Statistics

Exercise-15.1

Question 1:

Find the mean of the following frequency distribution :

Class	0-50	50-100	100-150	150-200	200-250	250-300	300-350
Frequency	10	15	30	20	15	8	2

Solution :

Let A = 175 and c = 50

Class	Frequency (f_i)	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
0 - 50	10	25	-3	-30
50 - 100	15	75	-2	-30
100 - 150	30	125	-1	-30
150 - 200	20	175 = A	0	0
200 - 250	15	225	1	15
250 - 300	8	275	2	16
300 - 350	2	325	3	6
Total	$\sum f_i = 100$			$\sum f_i u_i = -53$

Using the step-deviation method, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times c$

$\bar{x} = 175 + \dfrac{-53}{100} \times 50$

$= 175 - 26.5$

$= 148.5$

∴ The mean of the data is 148.5.

Question 2:

Find the mean wage of 200 workers of a factory where wages are classified as follows :

Class	100-150	150-200	200-250	250-300	300-350	350-400	400-450	450-500	500-550
Frequency	4	8	14	42	50	40	32	6	4

Solution :

Let A = 325 and c = 50

Class	Frequency (f_i)	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
100 - 150	4	125	-4	-16
150 - 200	8	175	-3	-24
200 - 250	14	225	-2	-28
250 - 300	42	275	-1	-42
300 - 350	50	325 = A	0	00
350 - 400	40	375	1	40
400 - 450	32	425	2	64
450 - 500	6	475	3	18
500 - 550	4	525	4	16
Total	$\sum f_i = 200$			$\sum f_i u_i = 28$

Using the step-deviation method,

$$\bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times c$$

$$\bar{x} = 325 + \frac{28}{200} \times 50$$

$$= 325 + 7$$

$$= 332$$

∴ The mean wages of the workers is Rs. 332.

Question 3:

Marks obtained by 140 students of class X out of 50 in mathematics are given in the following distribution. Find the mean by method of assumed mean method :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	20	24	40	36	20

Solution :

Let A = 25

Class	Frequency (f_i)	x_i	$d_i = x_i - A$	$f_i d_i$
0-10	20	5	-20	-400
10-20	24	15	-10	-240
20-30	40	25 = A	0	0
30-40	36	35	10	360
40-50	20	45	20	400
Total	$\sum f_i = 140$			$\sum f_i d_i = 120$

Using the assumed mean method, $\bar{x} = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$\bar{x} = 25 + \dfrac{120}{140}$

$= 25 + 0.857$

$= 25.857$

∴ The mean marks obtained by students in mathematics is 25.857.

Question 4:

Find the mean of the following frequency distribution by step-deviation method :

Class	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	5	10	20	9	6	2

Solution :

Let $A = 65$ and $c = 10$

Class	Frequency (f_i)	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
40 - 50	5	45	-2	-10
50 - 60	10	55	-1	-10
60 - 70	20	65=A	0	0
70 - 80	9	75	1	9
80 - 90	6	85	2	12
90 - 100	2	95	3	6
Total	$\Sigma f_i = 52$			$\Sigma f_i u_i = 7$

Using the step-deviation method, $\bar{x} = A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times c$

$\bar{x} = 65 + \dfrac{7}{52} \times 10$

$= 65 + 1.346$

$= 66.346$

∴ The mean of the given frequency distribution is 66.346.

Question 5:

Find the mean for the following frequency distribution :

Class	1-5	6-10	11-15	16-20	21-25	26-30	31-35
Frequency	18	32	30	40	25	15	40

Solution :

Let A = 18 and c = 5

Class	Frequency (f_i)	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
1 - 5	18	3	-3	-54
6 - 10	32	8	-2	-64
11 - 15	30	13	-1	-30
16 - 20	40	18 = A	0	0
21 - 25	25	23	1	25
26 - 30	15	28	2	30
31 - 35	40	33	3	120
Total	$\sum f_i = 200$			$\sum f_i u_i = 27$

Using the step-deviation method, $\overline{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times c$

$\overline{x} = 18 + \dfrac{27}{200} \times 5$

= 18 + 0.675

= 18.675

∴ The mean of the given frequency distribution is 18.675.

Question 6:

A survey conducted by a student Of B.B.A. for daily income Of 600 families is as follows, find the mean income of a family :

Income	200-299	300-399	400-499	500-599	600-699	700-799	800-899
Number of families	3	61	118	139	126	151	2

Solution :

Let A = 549.5 and c = 100

Income (in Rs.)	Number of families (f_i)	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
200 - 299	3	249.5	-3	-9
300 - 399	61	349.5	-2	-122
400 - 499	118	449.5	-1	-118
500 - 599	139	549.5 = A	0	0
600 - 699	126	649.5	1	126
700 - 799	151	749.5	2	302
800 - 899	2	849.5	3	6
Total	$\sum f_i = 600$			$\sum f_i u_i = 185$

Using the step-deviation method, $\overline{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times c$

$\overline{x} = 549.5 + \dfrac{185}{600} \times 100$

= 549.5 + 30.83

= 580.33

∴ The mean income of a family is Rs. 580.33.

Question 7:

The number of shares held by a person of various companies are as follows. Find the mean :

Number of shares	100-200	200-300	300-400	400-500	500-600	600-700
Number of companies	5	3	3	6	2	1

Solution :

Let $A = 450$ and $c = 100$

Number of shares	Number of companies	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
100 - 200	5	150	-3	-15
200 - 300	3	250	-2	-6
300 - 400	3	350	-1	-3
400 - 500	6	450 = A	0	0
500 - 600	2	550	1	2
600 - 700	1	650	2	2
Total	$\Sigma f_i = 20$			$\Sigma f_i u_i = -20$

Using the step-deviation method, $\bar{x} = A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times c$

$\bar{x} = 450 + \dfrac{-20}{20} \times 100$

$= 450 - 100$

$= 350$

∴ The mean of the data is 350 shares.

Question 8:

The mean of the following frequency distribution of 100 observations is 148. Find the missing frequencies f_1 and f_2 :

Class	0-49	50-99	100-149	150-199	200-249	250-299	300-349
Frequency	10	15	f_1	20	15	f_2	2

Solution :

Here, $n = \sum f_i = 100$. Let $A = 124.5$ and $c = 50$

Class	Frequency	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
0 - 49	10	24.5	-2	-20
50 - 99	15	74.5	-1	-15
100 - 149	f_1	124.5 = A	0	0
150 - 199	20	174.5	1	20
200 - 249	15	224.5	2	30
250 - 299	f_2	274.5	3	$3f_2$
300 - 349	2	324.5	4	8
Total	$\sum f_i = 62 + f_1 + f_2$			$\sum f_i u_i = 23 + 3f_2$

$n = \sum f_i = 100$

$\therefore 62 + f_1 + f_2 = 100$

$\therefore f_1 + f_2 = 38$

Using the step-deviation method, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times c$

$\therefore 148 = 124.5 + \dfrac{23 + 3f_2}{100} \times 50$

$\therefore 23.5 = \dfrac{23 + 3f_2}{2}$

$\therefore 47 = 23 + 3f_2$

$\therefore 3f_2 = 24$

$\therefore f_2 = 8$

Moreover, $f_1 + f_2 = 38$

$\therefore f_1 + 8 = 38$

$\therefore f_1 = 30$

$\therefore$ The missing frequencies are $f_1 = 30$ and $f_2 = 8$.

Question 9:

The table below gives the percentage of girls in higher secondary science stream of rural areas of various states of India. Find the mean percentage of girls by step-deviation method :

Percentage of girls	15-25	25-35	35-45	45-55	55-65	65-75	75-85
Number of states	6	10	5	6	4	2	2

Solution :

Let A = 50 and c = 10

Percentage of girls	Number of states	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
15 - 25	6	20	-3	-18
25 - 35	10	30	-2	-20
35 - 45	5	40	-1	-5
45 - 55	6	50 = A	0	0
55 - 65	4	60	1	4
65 - 75	2	70	2	4
75 - 85	2	80	3	6
Total	$\sum f_i = 35$			$\sum f_i u_i = -29$

Using the step-deviation method, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times c$

$\bar{x} = 50 + \dfrac{-29}{35} \times 10$

$= 50 - 8.29$

$= 41.71$

∴ The mean persentage of girls is 41.71.

Question 10:

The following distribution shows the number of out door patients in 64 hospitals as follows. If the mean is 18, find the missing frequencies f_1 and f_2 :

Number of patients	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of hospitals	7	6	f_1	13	f_2	5	4

Solution :

Here, n = $\sum f_i$ = 64. Let A = 16 and c = 2

Number of patients	Number of hospitals	x_i	$u_i = \dfrac{x_i - A}{c}$	$f_i u_i$
11 - 13	7	12	-2	-14
13 - 15	6	14	-1	-6
15 - 17	f_1	16 = A	0	0
17 - 19	13	18	1	13
19 - 21	f_2	20	2	$2f_2$
21 - 23	5	22	3	15
23 - 25	4	24	4	16
Total	$\sum f_i = 35 + f_1 + f_2$			$\sum f_i u_i = 24 + 2f_2$

n = $\sum f_i$ = 64

∴ 35 + f_1 + f_2 = 64

∴ f_1 + f_2 = 29

Using the step-deviation method, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times c$

∴ 18 = 16 + $\dfrac{24 + 2f_2}{64} \times 2$

∴ 2 = $\dfrac{24 + 2f_2}{32}$

∴ 24 + $2f_2$ = 64

∴ $2f_2$ = 40

∴ f_2 = 20

Moreover, f_1 + f_2 = 29

∴ f_1 + 20 = 29

∴ f_1 = 9

∴ The missing frequencies are f_1 = 9 and f_2 = 20.

Exercise-15.2

Question 1:

Find the mode for the following Frequency distribution :

Class	4-8	8-12	12-16	16-20	20-24	24-28
Frequency	9	6	12	7	15	1

Solution :

Here, the maximum class frequency is 15 and the class corresponding
to this frequency is 20 – 24.
So the modal class is 20 – 24.
The lower limit l of the modal class 20 – 24 = 20
and class size c = 4
Frequency of the modal class = f_1 = 15
Frequency of the class perceding the modal class = f_0 = 7
Frequency of the class succeeding the modal class = f_2 = 1
Now let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

$$Z = 20 + \left(\frac{15 - 7}{2(15) - 7 - 1} \right) \times 4$$

$$= 20 + \frac{8}{22} \times 4$$

$$= 20 + 1.45$$

$$= 21.45$$

So, the mode of above data is 21.45.

Question 2:

The data obtained for 100 shops for their daily profit per shop are as follows :

Daily profit per shop (in ₹)	0-100	100-200	200-300	300-400	400-500	500-600
Number of shops	12	18	27	20	17	6

Find the modal profit per shop.

Solution :

Here, the maximum class frequency is 27 and the class corresponding
to this frequency is 200 – 300
∴ The modal class is 200 – 300.
Lower limit l of the modal class = 200
Class size c = 100
Frequency of the modal class = f_1 = 27
Frequency of the class preceding the modal class = f_0 = 18
Frequency of the class succeeding the modal class = f_2 = 20
Now let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

$$Z = 200 + \left(\frac{27 - 18}{2(27) - 18 - 20} \right) \times 100$$

$$= 200 + \frac{9}{16} \times 100$$

$$= 200 + 56.25$$

$$= 256.25$$

So, the modal profit is Rs. 256.25.

Question 3:

Daily wages of 90 employees of a factory are as follows :

Daily wages (in ₹)	150-250	250-350	350-450	450-550	550-650	650-750	750-850	850-950
Number of employees	4	6	8	12	33	17	8	2

Find the modal wage of an employee.

Solution :

Here, the maximum frequency is 33 and the class corresponding
to this frequency is $550 - 650$.
∴ The modal class is $550 - 650$.
Lower limit l of the modal class $550 - 650 = 550$
Class size c = 100
Frequency of the modal class = f_1 = 33
Frequency of the class preceding the modal class = f_0 = 12
Frequency of the class succeeding the modal class = f_2 = 17
Now let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

$$Z = 550 + \left(\frac{33 - 12}{2(33) - 12 - 17} \right) \times 100$$

$$= 550 + \frac{21}{37} \times 100$$

$$= 550 + 56.76$$

$$= 606.76$$

So, the modal wage is Rs. 606.76.

Question 4:

Find the mode for the following data : (4 and 5)

Class	0-7	7-14	14-21	21-28	28-35	35-42	42-49	49-56
Frequency	26	31	35	42	82	71	54	19

Solution :

Here, the maximum frequency is 82 and the class corresponding
to this frequency is $28 - 35$.
∴ Modal class is $28 - 35$.
Lower limit l of the modal class = 28
Class size c = 7
Frequency of the modal class = f_1 = 82
Frequency of the class preceding the modal class = f_0 = 42
Frequency of the class succeeding the modal class = f_2 = 71
Now, let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

$$Z = 28 + \left(\frac{82 - 42}{2(82) - 42 - 71} \right) \times 7$$

$$= 28 + \frac{40}{51} \times 7$$

$$= 28 + 5.49$$

$$= 33.49$$

So, the mode of the data is 33.49.

Question 5:

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160	160-180
Frequency	11	14	18	21	31	27	12	11	10

Solution :

Here, the maximum frequency is 31 and the class corresponding to this frequency is 80 – 100.

∴ Modal class is 80 – 100 .

Lower limit l of modal class = 80

Class size c = 20

Frequency of the modal class = f_1 = 31

Frequency of the class preceding the modal class = f_0 = 21

Frequency of the class succeeding the modal class = f_2 = 27

Now, let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$$

$$Z = 80 + \left(\frac{31 - 21}{2(31) - 21 - 27}\right) \times 20$$

$$= 80 + \frac{10}{14} \times 20$$

$$= 80 + 14.29$$

$$= 94.29$$

So, the mode of the data is 94.29

Question 6:

The following data gives the information of life of 200 electric bulbs (in hours) as follows :

Life in hours	0-20	20-40	40-60	60-80	80-100	100-120
Number of electric bulbs	26	31	35	42	82	71

Find the modal life of the electric bulbs.

Solution :

Here, the maximum frequency is 82 and the class corresponding to this frequency is 80 – 100.

∴ The modal class is 80 – 100 .

Lower limit l of the modal class 80 – 100 = 80

Class size c = 20

Frequency of the modal class = f_1 = 82

Frequency of the class preceding the modal class = f_0 = 42

Frequency of the class succeeding the modal class = f_2 = 71

Now, let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$$

$$Z = 80 + \left(\frac{82 - 42}{2(82) - 42 - 71}\right) \times 20$$

$$= 80 + \frac{40}{51} \times 20 = 80 + 15.69 = 95.69$$

So, the modal life of the electric bulbs is 95.69 hours.

Exercise-15.3

Question 1:

Find the median for the following .

Value of variable	12	13	14	15	16	17	18	19	20
Frequency	7	10	15	18	20	10	9	8	3

Solution :

We will prepare the table containing the cumulative frequency as below:

Value of variable	Frequency (f)	Cumulative frequency (cf)
12	7	7
13	10	7 + 10 = 17
14	15	17 + 15 = 32
15	18	32 + 18 = 50
16	20	50 + 20 = 70
17	10	70 + 10 = 80
18	9	80 + 9 = 89
19	8	89 + 8 = 97
20	3	97 + 3 = 100

Here, the number of observations is 100 which is even.

$\therefore$ n = 100

$\dfrac{n}{2} = 50$

The median will be the average of the 50th and the 51st observations.
From the above table, we see that the
50th observation = 15 and the 51st observation = 16.

$\therefore$ Median M = $\dfrac{15 + 16}{2} = \dfrac{31}{2} = 15.5$

$\therefore$ The median of the data is 15.5.

Question 2:

Find the median for the following frequency distribution :

Class	4-8	8-12	12-16	16-20	20-24	24-28
Frequency	9	16	12	7	15	1

Solution :

Let us prepare the table containing the cumulative frequencies as below :

Class	Frequency (f)	Cumulative frequency (cf)
4 – 8	9	9

8 – 12	16	9 + 16 = 25
12 – 16	12	25 + 12 = 37
16 – 20	7	37 + 7 = 44
20 – 24	15	44 + 15 = 59
24 – 28	1	59 + 1 = 60

Here, the number of observations is 60.

$\therefore$ n = 60. So, $\frac{n}{2}$ = 30

This observation lies in the class 12 – 16

So the median class is 12 – 16.

So, l = 12, $\frac{n}{2}$ = 30, cf = 25, f = 12 and c = 4.

Using the formula M = l + $\dfrac{\left(\frac{n}{2} - cf\right)}{f}$ × c,

M = 12 + $\dfrac{30 - 25}{12}$ × 4

= 12 + $\dfrac{5}{3}$

= 12 + 1.67

= 13.67

So the median of the data is 13.67.

Question 3:

Find the median from following frequency distribution :

Class	0-100	100-200	200-300	300-400	400-500	500-600
Frequency	64	62	84	72	66	52

Solution :

We will prepare the table containing the cumulative frequencies as below :

Class	Frequency (f)	Cumulative frequency (cf)
0 – 100	64	64
100 – 200	62	64 + 62 = 126
200 – 300	84	126 + 84 = 210
300 – 400	72	210 + 72 = 282
400 – 500	66	282 + 66 = 348
500 – 600	52	348 + 52 = 400

Here, the number of observations is 400.

$\therefore$ n = 400. So, $\dfrac{n}{2}$ = 200

This observation lies in the class 200–300

So the median class is 200–300.

So, l = 200, $\dfrac{n}{2}$ = 200, cf = 126, f = 84 and c = 100.

Using the formula $M = l + \dfrac{\left(\dfrac{n}{2} - cf\right)}{f} \times c$,

$$= 200 + \dfrac{(200 - 126)}{84} \times 100$$

$$= 200 + \dfrac{74 \times 100}{84}$$

$$= 200 + 88.09$$

$$= 288.09$$

So, the median of the data is 288.09

Question 4:

The following frequency distribution represents the deposits (in thousand rupees) and the number of depositors in a bank. Find the median of the data :

Deposit (₹ in thousand)	0-10	10-20	20-30	30-40	40-50	50-60
Number of depositors	1071	1245	150	171	131	8

Solution :

We will prepare the table containing the cumulative frequencies as below:

Deposit (In thousand Rs.)	Number of depositors (f)	Cumulative frequency (cf)
0 – 10	1071	1071
10 – 20	1245	1071 + 1245 = 2316
20 – 30	150	2316+150 = 2466
30 – 40	171	2466 + 171 = 2637
40 – 50	131	2637 + 131 = 2768
50 – 60	8	2768 + 8 = 2776

Here, the number of observations is 2776

$\therefore$ n = 2776. So, $\frac{n}{2}$ = 1388

This observation lies in the class 10 – 20

So the median class is 10 – 20.

So, l = 100, $\frac{n}{2}$ = 1388, cf = 1071, f = 1245 and c = 10.

Using the formula M = l + $\dfrac{\left(\frac{n}{2} - cf\right)}{f} \times c$,

$$= 10 + \frac{1388 - 1071}{1245} \times 10$$

$$= 10 + 2.55$$

$$= 12.55$$

So, the median of the data is 12.55 (in thousand Rs.).

Question 5:

The median of the following frequency distribution is 38. Find the value of a and b if the sum of frequences is 400 :

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	42	38	a	54	b	36	32

Solution :

Let us prepare the table containing the cumulative frequency as below:

Class	Frequency (f)	Cumulative frequency (cf)
10 – 20	42	42
20 – 30	38	42 + 38 = 80
30 – 40	a	80 + a =80+a
40 – 50	54	80 + a + 54 = 134 + a
50 – 60	b	134 + a + b=134 + a + b
60 – 70	36	134 + a + b = 134 + a+b
70 – 80	32	170 + a + b +32 = 202 + a + b

Here, n = 400.

∴ 202 + a + b = 400

∴ a + b = 198

As n = 400, $\frac{n}{2}$ = 200.

This observation lies in the class 30– 40

So, the median class is 30 – 40.

∴ l = 30, $\frac{n}{2}$ = 200, cf = 80, f = a and c = 10.

Using the formula M = l + $\frac{\left(\frac{n}{2} - cf\right)}{f}$ × c

38 = 30 + $\frac{200 - 80}{a}$ × 10

∴ 8 = $\frac{120 \times 10}{a}$

∴ a = $\frac{1200}{8}$

∴ a = 150

Now, a + b = 198

∴ 150 + b = 198

∴ b = 48

So, a = 150 and b = 48.

Question 6:

The median of 230 observations of the following frequency distribution is 46. Find a and b :

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	a	65	b	25	18

Solution :

We will prepare the table containing the cumulative frequencies as below :

Class	Frequency (f)	Cumulative frequency (cf)
10 – 20	12	12
20 – 30	30	12+30 = 42
30 – 40	a	42 + a = 42 + a
40 – 50	65	42 + a + 65 =107 + a
50 – 60	b	107 + a + b =107 + a + b
60 – 70	25	107+a+b + 25 = 132 + a + b
70 – 80	18	132+ a + b + 18 = 150 + a + b

Here, n = 230

∴ 150 + a + b = 230

∴ a + b = 80

As n = 230, $\frac{n}{2}$ = 115.

This observation lies in the class 40 – 50.

So the median class is 40 – 50.

∴ l = 40, $\frac{n}{2}$ = 115, cf = 42 + a, f = 65 and c = 10.

Using the formula M = l + $\dfrac{\left(\frac{n}{2} - cf\right)}{f} \times c$,

$$46 = 40 + \frac{115 - (42 + a)}{65} \times 10$$

$$\therefore 6 = \frac{73 - a}{65} \times 10$$

$$\therefore \frac{6 \times 65}{10} = 73 - a$$

$$\therefore 39 = 73 - a$$

$$\therefore a = 34$$

Now, a + b = 80

∴ 34 + b = 80

∴ b = 46

So, a = 34 and b = 46.

Question 7:

The following table gives the frequency distribution of marks scored by 50 students of class X in mathematics examination of 80 marks. Find the median of the data :

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	5	8	16	9	5	3	2

Solution :

Let us prepare the table containing the cumulative frequencies as below:

Class	Frequency (f)	Cumulative frequency (cf)
0 – 10	2	2
10 – 20	5	2 + 5 = 7
20 – 30	8	7 + 8 = 15
30 – 40	16	15 + 16 = 31
40 – 50	9	31 + 9 = 40
50 – 60	5	40 + 5 = 45
60 – 70	3	45 + 3 = 48
70 – 80	2	48 + 2 = 50

Here, the number of observation is 50

$\therefore$ n = 50. So, $\dfrac{n}{2}$ = 25

This observation lies in the class 30– 40.

So the median class is 30 – 40.

So, l = 30, $\dfrac{n}{2}$ = 25, cf = 15, f = 16 and c = 10.

Using the formula M = l + $\dfrac{\left(\dfrac{n}{2} - cf\right)}{f} \times c$,

$$M = 30 + \dfrac{25 - 15}{16} \times 10$$
$$= 30 + 6.25$$
$$= 36.25$$

So, the median of the data is 36.25 marks.

Exercise-15

Question 1:

In a retail market, a fruit vendor was selling apples kept in packed boxes. These boxes contained varying number of apples. The following was the distribution of apples according to the number of boxes. Find the mean by the assumed mean number of apples kept in the box.

Number of apples	50-53	53-56	56-59	59-62	62-65
Number of boxes	20	150	115	95	20

Solution :

Let A = 57.5

Number of apples (Class)	Number of boxes (f_i)	Midpoint (x_i)	$d_i = x_i - A$	$f_i d_i$
50 – 53	20	51.5	-6	-120
53 – 56	150	54.5	-3	-450
56 – 59	115	57.5 = A	0	0
59 – 62	95	60.5	3	285
62 – 65	20	63.5	6	120
Total	Σf_i= 400			$\Sigma f_i d_i$= -165

Using the assumed mean method, $\bar{X} = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$,

$\bar{X} = 57.5 + \dfrac{-165}{400}$

= 57.5 – 04125

= 57.0875

Hence, the mean number of apples in a box is 57.0875.

Question 2 :

The daily expenditure of 50 hostel students are as follows :

Daily expenditure (in ₹)	100-120	120-140	140-160	160-180	180-200
Number of students	12	14	8	6	10

Find the mean daily expenditure of the students of hostel using appropriate method.

Solution :

Let A = 150 and c = 20

Daily expense (in Rs.) Class	No. of students (f_i)	Midpoint (x_i)		f_iu_i
100 – 120	12	110	-2	-24
120 – 140	14	130	-1	-14
140 – 160	8	150 = A	0	0
160 – 180	6	170	1	6
180 – 200	10	190	2	20
Total	$\sum f_i$= 50			$\sum f_iu_i$= -12

Using the step-deviation method $\bar{x} = A + \dfrac{\sum f_iu_i}{\sum f_i} \times c$,

$\bar{x} = 150 + \dfrac{-12}{50} \times 20$

$= 150 - 4.80$

$= 145.20$

So, the mean daily expenditure of the students is Rs. 145.20.

Question 3 :

The mean of the following frequency distribution of 200 observations is 332. Find the value of x and y.

Class	100-150	150-200	200-250	250-300	300-350	350-400	400-450	450-500	500-550
Frequency	4	8	x	42	50	y	32	6	4

Solution :

Let A = 375 and c = 50

Class	Frequency (f_i)	Midpoint (x_i)		f_iu_i
100-150	4	125	-5	-20

150-200	8	175	-4	-32
200-250	X	225	-3	-3x
250-300	42	275	-2	-84
300-350	50	325	-1	-50
350-400	Y	375 = A	0	0
400-450	32	425	1	32
450-500	6	475	2	12
500-550	4	525	3	12
Total	$\sum f_i$= 146 + x + y			$\sum f_iu_i$ = -130 – 3x

Here, n = 200

$\therefore \sum f_i = 200$

$\therefore 146 + x + y = 200$

$\therefore x + y = 54$ ....(1)

For the data, mean $\bar{x}$ = 332.

Using the step-deviation method, $\bar{x} = A + \dfrac{\sum f_iu_i}{\sum f_i} \times c$

$\therefore 332 = 375 + \dfrac{-130 - 3x}{200} \times 50$

$\therefore \dfrac{130 + 3x}{4} = 375 - 332$

$\therefore \dfrac{130 + 3x}{4} = 43$

$\therefore 130 + 3x = 172$

$\therefore 3x = 42$

$\therefore x = 14$

Now, x + y = 54 (by (1))

$\therefore 14 + y = 54$

$\therefore y = 40$

So, x = 14 and y = 40.

Question 4 :

Find the mode of the following frequency distribution :

Class	0-15	15-30	30-45	45-60	60-75	75-90	90-105
Frequency	8	16	23	57	33	23	13

Solution :

Here, the maximum frequency is 57 and the class corresponding to this frequency is 45 – 60.

∴ The modal class is 45 – 60 .

Now,

Class length = c = 15

Lower limit of the modal class = l = 45

Frequency of the modal class = f_1 = 57

Frequency of the class preceding the modal class = f_0 = 23

Frequency of the class succeeding the modal class = f_2 = 33

Now let us substitute these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c,$$

$$Z = 45 + \left(\frac{57 - 23}{2(57) - 23 - 23}\right) \times 15$$

$$= 45 + \frac{34}{58} \times 15$$

$$= 45 + 8.793$$

$$= 53.793$$

So, the mode of the data is 53.793.

Question 5 :

Find the mode of the following data :

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	12	17	28	23	7	8	5

Solution :

Here, the maximum frequency is 28 and the class corresponding to this frequency is 50 – 60.

∴ The modal class is 50 – 60 .

Now,

Class length = c = 10

Lower limit of the modal class = l = 50

Frequency of the modal class = f_1 = 28

Frequency of the modal class preceding the modal class = f_0 = 17

Frequency of the class succeeding the modal class = f_2 = 23

Substituting these values in the formula:

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c,$$

$$Z = 50 + \left(\frac{28 - 17}{2(28) - 17 - 23}\right) \times 10$$

$$= 50 + \frac{11}{16} \times 10$$

$$= 50 + 6.875$$

$$= 56.875$$

So the mode of the data is 56.875.

Question 6 :

The mode of the following frequency distribution of 165 observations is 34.5. Find the value of a and b.

Class	5-14	14-23	23-32	32-41	41-50	50-59	59-68
Frequency	5	11	a	53	b	16	10

Solution :

The total number of observations is 165.

$\therefore 5 + 11 + a + 53 + b + 16 + 10 = 165$

$\therefore a + b + 95 = 165$

$\therefore a + b = 70$ .....(1)

Here, the mode 34.5 lies in the class 32 – 41.

So, the modal class is 32 – 41.

Now,

Class length = c = 9

Lower limit of the modal class = l = 32

Frequency of the modal class = f_1 = 53

Frequency of the class preceding the modal class = f_0 = a

Frequency of the class succeeding the modal class = f_2 = b

Substituting these values in the formula :

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

$$34.5 = 32 + \left(\frac{53 - a}{2(53) - a - b} \right) \times 9$$

$$\therefore 2.5 = 9 \left(\frac{53 - a}{106 - (a + b)} \right)$$

$$\therefore 2.5 = 9 \left(\frac{53 - a}{106 - 70} \right)$$

$$\therefore 2.5 = 9 \left(\frac{53 - a}{36} \right)$$

$$\therefore 2.5 = \frac{53 - a}{4}$$

$\therefore 10 = 53 - a$

$\therefore a = 53 - 10$

$\therefore a = 43$

From (1), we have

$a + b = 70$

$\therefore 43 + b = 70$

$\therefore b = 70 - 43$

$\therefore b = 27$

So, a = 43 and b = 27.

Question 7 :

Find the mode of the following frequency distribution :

Class	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Frequency	14	56	60	86	74	62	48

Solution :

Here, the maximum frequency is 86 and the class corresponding to this frequency is 3000 – 3500.

$\therefore$ Modal class is 3000 – 3500.

Now,

Class length = c = 500

Lower limit of the modal class = l = 3000

Frequency of the modal class = f_1 = 86

Frequency of the modal class preceding the modal class = f_0 = 60

Frequency of the class succeeding the modal class = f_2 = 74

Substituting these values in the formula :

$$Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c,$$

$$Z = 3000 + \left(\frac{86 - 60}{2(86) - 60 - 74} \right) \times 500$$

$$= 3000 + \frac{26}{38} \times 500$$

$$= 3000 + 342.10$$

$$= 3342.10$$

So, the mode of the data is 3342.1.

Question 8 :

Find the median of the following frequency distribution :

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	9	11	15	24	19	9	8	5

Solution :

Class	Frequency (f)	Cumulative Frequency (cf)
10-20	9	9
20-30	11	9 + 11 = 20
30-40	15	20 + 15 = 35
40-50	24	35 + 24 = 59
50-60	19	59 + 19 = 78
60-70	9	78 + 9 = 87
70-80	8	87 + 8 = 95
80-90	5	95 + 5 = 100

Here, n=100

$\therefore \dfrac{n}{2} = 50$

The cumulative frequency just greater than 50 is 59
which is the cumulative frequency of class 40-50.

$\therefore l = 40$, $\dfrac{n}{2} = 50$, cf=35, f= 24 and c=10.

Substituting the values in $M = l + \dfrac{\left(\dfrac{n}{2} - cf\right)}{f} \times c$,

$M = 40 + \dfrac{50 - 35}{24} \times 10$

$= 40 + 6.25$

$= 46.25$

Thus, the median of the data is 46.25.

Question 9 :

The median of the following data is 525. Find the value of x and y, if the sum of frequency is 100 :

Class	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	3	4	x	12	17	20	9	y	8	3

Solution :

We will prepare the table containing cumulative frequencies as below:

Class	Frequency (f)	Cumulative frequency (cf)
0 – 100	3	3
100 – 200	4	3+4=7
200 – 300	x	7+x=7+x
300 – 400	12	7+x+12=19+x
400 – 500	17	19+x+17=36+x
500 – 600	20	36+x+20=56+x
600 – 700	9	56+x+9=65+x
700 – 800	y	65+x+y=65+x+y
800 – 900	8	65+x+y+8=73+x+y
900 – 1000	3	73+x+y+3=76+x+y

Here, n = 100 $\therefore$ 76 + x + y = 100 $\therefore$ x + y = 24 ....(1)

Since n = 100, $\frac{n}{2}$ = 50.

So, the median class is 500 – 600.

$\therefore$ l = 500, $\frac{n}{2}$ = 50, cf = 36 + x, f = 20 and c = 100.

Using the formula $M = l + \dfrac{\left(\frac{n}{2} - cf\right)}{f} \times c$,

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$\therefore 25 = (14 - x)5$$

$$\therefore 5 = 14 - x$$

$$\therefore x = 9$$

Now, x + y = 24

$\therefore$ 9 + y = 24

$\therefore$ y = 15

So, x = 9 and y = 15.

Question 10:

Select a proper option (a), (b), (c) or (d) from given options :

Question 10(1) :

For some data, if Z = 25 and $\overline{x}$ = 25, then M = ...

Solution :

a. 25

We know that,
Z = 3M − 2$\overline{x}$
∴ 25 = 3M − 2(25)
∴ 25 = 3M − 50
∴ 75 = 3M
∴ M = 25

Question 10(2) :

For some data Z − M = 2.5. If the mean of the data is 20, then Z = ...

Solution :

c. 23.75

We know that,
Z = 3M − 2$\overline{x}$
∴ Z = 3(Z − 2.5) − 2$\overline{x}$ (∵ Z − M = 2.5)
∴ Z = 3Z − 7.5 − 2(20)
∴ Z = 3Z − 7.5 − 40
∴ −2Z = − 47.5
∴ Z = $\frac{-47.5}{-2}$
∴ Z = 23.75

Question 10(3) :

If $\overline{x}$ − Z = 3 and $\overline{x}$ + Z = 45, then M = ...

Solution :

d. 23

$\bar{x} - Z = 3$ ....(i)
$\bar{x} + Z = 45$....(ii)
Adding (i) and (ii), we get,
$2\bar{x} = 48$
∴ $\bar{x} = 24$
From (ii), $\bar{x} + Z = 45$
∴ $24 + Z = 45$
∴ $Z = 21$
We know that,
$Z = 3M - 2\bar{x}$
∴ $21 = 3M - 2(24)$
∴ $21 + 48 = 3M$
∴ $3M = 69$
∴ $M = 23$

Question 10(4) :

If $Z = 24, \bar{x} = 18$, then M = ...

Solution :

b. 20

We know that,
$Z = 3M - 2\bar{x}$
∴ $24 = 3M - 2(18)$
∴ $24 + 36 = 3M$
∴ $3M = 60$
∴ $M = 20$

Question 10(5) :

If $M = 15, \bar{x} = 10$, then Z = ...

Solution :

c. 25

We know that,
$Z = 3M - 2\bar{x}$
$= 3(15) - 2(10)$
$= 45 - 20$
$= 25$

Question 10(6) :

If M = 22, Z = 16, then $\overline{x}$ = …….

Solution :

b. 25

We know that,

Z = 3M − 2$\overline{x}$

16 = 3(22) − 2($\overline{x}$)

16 = 66 − 2($\overline{x}$)

2($\overline{x}$)= 50

∴ $\overline{x}$ = 25

Question 10(7) :

If $\overline{x}$ = 21.44 and Z = 19.13, then M = ………

Solution :

d. 20.67

We know that,

Z = 3M − 2$\overline{x}$

19.13 = 3M − 2(21.44)

3M = 19.13 + 2(21.44)

3M = 19.13 + 42.88

3M = 62.01

∴ M = 20.67

Question 10(8) :

If M = 26, $\overline{x}$ = 36, then Z = ……..

Solution :

a. 6

We know that,

Z = 3M − 2$\overline{x}$

  = 3(26) − 2(36)

  = 78 − 72

  = 6

Question 10(9) :

The modal class of the frequency distribution given below is ……

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	15	13	17	10

Solution :

c. 30 – 40

Here, the maximum frequency is 17 and the corresponding class of this frequency is 30 – 40.

Hence, the modal class is 30 – 40.

Question 10(10) :

The cumulative frequency of class 20-30 of the frequency distribution given in (9) is ………

Solution :

b. 35

Cumulative frequency of class 20 – 30 = frequency of class 20 – 30 and the frequencies of all the classes preceding class 20 – 30 = 7 + 15 + 13 = 35

Question 10(11) :

The median class of the frequency distribution given in (9) is ………

Solution :

c. 20 – 30

Here, n = 7 + 15 + 13 + 17 + 10 = 62

$\therefore \dfrac{n}{2} = \dfrac{62}{2} = 31$

Now, the cumulative frequency of the given classes are 7, 22, 35, 52 and 62 respectively.

Among these, cumulative frequency 35 belongs to the class 20-30.

Hence, the median class is 20 – 30.