Speed Test-23

1. (b) $\frac{1}{f_R} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_v} = (1.45-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{f_v}{f_R} = \frac{0.5}{0.45} = \frac{10}{9}$

$f_R = \frac{9}{10} f_v = \frac{9}{10} \times 20 \text{ cm} = 18 \text{ cm}.$

2. (a) We have,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or $\sin\frac{A}{2}\cdot\cot\frac{A}{2} = \sin\left(\frac{A+\delta_m}{2}\right)$

or $\sin\frac{A}{2}\cdot\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \sin\left(\frac{A+\delta_m}{2}\right)$

or $\cos\frac{A}{2} = \cos\left[\frac{\pi}{2} - \left(\frac{A+\delta_m}{2}\right)\right]$

$\Rightarrow \frac{A}{2} = \frac{\pi}{2} - \left(\frac{A+\delta_m}{2}\right)$

or $A = \pi - A - \delta_m \Rightarrow \delta_m = \pi - 2A$.

3. (a) Let the distance between the lenses be d.

Then, equivalent power is

$P = P_1 + P_2 - d\,P_1 P_2$

Given $P_1 = P_2 = +5$ D

$\therefore$ $P = (10 - 25d)$ D

For P to be –ve,

$10 - 25d < 0 \Rightarrow d > \frac{2}{5}$ m

or, $d > 0.4$ m or $d > 40$ cm

4. (b) ${}^a\mu_g = \frac{\sin 60°}{\sin 35°}$ ... (i)

${}^a\mu_w = \frac{\sin 60°}{\sin 41°}$ ... (ii)

${}^w\mu_g = \frac{\sin 41°}{\sin\theta}$ ... (iii)

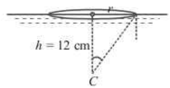
${}^a\mu_w \times {}^w\mu_g = {}^a\mu_g$

$\frac{\sin 60°}{\sin 41°} \times \frac{\sin 41°}{\sin\theta} = \frac{\sin 60°}{\sin 35°}$

(Using (i), (ii) and (iii))

$= \sin\theta = \sin 35°$ $\theta = 35°$

5. (d)



$\sin C = \frac{1}{\mu} = \frac{1}{4/3} = \frac{3}{4}.$

Now $r = h \tan C$

$= 12 \times \frac{3}{\sqrt{7}} = \frac{36}{\sqrt{7}}$ cm

6. (a) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

According to Cauchy relation

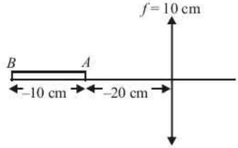
$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \cdots$ Hence $f \propto \lambda$.

Hence, red light having maximum wavelength has maximum focal length.

$\therefore$ $f_v < f_r$ and also $F_v > F_r$ as focal length is negative for a concave lens.

7. (c) To minimise spherical aberration in a lens, the total deviation should be equally distributed over the two surfaces.

8. (d)



The focal length of the mirror

$-\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

For A end of the rod the image distance

When $u_1 = -20$ cm

$\Rightarrow \frac{-1}{10} = \frac{1}{v_1} - \frac{1}{20}$

$$\frac{1}{v_1}=\frac{-1}{10}+\frac{1}{20}=\frac{-2+1}{20}$$

$v_1 = -20$ cm

For when $u_2 = -30$ cm

$$\frac{1}{f}=\frac{1}{v_2}-\frac{1}{30}$$

$$\frac{1}{v_2}=\frac{-1}{10}+\frac{1}{30}=\frac{-30+10}{300}=\frac{-20}{300}$$

$v_2 = -15$ cm

$L = v_2 - v_1 = -15-(-20)$

$L = 5$ cm

9. **(a)** Magnification

$$=\frac{f_0}{f_e}=\frac{\text{Angle subtended by final image on the eye}}{\text{Angle subtended by the object on eye (or objective)}}$$

$$\Rightarrow \frac{0.3\text{m}}{3\text{cm}}=\frac{\beta}{0.5^\circ} \Rightarrow \frac{30\text{ cm}}{3\text{cm}}=\frac{\beta}{0.5^\circ}$$

$\Rightarrow \beta = 5^\circ$

10. **(b)** Due to difference in refractive indices images obtained will be two. Two media will form images at two different points due to difference in focal lengths.

11. **(c)** For reading purposes :

$u = -25$ cm, $v = -50$ cm, $f = ?$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{50}+\frac{1}{25}=\frac{1}{50};$$

$$P=\frac{100}{f}=+2\text{ D}$$

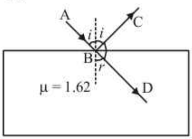
For distant vision, f' = distance of far point = –3 m

$$P=\frac{1}{f'}=-\frac{1}{3}\text{D}=-0.33\text{ D}$$

12. **(a)** Clearly,

$i + r + 90^\circ = 180^\circ$

$\Rightarrow i + r = 90^\circ$ ...(i)



Now, $\frac{\sin i}{\sin r}=\mu$

$\Rightarrow \frac{\sin i}{\sin(90^\circ - i)}=\mu$, from (1)

or $\frac{\sin i}{\cos i}=\mu \Rightarrow \tan i = \mu$

or $i=\tan^{-1}(\mu)$ i.e., $i=\tan^{-1}(1.62)$

13. **(b)** $f_0 = 100$ cm, $f_e = 5$ cm

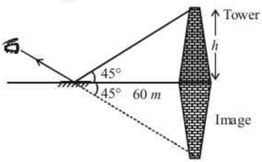
When final image is formed at least distance of distinct vision (d), then

$$M=\frac{f_0}{f_e}\left(1+\frac{f_e}{d}\right)=\frac{100}{5}\left(1+\frac{5}{25}\right) \quad [\because D = 25\text{ cm}]$$

$$M = 20\times\frac{6}{5}=24$$

14. **(b)** Secondary rainbow is formed by rays undergoing internal reflection twice inside the drop.

15. **(b)** $\tan 45^\circ = \frac{h}{60} \Rightarrow h = 60m$



16. **(c)** Using, $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}$

or $\frac{2}{v}-\frac{1}{\infty}=\frac{2-1}{R}$

$\therefore\ v = 2R$

17. **(a)** ${}_an_\ell = 1.6$, ${}_an_w = 1.33$

f = 20 cm

We have,

$$\frac{1}{f}=\left({}_an_\ell-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{20}=(1.6-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad .....(1)$$

Also, $\frac{1}{f'}=\left({}_wn_\ell-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$=\left(\frac{{}_an_\ell}{{}_an_w}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f'}=\left(\frac{1.6}{1.33}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad .....(2)$$

Dividing equation (1) by (2)

$$\Rightarrow \frac{f'}{20}=\frac{0.6}{(1.2-1)}$$

$$f'=\frac{0.6\times 20}{0.2}=60\text{ cm.}$$

Hence it's focal length is three times longer than in air.

18. **(a)** $m = \frac{v_0}{|u_0|}\left(1+\frac{d}{f_e}\right) = \frac{20}{5}\left(1+\frac{20}{10}\right)$

$= 4\left(\frac{10+20}{10}\right) = \frac{4\times 30}{10} = 12$

19. **(a)** Given $i = 60°$

$A = \delta = e$

$\delta = i + e - A \Rightarrow \delta = i \quad (\because e = A)$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Here angle of deviation is min. ($\because$ i = e)

$$\mu = \frac{\sin\left(\frac{60°+60°}{2}\right)}{\sin\frac{60°}{2}} = 1.73$$

20. **(b)** $u = -50\text{ cm} = -0.5\text{ m}$

$v = -30\text{ cm} = -0.3\text{ m}$

$P = \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{-1}{0.3} + \frac{1}{0.5} = \frac{-0.2}{0.15} = -1.33\text{ D}$.

21. **(b)** Object distance $u = -40$ cm

Focal length $f = -20$ cm

According to mirror formula

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

or $\frac{1}{v} + \frac{1}{-20} - \frac{1}{(-40)} = \frac{1}{-20} + \frac{1}{40}$

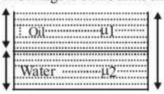
$\frac{1}{v} = \frac{-2+1}{40} = -\frac{1}{40}$ or $v = -40$ cm.

Negative sign shows that image is infront of concave mirror. The image is real.

Magnification, $m = \frac{-v}{u} = -\frac{(-40)}{(-40)} = -1$

The image is of the same size and inverted.

22. **(a)**



As refractive index, $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\therefore$ Apparent depth of the vessel when viewed from above is

$$d_{\text{apparent}} = \frac{x}{2\mu_1} + \frac{x}{2\mu_2} = \frac{x}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$

$$= \frac{x}{2}\left(\frac{\mu_2+\mu_1}{\mu_1\mu_2}\right) = \frac{x(\mu_1+\mu_2)}{2\mu_1\mu_2}$$

23. **(d)** As $r_1 < i_1$ i.e., the incident ray bends towards the normal $\Rightarrow$ medium 2 is denser than medium 1.

Or $r_2 < i_1 \Rightarrow$ medium 3 is denser than medium 1.

Also, $r_2 > r_1 \Rightarrow$ medium 2 is denser than medium 3.

24. **(d)** Here, $v_A = 1.8 \times 10^8$ m s^{-1}

$v_B = 2.4 \times 10^8$ m s^{-1}

Light travels slower in denser medium. Hence medium A is a denser medium and medium B is a rarer medium. Here, Light travels from medium A to medium B. Let C be the critical angle between them.

$\therefore \quad \sin C = {}^A\mu_B = \frac{1}{{}^B\mu_A}$

Refractive index of medium B w.r.t. to medium A is

$${}^A\mu_B = \frac{\text{Velocity of light in medium } A}{\text{Velocity of light in medium } B} = \frac{v_A}{v_B}$$

$\therefore \quad \sin C = \frac{v_A}{v_B} = \frac{1.8\times10^8}{2.4\times10^8} = \frac{3}{4}$ or $C = \sin^{-1}\left(\frac{3}{4}\right)$

25. **(a)** For a thin prism, $D = (\mu - 1)A$

Since $\lambda_b < \lambda_r \Rightarrow \mu_r < \mu_b \Rightarrow D_1 < D_2$

26. **(b)** Difference between apparent and real depth of a pond is due to the refraction of light, not due to the total internal reflection. Other three phenomena are due to the total internal reflection.

27. **(b)** Using the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Given $v = d$, for equal size image $|v| = |u| = d$

By sign convention $u = -d$

$\therefore \quad \frac{1}{f} = \frac{1}{d} + \frac{1}{d}$ or $f = \frac{d}{2}$

28. **(a)** Due to covering the reflection from lower part is not there so it makes the image less bright.

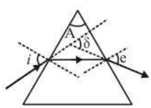
29. **(b)** From the fig.

Angle of deviation,

$\delta = i + e - A$

Here, $e = i$

and $e = \frac{3}{4}A$



$\therefore \delta = \frac{3}{4}A + \frac{3}{4}A - A = \frac{A}{2}$

For equilateral prism, $A = 60°$

$\therefore \delta = \frac{60°}{2} = 30°$

30. **(a)** Power of lens, P (in dioptre)

$$= \frac{100}{\text{focal length f (in cm)}}$$

$$\therefore \quad f = \frac{100}{10} = 10 \text{ cm}$$

By lens maker's formula, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For biconvex lens, $R_1 = +R$, and $R_2 = -R$

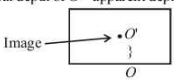
$$\therefore \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\frac{1}{10} = (\mu - 1)\left(\frac{2}{10}\right)$$

$$(\mu - 1) = \frac{1}{2} \text{ or } \mu = \frac{1}{2} + 1 = \frac{3}{2}$$

31. (d) In the later case microscope will be focussed for O'. So, it is required to be lifted by distance OO'.
OO' = real depth of O – apparent depth of O.



$$= 3 - \frac{3}{1.5} \quad \left[\mu = \frac{\text{real depth}}{\text{apparent depth}}\right]$$

$$= 3\left[\frac{1.5 - 1}{1.5}\right] = \frac{3 \times .5}{1.5} = 1 \text{ cm}$$

32. (d) The cause of chromatic aberration is that lens focusses different colours at different points.

33. (c) For the prism as the angle of incidence (i) increases, the angle of deviation (δ) first decreases goes to minimum value and then increases.

34. (d) $d_A : d_B = 6 : 4$

$\because$ Time taken $\propto$ thickness

and time taken $\propto \frac{1}{\text{velocity}}$

$\therefore$ Thickness $\propto \frac{1}{\text{velocity}}$

$$\therefore \quad \frac{d_A}{d_B} = \frac{v_B}{v_A}$$

Also, $\mu = \frac{c}{v} \quad \therefore \frac{\mu_A}{\mu_B} = \frac{v_B}{v_A}$

$$\therefore \quad \frac{d_A}{d_B} = \frac{\mu_A}{\mu_B} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$\therefore \quad {}_B\mu_A = 1.5$$

35. (b) Since $\frac{\text{Apparent depth}}{\text{Real depth}} = \frac{1}{\mu}$

$\Rightarrow$ Apparent depth = d/μ

So mark raised up = Real depth – Apparent depth

$$= d - \frac{d}{\mu} = d\left(1 - \frac{1}{\mu}\right) = \left(\frac{\mu - 1}{\mu}\right)d$$

36. (b) Dispersive power of a prism $\omega = \frac{\mu_V - \mu_R}{\mu_y - 1} = \frac{d\mu}{\mu - 1}$,

where $\mu = \mu_y = \frac{\mu_V + \mu_R}{2}$

37. (a) Considering refraction at the curved surface,

$u = -20,\ \mu_2 = 1$

$\mu_1 = 3/2,\ R = +20$

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1}{v} - \frac{3/2}{-20} = \frac{1 - 3/2}{20} \Rightarrow v = -10$$

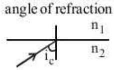
i.e., 10 cm below the curved surface or 10 cm above the actual position of flower.

38. (b) When $\theta = 90°$ then $\frac{360}{\theta} = \frac{360}{90} = 4$

is an even number. The number of images formed is given by

$$n = \frac{360}{\theta} - 1 = \frac{360}{90} - 1 = 4 - 1 = 3$$

39. (b) The critical angle of incidence is that angle at which angle of refraction is 90°.



$\sin i_c = \frac{n_1}{n_2}$ where $n_2 > n_1$

As, refractive index = $\frac{\text{velocity (air)}}{\text{velocity (medium)}}$

$$\therefore \sin i_c = \frac{2.2 \times 10^8 \text{ m/sec}}{2.4 \times 10^8 \text{ m/sec}} = \frac{11}{12}$$

$$\Rightarrow i_C = \sin^{-1}\left(\frac{11}{12}\right)$$

40. (b) $\frac{P_a}{P_l} = \frac{\left(\frac{\mu_g}{\mu_a} - 1\right)}{\left(\frac{\mu_g}{\mu_l} - 1\right)} = \frac{+5}{-100/100} = -5$

$$-5\left(\frac{\mu_g}{\mu_l} - 1\right) = \frac{\mu_g}{\mu_a} - 1$$

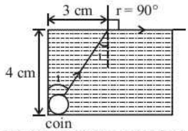
$$\frac{1.5}{\mu_1}-1=\frac{-1}{5}(1.5-1)=-0.1\,;\ \ \mu_1=\frac{1.5}{0.9}=\frac{5}{3}$$

41. (d) $\sin C=\frac{1}{\mu}=\frac{1}{\sqrt{2}}\quad\therefore\ C=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^{\circ}$

Now $\frac{\sin C}{\sin r}=\frac{1}{\mu}$ or $\frac{\sin 45^{\circ}}{\sin r}=\frac{1}{\sqrt{2}}$

$\sin r = 1$ or $r = 90^{\circ}$

42. (d)



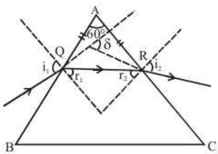
Hypotenuse comes out to be 5 cm.

Since, $\frac{1}{\mu}=\frac{\sin i}{\sin 90^{\circ}}$

$\mu=\frac{1}{\sin i}=\frac{5}{3}$

Speed, $v=\frac{c}{\mu}=\frac{3\times10^{8}}{5/3}=1.8\times10^{8}\ \text{m/s}$

43. (a)



Given AQ = AR and ∠A = 60°

$\therefore\ \angle AQR=\angle ARQ=60^{\circ}$

$\therefore\ r_1=r_2=30^{\circ}$

Applying Snell's law on face AB.

$\sin i_1=\mu\sin r_1$

$\Rightarrow\ \sin i_1=\sqrt{3}\sin 30^{\circ}=\sqrt{3}\times\frac{1}{2}=\frac{\sqrt{3}}{2}$

$\therefore\ i_1=60^{\circ}$

Similarly, $i_2=60^{\circ}$

In a prism, deviation

$\delta=i_1+i_2-A=60^{\circ}+60^{\circ}-60^{\circ}=60^{\circ}$

44. (a) $\frac{1}{f}=\left(\frac{\mu_g}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

If $\mu_g=\mu_m$, then $\frac{1}{f}=(1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow\quad \frac{1}{f}=0$

$$\boxed{f=\frac{1}{0}=\infty}$$

This implies that the liquid must have refractive index equal to glass.

45. (b) Minimum deviation of the prism when it is dipped in water $=\delta_m'=({}_w\mu_g-1)A$

$$=\left(\frac{{}_a\mu_g}{{}_a\mu_\omega}-1\right)A=\left(\frac{\frac{3}{2}}{\frac{4}{3}}-1\right)A=\frac{1}{8}A$$

Minimum deviation of the prism with respect to air

$$=\delta_m=(\mu-1)A=\left(\frac{3}{2}-1\right)A=\frac{1}{2}A$$

$$\frac{\delta_m'}{\delta_m}=\frac{\frac{1}{8}A}{\frac{1}{2}A}=\frac{1}{4}$$