JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEARS

1. The sum of all real values of x satisfying the equation [JEE Main-2016]

$(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is:

(a) 3 (b) –4
(c) 6 (d) 5

2. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + \dots + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to: [JEE Main-2017]

(a) 11 (b) 12
(c) 9 (d) 10

3. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to- [JEE Main-2018]

(a) 0 (b) 1
(c) 2 (d) –1

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:- [JEE Main-2019 (January)]

(a) –256 (b) 512
(c) –512 (d) 256

5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval [JEE Main-2019 (January)]

(a) (4, 5) (b) (3, 4)
(c) (5, 6) (d) (–5, –4)

6. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is: [JEE Main-2019 (January)]

(a) 2 (b) 5
(c) 3 (d) 4

7. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is: [JEE Main-2019 (January)]

(a) 18 (b) 12
(c) 10 (d) 11

8. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is: [JEE Main-2019 (January)]

(a) –81 (b) 100
(c) 144 (d) –300

9. The number of integral values of m for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is:- [JEE Main-2019 (January)]

(a) 3 (b) 8
(c) 7 (d) 6

10. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is [JEE Main-2019 (January)]

(a) $2-\sqrt{3}$ (b) $4-3\sqrt{2}$
(c) $-2+\sqrt{2}$ (d) $4-2\sqrt{3}$

11. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is: [JEE Main-2019 (April)]

(a) 2 (b) 3
(c) 4 (d) 5

12. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? [JEE Main-2019 (April)]

(a) d, e, f are in A.P. (b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
(c) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (d) d, e, f are in G.P.

13. The number of integral values of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is: [JEE Main-2019 (April)]

(a) infinitely many (b) 2
(c) 3 (d) 1

14. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then: [JEE Main-2019 (April)]

(a) $q^2 + 4p + 14 = 0$ (b) $p^2 - 4q - 12 = 0$
(c) $q^2 - 4p - 16 = 0$ (d) $p^2 - 4q + 12 = 0$

15. If m is chosen in the quadratic equation $(m^2+1)x^2 - 3x + (m^2+1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:- [JEE Main-2019 (April)]

(a) $8\sqrt{3}$ (b) $4\sqrt{3}$
(c) $10\sqrt{5}$ (d) $8\sqrt{5}$

16. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\dfrac{\alpha^{12} + \beta^{12}}{\left(\alpha^{-12} + \beta^{-12}\right)(\alpha - \beta)^{24}}$ is equal to:

[*JEE Main-2019* (*April*)]

(*a*) $\dfrac{2^6}{(\sin\theta + 8)^{12}}$ (*b*) $\dfrac{2^{12}}{(\sin\theta - 8)^{6}}$

(*c*) $\dfrac{2^{12}}{(\sin\theta - 4)^{12}}$ (*d*) $\dfrac{2^{12}}{(\sin\theta + 8)^{12}}$

17. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is : **[*JEE Main-2019* (*April*)]**

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 1

18. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \alpha^r + \lim_{n\to\infty} \sum_{r=1}^{n} \beta^r$ is equal to:

[*JEE Main-2019* (*April*)]

(*a*) $\dfrac{21}{346}$ (*b*) $\dfrac{29}{358}$

(*c*) $\dfrac{1}{12}$ (*d*) $\dfrac{7}{116}$

19. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to:

[*JEE Main-2019* (*April*)]

(*a*) $\beta\gamma$ (*b*) 0

(*c*) $\alpha\gamma$ (*d*) $\alpha\beta$

20. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \ge 1$, then which of the following statements is not true? **[*JEE Main-2020* (*January*)]**

(*a*) $p_5 = p_2 \cdot p_3$ (*b*) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(*c*) $p_3 = p_5 - p_4$ (*d*) $p_5 = 11$

21. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \dfrac{33}{2} = 2a$ has real roots is _______.

[*JEE Main-2020* (*January*)]

22. Let S be the set of all real roots of the equation,

[*JEE Main-2020* (*January*)]

$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S:

(*a*) contains exactly two elements

(*b*) is a singleton

(*c*) contains at least four elements

(*d*) is an empty set

23. The number of real roots of the equation,

[*JEE Main-2020* (*January*)]

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

(*a*) 3 (*b*) 1

(*c*) 4 (*d*) 2

24. Let a, b $\in$ R, a $\ne$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, $\alpha^2 + \beta^2$ is equal to:

[*JEE Main-2020* (*January*)]

(*a*) 25 (*b*) 26

(*c*) 24 (*d*) 28

25. Let f(x) be a quadratic polynomial such that f(–1) + f(*b*) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in: **[*JEE Main-2020* (*September*)]**

(*a*) (–1, 0) (*b*) (–3, –1)

(*c*) (0, 1) (*d*) (1, 3)

26. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, , n = 1, 2, 3, ..., then:

[*JEE Main-2020* (*September*)]

(*a*) $5S_6 + 6S_5 = 2S_4$

(*b*) $6S_6 + 5S_5 + 2S_4 = 0$

(*c*) $6S_6 + 5S_5 = 2S_4$

(*d*) $5S_6 + 6S_5 + 2S_4 = 0$

27. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0,1) is:

[*JEE Main-2020* (*September*)]

(*a*) (–3, –1) (*b*) (2, 4]

(*c*) (0, 2) (*d*) (1, 3]

28. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \dfrac{1}{\alpha}\right)\left(\beta - \dfrac{1}{\beta}\right)\left(\alpha + \dfrac{1}{\beta}\right)\left(\beta + \dfrac{1}{\alpha}\right)$ is equal to:

[*JEE Main-2020* (*September*)]

(*a*) $\dfrac{9}{4}(9 - q^2)$ (*b*) $\dfrac{9}{4}(9 + p^2)$

(*c*) $\dfrac{9}{4}(9 + q^2)$ (*d*) $\dfrac{9}{4}(9 - p^2)$

29. Let $\lambda \ne 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\dfrac{\beta\gamma}{\lambda}$ is equal to:

[*JEE Main-2020* (*September*)]

(*a*) 18 (*b*) 9

(*c*) 27 (*d*) 36

30. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is:

[*JEE Main-2020* (*September*)]

(*a*) 3 : 1 (*b*) 5 : 3

(*c*) 9 : 7 (*d*) 33 : 31

31. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to: **[*JEE Main-2020* (*September*)]**

(*a*) $\frac{1}{24}$ (*b*) $\frac{27}{32}$

(*c*) $\frac{3}{8}$ (*d*) $\frac{27}{16}$

32. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is: **[*JEE Main-2020* (*September*)]**

(*a*) $\frac{25}{9}$ (*b*) $\frac{25}{81}$

(*c*) $\frac{5}{9}$ (*d*) $\frac{5}{27}$

33. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to: **[*JEE Main-2020* (*September*)]**

(*a*) $2\alpha^2$ (*b*) $-2\alpha(\alpha + 1)$

(*c*) $2\alpha(\alpha - 1)$ (*d*) $2\alpha(\alpha + 1)$

34. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is: **[*JEE Main-2020* (*September*)]**

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

35. Let p and q be two positive number such that $p + q = 2$ and $p^4 + p^4 = 272$. Then p and q are roots of the equation: **[*JEE Main-2021* (*February*)]**

(*a*) $x^2 - 2x + 2 = 0$ (*b*) $x^2 - 2x + 8 = 0$

(*c*) $x^2 - 2x + 136 = 0$ (*d*) $x^2 - 2x + 16 = 0$

36. Let *a, b, c* be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is: **[*JEE Main-2021* (*February*)]**

(*a*) $\frac{71}{256}$ (*b*) $-\frac{69}{256}$

(*c*) $\frac{69}{256}$ (*d*) $-\frac{71}{256}$

37. The number of the real roots of the equation $(x + 1)^2 + |x - 5| = \frac{27}{4}$ is **[*JEE Main-2021* (*February*)]**

38. The coefficients *a, b* and *c* of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is **[*JEE Main-2021* (*February*)]**

(*a*) $\frac{1}{54}$ (*b*) $\frac{1}{72}$

(*c*) $\frac{1}{36}$ (*d*) $\frac{5}{216}$

39. The integer 'k'. for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R is: **[*JEE Main-2021* (*February*)]**

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 0

40. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + az + \beta = 0$, then $(\alpha - \beta)$ is equal to: **[*JEE Main-2021* (*February*)]**

(*a*) 7 (*b*) –3

(*c*) 3 (*d*) –7

41. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = a^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is: **[*JEE Main-2021* (*February*)]**

(*a*) 4 (*b*) 1

(*c*) 2 (*d*) 3

42. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is **[*JEE Main-2021* (*February*)]**

43. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ and $P_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of P_n^2 is: **[*JEE Main-2021* (*February*)]**

44. The number of elements in the set $\{x \in R: (|x| - 3)|x + 4| = 6\}$ is equal to **[*JEE Main-2021* (*March*)]**

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

45. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x - 2)$. Then the value of $9(b + c)$ is equal to: **[*JEE Main-2021* (*March*)]**

(*a*) 9 (*b*) 15

(*c*) 7 (*d*) 11

46. The value of $4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\ldots\infty}}}}$ **[*JEE Main-2021* (*March*)]**

(*a*) $2 + \frac{2}{5}\sqrt{30}$ (*b*) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$

(*c*) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$ (*d*) $5 + \frac{2}{5}\sqrt{30}$

47. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to— **[*JEE Main-2021* (*March*)]**

JEE-ADVANCED PREVIOUS YEARS

1. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[*JEE Advanced-2014*]**
   (*a*) only purely imaginary roots
   (*b*) all real roots
   (*c*) two real and two purely imaginary roots
   (*d*) neither real nor purely imaginary roots
2. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S? **[*JEE Advanced-2015*]**
   (*a*) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$
   (*b*) $\left(-\frac{1}{\sqrt{5}}, 0\right)$
   (*c*) $\left(0, \frac{1}{\sqrt{5}}\right)$
   (*d*) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Comprehension–1 (3 and 4)

Let p, q_n be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For n = 0, 1, 2,........let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$. **[*JEE Advanced-2017*]**

3. $a_{12} =$
   (*a*) $a_{11} + 2a_{10}$
   (*b*) $2a_{11} + a_{10}$
   (*c*) $a_{11} - a_{10}$
   (*d*) $a_{11} + a_{10}$
4. If $a_4 = 28$, then $p + 2q =$
   (*a*) 14 (*b*) 7
   (*c*) 21 (*d*) 12
5. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define
   $$a^n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1$$ **[*JEE Advanced-2019*]**
   $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$.
   Then which of the following options is/are correct?
   (*a*) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$
   (*b*) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$
   (*c*) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$
   (*d*) $b_n = \gamma^n + \beta^n$ for all $n \geq 1$
6. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define
   $a^n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$.
   Then which of the following options is/are correct? **[*JEE Advanced-2019*]**
   (*a*) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \geq 1$
   (*b*) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$
   (*c*) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$
   (*d*) $b_n = \gamma^n + \beta^n$ for all $n \geq 1$
7. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c,d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of
   $ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$ is **[*JEE(Advanced)-2020*]**
   (*a*) 0 (*b*) 8000
   (*c*) 8080 (*d*) 16000

JEE Mains & Advanced Past Years Questions

JEE-MAIN
PREVIOUS YEARS

1. (*a*) $(x^2-5x+5)^{x^2+4x-60}=1$

$x^2-5x+5=1 \Rightarrow x=1,4$

or, $x^2+4x-60=0$ $x=-10, 6$

or , $x^2-5x+5=-1 \Rightarrow x=2,3$

But for $x=3$, $x^2+4x-60$ is odd

So Required values of x are $1,4,-10,6,2$ Sum $=3$

2. (*a*) We have $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$\Rightarrow \sum_{r=1}^{n}\left(x^2+(2r-1)x+(r^2-r)\right)=10n$

$\therefore$ On solving, we get

$x^2+nx+\left(\frac{n^2-31}{3}\right)=0$ $\therefore (2\alpha+1)=-n$

/ \

α $\alpha+1$

$\Rightarrow \alpha=\frac{-(n+1)}{2}$ .....(1)

and $\alpha(\alpha+1)=\frac{n^2-31}{3}$ .....(2)

$\Rightarrow$ $n^2=121$ (using (1) in (2))

or $n=11$

3. (*b*) α, β are roots of $x^2-x+1=0$

$\therefore \alpha=-\omega$ and $\beta=-\omega^2$

where ω is non-real cube root of unity

so, $a^{101}+\alpha^{107}$

$\Rightarrow (-\omega)^{101}+(-\omega^2)^{107}$

$\Rightarrow -[\omega^2+\omega]$

$\Rightarrow -[-1]=1$

(As $1+\omega+\omega^2=0$ & $\omega^3=1$)

4. (*a*) $x^2+2x+2=0 \Rightarrow (x+1)^2=-1$

$x=-1\pm i=\sqrt{2}e^{i\left(\pm\frac{3\pi}{4}\right)}$

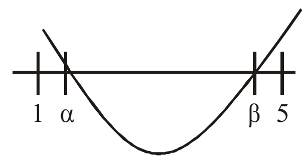
$\therefore \alpha^{15}, \beta^{15}=\left(\sqrt{2}\right)^{15}\times 2\cos\left(15.\frac{3\pi}{4}\right)$

$=2^8\sqrt{2}\times\left(-\frac{1}{\sqrt{2}}\right)=-256$

5. (Bonus)

$x^2-mx+4=0$

$\alpha, \beta \in [1, 5]$



(*a*) $D>0 \Rightarrow m^2-16>0$

$\Rightarrow m \in (-\infty,-4)\cup(4,\infty)$

(*b*) $f(1)\geq 0 \Rightarrow 5-m\geq 0 \Rightarrow m\in(-\infty, 5)$

(*c*) $f(5)\geq 0 \Rightarrow 29-5m\geq 0 \Rightarrow m\in\left(-\infty,\frac{29}{5}\right]$

(*d*) $1<\frac{-b}{2a}<5 \Rightarrow 1<\frac{m}{2}<5 \Rightarrow m\in(2, 10)$

$\Rightarrow m\in(4, 5)$

No option correct : Bonus

* If we consider $\alpha, \beta \in (1, 5)$ then option (1) is correct.

6. (*c*) D must be perfect square

$\Rightarrow 121-24\alpha=\lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha=1 \Rightarrow \lambda \notin 1$

$\alpha=2 \Rightarrow \lambda \notin 1$

$\alpha=3 \Rightarrow \lambda \in 1$

$\Rightarrow$ 3 integral values

$\alpha=4 \Rightarrow \lambda \in 1$

$\alpha=5 \Rightarrow \lambda \in 1$

7. (*d*) Case - 1

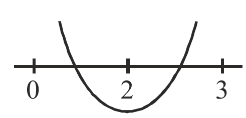
$c-5>0$ ....(i)

$f(0)>0$

$c-4>0$ ...(ii)

$f(2)<0$

$4(c-5)-4c+c-4<0$



$c<24$ ...(iii)

$f(2)>0$

$9(c-5)-6c+c-4>0$

$4c-49>0 \Rightarrow c>\frac{49}{4}$ ...(iv)

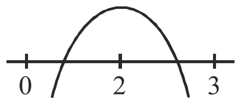
Here (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$c \in \left(\frac{49}{4}, 24\right)$

Case - II

$c - 5 < 0$ ...(i)

$f(0) < 0$



$c < 4$ ...(ii)

$f(2) > 0 \Rightarrow c > 24$ ...(iii)

$f(3) < 0 \Rightarrow c > 49$ ...(iv)

$4 \Rightarrow c \in \phi$

$c \in \left(\frac{49}{4}, 24\right)$.

8. (*d*) $\alpha + \alpha^3 = -\frac{K}{81}$ ....(1)

$\alpha^4 = \frac{256}{81}$

$\alpha = \pm\frac{4}{3}$ ....(2)

From (1) and (2)

$\frac{4}{3} + \frac{64}{27} = \frac{-K}{81}$

$K = -300$

9. (*c*) Expression is always positive if $2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$

and $D < 0 \Rightarrow m^2 - 6m - 3 < 0$

$3\sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ common interval is $3 - \sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ Integral value of $m\{0,1,2,3,4,5,6\}$.

10. (*b*) Let roots are α and β now

$\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 2\alpha\beta$

$\left(\frac{-m(m-4)}{3m^2}\right)^2 = 3 \cdot \frac{2}{3m^2}$

$m^2 - 8m - 2 = 0$

$m = 4 \pm 3\sqrt{2}$

So least value of $m = 4 - 3\sqrt{2}$

11. (*c*) $(x-1)^2 + 1 = 0 \Rightarrow x = 1 + i, 1 - i$

$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm 1)^n = 1$

$\therefore$ n (least natural number) = 4

12. (*c*) a, b, c, in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation

$d(-r)^2 + 2e(-r) + f = 0$

$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0 \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

13. (*a*) $D < 0$

$4(1 + 3m)^2 - 4(Hm^2)(1 + 8m) < 0$

$\Rightarrow m(2m - 1)^2 > 0 \Rightarrow m > 0$

14. (*b*) In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$x^2 - (\alpha + 2 - \sqrt{3})\,x + \alpha.\,(2 - \sqrt{3}) = 0$

Now, we can have none or any of the options can be correct depending upon 'α' Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2 + \sqrt{3}$

$\Rightarrow p = -(2 + \sqrt{3} - 2 - \sqrt{3}) = -4$

and $q = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

Option (*b*) is correct

15. (*d*) $SOR = \frac{3}{m^2 + 1} \Rightarrow (S.O.R)_{max} = 3$

when $m = 0$

$x^2 - 3x + 1 = 0$ $\nearrow \alpha$ $\searrow \beta$

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^3 - \beta^2| = ||\alpha - \beta|(\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha - \beta)^2 - \alpha\beta}\,((\alpha + \beta)^2 - \alpha\beta\right|$

$= \left|\sqrt{9 - 4}\,(9 - 1)\right|$

$= \sqrt{5} \times 8$

16. (*d*) $\frac{\alpha^{12} + \beta^{12}}{\left(\frac{1}{\alpha^{12}} + \frac{1}{\beta^{12}}\right)(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$

$= \frac{(\alpha\beta)^{12}}{\left[(\alpha + \beta)^2 - 4\alpha\beta\right]^{12}} = \left[\frac{\alpha\beta}{(\alpha + \beta)^2 - 4\alpha\beta}\right]^{12}$

$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12} = \frac{2^{12}}{(\sin\theta + 8)^{12}}$

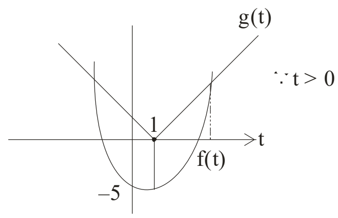
17. (*d*) Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t-1| = (t^2 - 2t - 5)$

g(t) f(t)

From the graph



So, number of real root is 1.

18. (*c*) $375x^2 - 25x - 2 = 0$

$\alpha + \beta = \dfrac{25}{375}, \alpha\beta = \dfrac{-2}{375}$

$\Rightarrow (\alpha + \alpha^2 + ...$ upto infinite terms) + $(\beta + \beta^2 + ...$ upto

infinite terms) $= \dfrac{\alpha}{1-\alpha} + \dfrac{\beta}{1-\beta} = \dfrac{1}{12}$

19. (*a*) $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t, \gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha t x + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0$ ....(1)

Now

$\alpha(\beta + \gamma) = \alpha^2 (t + t^2)$

Option 1 $\beta\gamma = \alpha t \,.\, \alpha t^2 = \alpha^2 t^3 = a^2 (t^2 + t)$

from equation 1

20. (*a*) $\alpha^5 = 5\alpha + 3$

$\beta^5 = 5\beta + 3$

$p_5 = 5(\alpha + \beta) + 6$

$= 5(1) + 6$

$P_5 = 11$ and $p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$

$P_2 = 3$ and $p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1 = 2(1) + 2 = 4$

$P_2 \times P_3 = 12$ and $P_5 = 11 \Rightarrow P_5 \neq P_2 \times P_3$

21. 8 $D \geq 0$

$(a-10)^2 - 4(2)\left(\dfrac{33}{2} - 2a\right) \geq 0$

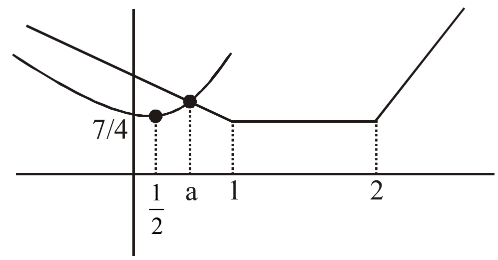
$(a-10)^2 - 4(33 - 4a) \geq 0$

$a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, -4] \cup [8, \infty)$.

22. (*b*) Let $3^x = t$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$



are positive solution

$t = a$

$3^x = a$

$x = \log_3$ is singleton set.

23. (*b*) Let $e^x = t \in (0, \infty)$

Given equation

$t^4 + t^3 - 4t^2 + t + 1 = 0$

$t^2 + t - 4 + \dfrac{1}{t} + \dfrac{1}{t^2} = 0$

$\left(t^2 + \dfrac{1}{t^2}\right) + \left(t + \dfrac{1}{t}\right) - 4 = 0$

Let $t + \dfrac{1}{t} = \alpha$

$(\alpha^2 - 2) + \alpha - 4 = 0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha = -3, 2 \quad \Rightarrow \quad \alpha = 2 \quad \Rightarrow \quad e^x + e^{-x} = 2$

$x = 0$ only solution

24. (*a*) $2\alpha = \dfrac{2b}{a} \Rightarrow \alpha = \dfrac{b}{a}$ and $\alpha^2 = \dfrac{5}{a} \quad \Rightarrow \quad \dfrac{b^2}{a^2} = \dfrac{5}{a}$

$\Rightarrow b^2 = 5a$ .....(i) $(a \neq 0)$

$\alpha + \beta = 2b$ ....(ii)

$\alpha\beta = -10$ ....(iii)

$\alpha = \dfrac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow 20a^2 = 5a$

$\Rightarrow a = \dfrac{1}{4}$ and $b^2 = \dfrac{5}{4}$

$\alpha^2 = 20$ and $\beta^2 = 5$

Now $\alpha^2 + \beta^2$

$= 5 + 20$

$= 25$

25. (*a*) Let $f(x) = ax^2 + bx + c$

Let roots are 3 and α

and $f(-1) + f(2) = 0$

$4a + 2b + c + a - b + c = 0$

$5a + b + 2c = 0$ ...(i)

$\because f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ...(ii)

From equation (i) and (ii)

$$\frac{a}{1-6} = \frac{b}{18-5} = \frac{c}{15-9} \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$$

$\therefore f(x) = k(-5x^2 + 13x + 6)$

$= -k(5x + 2)(x - 3)$

= Root are 3 and $-\frac{2}{5}$

$\therefore -\frac{2}{5}$ lies in interval $(-1, 0)$

26. (*a*) $\because$ α is a root of given equation, then

$5\alpha^2 + 6\alpha = 2$

$\Rightarrow 5\alpha^6 + 6\alpha^5 = 2\alpha^4$ ...(1)

Similarly $5\beta^6 + 6\beta^5 = 2\beta^4$ ...(2)

Adding (1) and (2), we get

$5S_6 + 6S_5 = 2S_4$

27. (*d*) $\because$ Equation is : $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$

$\because$ One root in interval (0,1)

$\therefore f(0).\ f(1) < 0$

$2.\ (\lambda^2 + 1 - 4\lambda + 2) < 0$

$(\lambda - 3)(\lambda - 1) < 0$

$\therefore \lambda \in (1, 3)$

If $\lambda = 3$, then roots are 1 and $\frac{1}{5}$

$\therefore \lambda \in (1, 3]$

28. (*d*) $\alpha.\beta = 2$ and $\alpha + \beta = -p$ also $\frac{1}{\alpha} + \frac{1}{\beta} = -q$

$\Rightarrow p = 2q$

Now $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$$= \left[\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right]\left[\alpha\beta + \frac{1}{\alpha\beta} + 1 + 1\right]$$

$$= \frac{9}{2}\left[\frac{5}{2} - \frac{\alpha^2 + \beta^2}{2}\right] = \frac{9}{4}\left[5 - (p^2 - 4)\right]$$

$$= \frac{9}{4}(9 - p^2)$$

29. (*a*) Roots of $x^2 - x + 2\lambda = 0$ are α and β
and roots of $3x^2 - 10x + 27\lambda = 0$ and α and γ
Here,

$3\alpha^2 - 10\alpha + 27\lambda = 0$ ....(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ ....(ii)

$\therefore \alpha = 3\lambda$

Now,

$3\lambda + \beta = 1$ and $3\lambda \cdot \beta = 2\lambda$

and, $3\lambda + \gamma = \frac{10}{3}$ and $3\lambda.\gamma = 9\lambda$

$\therefore \gamma = 3, \alpha = \frac{1}{3}$ and $\beta = \frac{2}{3}, \lambda = \frac{1}{9}$

$\frac{\beta\gamma}{\lambda} = 18$

30. (*c*) $\because$ $\alpha, \beta, \gamma, \delta$ are in G.P, so $\alpha\delta = \beta\gamma$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha - \beta}{\alpha + \beta}\right| = \left|\frac{\gamma - \delta}{\gamma + \delta}\right|$$

$$\Rightarrow \sqrt{\frac{9 - 4p}{3}} = \sqrt{\frac{36 - 4p}{6}}$$

$\Rightarrow 36 - 16p = 36 - 4q$

$\Rightarrow q = 4p$

So, $\frac{2q + p}{2q - p} = \frac{9p}{7p} = \frac{9}{7}$

31. (*d*) $7x^2 - 3x - 2 = 0 \Rightarrow \alpha + \beta = \frac{3}{7}, \alpha\beta = \frac{-2}{7}$

Now $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$

$$= \frac{\alpha - \alpha\beta(\alpha + \beta) + \beta}{1 - (\alpha^2 + \beta^2) + (\alpha\beta)^2}$$

$$= \frac{(\alpha + \beta) - \alpha\beta(\alpha + \beta)}{1 - (\alpha^2 + \beta^2) + 2\alpha\beta + (\alpha\beta)^2}$$

$$= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2 \times \frac{-2}{7} + \frac{4}{49}} = \frac{21 + 6}{49 - 9 - 28 + 4} = \frac{27}{16}$$

32. (*b*) Let $|x| = t$ we have

$9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$(3t - 1)(3t - 5) = 0$

$\Rightarrow t = \frac{1}{3}$ or $\frac{5}{3} \Rightarrow |x| = \frac{1}{3}$ or $\frac{5}{3}$

Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$

Product $= \frac{25}{81}$

33. (*b*) $\alpha+\beta=-\dfrac{1}{2}\Rightarrow -1=2\alpha+2\beta$

and $4\alpha^2+2\alpha-1=0$

$\Rightarrow 4\alpha^2+2\alpha+2\alpha+2\beta=0$

$\Rightarrow \beta=-2\alpha(\alpha+1)$

34. (*b*) $\dfrac{\alpha^{3/8}}{\beta^{5/8}}+\dfrac{\beta^{3/8}}{\alpha^{5/8}}=\dfrac{\alpha+\beta}{(\alpha\beta)^{5/8}}$

For $x^2-64x+256=0$

$\alpha+\beta=64$

$\alpha\beta=256$

$\therefore \dfrac{\alpha+\beta}{(\alpha\beta)^{5/8}}=\dfrac{64}{\left(2^8\right)^{5/8}}=\dfrac{64}{32}=2$

35. (*d*) Given, $p+q=2, p^4+q^4=272$

$\Rightarrow (p^2+q^2)-2p^2q^2=272$

$\Rightarrow ((p+q)^2-2pq)^2=2p^2q^2=272$

$\Rightarrow (4+2pq)^2-2(pq)^2=272$

$16+4(pq)^2-16pq-2(pq)^2=272$

$2(pq)^2-16(pq)-256=0$

$(pq)^2-8(pq)-128=0$

$pq=\dfrac{8\pm\sqrt{64+4.128}}{2}=\dfrac{8\pm 24}{2}=16$

As, p,q are $+ve_1$ So, $pq\neq -8$

$pq=16$

Quadrant equation is

$x^2-(p+q)x+pq=0$

$x^2-2x+16=0$

36. (*d*) $a,b,c\to \text{AP}\Rightarrow 2b=a+c$ ...(i)

Centroid is $\left(\dfrac{10}{3},\dfrac{7}{3}\right)$

$\dfrac{a+2+a}{3}=\dfrac{10}{3}$ & $\dfrac{c+b+b}{3}=\dfrac{7}{3}$

$2a+2=10$ & $c+2b=7$

$2a=8$

$a=4;$ ...(ii)

from (i) & (ii)

$2b=c+4$

$c+2b=7$

$c+c+4=7$

$2c=3$

$c=\dfrac{3}{2}$

$2b=c+4=\dfrac{11}{2}$

$b=\dfrac{11}{4}$

$ax^2+bx+4=0\Rightarrow 4x^2+\dfrac{11x}{4}+1=0$

$\alpha+\beta=\dfrac{-11}{4.4}=\dfrac{-11}{16};\ \alpha\beta=\dfrac{1}{4}x$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\alpha^2+\beta^2-\alpha\beta=(\alpha+\beta)^2-3\alpha\beta=\dfrac{121}{256}-\dfrac{3}{4}=\dfrac{-71}{256}$

37. (2) $(x+1)^2+|x-5|=\dfrac{27}{4}$

Case 1: if $x\geq 5$

$(x+1)^2+x-5=\dfrac{27}{4}$

$x^2+3x-4=\dfrac{27}{4}\Rightarrow x^2\,3x-\dfrac{43}{4}=0$

$4x^2+12x-43=0$

$x=\dfrac{-12\pm\sqrt{144-4.4.(-43)}}{2.4}=\dfrac{-12\pm\sqrt{832}}{8}$

$=\dfrac{-12\pm 28.8}{8}$

$=\dfrac{-3\pm 7.2}{2}=\dfrac{-10.2}{2},\dfrac{+4.2}{2}$

But $x\geq 5$, So, no real root.

Case 2: if $x<5$

$(x+1)^2+(-(x-5))=\dfrac{27}{4}\Rightarrow x^2+1+2x-x+5=\dfrac{27}{4}$

$x^2+x+6-\dfrac{27}{4}=0\Rightarrow 4x^2+4x-3=0$

$x=\dfrac{-4\pm\sqrt{16+4.4.3}}{2.4}=\dfrac{-4\pm 8}{8}=-\dfrac{12}{8},\dfrac{4}{8}$

Two real roots.

38. (*d*) $ax^2+bx+c=0$

$a,b,c\in\{1,2,3,4,5,6,\}$

$n(s)=6\times 6\times 6=216$

for equal roots, $D=0\Rightarrow b^2-4ac=0$

$\Rightarrow ac=\dfrac{b^2}{4}\Rightarrow b^2$ must be divisible by 4.

(*a*) $b^2=4\Rightarrow b=2$ & $ac=\dfrac{4}{4}=1\Rightarrow a=1, c=1$

(*b*) $b^2=16\Rightarrow b=4$ & $ac=\dfrac{16}{4}=4\Rightarrow a=4, c=1$

$a=2, c=2$

or

$a=1, c=4$

(*d*) $b^2=36\Rightarrow b=6$ & $oc=\dfrac{36}{4}=9$

$\Rightarrow a=3, c=3$

Total favourable case = 5

Req. Probability $=\dfrac{5}{216}$

39. (*a*) for $x^2-2(3k-1)x+8k^2-7>0$

$D<0$

$(-2(3k-1))^2-4.1.(8k^2-7)<0$

$4(9k^2+1-6k)-4(8k^2-7)<0$

$k^2-6k+8<0$

$(k-2)(k-4)<0$

$k\in(2,4)$

Integer value of $k=3$

40. (*d*) $z^2+\alpha z+\beta=0$

$z=1-2i$ is root

So, $(1-2i)^2+\alpha(1-2i)+\beta=0$

$1+4i^2-4i+\alpha-2\alpha i+\beta=0$

$(\alpha+\beta-3)+i(4+2\alpha)=0$

$\alpha+\beta-3=0$ & $4+2\alpha=0$

$2\alpha=-4$

$\alpha=-2$

$\alpha+\beta-3=0$

$-2+\beta-3=0$

$\beta=5$

$\alpha-\beta=-2-5=-7$

41. (*c*) $x^2-6x-2=0$

α, β roots

$\alpha+\beta=6;\ \alpha\beta=-2$ ...(i)

$$\left.\begin{array}{l}\alpha^2-6\alpha-2=0\\ \beta^2-6\beta-2=0\end{array}\right\}\quad ...(ii)$$

$$\frac{a_{10}-2a_8}{3a_9}=\frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{3(\alpha^9-\beta^9)}$$

$$=\frac{\alpha^8\left(\alpha^2-2\right)-\beta^8(\beta^2-2)}{3(\alpha^9-\beta^9)}=\frac{\alpha^8.6\alpha-\beta^8.6\beta}{3(\alpha^9-\beta^9)}\ \text{(from-(ii))}$$

$$=\frac{6(\alpha^9-\beta^9)}{3(\alpha^9-\beta^9)}=2$$

42. (1) $\log_4(x-1)=\log_2(x-3)$

$x>1$ & $x>3\Rightarrow x>3$ (according to definition of log)

$\Rightarrow\log_2(x-1)=\log_2(x-3)$

$\frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$\log_2(x-1)=\log_2(x-3)^2$

$(x-1)=(x-3)^2$

$x^2+9-6x=x-1$

$x^2-7x+10=0$

$(x-2)(x-5)=0$

$x=2$ or 5

But $x>B$

So, $x=5$

Only one solution.

43. (324) $\alpha+\beta=1;\ \alpha\beta=-1$

Quad. Eq$^{\underline{n}}$: $x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-x-1=0$

α, β roots

$\alpha^2-\alpha-1=0\Rightarrow\alpha^2=\alpha+1$

Multiplying both side by, α^{n-1}

$\alpha^{n-1},\ \alpha^2=\alpha^{n-1}(\alpha+1)$

$\alpha^{n+1}=\alpha^n+\alpha^{n-1}$ ...(i)

Similarly, $\beta^{n+1}=\beta^n+\beta^{n-1}$ ...(ii)

Adding (i) & (ii)

$\alpha^{n+1}+\beta^{n+1}=(\alpha^n+\beta^n)+(\alpha^{n-1}+\beta^{n-1})$

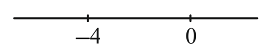
$P_{n+1}=P_n+P_{n-1}$

$29=P_n+11$

$P_n=18$

$P_n^2=18^2=324$

44. (*b*) $(|x|-3)(|x+4|)=6$



Case I:	**Case II:**	**Case III**
$x\geq 0$	$-4\leq x<0$	$x<-4$
$(x-3)(x+4)=6$	$(-(x)-3)(x+4)=6$	$(-x-3)(-x-3)(-(x+4))=6$
$x^2+x-18=0$	$(x+3)(x+4)=-6$	$(x+3)(x+4)=6$
$x=\frac{-1\pm\sqrt{1+4.18}}{2}$	$x^2+7x+18=0$	$x^2+7x+6=0$
$x=\frac{-1\pm\sqrt{73}}{2}$	$x=\frac{-7\pm\sqrt{49-72}}{2}$	$(x+1)(x+6)=0$
But $x\geq 0$	No real root	$x=-1$ or -6
$x=\frac{-1\pm\sqrt{73}}{2}$		But $x<-4$
Only 1 solution		$x=-6$
Only 1 solution		

In total, there are 2 real solution.

45. (*c*) $\int_0^1 P(x)dx=1\Rightarrow\int_0^1(x^2+bx+c)dx=1$

$$\left[\frac{x^3}{3}+\frac{bx^2}{2}+c\right]_0^1=1\Rightarrow\frac{1}{3}+\frac{b}{2}+c=1$$

$3b+6c=4$ ...(i)

Also, $P(2)=5$

$(2)^2+b(2)+c=5$

$12b+6c=6$ ...(ii)

from (i) & (ii)

$b=\frac{2}{9}$ & $c=\frac{5}{9}$

$$a(b+c)=\frac{9(2+5)}{9}=7$$

46. (*a*) Let $x = 4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\ldots\ldots\infty}}}}$

Then,

$$x = 4+\frac{1}{5+\frac{1}{x}} \Rightarrow x-4 = \frac{x}{5x+1}$$

$(x-4)(5x+1) = x$

$5x^2 - 20x - 4 = 0$

$$x = \frac{20 \pm \sqrt{400+4.5.4}}{2.5} = \frac{20 \pm \sqrt{480}}{10}$$

But x is $+ve$.

So, $x = \dfrac{20+\sqrt{480}}{10} = \dfrac{2+4\sqrt{30}}{10}$

$$2+\frac{2\sqrt{30}}{5}$$

47. (0) $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(i)

$x^2 + x + 1 = 0$

$x = \omega,\ \omega^2$ (cube root of unity)

$P(x)$ is divisible by $x^2 + x + 1$

$P(\omega) = 0;\ P(\omega^2) = 0$

$P(\omega) = 0 \Rightarrow f(\omega^3) + \omega g(\omega^3) = 0$

$f(1) + \omega g(1) = 0$ ...(ii)

$P(\omega^2) = 0 \Rightarrow f(\omega^6) + \omega^2(g(\omega^6)) = 0$

$f(1) + \omega^2(g(1)) = 0$ ...(iii)

(ii) – (iii)

$(\omega - \omega^2)\, g(1) = 0$

As $(\omega - \omega^2) \neq 0$, So, only possibility is $g(1) = 0$

Putting value of $g(1)$ in –(*ii*),

$f(1) + 0 = 0 \Rightarrow f(1) = 0$

$P(1) = f(1) + g(1) = 0 + 0 = 0$

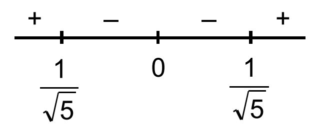
JEE-ADVANCED PREVIOUS YEARS

1. (*d*) p(x) will be of the form $ax^2 + c$. Since it has purely imaginary roots only.
Since p(x) is zero at imaginary values while $ax^2 + c$ takes real value only at real 'x', no root is real.
Also $p(p(x)) = 0 \Rightarrow p(x)$ is purely imaginary
$\Rightarrow$ $ax^2 + c$ = purely imaginary
Hence x can not be purely imaginary since x^2 will be negative in that case and $ax^2 + c$ will be real.
Thus. (*d*) is correct.

2. (*a*, *d*)

$(x_1 + x_2)^2 - 4x_1x_2 < 1$

$$\frac{1}{\alpha^2} - 4 < 1 \quad \Rightarrow \quad 5 - \frac{1}{\alpha^2} > 0 \Rightarrow \frac{5\alpha^2 - 1}{\alpha^2} > 0$$



$$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right) \quad ...(1)$$

$D > 0$

$1 - 4\alpha^2 > 0$

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right) \quad ...(2)$$

(a) & (2)

$$\alpha \in \left(-\frac{1}{2}, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$$

3. (*d*) As α and β are roots of equation $x^2 - x - 1 = 0$, we get:

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^2 = \beta + 1$

$\therefore a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$

$= p\alpha^{10}(\alpha + 1) + q\beta^{10}(\beta + 1)$

$= p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2$

$= p\alpha^{12} + q\beta^{12} = \alpha_{12}$

4. (*d*) $a_{n+2} = a_{n+1} + a_n$

$a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \dfrac{1+\sqrt{5}}{2}, \beta = \dfrac{1-\sqrt{5}}{2}$, we get

$$a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$$

$$\Rightarrow \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0 \quad ...(i)$$

$$\text{and} \Rightarrow \frac{3p}{2} - \frac{3q}{2} = 0. \quad ....(ii)$$

$\Rightarrow$ $p = q$ (from (ii))

$\Rightarrow$ $7p = 28$ (from (i) and (ii))

$\Rightarrow$ $p = 4$

$\Rightarrow$ $q = 4$

$\Rightarrow$ $p + 2q = 12$

5. (*a*,*b*,*c*)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta}$$

$$= \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} = a_{r+1}$$

$$\Rightarrow a_{r+2} - a_{r+1} = a_r$$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\dfrac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta}}{(\alpha - \beta)} = \frac{10}{(10 - \alpha)(10 - \beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10} +}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

6. (*a*, *b*, *c*)

α, β are roots of $x^2 - x - 1$

$$a_{r+2} - a_r = \frac{(\alpha^{r+2} - \beta^{r+2}) - (\alpha^r - \beta^r)}{\alpha - \beta}$$

$$= \frac{(\alpha^{r+2} - \alpha^r) - (\beta^{r+2} - \beta^r)}{\alpha - \beta}$$

$$= \frac{\alpha^r(\alpha^2 - 1) - \beta^r(\beta^2 - 1)}{\alpha - \beta} = \frac{\alpha^r\alpha - \beta^r\beta}{\alpha - \beta} = \frac{\alpha^{r+1} - \beta^{r+1}}{\alpha - \beta} =$$

$$a_{r+1}$$

$$\Rightarrow a_{r+2} - a_{r+1} = a_r$$

$$\Rightarrow \sum_{r=1}^{n} a_r = a_{n+2} - a_2 = a_{n+2} - \frac{\alpha^2 - \beta^2}{\alpha - \beta}$$

$$= a_{n+2} - (\alpha + \beta) = a_{n+2} - 1$$

$$\text{Now } \sum_{r=1}^{\infty} \frac{a_n}{10^n} = \frac{\sum_{r=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{r=1}^{\infty}\left(\frac{\beta}{10}\right)^n}{\alpha - \beta}$$

$$\frac{\dfrac{\frac{\alpha}{10}}{1 - \frac{\alpha}{10}} - \dfrac{\frac{\beta}{10}}{1 - \frac{\beta}{10}}}{\alpha - \beta} = \frac{\frac{\alpha}{10 - \alpha} - \frac{\beta}{10 - \beta}}{(\alpha - \beta)} = \frac{10}{(10 - \alpha)(10 - \beta)} = \frac{10}{89}$$

$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_{n-1} + a_{n+1}}{10^n} = \frac{\frac{\alpha}{10} +}{1 - \frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1 - \frac{\beta}{10}} = \frac{12}{89}$$

Further, $b_n = a_{n-1} + a_{n+1}$

$$= \frac{(\alpha^{n-1} - \beta^{n-1}) + (\alpha^{n+1} - \beta^{n+1})}{\alpha - \beta}$$

(as $\alpha\beta = -1 \Rightarrow \alpha^{n-1} = -\alpha^n\beta$ & $\beta^{n-1} = -\alpha\beta^n$)

$$= \frac{\alpha^n(\alpha - \beta) + (\alpha - \beta)\beta^n}{\alpha - \beta} = \alpha^n + \beta^n$$

7. (*d*) $x^2 + 20x - 2020 = 0$ has two roots $a, b \in R$

$x^2 - 20x + 2020 = 0$ has two roots $c, d \in$ complex

$ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c + d) + b^2(c + d) - c^2(a + b) - d^2(a + b)$

$= (c + d)(a^2 + b^2) - (a + b)(c^2 + d^2)$

$= (c + d)((a + b)^2 - 2ab) - (a + b)((c + d)^2 - 2cd)$

$= 20[(20)^2 + 4040] + 20[(20)^2 - 4040]$

$= 20[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$