4. POLYNOMIALS

Polynomial in one variable

An algebraic expression of the form

$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x^1 + a_0 x^0$, where

(i) $a_n \neq 0$

(ii) $a_0, a_1, a_2, ... a_n$ are real numbers

(iii) power of x is a positive integer, is called a polynomial in one variable.

Hence, $a_n, a_{n-1}, a_{n-2}, ..., a_0$ are coefficients of $x^n, x^{n-1}, ..., x^0$ respectively and $a_n x^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ...$ are terms of the polynomial. Here the term $a_n x^n$ is called the ***leading term*** and its coefficient a_n, the ***leading coefficient.***

Degree of polynomials

Degree of the polynomial in one variable is the largest exponent of the variable. For example, the degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7 and the degree of the polynomial $5x^6 - 4x^2 - 6$ is 6.

Classification of polynomials

- **Polynomials classified by degree**

Degree	Name	General form	Example
$-\infty$ (undefined)	Zero polynomial	0	0
0	(Non-zero) constant polynomial	a; $(a \neq 0)$	1
1	Linear polynomial	$ax + b$; $(a \neq 0)$	$x + 1$
2	Quadratic polynomial	$ax^2 + bx + c$; $(a \neq 0)$	$x^2 + 1$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d$; $(a \neq 0)$	$x^3 + 1$

A polynomial of degree n, for n greater than 3, is called a polynomial of degree n.

- **Polynomials classified by terms**

Monomials : Polynomials having only one term are called monomials. E.g. 2, 2x, $7y^5$, $12t^7$ etc.

Binomials : Polynomials having exactly two terms are called binomials. E.g. $p(x) = 2x + 1$, $r(y) = 2y^7 + 5y^6$. etc.

Trinomials : Polynomials having exactly three terms are called trinomials. E.g. $p(x) = 2x^2 + x + 6$,

$q(y) = 9y^6 + 4y^2 + 1$ etc.

Zeros / Roots of a polynomial / equation

The value of the variable x, for which the polynomial f(x) becomes zero is called zero of the polynomial.

E.g. : consider, a polynomial $p(x) = x^2 - 5x + 6$; replace x by 2 and 3.

$p(2) = (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$,

$p(3) = (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

$\therefore$ 2 and 3 are the zeros of the polynomial p(x).

Roots of a polynomial equation

An expression $f(x) = 0$ is called a polynomial equation if f(x) is a polynomial of degree $n \geq 1$.

A real number α is a root of a polynomial $f(x) = 0$ if $f(\alpha) = 0$ i.e. α is a zero of the polynomial f(x).

E.g. consider the polynomial $f(x) = 3x - 2$, then $3x - 2 = 0$ is the corresponding polynomial equation.

Here, $f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) - 2 = 0$

i.e. $\frac{2}{3}$ is a zero of the polynomial $f(x) = 3x - 2$

or $\frac{2}{3}$ is a root of the polynomial equation $3x - 2 = 0$

Important concepts

- A non-zero constant is a polynomial of degree zero, but the degree of zero polynomial is not defined.
- If the sum of the co-efficients of polynomial is zero, then $(x - 1)$ is a factor of the polynomial.
- A polynomial in x is said to be a polynomial in standard form, if the powers of x are either in ascending order or in descending order.
- A polynomial of degree $n \geq 1$ can have at the most n real zeros.
- A non-zero constant polynomial has no zero.
- Every linear polynomial has one and only one zero.
- A quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ can have at most two real zeros. In some cases, it may not have any real zero.
- Zero of a polynomial is actually the solution of the curve, $y = f(x)$ and the line $y = 0$.

Remainder theorem

- **Statement :** Let p(x) be a polynomial of degree ≥ 1 and 'a' is any real number. If p(x) is divided by $(x - a)$, then the remainder is p(a).
  - $p(-a)$ is remainder on dividing p(x) by $(x + a)$ $[\because x + a = 0 \Rightarrow x = -a]$
  - $p\left(\frac{b}{a}\right)$ is remainder on dividing p(x) by $(ax - b)$ $[\because ax - b = 0 \Rightarrow x = b/a]$
  - $p\left(\frac{-b}{a}\right)$ is remainder on dividing p(x) by $(ax + b)$ $[\because ax + b = 0 \Rightarrow x = -b/a]$
  - $p\left(\frac{b}{a}\right)$ is remainder on dividing p(x) by $(b - ax)$ $[\because b - ax = 0 \Rightarrow x = b/a]$

Factor theorem

- **Statement :** Let f(x) be a polynomial of degree ≥ 1 and a be any real constant such that $f(a) = 0$, then $(x - a)$ is a factor of f(x). Conversely, if $(x - a)$ is a factor of f(x), then $f(a) = 0$.
- **P(x) is a polynomial of degree ≥ 1 and "a" is a real number then**

  $p(a) = 0 \Rightarrow (x - a)$ is a factor of p(x)
  - $(x - a)$ is a factor of p(x) then $p(a) = 0$
  - $ax - b$ is a factor of p(x) then $P\left(\frac{b}{a}\right) = 0$
  - $ax + b$ is a factor of p(x) then $P\left(\frac{-b}{a}\right) = 0$
  - $(x - a)$ is a factor of $x^n - a^n$ where "n" is an odd positive integer
  - $(x + a)$ is a factor of $x^n - a^n$ where "n" is an odd positive integer
  - $(x + a)$ is factor of $x^n - a^n$ where "n" is positive even integer.
  - $(x^n + a^n)$ is not divisible by $x + a$ when "n" is even
  - $x^n + a^n$ is not divisible by $x - a$ for any "n"
  - If $x - 1$ is a factor of polynomial of degree 'n' then the condition is sum of the coeffecients is zero.
  - If $(x + 1)$ is a factor of polynomial of degree 'n' then the condition is sum of the coefficients of even terms is equal to the coefficients of odd terms.

Relationship between the zeros and coefficients of a polynomial

For a linear polynomial $ax + b$, $(a \neq 0)$, we have,

$$\boxed{\text{zero of a linear polynomial} = -\frac{b}{a} = -\frac{(\text{constant term})}{(\text{coefficient of } x)}}$$

For a quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$, with α and β as it's zeros, $(x - \alpha)$ and $(x - \beta)$ are the factors of $ax^2 + bx + c$.

Therefore, $ax^2 + bx + c = K(x - \alpha)(x - \beta)$, (where K is a constant to balance the equation of the coefficient of x^2 i.e. $a \neq 1$.)

$$= K x^2 - K(\alpha + \beta) x + K \alpha \beta$$

comparing the coefficients of x^2, x and constant terms on both the sides, we get

$$a = K,\ b = -K(\alpha + \beta) \text{ and } c = K \alpha \beta$$

This gives

$$\text{Sum of zeros} = \alpha + \beta = -\frac{b}{a} = -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeros} = \alpha\beta = \frac{c}{a} = \frac{(\text{constant term})}{(\text{coefficient of } x^2)}$$

If α and β are the zeros of a quadratic polynomial f(x). Then polynomial f(x) is given by

$f(x) = K\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

or $f(x) = K\{x^2 -$ (sum of the zeros) x + product of the zeros$\}$

where K is a constant.

Symmetric functions of the zeros

Let α,β be the zeros of a quadratic polynomial, then the expression of the form $\alpha + \beta$; $(\alpha^2 + \beta^2)$; $\alpha\beta$ are called the functions of the zeros. By symmetric function we mean that the function remain invariant (unaltered) in values when the roots are changed cyclically. In other words, an expression involving α and β which remains unchanged by interchanging α and β is called a symmetric function of α and β.

Some useful relations involving α and β are :-

- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
- $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
- $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
- $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$
- $\alpha^4 - \beta^4 = (\alpha^2 + \beta^2)(\alpha + \beta)(\alpha - \beta) = [(\alpha + \beta)^2 - 2\alpha\beta](\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
- $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$
- $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)(\alpha^2 + \beta^2) - \alpha^2\beta^2(\alpha + \beta) = [(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)][(\alpha + \beta)^2 - 2\alpha\beta] - (\alpha\beta)^2(\alpha + \beta)$

Division algorithm for polynomials

If f(x) is a polynomial and g(x) is a non-zero polynomial, then there exist two polynomials q(x) and r(x) such that $f(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree r(x) < degree g(x). In other words,

Dividend = Divisor × Quotient + Remainder

Remark : If r(x) = 0 , then polynomial g(x) is a factor of polynomial f(x).

Algebraic identities

An algebraic identity is an algebraic equation that is true for all values of the variables present in the equation.

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $x^2 - y^2 = (x + y)(x - y)$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
- $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
- If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

1. If $x + \frac{1}{x} = 5$, then the value of $x^3 + \frac{1}{x^3}$ is

(1) 110 (2) 90

(3) 80 (4) 50

2. If $x^3 - (x + 1)^2 = 2001$ then the value of x is

(1) 14 (2) 13

(3) 10 (4) None

3. The square root of $\frac{x^2}{y^2} + \frac{y^2}{4x^2} - \frac{x}{y} + \frac{y}{2x} - \frac{3}{4}$ is

(1) $\frac{x}{y} - \frac{1}{2} - \frac{y}{2x}$ (2) $\frac{x}{y} + \frac{1}{2} - \frac{y}{2x}$

(3) $\frac{x}{y} + \frac{1}{2} + \frac{y}{2x}$ (4) $\frac{x}{y} - \frac{1}{4} - \frac{y}{2x}$

4. If the zeros of the polynomial $ax^2 + bx + c$ be in the ratio m : n, then

(1) $b^2 mn = (m^2 + n^2) ac$

(2) $(m + n)^2 ac = b^2 mn$

(3) $b^2 (m^2 + n^2) = mnac$

(4) None of these

5. If $\alpha \neq \beta$ and the difference between the roots of the polynomials $x^2 + ax + b$ and $x^2 + bx + a$ is the same, then

(1) $a + b + 4 = 0$

(2) $a + b - 4 = 0$

(3) $a - b + 4 = 0$

(4) $a - b - 4 = 0$

6. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the polynomial whose zeros are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is :

(1) $3x^2 - 25x + 3$ (2) $x^2 - 5x + 3$

(3) $x^2 + 5x - 3$ (4) $3x^2 - 19x + 3$

7. The factors of $a^2(b^3 - c^3) + b^2(c^3 - a^3) + c^2(a^3 - b^3)$ are

(1) (a – b) (b – c) (c – a) (ab + bc + ca)

(2) (a + b) (b + c) (c + a) (ab + bc + ca)

(3) (a – b) (b – c) (c – a) (ab – bc – ca)

(4) None of these

8. If p,q are zeros of $x^2 + px + q$, then

(1) p = 1 (2) p = 1 or 0

(3) p = – 2 (4) p = – 2 or 0

9. On simplifying $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$ we get

(1) $8a^2$ (2) $8a^2b$

(3) $8a^3b$ (4) $8a^3$

10. Factors of $(42 - x - x^2)$ are

(1) (x – 7)(x – 6) (2) (x + 7)(x – 6)

(3) (x + 7)(6 – x) (4) (x + 7)(x + 6)

11. Factors of $\left(x^2 + \frac{x}{6} - \frac{1}{6}\right)$ are

(1) $\frac{1}{6}(2x+1)(3x+1)$ (2) $\frac{1}{6}(2x+1)(3x-1)$

(3) $\frac{1}{6}(2x-1)(3x-1)$ (4) $\frac{1}{6}(2x-1)(3x+1)$

12. Value of $\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$, when a = –5, b = –6, c = 10 is

(1) 1 (2) –1

(3) 2 (4) –2

13. If (x + y + z) = 1, xy + yz + zx = –1, xyz = –1, then the value of $x^3 + y^3 + z^3$ is

(1) –1 (2) 1

(3) 2 (4) –2

14. In method of factorization of an algebraic expression, Which of the following statements is false?

(1) Taking out a common factor from two or more terms

(2) Taking out a common factor from a group of terms

(3) By using remainder theorem

(4) By using standard identities

15. Factors of $(a + b)^3 - (a - b)^3$ are

(1) $2ab(3a^2 + b^2)$ (2) $ab(3a^2 + b^2)$

(3) $2b(3a^2 + b^2)$ (4) $3a^2 + b^{20}$

16. The homogeneous function of the second degree in x and y having $2x - y$ as a factor, taking the value 2 when $x = y = 1$ and vanishing if $x = -1, y = 1$ is

(1) $2x^2 + xy - y^2$ (2) $3x^2 - 2xy + y^2$

(3) $x^2 + xy - 2y^2$ (4) None of these

17. The common quantity that must be added to each term of $a^2 : b^2$ to make it equal to a : b is

(1) ab (2) a + b (3) a – b (4) $\frac{a}{b}$

18. If the polynomial $16x^4 - 24x^3 + 41x^2 - mx + 16$ be a perfect square,then the value of "m" is

(1) 12 (2) –12 (3) 24 (4) –24

19. If $a - b = 3$, $a + b + x = 2$, then the value of $(a - b)[x^3 - 2ax^2 + a^2x - (a + b)b^2]$ is

(1) 84 (2) 48 (3) 32 (4) 36

20. If $abx^2 = (a - b)^2(x + 1)$, then the value of $1 + \frac{4}{x} + \frac{4}{x^2}$ is:-

(1) $\left(\frac{a-b}{a+b}\right)^2$ (2) $\left(\frac{a+b}{a-b}\right)^2$

(3) $\left(\frac{a}{a+b}\right)^2$ (4) $\left(\frac{b}{a+b}\right)^2$

21. Let α,β be the zeros of the polynomial $(x - a)(x - b) - c$ with $c \neq 0$. Then the zeros of the polynomial $(x - \alpha)(x - \beta) + c$ are

(1) a, c (2) b, c

(3) a, b (4) a + c, b + c

22. A homogeneous expression of second degree in x & y is

(1) $ax^2 + bx + c$ (2) $ax^2 + bx + cy$

(3) $ax^2 + bx + cy^2$ (4) $ax^2 + bxy + cy^2$

23. If the sum of the zeros of the polynomial $x^2 + px + q$ is equal to the sum of their squares, then

(1) $p^2 - q^2 = 0$ (2) $p^2 + q^2 = 2q$

(3) $p^2 + p = 2q$ (4) None of these

24. The G.C.D of $x^2 - 3x + 2$ and $x^2 - 4x + 4$ is

(1) $x - 2$ (2) $(x - 2)(x - 1)$

(3) $(x - 2)^2$ (4) $(x - 2)^3(x - 1)$

25. The L.C.M. of $22x(x + 1)^2$ and $36x^2(2x^2 + 3x + 1)$ is

(1) $2x(x + 1)$

(2) $396x^2(x + 1)^2(2x + 1)$

(3) $792x^3(x + 1)^2 (2x^2 + 3x + 1)$

(4) None of these

26. The L.C.M of $x^3 - 8$ and $x^2 - 5x + 6$ is

(1) $x - 2$

(2) $x^2 + 2x + 4$

(3) $(x - 2)(x^2 + 2x + 4)$

(4) $(x - 2)(x - 3)(x^2 + 2x + 4)$

27. If the G.C.D. of the polynomials $x^3 - 3x^2 + px + 24$ and $x^2 - 7x + q$ is $(x - 2)$, then the value of $(p + q)$ is:

(1) 0 (2) 20 (3) –20 (4) 40

28. If the L.C.M. of two polynomials p(x) and q(x) is $(x + 3)(x - 2)^2(x - 6)$ and their H.C.F. is $(x - 2)$.If $p(x) = (x + 3)(x - 2)^2$, then q(x) =________

(1) $(x + 3)((x - 2)$ (2) $x^2 - 3x - 18$

(3) $x^2 - 8x + 12$ (4) none of these

29. The G.C.D. of two polynomials is $(x - 1)$ and their L.C.M. is $x^6 - 1$. If one of the polynomials is $x^3 - 1$, then the other polynomial is_____.

(1) $x^3 - 1$ (2) $x^4 - x^3 + x - 1$

(3) $x^2 - x + 1$ (4) None of these

30. The L.C.M. of 2x and 8 is

(1) 2x (2) 4x (3) 8x (4) 16x

31. If $x^2 + \frac{1}{x^2} = 38$, then the value of $x - \frac{1}{x}$ is

(1) 6 (2) 4 (3) 0 (4) None

32. The simplest form of $(2x + 3)^3 - (2x - 3)^3$ is

(1) $54 + 72x^2$ (2) $72 + 54x^2$

(3) $54 + 54x^2$ (4) None of these

33. The simplest form of $(p - q)^3 + (q - r)^3 + (r - p)^3$ is

(1) $4(p - q)(q - r)(r - p)$ (2) $2(p - q)(q - r)(r - p)$

(3) $(p - q)(q - r)(r - p)$ (4) None of these

34. The square root of $x^4 + 6x^3 + 17x^2 + 24x + 16$ is

(1) $x^2 + 3x + 4$ (2) $2x^2 + 3x + 4$

(3) $3x^2 + 3x + 4$ (4) None of these

35. The square root of $x^4 - 2x^3 + 3x^2 - 2x + 1$ is

(1) $x^2 + x + 1$ (2) $x^2 - x + 1$

(3) $x^2 + x - 1$ (4) $x^2 - x - 1$

36. The value of λ for which one zero of $3x^2 - (1 + 4\lambda) x + \lambda^2 + 2$ may be one-third of the other is

(1) 4 (2) $\frac{33}{8}$

(3) $\frac{17}{4}$ (4) $\frac{31}{8}$

37. The factors of $a^3(b - c) + b^3(c - a) + c^3(a - b)$ are

(1) $(a + b + c) (a - b) (b - c) (c - a)$

(2) $- (a + b + c) (a - b) (b - c) (c - a)$

(3) $2 (a + b + c) (a - b) (b - c) (c - a)$

(4) $- 2 (a + b + c) (a - b) (b - c) (c - a)$

38. The value of 'a', for which one root of the quadratic polynomial $(a^2 - 5a + 3) x^2 + (3a - 1) x + 2$ is twice as large as the other, is

(1) $-\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $-\frac{2}{3}$ (4) $\frac{1}{3}$

39. If the polynomial

$x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 + x^3$

is divided by $(x^2 + 1)$, then the remainder is

(1) 1 (2) $x^2 + 4$

(3) $-x$ (4) x

40. If $(x - 2)$ is a common factor of $x^3 - 4x^2 + ax + b$ and $x^3 - ax^2 + bx + 8$, then the values of a and b are respectively

(1) 3 and 5 (2) 2 and –4

(3) 4 and 0 (4) 0 and 4

41. If the expressions $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ on dividing by $x - 4$ leave the same remainder, then the value of a is

(1) 1 (2) 0 (3) 2 (4) –1

42. If the polynomial $x^6 + px^5 + qx^4 - x^2 - x - 3$ is divisible by $x^4 - 1$, then the value of $p^2 + q^2$ is

(1) 1 (2) 5 (3) 10 (4) 13

43. If $3x^3 + 2x^2 - 3x + 4 = (Ax + B)(x - 1)(x + 2) + C(x - 1) + D$ for all values of x, then $A + B + C + D$ is

(1) 0 (2) 14

(3) 10 (4) All

44. The expression $x^3 + gx^2 + hx + k$ is divisible by both x and $x - 2$ but leaves a remainder of 24 when divided by $x + 2$ then the values of g, h and k are

(1) $g = 10, h = -3, k = 0$

(2) $g = 3, h = -10, k = 0$

(3) $g = 10, h = -2, k = 3$

(4) None of these

45. The value of m if $2x^m + x^3 - 3x^2 - 26$ leaves a remainder of 226 when it is divided by $x - 2$.

(1) 0 (2) 7

(3) 10 (4) All of these

46. The expression $Ax^3 + x^2 + Bx + C$ leaves remainder of $\frac{21}{4}$ when divided by $1 - 2x$ and 18 when divided by x. Given also the expression has a factor of $(x - 2)$, the values of A, B and C are

(1) $A = 5, B = -9, C = 3$

(2) $A = 27, B = -18, C = 4$

(3) $A = 4, B = -27, C = 18$

(4) None of these

47. If $h(x) = 2x^3 + (6a^2 - 10) x^2 + (6a + 2) x - 14a - 2$ is exactly divisible by $x - 1$ but not by $x + 1$, then the value of a is

(1) 0 (2) –1

(3) 10 (4) 2

48. Given the polynomial is exactly divided by $x + 1$, and when it is divided by $3x - 1$, the remainder is 4. The polynomial gives a remainder $hx + k$ when divided by $3x^2 + 2x - 1$ then the values of h and k are

(1) $h = 2, k = 3$ (2) $h = 3, k = 3$

(3) $h = 3, k = 2$ (4) None of these

49. The remainder when $f(x) = (x^4 - x^3 + 2x - 3) g(x)$ is divided by $x - 3$, given that $x - 3$ is a factor of $g(x) + 3$, where g(x) is a polynomial is

(1) 0 (2) –171

(3) 10 (4) 2

50. If $x^3 - hx^2 + kx - 9$ has a factor of $x^2 + 3$, then the values of h and k are

(1) $h = 3, k = 3$

(2) $h = 2, k = 2$

(3) $h = 2, k = 1$

(4) None of these

51. The polynomial f(x) has roots of equations 3, –3, –k. Given that the coefficient of x^3 is 2, and that f(x) has a remainder of 8 when divided by x + 1, the value of k is

(1) 1/2 (2) 1/4 (3) 1/5 (4) 2

52. One of the factors of $x^3 + 3x^2 - x - 3$ is

(1) x + 1 (2) x + 2
(3) x – 2 (4) x – 3

53. If $ax^2 + 2a^2x + b^3$ is divisible by x + a, then _____.

(1) a = b (2) a + b = 0
(3) $a^2 - ab + b^2 = 0$ (4) $a^2 + 2ab + b^2 = 0$

54. If $x^3 + 2x^2 + ax + b$ is exactly divisible by (x + a) and (x – 1), then _____.

(1) a = –2 (2) b = –1
(3) a = –1 (4) b = 1

55. If $f(x) = ax^2 + bx + c$ is divided by (bx + c), then the remainder is_____.

(1) $\frac{c^2}{b^2}$ (2) $\frac{ac^2}{b^2}+2c$
(3) $f\left(-\frac{c}{b}\right)$ (4) $\frac{ac^2+2b^2c}{b^2}$

56. $ax^4 + bx^3 + cx^2 + dx + e$ is exactly divisible by $x^2 - 1$, when:

(1) a + b + c + e = 0 (2) a + c + e = 0
(3) a + b = 0 (4) a + c + e = b + d = 1

57. The remainder of $x^4 + x^3 - x^2 + 2x + 3$ when divided by x – 3 is

(1) 105 (2) 108 (3) 10 (4) None

58. If x – 3 is a factor of $x^3 + 3x^2 + 3x + p$, then the value of p is

(1) 0 (2) –63 (3) 10 (4) None

59. The value of $ax^2 + bx + c$ when x = 0 is 6. The remainder when dividing by x + 1 is 6. The remainder when dividing by x + 2 is 8. Then the sum of a, b and c is

(1) 0 (2) –1 (3) 10 (4) None

60. $x^n - y^n$ is divisible by x + y, when n is_______.

(1) An odd positive integer
(2) An even positive integer
(3) An integer
(4) None of these

61. If α,β are the zeros of the quadratic polynomial $4x^2 - 4x + 1$, then $\alpha^3 + \beta^3$ is –

(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) 16 (4) 32

62. If α,β,γ are the zeros of the polynomial x^3+4x+1, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} =$

(1) 2 (2) 3 (3) 4 (4) 5

63. The remainder when x^{1999} is divided by $x^2 - 1$ is

(1) – x (2) 3x (3) x (4) None

64. For the expression $f(x) = x^3 + ax^2 + bx + c$, if f(1) = f(2) = 0 and f(4) = f(0). The values of a, b & c are

(1) a = – 9, b = 20, c = – 12
(2) a = 9, b = 20, c = 12
(3) a = – 1, b = 2, c = – 3
(4) None of these

65. If x + 1 is a factor of $ax^4+bx^3+cx^2+dx+e=0$ then ____

(1) a + c + e = b + d
(2) a + b = c + d
(3) a + b + c + d + e = 0
(4) a + c + b = d + e

66. If (x – 3) is the factor of $3x^3 - x^2 + px + q$ then___

(1) p + q = 72
(2) 3p + q = 72
(3) 3p + q = –72
(4) q – 3p = 72

67. For what values of n, (x + y) is a factor of $(x - y)^n$.

(1) for all values of n
(2) 1
(3) only for odd numbers
(4) none of these

68. $f(x) = 3x^5 + 11x^4 + 90x^2 - 19x + 53$ is divided by x + 5 then the remainder is ______.

(1) 100 (2) –100
(3) –102 (4) 102

69. If (x – 3), (x – 3) are factors of $x^3 - 4x^2 - 3x + 18$; then the other factor is

(1) x + 2 (2) x + 3
(3) x – 2 (4) x + 6

70. If $f\left(\frac{-3}{4}\right)=0$; then for f(x), which of the following is a factor?

(1) 3x – 4 (2) 4x + 3
(3) –3x + 4 (4) 4x – 3

71. $f(x) = 16x^2 + 51x + 35$ then one of the factors of f(x) is

(1) $x - 1$ (2) $x + 3$

(3) $x - 3$ (4) $x + 1$

72. If $ax^3 + 9x^2 + 4x - 1$ is divided by $(x + 2)$, the remainder is –6; then the value of 'a' is

(1) –3 (2) –2 (3) 0 (4) $\frac{33}{8}$

73. If $a^3 - 3a^2b + 3ab^2 - b^3$ is divided by $(a - b)$, then the remainder is

(1) $a^2 - ab + b^2$ (2) $a^2 + ab + b^2$

(3) 1 (4) 0

74. If $\alpha + \beta = 4$ and $\alpha^3 + \beta^3 = 44$, then α,β are the zeros of the polynomial.

(1) $2x^2 - 7x + 6$ (2) $3x^2 + 9x + 11$

(3) $9x^2 - 27x + 20$ (4) $3x^2 - 12x + 5$

75. If $y = f(x) = mx + c$; then f(y) in terms of x is

(1) $mx + m + c$ (2) $m + mc + c$

(3) $m^2x + mc + c$ (4) $m^2x + m^2c$

76. If $7 + 3x$ is a factor of $3x^3 + 7x$, then the remainder is

(1) $\frac{490}{9}$ (2) $\frac{-490}{9}$ (3) $\frac{470}{9}$ (4) None

77. The remainder when $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$ is

(1) –1 (2) 1 (3) 0 (4) –2

78. The remainder when $1 + x + x^2 + x^3 + \dots\dots + x^{2006}$ is divided by $x - 1$ is

(1) 2005 (2) 2006

(3) 2007 (4) 2008

79. If $(x - 1)$, $(x + 1)$ and $(x - 2)$ are factors of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then the value of p is

(1) 1 (2) 2

(3) 3 (4) 4

80. If the remainder when the polynomial f(x) is divided by $x - 1$, $x + 1$ are 6, 8 respectively then the remainder when f(x) is divided by $(x - 1)(x + 1)$ is

(1) $7 - x$ (2) $7 + x$

(3) $8 - x$ (4) $8 + x$

81. Find the remainder obtained when x^{2007} is divisible by $x^2 - 1$.

(1) x^2 (2) x (3) $x + 1$ (4) –x

82. If a polynomial $2x^3 - 9x^2 + 15x + p$, when divided by $(x - 2)$, leaves –p as remainder, then p is equal to

(1) –16 (2) –5 (3) 20 (4) 10

83. If α, β and γ are the zeros of the polynomial $2x^3 - 6x^2 - 4x + 30$, then the value of $(\alpha\beta + \beta\gamma + \gamma\alpha)$ is

(1) – 2 (2) 2

(3) 5 (4) – 30

84. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} =$

(1) $-\frac{b}{a}$ (2) $\frac{c}{d}$ (3) $-\frac{c}{d}$ (4) $-\frac{c}{a}$

85. If α, β and γ are the zeros of the polynomial $f(x) = ax^3 - bx^2 + cx - d$, then $\alpha^2 + \beta^2 + \gamma^2 =$

(1) $\frac{b^2 - ac}{a^2}$ (2) $\frac{b^2 + 2ac}{b^2}$

(3) $\frac{b^2 - 2ac}{a}$ (4) $\frac{b^2 - 2ac}{a^2}$

86. If α, β and γ are the zeros of the polynomial $f(x) = x^3 + px^2 - pqrx + r$, then $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} =$

(1) $\frac{r}{p}$ (2) $\frac{p}{r}$ (3) $-\frac{p}{r}$ (4) $-\frac{r}{p}$

87. The coefficient of x in $x^2 + px + q$ was taken as 17 in place of 13 and it's zeros were found to be – 2 and – 15. The zeros of the original polynomial are

(1) 3, 7 (2) – 3, 7

(3) – 3, – 7 (4) –3, – 10

88. Let α, β be the zeros of the polynomial $x^2 - px + r$ and $\frac{\alpha}{2}, 2\beta$ be the zeros of $x^2 - qx + r$. Then the value of r is –

(1) $\frac{2}{9}(p - q)(2q - p)$ (2) $\frac{2}{9}(q - p)(2p - q)$

(3) $\frac{2}{9}(q - 2p)(2q - p)$ (4) $\frac{2}{9}(2p - q)(2q - p)$

89. When $x^{200} + 1$ is divided by $x^2 + 1$, the remainder is equal to –

(1) $x + 2$ (2) $2x - 1$

(3) 2 (4) – 1

90. If $a(p+q)^2 + 2bpq + c = 0$ and also $a(q+r)^2 + 2bqr + c = 0$ then pr is equal to –

(1) $p^2 + \frac{a}{c}$ (2) $q^2 + \frac{c}{a}$

(3) $p^2 + \frac{a}{b}$ (4) $q^2 + \frac{a}{c}$

91. If a,b and c are not all equal and α and β be the zeros of the polynomial $ax^2 + bx + c$, then value of $(1+\alpha+\alpha^2)(1+\beta+\beta^2)$ is :

(1) 0 (2) positive

(3) negative (4) non-negative

92. If 2 and 3 are the zeros of $f(x) = 2x^3 + mx^2 - 13x + n$, then the values of m and n are respectively –

(1) –5, – 30 (2) –5, 30

(3) 5, 30 (4) 5, – 30

93. If α,β are the zeros of the polynomial $6x^2 + 6px + p^2$, then the polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is –

(1) $3x^2 + 4p^2x + p^4$ (2) $3x^2 + 4p^2x - p^4$

(3) $3x^2 - 4p^2x + p^4$ (4) None of these

94. If c, d are zeros of $x^2 - 10ax - 11b$ and a, b are zeros of $x^2 - 10cx - 11d$, then value of a + b + c + d is

(1) 1210 (2) – 1 (3) 2530 (4) – 11

95. If the ratio of the roots of polynomial $x^2 + bx + c$ is the same as that of the ratio of the roots of $x^2 + qx + r$, then

(1) $br^2 = qc^2$ (2) $cq^2 = rb^2$

(3) $q^2c^2 = b^2r^2$ (4) $bq = rc$

96. The quadratic polynomial whose zeros are twice the zeros of $2x^2 - 5x + 2 = 0$ is –

(1) $8x^2 - 10x + 2$ (2) $x^2 - 5x + 4$

(3) $2x^2 - 5x + 2$ (4) $x^2 - 10x + 6$

97. If α,β,γ are the zeros of the polynomial $x^3 - 3x + 11$, then the polynomial whose zeros are $(\alpha+\beta)$, $(\beta+\gamma)$ and $(\gamma+\alpha)$ is –

(1) $x^3 + 3x + 11$ (2) $x^3 - 3x + 11$

(3) $x^3 + 3x - 11$ (4) $x^3 - 3x - 11$

98. If α,β,γ are such that $\alpha+\beta+\gamma = 2$, $\alpha^2+\beta^2+\gamma^2 = 6$, $\alpha^3+\beta^3+\gamma^3 = 8$, then $\alpha^4+\beta^4+\gamma^4$ is equal to

(1) 10 (2) 12

(3) 18 (4) None

99. If α,β are the roots of $ax^2 + bx + c$ and $\alpha + k$, $\beta + k$ are the roots of $px^2 + qx + r$, then k =

(1) $-\frac{1}{2}\left[\frac{a}{b} - \frac{p}{q}\right]$ (2) $\left[\frac{a}{b} - \frac{p}{q}\right]$

(3) $\frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$ (4) (ab – pq)

100. The condition that $x^3 - ax^2 + bx - c = 0$ may have two of the roots equal to each other but of opposite signs is :

(1) ab = c (2) $\frac{2}{3}a = bc$ (3) $a^2b = c$ (4) None

101. If one zero of the polynomial $ax^2 + bx + c$ is positive and the other negative then $(a,b,c \in R, a \neq 0)$

(1) a and b are of opposite signs.

(2) a and c are of opposite signs.

(3) b and c are of opposite signs.

(4) a,b,c are all of the same sign.

102. If α,β are the zeros of the polynomial $x^2 - px + q$, then $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$ is equal to –

(1) $\frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$ (2) $\frac{p^4}{q^2} - 2 + \frac{4p^2}{q}$

(3) $\frac{p^4}{q^2} + 2q^2 - \frac{4p^2}{q}$ (4) None of these

103. If α,β are the zeros of the polynomial $x^2 - px + 36$ and $\alpha^2 + \beta^2 = 9$, then p =

(1) ± 6 (2) ± 3 (3) ± 8 (4) ± 9

104. If α,β are zeros of $ax^2 + bx + c$, $ac \neq 0$, then zeros of $cx^2 + bx + a$ are –

(1) $-\alpha, -\beta$ (2) $\alpha, \frac{1}{\beta}$

(3) $\beta, \frac{1}{\alpha}$ (4) $\frac{1}{\alpha}, \frac{1}{\beta}$

105. A real number is said to be algebraic if it satisfies a polynomial equation with integral coefficients. Which of the following numbers is not algebraic :

(1) $\frac{2}{3}$ (2) 2 (3) 0 (4) π

106. The cubic polynomials whose zeros are 4, $\frac{3}{2}$ and –2 is :

(1) $2x^3 + 7x^2 + 10x - 24$

(2) $2x^3 + 7x^2 - 10x - 24$

(3) $2x^3 - 7x^2 - 10x + 24$

(4) None of these

107. If the sum of zeros of the polynomial $p(x) = kx^3 - 5x^2 - 11x - 3$ is 2, then k is equal to

(1) $k = -\frac{5}{2}$

(2) $k = \frac{2}{5}$

(3) $k = 10$

(4) $k = \frac{5}{2}$

108. If $f(x) = 4x^3 - 6x^2 + 5x - 1$ and α, β and γ are its zeros, then $\alpha\beta\gamma =$

(1) $\frac{3}{2}$ (2) $\frac{5}{4}$

(3) $-\frac{3}{2}$ (4) $\frac{1}{4}$

109. Consider $f(x) = 8x^4 - 2x^2 + 6x - 5$ and $\alpha, \beta, \gamma, \delta$ are it's zeros then $\alpha + \beta + \gamma + \delta =$

(1) $\frac{1}{4}$ (2) $-\frac{1}{4}$

(3) $-\frac{3}{2}$ (4) None

110. If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common and the second equation has equal roots, then

(1) $b + q = 2ap$

(2) $b + q = \frac{ap}{2}$

(3) $b + q = ap$

(4) None of these

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	2	1	2	1	4	1	1	4	3	2	1	2	3	3	1	1	3	2	
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	3	4	3	1	2	4	1	3	2	3	1	1	3	1	2	4	2	2	3	
Que.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	1	3	2	2	2	3	4	2	2	1	1	1	1	3	3	2	2	2	1	
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans.	1	3	3	1	1	3	4	3	1	2	4	4	4	4	3	2	3	3	4	
Que.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
Ans.	2	2	1	3	4	2	4	4	3	2	4	2	3	1	2	2	4	3	3	
Que.	**101**	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**										
Ans.	2	1	4	4	4	3	4	4	4	2										