Chapter—18

OPERATIONS ON RATIONAL NUMBER

You have learnt that the numbers which can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ o, are called rational numbers.

In class 6, we learnt addition, subtraction, multiplication and division of positive fractions. Let us learn all these operations in detail.

Addition of Rational Numbers

Figure 18.1

A watermelon seller divided one watermelon into 10 equal parts. From these, Sujeet bought 2 parts, Uma bought 3 parts and Akanksha bought 3 parts, how many parts have been sold?

From a total of 10 parts, Sujeet has taken 2 parts = 2/10

From a total of 10 parts, Uma has taken 3 parts = 3/10

Akanksha has taken 3 parts =3/10

Therefore, total parts that Sujeet, Uma and Akanksha have taken = 2/10 +3/10 +3/10 = (2+3+3)/10 =8/10

The seller sold 8 parts out of 10 or 8/10 parts of the watermelon.

Let us learn to add two rational numbers with the help of a figure.

X 0	X	X
X 0	X	X
X 0	X	X
0		
0		

Figure 18.2

Example 1: Add $\frac{3}{5}$ and $\frac{1}{3}$

Take a rectangle, to represent 3/5 draw 4 horizontal lines spaced such that the rectangle is divided into 5 equal parts. From these, label three parts by the symbol 'x'. Now to represent 1/3, draw 2 vertical lines spaced such that the rectangle is divided into 3 equal parts. From these 3 parts, label one by '0'. Now the rectangle has been divided into 15 parts. Add the parts labeled by 'x' and the parts labeled by '0'.

Number of 'x' blocks + Number of '0' blocks = 9+5 = 14

14 parts out of 15 are marked or the marked blocks are $=\frac{14}{15}$

And 3/5+1/3 = (9 +5)/15 = 14/15

In the same way to find $\frac{3}{5}-\frac{1}{3}$, subtract the number of '0' blocks from the number of 'x' blocks or 9-5 = 4 parts and the total no of blocks is 15.

Therefore $\frac{3}{5}-\frac{1}{3}=\frac{9-5}{15}=\frac{4}{15}$

Similarly, add and subtract the following rational numbers using figures. Write the answer in lowest form.

(i) $\frac{3}{7}+\frac{1}{4}$ (ii) $\frac{2}{5}+\frac{1}{3}$ (iii) $\frac{3}{7}-\frac{1}{4}$ (iv) $\frac{2}{5}-\frac{1}{3}$

(v) $\frac{5}{6}+\frac{2}{3}$ (vi) $\frac{1}{4}-\frac{2}{3}$

Let us discuss the answers of the questions solved by you.

Ans (i) In solving this question, you drew horizontal lines in a rectangle to divide it into 7 equal parts. Of these 7 parts, you label 3 by 'X' .Now, by drawing 3 vertical lines you divide the rectangle into 4 equal parts and label one of them by '✓'. In this way the rectangle has been divided into 28 equal parts of which 12 blocks are marked by 'X' and 7 blocks are marked by '✓'.

Therefore, the sum of 3/7 and 1/4 will contain 12+7 = 19 blocks out of 28 blocks or $\frac{3}{7}+\frac{1}{4}=\frac{12}{28}+\frac{7}{28}=\frac{12+7}{28}=\frac{19}{28}$

Similarly, for $\frac{3}{7}-\frac{1}{4}=\frac{12}{28}-\frac{7}{28}=\frac{12-7}{28}=\frac{5}{28}$.

Ans (V): To solve this question, you divided the rectangle into 6 equal parts by drawing horizontal or vertical lines. You marked 5 parts with 'X' signs. Then you divided the rectangle into 3 equal parts as in previous questions and marked 2 of these with '✓' sign. Now the rectangle has been divided into 18 parts. It has 15 parts marked with 'X' and 12 parts marked with '✓'. Total number blocks marked by 'X' and '✓' =15 + 12 = 27

Therefore $\frac{5}{6}+\frac{2}{3}=\frac{15}{18}+\frac{12}{18}=\frac{27}{18}$

The lowest form of this will be $\frac{3}{2}$.

While solving this questions, Fatima told Raju, that last year, while adding and subtracting fractions, we converted the given fractions changed into common denominator forms.The denominator of the sum was always the product of the denominators of the two fractions.

In this method also the denominator of the sum is the product of the denominators of the two rational numbers. Raju said, that in the previous chapter we learnt that rational numbers can be expressed as $\frac{p}{q}$ or $\frac{r}{s}$, where p, q, r, s are integer and q ≠ o, s ≠ 0.

fractions with small denominators can be added by making equivalent fractions for eg $\frac{5}{6}+\frac{3}{8}$

$$\frac{5}{6}=\frac{5}{6},\frac{10}{12},\frac{15}{18},\frac{20}{24},\frac{25}{30}$$
$$\frac{3}{8}=\frac{3}{8},\frac{6}{16},\frac{9}{24},\frac{12}{32},\frac{15}{40}$$

Here equivalent fractions with same denominator are

$$\frac{5}{6} = \frac{20}{24} \;;\; \frac{3}{8} = \frac{9}{24}$$

$$\frac{5}{6}+ \frac{3}{8} = \frac{9}{24}+\frac{20}{24} = \frac{29}{24}$$

“Can we add or subtract these numbers by using the “common denominator method” Fatima said, “Let us try and find out”.

For getting the common denominator of $\frac{p}{q}+\frac{r}{s}$, we multiplying the numerator and denominator of p/q by s and multiply the numerator and denominator of r/s by q.

(p/q) x (s/s) + (r/s) x (q/q)= (ps/qs)+ (rq/sq) = (ps+rq)/sq

In the same way find the sum of the following rational numbers.

(i) $\frac{m}{n}+\frac{r}{\ell}$ (ii) $\frac{a}{b}+\frac{q}{n}$ (iii) $\frac{s}{t}+\frac{c}{d}$

Add $\frac{3}{5}+\frac{-4}{7}$ by using common denominator method.

Here, the denominators are 5 and 7.Therefore, to reduce them into common denominator form, multiply the numerator and denominator of the first rational number by 7 and multiply the numerator and denominator of the second rational number by 5.

Therefore, $\frac{3}{5}=\frac{3}{5}\times\frac{7}{7}=\frac{21}{35}$ and $\frac{-4}{7}=\frac{-4\times5}{7\times5}=\frac{-20}{35}$

Thus, $\frac{3}{5}+\frac{-4}{7}=\frac{21}{35}+\frac{-20}{35}=\frac{21-20}{35}=\frac{1}{35}$

Some-times while solving by reducing into the common denominator form, we may get a common factor in the denominator.

Can you find the value of $\frac{5}{6}+\frac{3}{8}$

Radha started solving the problem. $\frac{5}{6}\times\frac{3}{8}+\frac{3}{8}\times\frac{6}{6}$

But Fatima did not like this method. She said, since 2 is a common factor between the denominators, there is no need to multiply the numerator and denominator by 2, it means we will multiply the numerator and denominator of $\frac{5}{2\times3}$ by $\frac{4}{4}$ and the numerator and denominator of $\frac{3}{2\times4}$ is multiplied by $\frac{3}{3}$ $\frac{5}{6}\times\frac{4}{4}+\frac{3}{8}\times\frac{3}{3}=\frac{20}{24}+\frac{9}{24}=\frac{29}{24}$

We can get the common denominator form of two fractions this way also.

Radha said that on reducing $\frac{3}{2\times5}+\frac{5}{2\times7}$ to the common denominator form the denominator will be 2 × 5 ×7. This is also the L.C.M. of the denominators.

Activity 1

Find the LCM of the denominators and perform the addition and subtraction of the rational numbers according to the operations mentioned in the activity.

Table 18.1

S, No	Rational number I	Rational number II	LCM of denomi -nators	$\dfrac{\frac{\text{Numerator of I}\times\text{LCM}}{\text{Denominator of I}}+\frac{\text{Numerator of II}\times\text{LCm}}{\text{Denominator of II}}}{\text{L C M of denominator}}$	Answer
1.	$\frac{4}{15}$	$\frac{7}{12}$	60	$\dfrac{\frac{4\times60}{15}+\frac{7\times60}{12}}{60}=\frac{4\times4+7\times5}{60}=\frac{16+35}{60}$	$\frac{51}{60}$ or $\frac{17}{20}$
2.	$\frac{7}{20}$	$\frac{3}{10}$	....	....	....
3.	$\frac{-7}{3}$	$\frac{11}{12}$	....	....	....
4.	$\frac{15}{8}$	$\frac{13}{12}$	....	....	....
5.	$\frac{6}{7}$	$\frac{5}{21}$	....	....	....

The sum obtained by the addition of two rational numbers follows certain rules. Let us learn them through the following examples. Fill the blanks in the following examples and examine table.

Activity

Closure Property

Table 18.2

S.No.	Rational Numbers	Add	Steps for Adding	Sum	Is this a rational Number?
1	$\frac{5}{7}$ and $\frac{4}{7}$	$\frac{5}{7}+\frac{4}{7}$	$\frac{5+4}{7}$	$\frac{9}{7}$	Yes
2	3 and $-\frac{6}{5}$	$\frac{3}{1}+\frac{-6}{5}$	$\frac{3\times5+(-6)\times1}{5}$	$\frac{9}{5}$	Yes
3	$-\frac{5}{13}$ and $\frac{5}{13}$	–	–	–	–
4	$\frac{1}{8}$ and $\frac{7}{8}$	–	–	–	–

It is clear from the above table that the **sum of two rational numbers is always a rational number. This is called Closure Property of addition.**

Take any two rational numbers and check whether their sum is a rational number or not?

Commutative Property

Suppose two rational numbers are

$$\frac{-5}{6} \text{ and } \frac{3}{4}, \text{ then } \frac{-5}{6}+\frac{3}{4}=\frac{-5\times2+3\times3}{12}=\frac{-10+9}{12}=\frac{1}{12}$$

$$\text{and } \frac{3}{4}+\left(\frac{-5}{6}\right)=\frac{3\times3+(-5)\times2}{12}=\frac{9+(-10)}{12}=\frac{1}{12}$$

Therefore, -5/6+3/4 = 3/4 + (-5/6)

Fill up the blanks of the following table

Table 18.3

S.No.	Rational Numbers	Sum of rational numbers	Sum of rational numbers on changing their order	Are their sums equal in both the situations
1.	$\frac{1}{8}$ and $\frac{7}{8}$	$\frac{1}{8}+\frac{7}{8}=\frac{1+7}{8}=\frac{8}{8}$	$\frac{7}{8}+\frac{1}{8}=\frac{7+1}{8}=\frac{8}{8}$	Yes
2.	$\frac{-3}{8}$ and $\frac{5}{16}$	$-\frac{3}{8}+\frac{5}{16}=$ ------	$\frac{5}{16}+\left(\frac{-3}{8}\right)=$ ------	------
3.	$-\frac{7}{15}$ and $-\frac{8}{25}$	$-\frac{7}{15}+\frac{-8}{25}=$ ------	$\frac{-8}{25}+\frac{-7}{15}=$ ------	------
4.	$\frac{p}{q}$ and $\frac{r}{s}$	$\frac{p}{q}+\frac{r}{s}=$ ------	$\frac{r}{s}+\frac{p}{q}=$ ------	------

From the above table we get that sum of two rational numbers is equal to the sum of the rational numbers obtained by changing the order of addition of the rational number.

The sum of two rational numbers remains same if the order of their addition is changed. This property is called Commutative Property of addition in rational numbers.

So, if $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers then $\frac{p}{q}+\frac{r}{s} = \frac{r}{s}+\frac{p}{q}$

If $\frac{3}{4}+\frac{-5}{8} = x+\frac{3}{4}$ then, what is the value of x ?

Associative Property

Let the three rational numbers be $\frac{4}{5},\frac{2}{7}$ and $\frac{-3}{8}$. We can add these numbers in two ways.

First Method

$$\frac{4}{5}+\left(\frac{2}{7}+\frac{-3}{8}\right)=\frac{4}{5}+\left(\frac{2\times8-3\times7}{56}\right)$$

$$= \frac{4}{5} + \left(\frac{16-21}{56}\right) = \frac{4}{5} - \frac{5}{56}$$

$$= \frac{4\times56-5\times5}{280} = \frac{224-25}{280} = \frac{199}{280}$$

Second Method

$$\left(\frac{4}{5}+\frac{2}{7}\right)+\left(\frac{-3}{8}\right)=\left(\frac{4\times7+2\times5}{35}\right)+\left(\frac{-3}{8}\right)$$

$$=\left(\frac{28+10}{35}\right)-\left(\frac{3}{8}\right)$$

$$= \frac{38}{35}-\frac{3}{8} = \frac{38\times8-3\times35}{280}$$

$$= \frac{304-105}{280} = \frac{199}{280}$$

(Addition of rational number follows Associative Property)

Here $\frac{4}{5}+\left(\frac{2}{7}+\frac{-3}{8}\right)=\left(\frac{4}{5}+\frac{2}{7}\right)+\left(\frac{-3}{8}\right)$

In adding three rational numbers, if we add third number to the sum of first two numbers, we get the same value as we obtain when we add first to sum of second and third number. This rule is called Associative Property of addition of rational numbers.

Activity 3

Find the value of the following:-

(1) $\frac{1}{11}+\left(\frac{5}{6}+\frac{7}{12}\right)$ and $\left(\frac{1}{11}+\frac{5}{6}\right)+\frac{7}{12}$

(2) $\frac{3}{4}+\left(\frac{-5}{3}+\frac{4}{5}\right)$ and $\left(\frac{3}{4}+\frac{-5}{3}\right)+\frac{4}{5}$

(3) $\frac{-2}{3}+\left(\frac{1}{5}+\frac{3}{4}\right)$ and $\left(\frac{-2}{3}+\frac{1}{5}\right)+\frac{3}{4}$

Do we get the same value in both the situations?

From the above activity, we find that solution obtained in both the situations is the same. Therefore, we can say that addition of rational numbers follows Associative Property.

Sum of Zero with other rational numbers

We know that on adding 0 to another integer, there is no change in the value of the integer. Let us add 0 to rational numbers-

$$\frac{3}{5}+0=\frac{3}{5}+\frac{0}{5}=\frac{3+0}{5}=\frac{3}{5}$$

Similarly, $0+\frac{-4}{9}=\frac{-4}{9}$

Does there exist any number other than '0' which when added to another rational number does not change its value?

Thus we know that there is no number except 0 which when added to a rational number leaves the value of the rational number unchanged.

Because of this property '0' is called the **Additive Identity** for rational numbers. If $\frac{p}{q}$ is any rational number, $\frac{p}{q}+0=\frac{p}{q}$.

Additive inverse

$\frac{11}{15}$ and $\frac{-11}{15}$ are two rational numbers. Their sum $\frac{11}{15}+\left(\frac{-11}{15}\right)=\frac{11-11}{15}=0$

Given below are two rational numbers, one is positive and the other one is negative. Find the sum of these rational numbers.

(i) $\frac{-13}{36}+\frac{13}{36}=$ ______

(ii) $\frac{289}{295}+\frac{-289}{295}=$ ______

For each rational number, there always exists a rational number such that the sum of the two numbers is zero. That number is called the **Additive inverse** of the given number.

For example, the additive inverse of $\frac{3}{5}$ is $\frac{-3}{5}$

additive inverse of $\frac{17}{19}$ is $\frac{-17}{19}$

To obtain the additive inverse of any number we add such a number to the given number that the sum is zero. For example

$\frac{-5}{7} + x = 0$ or $x = \frac{5}{7}$.

Therefore, the additive inverse of $\frac{-5}{7}$ is $\frac{5}{7}$

Exercise 18.1

1. Add the following rational numbers –

   (i) $\frac{3}{2}, \frac{13}{17}$ (ii) $\frac{-7}{9}, \frac{-3}{4}$ (iii) $\frac{3}{4}, \frac{-2}{5}$

2. Use commutative property to fill up the following blanks:-

   (i) $\frac{-5}{9} + \frac{4}{7} = \frac{4}{7} + \_\_\_\_$ (iii) $\frac{-11}{29} + \frac{6}{31} = \_\_\_ + \_\_\_$

   (iii) $\frac{-15}{7} + \_\_\_ = \frac{13}{19} \_\_\_$ (iv) $\frac{5}{6} + \left(-\frac{7}{9}\right) = -\frac{7}{9} + \_\_\_\_$

3. Show that $\left(\frac{-2}{5} + \frac{4}{9}\right) + \frac{-3}{4} = \frac{-2}{5} + \left(\frac{4}{9} + \frac{-3}{4}\right)$. Which property is used in this?

4. Simplify -

   (i) $\frac{3}{7} + \frac{4}{9} + \frac{-6}{11}$ (ii) $\frac{-1}{6} + \frac{-2}{3} + \frac{-1}{3}$

   (iii) $\frac{5}{14} + \frac{2}{-7} + \frac{-3}{2}$

5. What should be added with $\frac{-7}{12}$ so that the sum is 0 ?

6. Fill up the blanks:–

   (i) Additive inverse of $\frac{-5}{7} = \_\_\_\_\_\_$

   (ii) $\frac{4}{17} + \frac{-4}{17} = \_\_\_\_$

(iii) $0+\frac{39}{51}=$ ______

(iv) Additive inverse of $\frac{42}{17}=$ ______

7. The following problems are related to some rule. Write the appropriate property that is used in the each blank-

(i) $\frac{13}{15}+\frac{4}{8}=\frac{4}{8}+\frac{13}{15}$ (..........................)

(ii) $\frac{2}{19}+\left(\frac{-3}{17}+\frac{4}{13}\right)=\left(\frac{2}{19}+\frac{-3}{17}\right)+\frac{4}{13}$ (..........................)

(iii) $\frac{p}{q}+0=\frac{p}{q}$ (..........................)

(iv) $\frac{-r}{s}+\frac{r}{s}=0$ (..........................)

8. Think of some rational numbers, verify the use of commutative and associative laws of addition for these rational numbers.

Subtraction of Rational Numbers

In class 6, while subtracting one fraction from the other, we found the answer by making the denominators equal. The operation of subtraction is opposite of the operation of addition. Subtraction of one number from the other number is the sum of the number and the additive inverse of the second number. Let us understand this through the following examples-

Examples 2: Subtract $\frac{1}{4}$ from $\frac{3}{8}$

$$\frac{3}{8}-\frac{1}{4}=\frac{3\times1-1\times2}{8}=\frac{3-2}{8}=\frac{1}{8} \qquad \text{(LCM of 4, 8 is 8)}$$

Addition of the given number with the additive inverse of $\frac{1}{4}$ is

$$\frac{3}{8}+\left(\frac{-1}{4}\right)=\frac{3}{8}+\frac{(-1)}{4}=\frac{3\times1+(-1)\times2}{8}=\frac{3-2}{8}=\frac{1}{8}$$

Thus, both the solutions are equal.

Now, subtract $\frac{11}{13}$ from $\frac{7}{19}$ and add $\frac{7}{19}$ to the additive inverse of $\frac{11}{13}$, then examine your answers.

We can even subtract rational numbers by using a number line. Let us see-

Example 3: Subtract $\frac{7}{12}$ from $\frac{5}{6}$

Solution: Here, the denominators are not equal. Therefore, before solving we will make the denominators equal.

$$\frac{5}{6} = \frac{5\times 2}{6\times 2} = \frac{10}{12} \; (\because \text{LCM of 6 and 12 is 12})$$

$$\frac{7}{12} = \frac{7\times 1}{12\times 1} = \frac{7}{12}$$

We divide 1 unit on the number line into 12 equal parts. Firstly, to show $\frac{10}{12}$ we move 10 parts to the right of 0. Since we have to now subtract $\frac{7}{12}$ from, we return 7 parts to the left of the10th part.

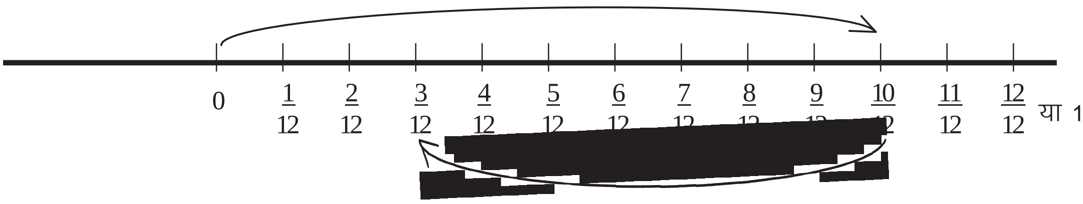


Figure 18.3

We reach $\frac{3}{12}$. Similarly, we get $\frac{3}{12}$ after subtracting $\frac{7}{12}$ from $\frac{5}{6}$

$$\frac{5}{6} - \frac{7}{12} = \frac{10}{12} - \frac{7}{12}$$

$$= \frac{10-7}{12} = \frac{3}{12} = \frac{1}{4}$$

Example 4: Subtract $\frac{-3}{9}$ from $\frac{4}{9}$

Solution: Since the subtraction of a rational number means the addition of the additive inverse of that rational number.

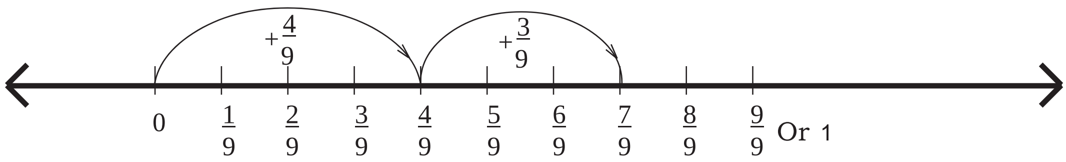


Figure 18.4

Therefore, subtraction of $\frac{-3}{9}$ means the addition of the additive inverse of $\frac{-3}{9}$,that is 3/9

$$\frac{4}{9}-\left(\frac{-3}{9}\right)=\frac{4}{9}+\frac{3}{9}=\frac{4+3}{9}=\frac{7}{9}$$

We divide the number line between 0 and 1 into 9 equal parts. We move 4 parts to the right of 0, and then again move 3 parts in the same direction. Thus we reach the 7^{th} part, equal to $\frac{7}{9}$.

Therefore, $\frac{4}{9}-\left(\frac{-3}{9}\right)=\frac{7}{9}$

Example 5: What should be added to $\frac{5}{9}$ so that the sum is $\frac{2}{3}$?

Solution: Let the sum of $\frac{5}{9}$ and $\frac{p}{q}$ be $\frac{2}{3}$

$$\frac{5}{9}+\frac{p}{q}=\frac{2}{3}$$

Add the additive inverse of $\frac{5}{9}$ to the both sides.

$$\frac{5}{9}+\frac{p}{q}+\left(\frac{-5}{9}\right)=\frac{2}{3}+\left(\frac{-5}{9}\right)$$

Or $\frac{p}{q}=\frac{2}{3}+\left(\frac{-5}{9}\right)$

Or $\frac{p}{q}=\frac{2\times3}{3\times3}+\frac{-5\times1}{9\times1}$ (LCM of 3, 9 is 9)

Or $\frac{p}{q}=\frac{6}{9}+\frac{-5}{9}$

Or $\frac{p}{q}=\frac{6-5}{9}$

Or $\frac{p}{q}=\frac{1}{9}$

Thus on adding 1/9 to 5/9 the sum is 2/3.

Example 6: What should be subtracted from $\frac{11}{13}$ to get $\frac{5}{26}$?

Solution: Subtracting p/q from 11/13 yields 5/26.

$$\frac{11}{13}-\frac{p}{q}=\frac{5}{26}$$

Add the additive inverse of $\frac{11}{13}$ to the both sides

Or $\frac{11}{13}-\frac{p}{q}+\left(-\frac{11}{13}\right)=\frac{5}{26}+\left(-\frac{11}{13}\right)$

Or $-\frac{p}{q}=\frac{5}{26}+\left(\frac{-11}{13}\right)$

Or $-\frac{p}{q}=\frac{5\times1}{26\times1}+\left(\frac{-11\times2}{13\times2}\right)$ (L.C.M. of13, 26 is 26)

Or $-\frac{p}{q}=\frac{5}{26}+\left(\frac{-22}{26}\right)$

Or $-\frac{p}{q}=\frac{5-22}{26}$

Or $-\frac{p}{q}=\frac{-17}{26}$

Or $\frac{p}{q}=\frac{17}{26}$ (Multiplying both sides by -1)

Thus we get $\frac{5}{26}$ after subtracting $\frac{17}{26}$ from $\frac{11}{13}$

Example 7: Simplify $\frac{1}{4}+\left(\frac{-5}{9}\right)-\left(\frac{-7}{12}\right)$.

Solution: Here, we are given three rational numbers, where the operations of addition and subtraction are to be done simultaneously.

For solving such questions, we make the denominators of all the rational numbers in the question equal.

$$\frac{1}{4}=\frac{1\times 9}{4\times 9}=\frac{9}{36}$$ Here the L.C.M. of 4, 9 and 12 is 36.

$$\frac{-5}{9}=\frac{-5\times 4}{9\times 4}=\frac{-20}{36}$$

$$\frac{-7}{12}=\frac{-7\times 3}{12\times 3}=\frac{-21}{36}$$

$$\frac{1}{4}+\left(\frac{-5}{9}\right)-\left(\frac{-7}{12}\right)=\frac{9}{36}+\frac{-20}{36}-\left(\frac{-21}{36}\right)$$

$$=\frac{9-20+21}{36}=\frac{30-20}{36}$$

$$=\frac{10}{36}=\frac{5}{18}$$

Properties of subtraction in Rational Number

1. Closure property- We have seen the properties of addition of rational numbers. In the subtraction of rational numbers also some properties apply. Let us see the following example:-

Subtract $\frac{25}{36}$ from $\frac{11}{21}$

Here $\frac{11}{21}-\frac{25}{36}=\frac{11\times 12-25\times 7}{252}=\frac{132-175}{252}$ (Here, LCM of 21 and 36 is 252)

$=\frac{-43}{252}$,which is a rational number.

Here, $\frac{11}{21}, \frac{25}{36}$ and $\frac{-43}{252}$ all three are rational numbers. The operation of subtraction between any two rational numbers gives a rational number. Check this property by taking some rational numbers.

2. Subtraction of zero from rational number- If zero is subtracted from a rational number, the value of the rational number does not change.

For example $\frac{-21}{45} - 0 = \frac{-21}{45}$ and $\frac{5}{17} - 0 = \frac{5}{17}$

$$\frac{P}{Q} - 0 = \frac{P}{Q}$$

3. Commutative property :-

Find the value of the following:-

(i) $\frac{5}{12} - \frac{6}{13}$ and (ii) $\frac{6}{13} - \frac{5}{12}$

Here $\frac{5}{12} - \frac{6}{13} = \frac{5 \times 13}{12 \times 13} - \frac{6 \times 12}{13 \times 12}$

$= \frac{65}{156} - \frac{72}{156}$ (L.C.M. of 12 and 13 is 156)

$= \frac{65-72}{156} = \frac{-7}{156}$

And $\frac{6}{13} - \frac{5}{12} = \frac{6 \times 12}{13 \times 12} - \frac{5 \times 13}{12 \times 13}$ = (72/156) – (65/156)

$= \frac{72-65}{156} = \frac{7}{156}$

Is $\frac{-7}{156}$ equal to $\frac{7}{156}$?

Therefore, (5/12) – (6/13) ≠ (6/13) – (5/12).

Hence, subtraction of rational numbers does not follow the commutative property.

Exercise 18.2

Q1. Subtract the first rational number from the second rational number.

(i) $\frac{4}{5}$ from $\frac{3}{4}$ (ii) $\frac{-1}{8}$ from $\frac{1}{4}$

(iii) $\frac{13}{24}$ from $\frac{-5}{12}$ (iv) $\frac{-7}{13}$ from $\frac{-8}{13}$

Q2. Solve

(i) $\frac{2}{9}+\frac{1}{3}-\frac{5}{9}$ (ii) $\frac{1}{5}-\frac{3}{7}+\frac{1}{2}$ (iii) $\frac{-1}{12}+\frac{3}{5}-6$

Q3. What should be added to $\frac{3}{8}$ so that the sum is $\frac{11}{12}$?

Q4. What should be subtracted from $\frac{13}{25}$ so that the difference is $\frac{19}{25}$?

Q5. Write true or false and give right statements for the false statements

(i) Additive inverse of $\frac{-3}{5}$ is $\frac{5}{3}$

(ii) $\frac{4}{5}-\frac{7}{9}=\frac{7}{9}-\frac{4}{5}$

(iii) The value of a number does not change after subtracting 0 from it.

(iv) Subtraction of a rational number means the addition of the additive inverse of that number.

Multiplication of Rational Number

While multiplying two fractions, you have seen that we multiply the denominator with the denominator and the numerator with the numerator. Since rational numbers are also composed of numerator and denominator, we multiply rational numbers in a similar way. Let us discuss the multiplication of two rational numbers through examples.

Example 8: Multiply $\frac{3}{4}$ and $\frac{7}{16}$ and write the value.

Solution: $\frac{3}{4}\times\frac{7}{16}=\frac{3\times 7}{4\times 16}=\frac{21}{64}$

Example 9: Multiply $\frac{-5}{7}$ and $\frac{13}{17}$ and write the value.

Solution: $\frac{-5}{7}\times\frac{13}{17}=\frac{-5\times 13}{7\times 17}=\frac{-65}{119}$

Example 10: Multiply $\frac{-9}{11}$ and $\frac{22}{27}$ and write the value.

Solution: $\frac{-9}{11} \times \frac{22}{27} = \frac{-9 \times 22}{11 \times 27} = \frac{-1 \times 2}{1 \times 3} = \frac{-2}{3}$

It is clear from the above examples that to find the product of two rational numbers, we multiply the numerator with the numerator and the denominator with the denominator.

If (p/q) and (r/s) are two rational numbers then (p/q) x (r/s) = (p x r)/ (q x s)

Example 11: Multiply $\frac{2}{3}, \frac{-6}{7}$ and $\frac{8}{15}$

Solution: $\frac{2}{3} \times \frac{-6}{7} \times \frac{8}{15} = \frac{2 \times -6 \times 8}{3 \times 7 \times 15} = \frac{-32}{105}$

In finding the product of more than two rational numbers, we multiply all numerators with each other and all denominators with the other denominators. If the rational numbers $\frac{p}{q}, \frac{r}{s}, \frac{u}{v}$ and $\frac{w}{z}$ etc are multiplied, then $\frac{p}{q} \times \frac{r}{s} \times \frac{u}{v} \times \frac{w}{z} = \frac{p \times r \times u \times w}{q \times s \times v \times z}$ and the product is written in its simplest form

Activity 4

Fill up the blanks in the table given below as directed:-

Table 18.4

S.No.	Rational Numbers	Multiplication of rational numbers	Product	On Changing the order of multiplication	Product	The obtained number is rational or not?
1.	$\frac{11}{15}, \frac{1}{4}$	$\frac{11}{15} \times \frac{1}{4}$	$\frac{11}{60}$	$\frac{1}{4} \times \frac{11}{15}$	$\frac{11}{60}$	Yes
2.	$\frac{-5}{8}, \frac{-7}{4}$	$\frac{-5}{8} \times \frac{-7}{4}$	---------	-------------		
3.	$\frac{-19}{12}, \frac{5}{13}$	$\frac{-19}{12} \times \frac{5}{13}$	---------	-------------		
4.	$\frac{4}{9}, \frac{-18}{5}$	------------	---------	------------		
5.	$\frac{31}{-6}, \frac{24}{7}$	-----------	---------	----------		

From the above, you find that on multiplying rational numbers, we get another rational number. Thus **the set of rational number is closed under multiplication.**

The change in the order of numbers being multiplied does not affect the product. This property is the commutative property of multiplication. Hence, if we have two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, $\frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$

Think of two other rational numbers and verify whether their multiplication follows Commutative Property of multiplication or not.

Distributive property

Integers follow distributive property. Is this also applicable to rational numbers? Let us see through some examples.

Example 12: Simplify $\frac{2}{5}\times\left(\frac{3}{4}+\frac{1}{7}\right)$

Solution: First Method;

$$\frac{2}{5}\times\left(\frac{3}{4}+\frac{1}{7}\right)=\frac{2}{5}\left(\frac{3\times7+1\times4}{28}\right)$$

$$=\frac{2}{5}\left(\frac{21+4}{28}\right)$$

$$=\frac{2}{5}\left(\frac{25}{28}\right)=\frac{5}{14}$$

We can also solve the above problems in the following way.

Second Method:

$$\frac{2}{5}\times\left(\frac{3}{4}+\frac{1}{7}\right)=\frac{2}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{1}{7}=\frac{2\times3}{5\times4}+\frac{2\times1}{5\times7}$$

$$=\frac{6}{20}+\frac{2}{35}=\frac{6\times7+2\times4}{140}=\frac{42+8}{140}=\frac{50}{140}=\frac{5}{14}$$

The solutions of the first and the second method are the same, this means that

$$\frac{2}{5}\left(\frac{3}{4}+\frac{1}{7}\right)=\frac{2}{5}\times\frac{3}{4}+\frac{2}{5}\times\frac{1}{7}$$

This is distributive law for rational numbers

Think of any other three similar rational numbers and verify whether they follow the distributive property.

Therefore, if p/q, r/s, and u/v are three rational numbers then (p/q) (r/s + u/v) = (p/q) × (r/s) + (p/q) × (u/v)

Example 13: If x, y and z are three rational numbers then

$$x \times (y+z) = x \times y + x \times z$$

Where $x = \frac{-5}{8}, y = \frac{7}{9}, z = \frac{11}{12}$

L.H.S. = x × (y +z)

$$= \frac{-5}{8} \times \left(\frac{7}{9} + \frac{11}{12}\right)$$ (substituting the values of x, y and z)

$$= \frac{-5}{8} \times \left(\frac{7 \times 4 + 11 \times 3}{36}\right)$$

$$= \frac{-5}{8} \times \left(\frac{28 + 33}{36}\right)$$

$$= \frac{-5}{8} \times \frac{61}{36} = \frac{-305}{288}$$

R.H.S. =x × y + x × z

$$= \frac{-5}{8} \times \frac{7}{9} + \left(\frac{-5}{8}\right) \times \frac{11}{12}$$

$$= \frac{-35}{72} + \frac{-55}{96}$$

R.H.S. $= \frac{-35 \times 4 - 55 \times 3}{288}$ (L.C.M. of 72 and 96 is 288)

$$= \frac{(-140 - 165)}{288}$$

$$= \frac{-305}{288}$$

L.H.S. = R.H.S.

Multiplication of rational number with zero

Zero is a rational number, You can write it in many ways like 0/1, 0/-27, 0/q where q is an integer and $q \neq 0$

Multiplying 0 with any rational number like:-

$$\frac{-27}{84} \times \frac{0}{q} = \frac{0}{84q} = 0$$

Similarly, the multiplication of 0 with any other rational number yields 0.

Multiplicative identity

Can you think of a rational number which when multiplied with a rational number $\frac{p}{q}$ yields the product $\frac{p}{q}$

Radha said to Fatima, "We know that when we multiply a number with 1, the value of the number does not change and since one is also a rational number, which can be written as 1/1, -2/-2, 57/57 etc. Therefore, 1 is the rational number whose multiplication with $^{p}/_{q}$ (where $q \neq 0$) gives the product $^{p}/_{q}$."

Here '1' is called the **Multiplicative identity**.

Multiplicative inverse

$\frac{1}{3} \times \square = 1$, which number should be put in the blank box so that it's product is 1.

Your answer will be $\frac{3}{1}$.

Activity 5

Some problems are given below. Fill up the blank boxes with appropriate numbers.

(i) $\frac{1}{7} \times \square = 1$ (ii) $\square \times \frac{1}{7} = 1$

(iii) $\frac{1}{13} \times \square = 1$ (iv) $\square \times \frac{1}{13} = 1$

(v) $\frac{7}{13} \times \square = 1$ (vi) $\frac{13}{7} \times \square = 1$

You are observing that two rational numbers whose product is 1 (a multiplicative identity) are being multiplied. You also write below pairs of rational numbers whose product is 1 (the multiplicative identity.)

(1) $\boxed{\frac{\quad}{\quad}} \times \boxed{\frac{\quad}{\quad}} = 1$

(2) $\boxed{\frac{\quad}{\quad}} \times \boxed{\frac{\quad}{\quad}} = 1$

(3) $\boxed{\frac{\quad}{\quad}} \times \boxed{\frac{\quad}{\quad}} = 1$

(4) $\boxed{\frac{\quad}{\quad}} \times \boxed{\frac{\quad}{\quad}} = 1$

While filling up the above, Raju was thinking that to obtain the additive identity, we added the number to its additive inverse. Then similarly to get the multiplicative identity do we multiply the number with its multiplicative inverse? If this is so, then the numbers given above would be the multiplicative inverses of each other.

Thus, **if the product of two numbers is one then both numbers are the multiplicative inverses of each other.**

Let us see, how to find multiplicative inverse.

Example 14: What is the multiplicative inverse of $\frac{p}{q}$?

Solution. Let x be the multiplicative inverse of $\frac{p}{q}$.

$$\frac{p}{q} \times x = 1$$

Or $px = q$

Or $x = \frac{q}{p}$.

Thus, the multiplicative inverse of $\frac{p}{q}$ is $\frac{q}{p}$.

That is, **we can obtain the multiplicative inverse of a number by changing the numerator into denominator and denominator into numerator.**

Let us see some examples:-

(1) $\frac{4}{3} \times \frac{3}{4} = 1$

(2) $\frac{-27}{53} \times \frac{53}{-27} = 1$

(3) $\frac{a}{b} \times \frac{b}{a} = 1$ or $\frac{b}{a} \times \frac{a}{b} = 1$

Thus, $\frac{b}{a}$ is the multiplicative inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is also known as the reciprocal of reciprocal of $\frac{b}{a}$.

Write the multiplicative inverse or the reciprocal of the following:-

$$\frac{-4}{9}, \frac{2}{-7}, \frac{8}{15}, \frac{c}{d}, 4, -5$$

Does a multiplicative inverse exist for each rational number?

What will be the multiplicative inverse of 0? Think about it?

The multiplicative inverse of 0 does not exist, since we do not get 1 on multiplying 0 with any rational number.

Thus, 0 does not have any multiplicative inverse.

Exercise 18.3

Q1. Substitute the values given below and check if $\frac{p}{q} \times \frac{r}{s} = \frac{r}{s} \times \frac{p}{q}$.

(i) $\frac{p}{q} = \frac{-3}{7}, \frac{r}{s} = \frac{11}{15}$ (ii) $\frac{p}{q} = 2, \frac{r}{s} = \frac{13}{17}$

(iii) $\frac{p}{q} = \frac{-105}{13}, \frac{r}{s} = \frac{-5}{8}$ (iv) $\frac{p}{q} = \frac{-16}{3}, \frac{r}{s} = 0$

Q2. Substitute x, y and z and check whether $x \times (y + z) = x \times y + x \times z$

(i) $x = \frac{-1}{2}, y = \frac{5}{7}, z = \frac{-7}{4}$ (ii) $x = \frac{3}{2}, y = \frac{-8}{5}, z = \frac{17}{6}$

(iii) $x = 1, y = \frac{9}{5}, z = 0$

Q3. Fill in the blanks using commutative property :-

(i) $\frac{2}{3} \times 4 = 4 \times$ ___ (ii) $\frac{11}{19} \times$ ___ $= \frac{1}{2} \times$ ___

(iii) ___ $\times \frac{7}{9} =$ ___ $\times \frac{-3}{17}$

Q4. Use associative property and fill the blanks.

(i) $\frac{1}{2} \times \left(\frac{17}{6} \times \frac{2}{9}\right) = \left(\frac{1}{2} \times \frac{17}{6}\right) \times$ ___

(ii) $\frac{-1}{8} \times \left(\frac{-2}{5} \times \frac{1}{4}\right) = (\text{\_\_\_} \times \text{\_\_\_}) \times \frac{1}{4}$ (iii) $\frac{4}{7} \times \left(\frac{-25}{3} \times \frac{1}{5}\right) = (\text{\_\_\_} \times \text{\_\_\_}) \times \text{\_\_\_}$

Q5. Each of the questions given below relate to some property .Write the property in the blank against each.

		Property
(i)	$\frac{7}{12} \times \left(\frac{1}{9} + \frac{5}{3}\right) = \frac{7}{12} \times \frac{1}{9} + \frac{7}{12} \times \frac{5}{3}$	(____________________)
(ii)	$\frac{5}{7} \times \left(\frac{25}{3} + \frac{4}{3}\right) = \frac{5}{7} \times \frac{25}{3} + \frac{5}{7} \times \frac{4}{3}$	(____________________)
(iii)	$\frac{8}{11} \times \frac{3}{7} = \frac{3}{7} \times \frac{8}{11}$	(____________________)
(iv)	$\frac{5}{3} \times \frac{3}{5} = 1$	(____________________)
(v)	$\frac{-3}{12} \times 1 = 1 \times \left(\frac{-3}{12}\right) = \frac{-3}{12}$	(____________________)

Q6. Write the reciprocal of the following:-

(i) 4 (ii) $\frac{17}{5}$ (iii) $\frac{-6}{29}$ (iv) $\frac{p}{q}$

Q7. Write true or false:

(i) The product of a rational number and its reciprocal is one.

(ii) If the reciprocal of x is y, the reciprocal of y is 1/x.

(iii) The multiplicative inverse of a positive rational number is a negative rational number.

(iv) 0 is not the multiplicative inverse of any number.

Division of Rational Number

Radha and Fatima were playing the game find the multiplicative inverse. They were giving numbers to each other to find its multiplicative inverse. Radha noticed something new. She told Fatima “See in these examples multiplication of a number with its multiplicative inverse is in fact equivalent to dividing the number by itself.

Like $4\times\frac{1}{4}=\frac{4}{4}=4\div 4$

$2\times\frac{1}{2}=\frac{2}{2}=2\div 2$

Fatima said that this means that dividing by a number is the same as multiplying with the multiplicative inverse of the number.

$3 \div 4 = 3 \times$ (multiplicative inverse of 4)

$=3\times\frac{1}{4}$

Activity 6

Convert division into multiplication using multiplicative inverse.

e.g $\frac{2}{3}\div\frac{1}{3}=\frac{2}{3}\times\frac{3}{1}$

(i) $\frac{4}{3}\div\frac{3}{4}=$ (ii) $\frac{7}{9}\div\frac{8}{7}=$ (iii) $\frac{a}{x}\div\frac{b}{y}=$ (iv) $\frac{p}{q}\div\frac{r}{s}=$

We can conclude from the above that if we have to divide $\frac{x}{y}$ by $\frac{a}{b}$. Then by writing it as $\frac{x}{y}\times$ (Multiplicative inverse of $\frac{a}{b}$), we can get the answer.

$\frac{x}{y}\div\frac{a}{b}=\frac{x}{y}\times$ (Multiplicative inverse of $\frac{a}{b}$) $=\frac{x}{y}\times\frac{b}{a}$

Example 15: Solve the following

(i) $2\div\frac{-2}{3}$ (ii) $\frac{-5}{4}\div\frac{15}{14}$ (iii) $\frac{23}{12}\div\frac{46}{36}$

Solution: (i) $2\div\frac{-2}{3} = 2\times\frac{3}{-2}=-3$ (ii) $\frac{-5}{4}\div\frac{15}{14}=\frac{-5}{4}\times\frac{14}{15}=\frac{-7}{6}$

(iii) $\frac{23}{12}\div\frac{46}{36}=\frac{23}{12}\times\frac{36}{46}=\frac{3}{2}$

Example 16: The product of two rational numbers is -21. If one of the number is $\frac{3}{10}$, find the second number ?

Solution: Let the second rational number be $\frac{p}{q}$

According to the question $\frac{3}{10} \times \frac{p}{q} = -21$

Multiplying both sides by $\frac{10}{3}$, (the multiplicative inverse of 3/10)

$$\frac{3}{10} \times \frac{p}{q} \times \frac{10}{3} = -21 \times \frac{10}{3}$$

$$\text{Or } \frac{p}{q} = \frac{-210}{3} = \frac{-70}{1}$$

Thus, the second number is $-\frac{210}{3}$ or $\frac{-70}{1}$

Exercise 18.4

Q.1. Divide -

(i) $\frac{1}{6}$ by $\frac{3}{4}$ (ii) $\frac{-8}{11}$ by $\frac{5}{9}$ (iii) -9 by $\frac{4}{7}$

(iv) $\frac{-102}{38}$ by $\frac{-17}{19}$ (v) $\frac{6}{15}$ by $\frac{8}{-35}$ (vi) $\frac{-60}{9}$ by -10

Q2. Simplify

(i) $\frac{4}{5} \div (-1)$ (ii) $\frac{95}{16} \div \frac{8}{19}$ (iii) $\left(\frac{-7}{8}\right) \div \left(\frac{-2}{15}\right)$

(iv) $\frac{21}{5} \div \frac{7}{-5}$ (v) $\frac{-6}{7} \div (-15)$ (vi) $-7 \div (-5)$

Q3. The product of two numbers is 12. If one number is $\frac{3}{5}$, find the second number?

Q4. Which number multiplied by $\frac{-9}{5}$ gives the product -11?

Q5. What should be multiplied by $\frac{-28}{39}$ so that the product is the multiplicative inverse of $\frac{3}{7}$?

Q6. There are 540 students in a school out of them $\frac{5}{9}$ are boys. How many girls are in the school

How many numbers are there between two rational numbers

Fatima and Kartik were solving questions related to ordering of rational numbers. Fatima said, "Kartik, we can write integers between – 15 and -8. These are -14, -13, -12, -11, -10, -9. Similarly, can we write rational numbers between two rational numbers?"
Kartik said, "Definitely, we can write many rational numbers between two rational numbers".

Fatima Said, "Yes! between $\frac{-15}{1}$ and $\frac{-8}{1}$ we have all the integers but besides that between $\frac{-15}{1}$ and $\frac{-14}{1}$ we also have $\frac{-29}{2}$. Kartik said, "There are many more rational numbers." Fatima, "Oh! Yes. There would be many, that it would be difficult to count them".
Do you agree with Kartik and Fatima? Is the statement of Fatima that there are countless rational numbers between $\frac{-15}{1}$ and $\frac{-8}{1}$ correct? Radha said, "How can this be? There is no fraction between $\frac{2}{5}$ and $\frac{3}{5}$."

Can you think which fractions lie between $\frac{2}{5}$ and $\frac{3}{5}$?

Kartik said, "Let us find out-

$$\frac{2}{5} = \frac{2\times 2}{5\times 2} = \frac{4}{10}$$

$$\frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10}$$

$\frac{5}{10}$ lies between the two rational numbers.

Immediately, Ramesh and Meena together said.

$$\frac{2}{5} = \frac{2\times 3}{5\times 3} = \frac{6}{15}$$

$$\frac{3}{5} = \frac{3\times 3}{5\times 3} = \frac{9}{15}$$

Now $\frac{7}{15}, \frac{8}{15}$ are between these numbers.

Can you find at least 20 rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$?

So, how many rational numbers do we have between $\frac{5}{7}$ and $\frac{6}{7}$?

Anu saw a special thing that if we have two rational numbers with equal denominators and consecutive numerators like $\frac{5}{7}$ and $\frac{6}{7}$, then if these are multiplied by $\frac{2}{2}$, we get $\frac{10}{14}$ & $\frac{12}{14}$, $\frac{11}{14}$ is between $\frac{10}{14}$ and $\frac{12}{14}$. If we multiply them by $\frac{3}{3}$, we get $\frac{15}{21}$ and $\frac{18}{21}$ thus $\frac{16}{21}$, $\frac{17}{21}$, are two other rational numbers between them.

If we multiply the chosen numbers by $\frac{5}{5}$, We get 4 new rational numbers between $\frac{25}{35}$ and $\frac{30}{35}$.

Anu said - if I multiply them by $\frac{17}{17}$, we will get 16 new rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$.

Do you agree with Anu's statement?

If we consider the above examples we can see that we have found many rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$. Can you imagine how many rational numbers exist between $\frac{5}{7}$ and $\frac{6}{7}$?

Activity 7

1. Write 25 rational numbers between $\frac{1}{3}$ and $\frac{2}{3}$.
2. Can you write any two fractions such that there is no fraction between them?

Example 4: Write 10 rational numbers between $\frac{-4}{3}$ and $\frac{3}{4}$

Solution: Given rational numbers do not have equal denominator.

So, we make all the denominators equal than, $\frac{-4}{3} = \frac{-4\times 4}{3\times 4} = \frac{-16}{12}$

and $\frac{3}{4} = \frac{3\times 3}{4\times 3} = \frac{9}{12}$. Now $\frac{-16}{12}$ and $\frac{9}{12}$ are rational numbers with equal denominators. The difference between their numerators is 25.

So, there will be 24 rational numbers between $\frac{-4}{3}$ and $\frac{3}{4}$

$$\frac{-15}{12},\frac{-14}{12},\frac{-13}{12}\text{..........}\frac{-2}{12},\frac{-1}{12},\frac{0}{12},\frac{1}{12},\frac{2}{12}\text{...}\frac{7}{12},\frac{8}{12}$$

We can choose any 10 rational numbers from the above numbers.

If you have to find 25 rational numbers between $\frac{-4}{3}$ and $\frac{4}{3}$ what will you do?

Another Method

Example 5: Write 5 rational numbers between $\frac{1}{8}$ and $\frac{1}{2}$

Solution: $\frac{1}{2}$ is bigger and $\frac{1}{8}$ is smaller among them.

If we add both the two numbers and divide them by 2. Then, the resulting number is the central number (midpoint) between them.

First mid point between them is:-

$$=\frac{\frac{1}{8}+\frac{1}{2}}{2}=\frac{1}{2}\left(\frac{1}{8}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1\times1}{8\times1}+\frac{1\text{x}4}{2\text{x}4}\right)$$

$$=\frac{1}{2}\left(\frac{1+4}{8}\right)=\frac{5}{16}$$

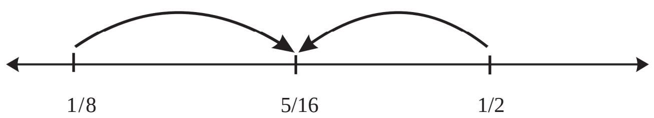


Figure 1.10

As shown in figure 1.10. This is exactly in the middle of $\frac{1}{8}$ and $\frac{1}{2}$.

To find more rational numbers we can find two more middle numbers between $\frac{1}{8}$ & $\frac{5}{16}$ and $\frac{5}{16}$ & $\frac{1}{2}$

The middle number between $\frac{1}{8}$ and $\frac{5}{16}$ is –

$$\frac{1}{2}\left(\frac{1}{8}+\frac{5}{16}\right)=\frac{1}{2}\left(\frac{1\times2}{2\times8}+\frac{5\times1}{16\times1}\right)$$

$$=\frac{1}{2}\left(\frac{2+5}{16}\right)=\frac{1}{2}\left(\frac{7}{16}\right)=\frac{7}{32}$$

The middle number between $\frac{5}{16}$ and $\frac{1}{2}$ is $\frac{1}{2}\left(\frac{5}{16}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{5\times1}{16\times1}+\frac{1\times8}{2\times8}\right)=\frac{1}{2}\left(\frac{5+8}{16}\right)=\frac{13}{32}$

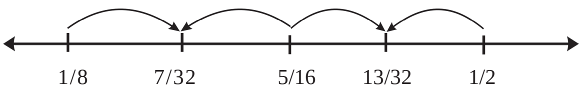


Figure 18.6

Now, We find the middle number between $\frac{1}{8}$ and $\frac{7}{32}$

The middle number between $\frac{1}{8}$ and $\frac{7}{32}$

$$\frac{1}{2}\left(\frac{1}{8}+\frac{7}{32}\right)=\frac{1}{2}\left(\frac{1\times4}{8\times4}+\frac{7\times1}{32\times1}\right)=\frac{1}{2}\left(\frac{4}{32}+\frac{7}{32}\right)=\frac{1}{2}\left(\frac{11}{32}\right)=\frac{11}{64}$$

The middle number between $\frac{13}{32}$ and $\frac{1}{2}$

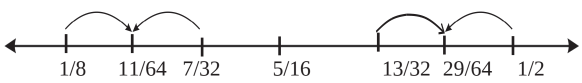


Figure 18.7

$$\frac{1}{2}\left(\frac{13}{32}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{13\times1}{32\times1}+\frac{1\times16}{2\times16}\right)=\frac{1}{2}\left(\frac{13}{32}+\frac{16}{32}\right)=\frac{29}{64}$$

Thus, five rational numbers between $\frac{1}{8}$ and $\frac{1}{2}$ are:- $\frac{11}{64}, \frac{7}{32}, \frac{5}{16}, \frac{13}{32}, \frac{29}{64}$

Rajni said, "This means that we can find at least one rational number between any two rational numbers." Rahul said, "That is not all, if we continue we can find as many rational numbers as we wish".

What do you think about this? Discuss among yourselves.

Example 6: Write three rational numbers between $\frac{-7}{3}$ and $\frac{5}{8}$

Solution : The middle rational number between $\frac{-7}{3}$ and $\frac{5}{8}$

$$=\frac{1}{2}\left(\frac{-7}{3}+\frac{5}{8}\right)$$

$$=\frac{1}{2}\left(\frac{-7\times8}{3\times8}=\frac{5\times3}{8\times3}\right)=\frac{1}{2}\left(\frac{-56+15}{24}\right)=\frac{-41}{48}$$

The middle rational number between $\frac{-7}{3}$ and $\frac{-41}{48}$

$$=\frac{1}{2}\left(\frac{-7}{3}+\frac{-41}{48}\right)=\frac{1}{2}\left(\frac{-7\times16}{3\times16}+\frac{-41\times1}{48\times1}\right)=\frac{1}{2}\left(\frac{-112+(-41)}{48}\right)=\frac{1}{2}\left(\frac{-153}{48}\right)=\frac{-153}{96}$$

The middle rational number between $\frac{-41}{48}$ and $\frac{5}{8}$

$$=\frac{1}{2}\left(\frac{-41}{48}+\frac{5}{8}\right)=\frac{1}{2}\left(\frac{-41\times1}{48\times1}+\frac{5\times6}{8\times6}\right)$$

$$=\frac{1}{2}\left(\frac{-41+30}{48}\right)$$

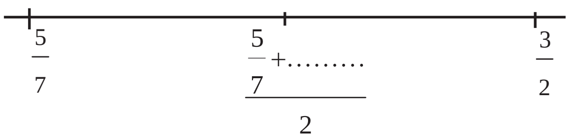
$$=\frac{1}{2}\left(-\frac{11}{48}\right)=\frac{-11}{96}$$

Thus three numbers between $\frac{-7}{3}$ and $\frac{5}{8}$ are $\frac{-153}{96}, \frac{-41}{48}$ and $\frac{-11}{96}$

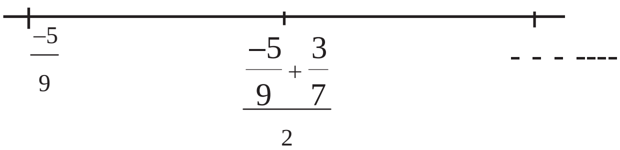
EXERCISE 18.5

1. Fill up the blanks in following figure.

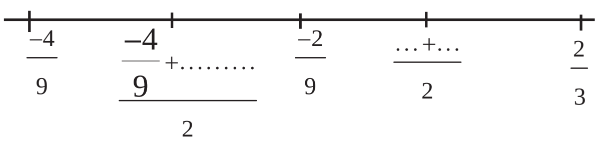
i)



ii)



iii)



2. How many rational numbers can be written between two rational numbers? Explain.

3. Write 5 rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

4. Write 4 rational numbers between $\frac{1}{3}$ and $\frac{-2}{7}$.

5. Write 6 rational numbers between $\frac{-1}{6}$ and $\frac{3}{4}$

6. True or false -

(i) $\frac{1}{10}$ is the middle number of $\frac{-1}{2}$ and $\frac{3}{5}$

(ii) There is no rational number between $\frac{4}{5}$ and $\frac{6}{5}$

(iii) There are only 3 rational numbers between 3 and 7

7. Write more questions in which you have to find rational numbers between two rational numbers. Give these questions to your friends ?

8. Write in your words what you have learnt in this chapter about rational numbers ?

We have learnt

1. If x and y are two rational numbers,

(i) x + y is also a rational number

(ii) x - y is also a rational number

(iii) x × y is also a rational number

(iv) x ÷ y is also a rational number (If y is non – zero)

2. If x and y are two rational numbers then

(i) $x + y = y + x$

(ii) $x \times y = y \times x$

(iii) $x-y \neq y-x$ (except when $x = y$)

(iv) $x \div y \neq y \div x$(except when $x = y$ and $x \neq 0, y \neq 0$)

3. If x, y and z are three rational numbers then

(i) $(x+y)+z = x+(y+z)$

(ii) $(x \times y) \times z = x \times (y \times z)$

4. If x, y and z are three rational numbers then

(i) $x \times (y+z) = x \times y + x \times z$

(ii) x × (y-z) = x × y – x × z

5. If x is any rational number, the following statements are true.
(i) x + 0 = 0+ x = x (ii) x – 0 = x

(iii) x × 0= 0 × x = 0 (iv) x × 1 = 1 × x = x (v) x ÷1 = x

6. If $x = \frac{p}{q}$ is a non – zero rational number, the multiplicative inverse of x is $\frac{1}{x} = \frac{q}{p}$ is also a rational number ?

7. Dividing a rational number x by another rational number y is the same as multiplying x with the multiplicative inverse of y ?

(x/y)/(a/b) = (x/y) × (multiplicative inverse of a/b) = (x/y) × (b/a)

Or divisor ÷ dividend = Divisor × (reciprocal of dividend).

8. In adding the two rational numbers, we convert them into common denominator fractions and then add them.

9. There are infinite rational numbers between two rational numbers.

10. If we have two rational numbers with equal denominators, then we can easily find rational numbers whose number is one less than the difference of their numerators.