefore \sum_{n=0}^{\infty}\left(\frac{1}{1} - \frac{1}{7}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = \frac{1}{1 - \frac{6}{7}} = 7$$

H. Assertion & Reason Type Questions

1. **(a)** The given equations are

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1$$

Here $D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$

and $D_2 = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = k - 3 \neq 0$ if $k \neq 3$

$\therefore$ If $k \neq 3$, the system has no solutions.

Hence statement-1 is true and statement-2 is a correct explanation for statement - 1.

I. Integer Value Correct Type

1. **(0)** We have $\omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \frac{-1+i\sqrt{3}}{2}$

$\therefore 1+\omega+\omega^2 = 0$ and $\omega^3 = 1$

Then $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$C_1 \leftrightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} z+1+\omega+\omega^2 & \omega & \omega^2 \\ z+1+\omega+\omega^2 & z+\omega^2 & 1 \\ z+1+\omega+\omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$$\Rightarrow z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

$$\Rightarrow z\left[1\left(z^2+z\omega+z\omega^2+\omega^3-1\right)-\omega\left(z+\omega-1\right)+\omega^2\left(1-z-\omega^2\right)=0\right]$$

$$\Rightarrow z\left[z^2+z\omega+z\omega^2-z\omega-\omega^2+\omega+\omega^2-z\omega^2-\omega^4\right]=0$$

$$\Rightarrow z\left[z^2\right]=0 \Rightarrow z^3=0 \Rightarrow z=0$$

$\therefore z = 0$ is the only solution.

2. **(4)** $|A| = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix}, C_2 \to C_2 - C_3$$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix}, R_2 \to R_2 - R_3$$

$= (1+2k)(8k-4k+4k^2+1) = (2k+1)^3$

Also $|B| = 0$ as B is skew symmetric of odd order.

$\therefore |\text{Adj } A| + |\text{Adj } B| = |A|^2 + |B|^2 = 10^6$

$\Rightarrow (2k+1)^6 = 10^6 \Rightarrow 2k+1 = 10 \Rightarrow k = 4.5$

$\therefore [k] = 4$

3. **(9)**

Let $M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

then $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{matrix} b_1 = -1 \\ b_2 = 2 \\ b_3 = 3 \end{matrix}$

$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & c_3 & c_3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow \begin{matrix} a_1 - b_1 = 1 \\ a_2 - b_2 = 1 \\ a_3 - b_3 = -1 \end{matrix}$

$\Rightarrow a_1 = 0, a_2 = 3, a_3 = 2$

and $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow a_3 + b_3 + c_3 = 12$
$\Rightarrow c_3 = 7$

$\therefore$ Sum of diagonal elements $= a_1 + b_2 + c_3 = 0 + 2 + 7 = 9$

4. **(2)** $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$

$$\Rightarrow x^3\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

Operating $C_2 - C_1, C_3 - C_1$ for both the determinants, we get

$$\Rightarrow x^3\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$$

$\Rightarrow x^3(-4+6) + x^6(48-36) = 10$
$\Rightarrow 2x^3 + 12x^6 = 10 \Rightarrow 6x^6 + x^3 - 5 = 0$

$\Rightarrow (6x^3-5)(x^3+1) = 0 \Rightarrow x = \left(\frac{5}{6}\right)^{\frac{1}{3}}, -1$

5. **(1)** $z = \frac{-1+i\sqrt{3}}{2} \Rightarrow z^3 = 1$ and $1 + z + z^2 = 0$

$$P^2 = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}\begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$$

$$= \begin{bmatrix} z^{2r}+z^{4s} & z^{2s}\left((-z)^r+z^r\right) \\ z^{2s}\left((-z)^r+z^r\right) & z^{4s}+z^{2r} \end{bmatrix}$$

For $P^2 = -I$ we should have
$z^{2r} + z^{4s} = -1$ and $z^{2s}((-z)^r + z^r) = 0$
$\Rightarrow z^{2r} + z^s + 1 = 0$ and $(-z)^r + z^r = 0$
$\Rightarrow$ r is odd and s = r but not a multiple of 3.
Which is possible when s = r = 1
$\therefore$ only one pair is there.

Section-B JEE Main/ AIEEE

1. **(c)** We have $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

By $R_3 \to R_3 - (xR_1 + R_2)$;

$$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+C) \end{vmatrix}$$

$= (ax^2+2bx+c)(b^2-ac) = (+)(-) = -$ve.

2. **(d)** For homogeneous system of equations to have non zero solution, $\Delta = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \quad C_2 \to C_2 - 2C_3$$

$$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad R_3 \to R_3 - R_2, R_2 \to R_2 - R_1$$

$$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$$

$b(c-b) - (b-a)(2c-b) = 0$

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore a, b, c$ are in Harmonic Progression.

3. **(b)** $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$$= 1\left(\omega^{3n} - 1\right) - \omega^n\left(\omega^{2n} - \omega^{2n}\right) + \omega^{2n}\left(\omega^n - \omega^{4n}\right)$$

$= \omega^{3n} - 1 - 0 + \omega^{3n} - \omega^{6n}$

$= 1 - 1 + 1 - 1 = 0 \left[\because \omega^{3n} = 1\right]$

4. **(c)** $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$$= \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$$

$\alpha = a^2 + b^2; \beta = 2ab$

5. **(a)** $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$\therefore A^{-1}$ exists, further $(-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

Also $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

6. **(a)** Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Also since, $B = A^{-1} \Rightarrow AB = I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$$

7. **(d)** Let r be the common ratio, then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

$$= 0 \left[\text{Apply } c_2 \to c_2 - \frac{1}{2}c_1 - \frac{1}{2}c_3\right]$$

8. (d) Given $A^2 - A + I = 0$

$A^{-1}A^2 - A^{-1}A + A^{-1}.I = A^{-1}.0$

(Multiplying A^{-1} on both sides)

$\Rightarrow A - 1 + A^{-1} = 0$ or $A^{-1} = 1 - A$.

9. (a) $\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1;$

$x + y + z\alpha = \alpha - 1$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$

$= \alpha\,(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$

For infinite solutions, $\Delta = 0$

$\Rightarrow (\alpha\ - 1)[\alpha^2 + \alpha - 1 - 1] = 0$

$\Rightarrow (\alpha\ - 1)[\alpha^2 + \alpha - 2] = 0 \Rightarrow \alpha = -2, 1;$

But $\alpha \neq 1$. $\therefore \alpha = -2$

10. (d) Applying, $C_1 \to C_1 + C_2 + C_3$ we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2x) \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2x) \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

[As given that $a^2 + b^2 + c^2 = -2$]

$\therefore a^2 + b^2 + c^2 + 2 = 0$

Applying $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\therefore f(x) = \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$f(x) = (x-1)^2$ Hence degree = 2.

11. (b) $\because a_1, a_2, a_3, ...$ are in G.P.

$\therefore$ Using $a_n = ar^{n-1}$, we get the given determinant, as

$$\begin{vmatrix} \log ar^{n-1} & \log ar^{n} & \log ar^{n+1} \\ \log ar^{n+2} & \log ar^{n+3} & \log ar^{n+4} \\ \log ar^{n+5} & \log ar^{n+6} & \log ar^{n+7} \end{vmatrix}$$

Operating $C_3 - C_2$ and $C_2 - C_1$ and using

$\log m - \log n = \log \dfrac{m}{n}$ we get

$$= \begin{vmatrix} \log ar^{n-1} & \log r & \log r \\ \log ar^{n+2} & \log r & \log r \\ \log ar^{n+5} & \log r & \log r \end{vmatrix}$$

= 0 (two columns being identical)

12. (b) $A^2 - B^2 = (A - B)(A + B)$

$A^2 - B^2 = A^2 + AB - BA - B^2 \quad \Rightarrow AB = BA$

13. (d) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

Hence, $AB = BA$ only when $a = b$

$\therefore$ There can be infinitely many B's for which $AB = BA$

14. (d) Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Apply $R_2 \to R_2 - R_1$ and $R \to R_3 - R_1$

$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y

15. (a) $|A^2| = 25 \Rightarrow |A|^2 = 25 \Rightarrow (25\alpha)^2 = 25 \Rightarrow |\alpha| = \dfrac{1}{5}$

16. (d) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 = I$

$\Rightarrow a^2 + bc = 1 \quad ab + bd = 0$

$ac + cd = 0 \quad bc + d^2 = 1$

From these four relations,

$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$

and $b(a + d) = 0 = c(a + d) \Rightarrow a = -d$

We can take $a = 1, b = 0, c = 0, d = -1$ as one possible set of values, then $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Clearly $A \neq I$ and $A \neq -I$ and $\det A = -1$

$\therefore$ Statement 1 is true.

Also if $A \neq I$ then $tr(A) = 0$

$\therefore$ Statement 2 is false.

17. (d) The given equations are

$-x + cy + bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

$\because x, y, z$ are not all zero

$\therefore$ The above system should not have unique (zero) solution

$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow -1(1-a^2) - c(-c-ab) + b(ac+b) = 0$
$\Rightarrow -1 + a^2 + b^2 + c^2 + 2abc = 0$
$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

18. (c) $\because$ All entries of square matrix A are integers, therefore all cofactors should also be integers.
If det $A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

19. (d) We know that $|adj\,(adj\,A)| = |\text{adj A}|^{2-1}$
$= |A|^{2-1} = |A|$
$\therefore$ Both the statements are true and statement -2 is a correct explantion for statement-1 .

20. (b) $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$

(Taking transpose of second determinant)
$C_1 \Leftrightarrow C_3$

$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$

$C_2 \Leftrightarrow C_3$

$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$

$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$

$C_2 - C_1,\ C_3 - C_1$

$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$

$R_1 + R_3$

$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $[1+(-1)^{n+2}](a+c)(2b+1+2b-1) = 0$
$\Rightarrow$ $4b(a+c)[1+(-1)^{n+2}] = 0$
$\Rightarrow$ $1+(-1)^{n+2} = 0$ as $b(a+c) \neq 0$
$\Rightarrow$ n should be an odd integer.

21. (c) $\begin{bmatrix} 1 & \dots & \dots \\ \dots & 1 & \dots \\ \dots & \dots & 1 \end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

Similarly, $\begin{bmatrix} \dots & \dots & 1 \\ \dots & 1 & \dots \\ 1 & \dots & \dots \end{bmatrix}$ are 6 non-singular matrices.

So, required cases are more than 7, non-singular 3×3 matrices.

22. (b) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d $\neq 0$

$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$
$ab + bd = ac + cd = 0$
$c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0 \Rightarrow \text{Tr(A)} = 0$
$|A| = ad - bc = -a^2 - bc = -1$

23. (c) $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0 \qquad D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

$\Rightarrow$ Given system, does not have any solution.
$\Rightarrow$ No solution

24. (a) $\Delta = 0 \Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 4(4-2) - k(k-2) + 2(2k-8) = 0$
$\Rightarrow 8 - k^2 + 2k + 4k - 16 = 0 \quad k^2 - 6k + 8 = 0$
$\Rightarrow (k-4)(k-2) = 0,\ k = 4, 2$

25. (a) $\therefore$ A′ = A, B′ = B
Now (A(BA))′ = (BA)′A′
= (A′B′)A′ = (AB)A = A(BA)
Similarly ((AB)A)′ = (AB)A
So, A(BA) and (AB)A are symmetric matrices.
Again (AB)′ = B′A′ = BA
Now if BA = AB, then AB is symmetric matrix.

26. (d) Let $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Then, $Au_1 + Au_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\Rightarrow \quad A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ...(1)

Also, $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix} \Rightarrow |A|=1(1)-0(2)+0(4-3)=1$

We know,

$A^{-1}=\frac{1}{|A|}adjA \Rightarrow A^{-1}=adj(A) \quad (\because |A|=1)$

Now, from equation (1), we have

$u_1+u_2=A^{-1}\begin{pmatrix}1\\1\\0\end{pmatrix} \Rightarrow u_1+u_2=A^{-1}$

$=\begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$

27. (c) Given $P^3=Q^3$ ...(1)
and $P^2Q=Q^2P$ ...(2)
Subtracting (1) and (2), we get
$P^3-P^2Q=Q^3-Q^2P$
$\Rightarrow P^2(P-Q)+Q^2(P-Q)=0$
$\Rightarrow (P^2+Q^2)(P-Q)=0 \Rightarrow |P^2+Q^2|=0$ as $P\neq Q$

28. (b) $|P|=1(12-12)-\alpha(4-6)+3(4-6)=2\alpha-6$
Now, adj A = P $\Rightarrow |\text{adj } A|=|P|$
$\Rightarrow |A|^2=|P| \Rightarrow |P|=16$
$\Rightarrow 2\alpha-6=16 \Rightarrow \alpha=11$

29. (a) Consider

$$\begin{vmatrix}3&1+f(1)&1+f(2)\\1+f(1)&1+f(2)&1+f(3)\\1+f(2)&1+f(3)&1+f(4)\end{vmatrix}$$

$$=\begin{vmatrix}1+1+1&1+\alpha+\beta&1+\alpha^2+\beta^2\\1+\alpha+\beta&1+\alpha^2+\beta^2&1+\alpha^3+\beta^3\\1+\alpha^2+\beta^2&1+\alpha^3+\beta^3&1+\alpha^4+\beta^4\end{vmatrix}$$

$$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}\times\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}$$

$$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}^2=[(1-\alpha)(1-\beta)(\alpha-\beta)]^2$$

So, $\boxed{k=1}$

30. (d) $BB'=B(A^{-1}A')'=B(A')'(A^{-1})'=BA(A^{-1})'$
$=(A^{-1}A')(A(A^{-1})')$
$=A^{-1}A.A'.(A^{-1})'$ {as $AA'=A'A$}
$=I(A^{-1}A)'$
$=I.I=I^2=I$

31. (a) $\left.\begin{aligned}2x_1-2x_2+x_3&=\lambda x_1\\2x_1-3x_2+2x_3&=\lambda x_2\\-x_1+2x_2&=\lambda x_3\end{aligned}\right\}$

$\Rightarrow (2-\lambda)x_1-2x_2+x_3=0$
$2x_1-(3+\lambda)x_2+2x_3=0$
$-x_1+2x_2-\lambda x_3=0$
For non-trivial solution, $\Delta=0$

i.e. $\begin{vmatrix}2-\lambda&-2&1\\2&-(3+\lambda)&2\\-1&2&-\lambda\end{vmatrix}=0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda)-4]+2[-2\lambda+2]+1[4-(3+\lambda)]=0$
$\Rightarrow \lambda^3+\lambda^2-5\lambda+3=0 \Rightarrow \lambda=1,1,3$
Hence, λ has 2 values.

32. (b) $\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}\begin{bmatrix}1&2&a\\2&1&2\\2&-2&b\end{bmatrix}=\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$

$\Rightarrow\begin{bmatrix}1+4+4&2+2-4&a+4+2b\\2+2-4&4+1+4&2a+2-2b\\a+4+2b&2a+2-2b&a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$

$\Rightarrow a+4+2b=0 \Rightarrow a+2b=-4$ ...(i)
$2a+2-2b=0 \Rightarrow 2a-2b=-2$
$\Rightarrow a-b=-1$ ...(ii)
On solving (i) and (ii) we get
$-1+b+2b=-4$ ...(i)
$b=-1$ and $a=-2$
$(a,b)=(-2,-1)$

33. (b) For trivial solution,

$\begin{vmatrix}1&\lambda&-1\\\lambda&-1&-1\\1&1&-\lambda\end{vmatrix}=0$

$\Rightarrow -\lambda(\lambda+1)(\lambda-1)=0$
$\Rightarrow \lambda=0,+1,-1$

34. (d) $A(\text{adj } A)=A\,A^T$
$\Rightarrow A^{-1}A\,(\text{adj } A)=A^{-1}A\,A^T$
$\text{adj } A=A^T$
$\Rightarrow\begin{bmatrix}2&b\\-3&5a\end{bmatrix}=\begin{bmatrix}5a&3\\-b&2\end{bmatrix}$
$\Rightarrow a=\frac{2}{5}$ and $b=3$
$\Rightarrow 5a+b=5$