Work, Energy and Power

1. Introduction

In our daily life 'work' implies an activity resulting in muscular or mental exertion. However, in physics the term 'work' is used in a specific sense which involves the displacement of a particle or body under the action of a force.

2. Work: An Overview

In general, we can say "Any activity involving mental or physical efforts done, in order to achieve a result", is called as work

Whenever a force acting on a body, displaces it, work is said to be done by the force.

Work done by a force is equal to the scalar product of the force applied and the displacement of the point of application, $W = \vec{F}\cdot\vec{s}$.

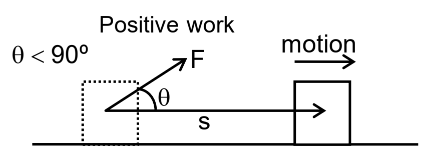
Work is a scalar quantity.

This can be termed as physical work/mechanical work.

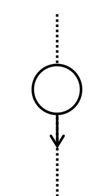
To feed the energy to a system (ball or spring) you need to deliver a speed to the ball or deform the spring. While doing so, we need to apply a force on them. As a result, the point at which you push (point of application of force F) undergoes a displacement x. This process (or act) is called work done, denoted as "W".

2.1 Nature of Work

(i) Positive Work: $W = Fs\cos\theta$



If the angle θ is acute ($\theta < 90°$) then the work is said to be positive. Positive work signifies that the external force favours the motion of the body.



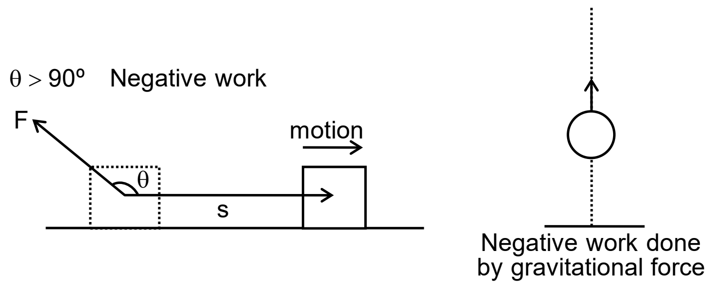
Positive work done by gravitational force

- When a body falls freely under the action of gravity ($\theta = 0°$), the work done by gravity is positive.
- When a spring is stretched, stretching force and the displacement both are in the same direction. So work done by stretching force is positive.

(ii) Negative Work: W = Fs cosθ

If the angle θ is obtuse ($\theta > 90°$).

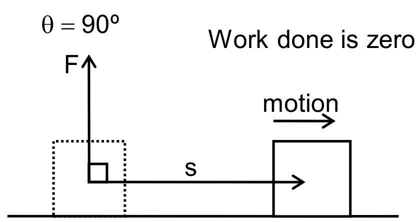
Then the work is said to be negative.



It signifies that the direction of force is such that It opposes the motion of the body.

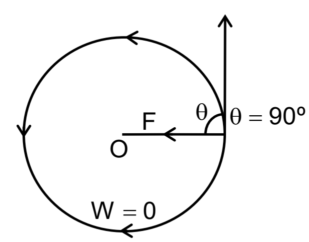
- Work done by frictional force is negative when it opposes the motion.
- Work done by braking force on the car is negative.

(iii) Zero Work: W = Fs cosθ



Work done will be zero if

F = 0 or s = 0 or $\theta = 90°$



- Body moving with uniform velocity, W.D. by net force = 0
- If net force on the particle is zero then W.D. on the body is also zero.
- When we push the wall and it remains at rest. W = 0.
- When a pendulum is oscillating, the W.D. by tension = 0.
- When electron is moving round the nucleus W.D. by attractive force = 0.
- When satellite is moving around the earth, work done by gravitational force is zero.
- Work done by coolie against gravity is zero.

2.2 Unit and Dimensions

- **Unit**

SI Unit : joule (J).

Joule : One joule of work is said to be done when a force of one newton displaces a body by one meter in the direction of force.

$$\mathbf{1\ joule = 1\ newton \times 1\ meter = 1\ kg.m^2s^{-2}}$$

erg : One erg of work is said to be done when a force of one dyne displaces a body by one centimeter in the direction of force.

$$\mathbf{1\ erg = 1\ dyne \times 1\ cm = 1\ g.cm^2s^{-2}}$$

Other Units

(a) 1 joule = 10^7 ergs

(b) 1 erg = 10^{-7} joules

(c) 1 eV = 1.6×10^{-19} joules

(d) 1 joule = 6.25×10^{18} eV

(e) 1 MeV = 1.6×10^{-13} J

(f) 1 J = 6.25×10^{12} MeV

(g) 1 kilowatt-hour (kWh) = 3.6×10^6 joules

- **Dimensions :** [Work] = [Force] [Displacement]

$$= [MLT^{-2}][L] = [ML^2T^{-2}].$$

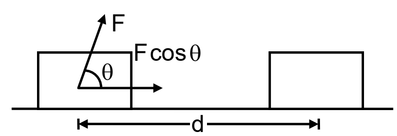
3. Work Done by a Constant Force

If the direction and magnitude of a force applied on a body is constant then the force is said to be constant. Work done by a constant force,

W = Force × component of displacement along the force

= displacement × component of force along the displacement

i.e., work done will be, $W = (F\cos\theta)d = F(d\cos\theta)$



In vector form, $W = \vec{F}\cdot\vec{d}$

Note:

The force of gravity within small altitudes is an example of constant force; consequently, work done by it is an example of work done by a constant force.

In Cartesian Form

- If a constant force 'F' denoted by

  $\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\hat{k}$

  Displaces a particle from point (x_1, y_1, z_1) to (x_2, y_2, z_2) then displacement

  $\hat{r} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$

  The work done by this force will be

  $W = \vec{F}\cdot\vec{r}$

  $W = F_x(x_2 - x_1) + F_y(y_2 - y_1) + F_z(z_2 - z_1)$

Example 1:

A particle undergoes a displacement from a point P(1, 1, 1) m to Q(−2, −1, 2)m by the action of a force $(2\hat{i} + 3\hat{j} - \hat{k})$ N. Find the work done by the force in the corresponding displacement.

Solution:

The work done is $W = \vec{F}\cdot\vec{s} = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1)$

$[2\hat{i} + 3\hat{j} - \hat{k}]\cdot[(-2\hat{i} - \hat{j} + 2\hat{k}) - (\hat{i} + \hat{j} + \hat{k})]$.

$= (2\hat{i} + 3\hat{j} - \hat{k}).\,(-3\hat{i} - 2\hat{j} + \hat{k}) = -6 - 6 - 1 = -13$ J

Example 2:

A particle is shifted from point (0, 0, 1 m) to point (1m, 1m, 2m), under the simultaneous action of several forces. Two of them are $\vec{F}_1(2\hat{i}+3\hat{j}-\hat{k})$ and $\vec{F}_2(\hat{i}-2\hat{j}+2\hat{k})$ N. Find the work done by the resultant of these two forces.

Solution:

Work done by a constant force equals to dot product of the force and displacement vectors.

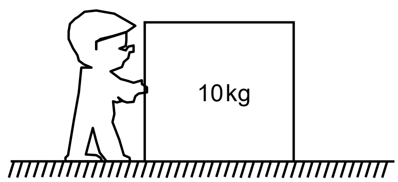
$W = \vec{F}\cdot\Delta\vec{r} \quad \Rightarrow \quad W = (\vec{F}_1+\vec{F}_2)\cdot\Delta\vec{r}$

Substituting the given values, we have

$W = (3\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 3 + 1 + 1 = 5J$

Example 3:

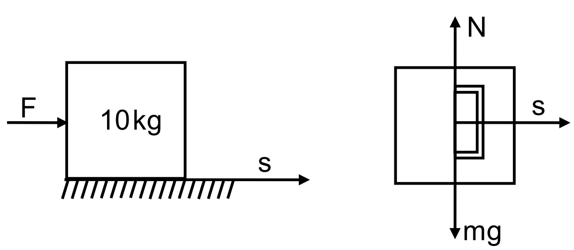
If you push a smooth box of mass 10 kg by a force of 10 N through a distance of 5 m, Find the work done by you.



Solution:

The work done by you on the box is

$W = (F\cos 0°)s = Fs = (10\ N)(5\ m) = 50\ Nm = 50\ J$



Example 4:

Calculate the work done to move a body of mass 10 kg along a smooth inclined plane (θ = 30°) with constant velocity through a distance of 10 m.

Solution:

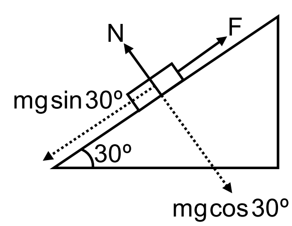
Here the motion is not accelerated, the resultant force parallel to the plane must be zero.

So $\quad F - Mg\sin 30° = 0 \Rightarrow F = Mg\sin 30°$

& $\quad d = 10$ m

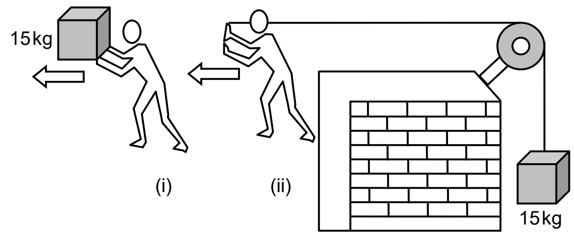
$W = Fd\cos\theta = (Mg\sin 30°)\ d\cos 0°$

$= 10 \times 10 \times \frac{1}{2} \times 10 \times 1 = 500$ J



Example 5:

In Fig. (i), the man walks 2 m carrying a mass of 15 kg on his hand. In Fig. (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



Solution:

In first case, the person need to apply a horizontal force to move the block horizontally & a vertical force to balance the weight. Work done by vertical force to move the block horizontally will be zero so net work in first case by person will be $F_{hor.}$ x.

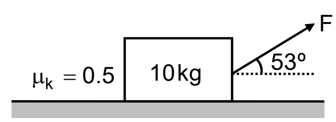
In second case, work done by person will be equal to work done by tension which will be mg x.

To move the block horizontal in first case, a very minimum force can cause this displacement

So, $\quad W_{(case\ I)} < W_{(case\ II)}$

Example 6:

A man pulls a 10 kg block 20 m along a horizontal surface at a constant speed with a force F as shown in figure. If μ_k = 0.5 then find the work done by the man on the block.



Solution:

As acceleration = 0 $\quad$ So, F cos 53° = μ_kN

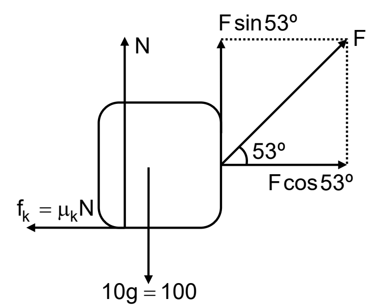
But $\quad$ N = 100 − F sin 53°

Therefore $\quad F\left(\frac{3}{5}\right) = 0.5\left[100 - F\left(\frac{4}{5}\right)\right]$

$\Rightarrow \quad$ F = 50 N

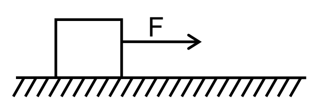
Work done = (F cos 53°) (s)

$$= \left(50 \times \frac{3}{5}\right)(20) = 600 \text{ J}$$



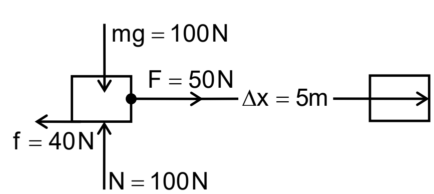
Example 7:

A 10 kg block placed on a rough horizontal floor is being pulled by a constant force of 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find the work done by each individual force acting on the block over a displacement of 5 m.



Solution:

Forces acting on the block are its weight (mg = 100 N), normal reaction (N = 100 N) by the ground, force of kinetic friction (f = 40 N) and the applied force (F = 50 N). All these forces and the displacement of the block are shown in the figure.



So, $\quad W_{mg} = 0 = W_N$

$\quad W_F = 50(5) = 250$ J

$\quad W_{friction} = -40(5) = -200$ J

Concept Builder-1

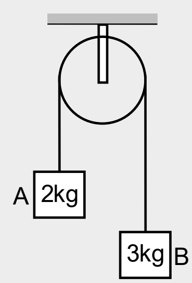
Q.1 A block of mass 5 kg is displaced from (0, 0, 0) to (1m, 2m, 3m) under a constant force $\vec{F} = (6\hat{i} - 2\hat{j} + \hat{k})$ N. Find the work done by this force.

Q.2 Calculate the work done by the force $\vec{F} = (3\hat{i} - 2\hat{j} + 4\hat{k})$ N in carrying a particle from point (–2m, 1m, 3m) to point (3m, 6m, –2m).

Q.3 A body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done is 25 joules, find the angle which the force makes with the direction of motion of the body.

Q.4 Calculate the amount of work done in raising a glass of water weighing 0.5 kg through a height of 20 cm. ($g = 10\ m/s^2$)

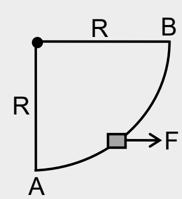
Q.5 If the given system is released from rest, find the work done by tension force on block B in first one second. ($g = 10\ m/s^2$)



(1) $\frac{200}{9}$ J (2) 16 J (3) – 24 J (4) – 36 J

Q.6 Find the work done by gravity when we lowered a point mass upto height 'h' ?

Q.7 A block of mass m is taken from point A to point B under the action of a constant force F along a quarter circle of radius R. Work done by this force is :



(1) FR (2) $\frac{\pi}{2}$ FR (3) $\frac{FR}{4}$ (4) $\sqrt{2}$ FR

4. Work Done by a Variable Force

If the force applied on a body is changing its direction or magnitude or both, the force is said to be variable. Suppose a variable force causes displacement of a body from position P_1 to position P_2. To calculate the work done by the force, the path from P_1 to P_2 can be divided into infinitesimal element; each element is so small that during the displacement of the body through it, the force is supposed to be constant. If be the small displacement of point of application and $\vec{F}$ be the force acting on the body, the work done by force is $dW = \vec{F} \cdot d\vec{r}$.

The total work done in displacing the body from P_1 to P_2 is given by = $\int dW = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$

$$\Rightarrow W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}.$$

If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the point P_1 and P_2 respectively then the total work done

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$$

Cartesian Form

When magnitude and direction of the force varies with position, work done by the force for infinitesimal displacement is $dW = \vec{F} \cdot d\vec{r}$

The total work done for the displacement from position A to B is $W_{AB} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (F\cos\theta)ds$

In terms of rectangular components4

$$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}, \; d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W_{AB} = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

Example 8:

A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joules.

Solution: :

$$W = \int_{x_1}^{x_2} F dx \Rightarrow W = \int_0^5 (7 - 2x + 3x^2) dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = 135 \text{ J}$$

Example 9:

The displacement x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation :

$$x = (t - 3)^2$$

Where x is in metre and t in sec. Calculate the work done by the force in the first 6 sec.

Solution:

as $x = (t-3)^2$

$v = \frac{dx}{dt} = 2(t-3) \qquad \Rightarrow dx = 2(t-3)dt$

$a = \frac{dv}{dt} = 2$

So F = ma = 2m

Work = $\int F\ dx$

$= \int_0^6 2m[2(t-3)]\,dt$

$= 4m\left[\frac{t^2}{2} - 3t\right]_0^6$

Work = 0

Alternate :

Since F = 2m = constant

Work done by a constant

Force = Fd cos θ

displacement = $X_{at\ t=6} - X_{at\ t=0} = 9 - 9 = 0$

So, work = 0

Example 10:

A block is constrained to move along x-axis under a force $F = 2x + 3x^2$. Here, F is in newton and x is in meter. The work done by this force when block is displaced from x = 1m to x = 2 m, is :

(1) 10 J (2) 15 J (3) 20 J (4) 40 J

Solution: (1)

Work done, $W = \int F\,dx = \int_1^2 (2x + 3x^2)\,dx = \left[2\left(\frac{x^2}{2}\right) + 3\left(\frac{x^3}{3}\right)\right]_1^2$

$= (x^2 + x^3)_1^2 = (2^2 + 2^3) - (1^2 + 1^3) = (4 + 8) - (1 + 1) = 10$ J

Example 11:

A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m ?

Solution:

m = 0.5 kg

$a = v\cdot\frac{dv}{dx} = (ax^{3/2})\left(\frac{3}{2}ax^{1/2}\right)$

$F = ma = \frac{3}{4}\ a^2x^2$

Work done = $\int_0^2 F\cdot dx = \int_0^2 \frac{3}{4}a^2x^2\,dx = \frac{3}{4}a^2\ \left(\frac{x^3}{3}\right)_0^2 = 2a^2$

Concept Builder-2

Q.1 A particle moves along the x-axis from $x = x_1$ to $x = x_2$ under the influence of a force given by $F = 2x$. Find the work done in the process.

Q.2 Position–time variation of a particle of mass 1 kg is $x = (2t^3 + 3)$ m. Calculate the work done by the force in first 2 seconds.

Q.3 A force $F = (10 + 0.5x)$ N acts on a particle in x direction, where x is in metres. Find the work done by this force during a displacement from x = 0 to x = 2.

Q.4 $\vec{F} = \left(2x\hat{i} + 3y\hat{j}\right)$ is working on a particle which moves from (0, 0) to (2, 0) along x axis and (2, 0) to (2, 2) along y axis. Find the work done by the force.

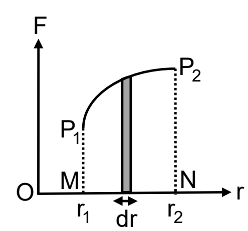
Q.5 A body of mass 6 kg is under a force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. Find the work done by the force in 2 seconds.

Q.6 A body of mass 4 kg travels in straight line with velocity $v = \frac{1}{2}x^2$. What is the work done by net force during its displacement from x = +1 to x = 3 m ?

5. Graphical Methods of Work Calculation:

5.1 Force – Displacement Curve

Suppose a body, whose initial position is $\vec{r}_1$, is acted upon by a variable force $\vec{F}$ and consequently the body acquires its final position $\vec{r}_2$. From position $\vec{r}$ to $\vec{r} + d\vec{r}$ or for small displacement $d\vec{r}$, the work done will be $\vec{F} + d\vec{r}$ whose value will be the area of the shaded strip of width $d\vec{r}$. The work done on the body in displacing it from position $\vec{r}_1$ to $\vec{r}_2$ will be equal to the sum of areas of all such strips.



Thus, total work done,

$$W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} \vec{F}.d\vec{r} = \text{Area of } P_1P_2NM$$

The area of the graph between curve and displacement axis is equal to the work done.

Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cosθ and r.

5.2 Acceleration – Displacement Curve

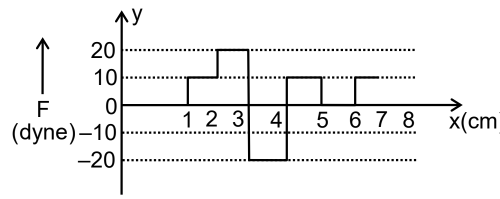
For any body subjected to a force, if it's mass does not change with time,

$F = ma$ & $dW = \vec{F}\cdot d\vec{s} = m\vec{a}\cdot d\vec{s}$

so, Work = $m\int \vec{a}\cdot d\vec{s}$ = mass (Area under a-s curve)

Example 12:

Corresponding to the force-displacement diagram shown in adjoining diagram, calculate the work done by the force in displacing the body from x = 1 cm to x = 5 cm.



Solution:

Work = Area between the curve and displacement axis = 10 + 20 – 20 + 10 = 20 ergs.

6. Dependency on Frame of Reference

A force does not depend on frame of reference and is assume to have same value in all frame of references, but displacement depends on frame of reference and may assume different values relative to different reference frames. Therefore, work of a force depends on choice of reference frame. For example, consider a man with a suitcase standing in a lift that is moving up. In the reference frame attached to the lift, the man applies some force on the bag but the displacement of the bag is zero, therefore work due to this force on the bag is zero. However, in a reference frame attached to the ground the bag has displacement equal to that of the lift and the force applied by the man does a non zero work.

With change of frame of reference (inertial) force does not change while displacement may change; so, the work done by a force will be different in different frame, e.g.,

(a) If a Porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero (as displacement relative to him is zero) while relative to a person on the ground will be (W) = mgh (as F = mg and s = h).

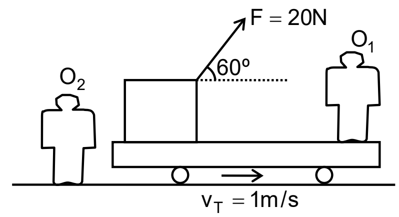
(b) If a person is pushing a box inside a moving train, the work done in the frame of train will be $(\vec{F}\cdot\vec{s})$ while in the frame of earth will be $\vec{F}\cdot(\vec{s}-\vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

Key Points

- Work is defined for an interval or displacement; there is no term like instantaneous work similar to instantaneous velocity.
- For a particular displacement, work done by a given force is independent of time.
- When several forces act, work done by a force, for a particular displacement, is independent of other forces.
- Displacement depends on reference frame so work done by a force is reference frame dependent, Work done by a force can be different in different reference frames.
- Work is done by an energy source or agent that applies the force.
- When $\theta = 0°$, A force does maximum positive work.
- When $\theta = 180°$, A force does maximum negative work.

Example 13:

As shown in figure the box is being pulled by man on the trolley with a constant speed of 0.5 m/s relative to trolley. Find the work done by the man in 4 seconds.



(a) As seen by observer O_1

(b) As seen by observer O_2

Solution:

Displacement of trolley = (1) × (4) = 4 m

Displacement of box relative to trolley = 0.5 × 4 = 2 m

(a) As seen by O_1

Displacement = 2 m

work = $F\, d \cos\theta = 20\,(2) \cos 60°$

work = 20 J

(b) As seen by O_2

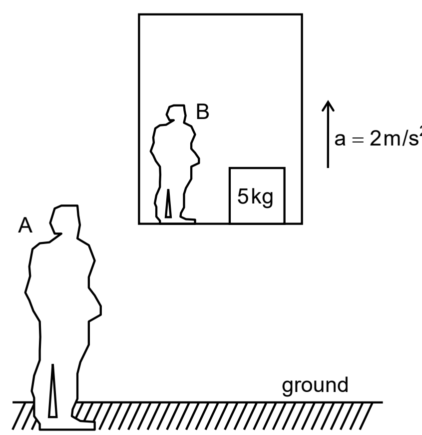
displacement = 2 + 4 = 6 m

work = 20 (6) cos 60° = 60 J

Example 14:

Paragraph

A block of mass 5 kg is kept in elevator (lift) which starts moving upward with constant acceleration 2 m/s^2 as shown in figure. Initially elevator at rest. The block is observed by two observers A and B for a time interval t = 0 to t = 5 sec. Observer B is at rest with respect to elevator and observer A is standing on the ground. (g = 10 m/s^2)



(i) The observer A finds that the work done by gravity on the block is

(1) 1000 J (2) –1300 J

(3) 1200 J (4) –1250 J

(ii) The observer A finds that work done by normal reaction acting on the block is

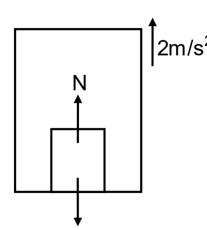
(1) 1400 J (2) 1550 J (3) 1500 J (4) 1450 J

(iii) The observer B finds that the work done by gravity on the block is

(1) 100 J (2) 600 J (3) 0 (4) 900 J

Solution:

FBD



(i) Displacement of block = $h = ut + \frac{1}{2}at^2$

$h = \frac{1}{2}(2)(5)^2 = 25\text{m}$

Work done = Force of gravity × Displacement

$= -mg \times 25 = -1250$ J

(ii) Net acceleration on the block

$g_{eff} = g + a = 12\ m/s^2$

Normal force = m (g + a) = 60 N

Now displacement of block w.r.t. ground

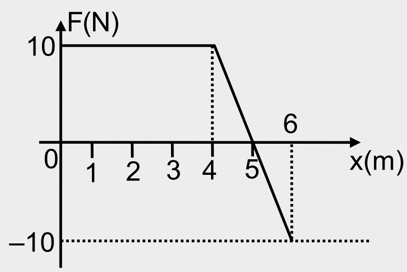
$S = \frac{1}{2} at^2 = 25\ m$

Work done by normal reaction = N × (S) = 60(25) = 1500 J

(iii) The Observer B finds that the work by gravity on the block is zero as the displacement of the block with respect to lift is zero.

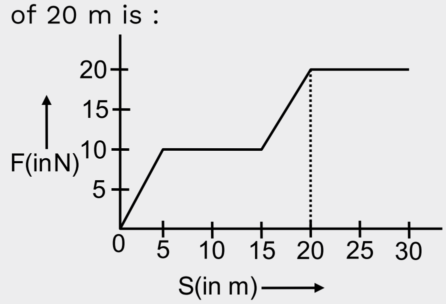
Concept Builder-3

Q.1 Force (F) acting on a particle varies with displacement (x) as shown in figure. The work done by this force on the particle from x = 0 to x = 6 m, is :



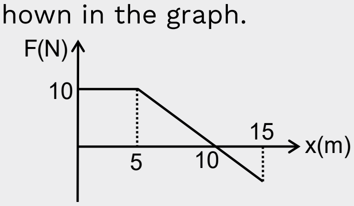
(1) 20 J (2) 40 J (3) 50 J (4) 60 J

Q.2 The work done by a force acting on a body is as shown; in the graph. The total work done in covering an initial distance of 20 m is :



(1) 225 J (2) 200 J (3) 400 J (4) 175 J

Q.3 A horizontal force F is used to pull a box placed on a floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.



(a) Calculate the work done by the force in moving the box from x = 0 m to x = 10 m.

(b) Calculate the work done by the force in moving the box from x = 10 m to x = 15 m.

(c) Calculate the work done by the force in moving the box from x = 0 m to x = 15 m.

7. Energy and It's Various Forms

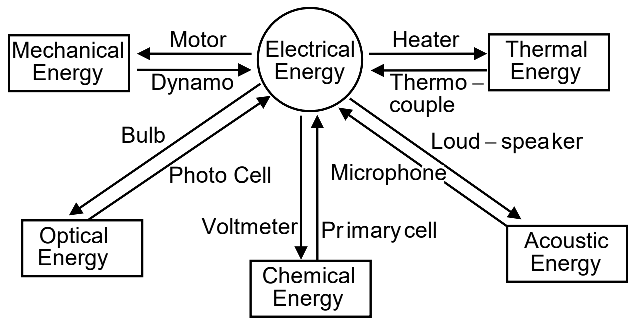
Energy is defined as the internal capacity to do work.

When we say that a body has energy it means that it can do work.

Different Forms of Energy

Mechanical energy, electrical energy, optical (light) energy, acoustical (sound), molecular, atomic and nuclear energy etc, are various forms of energy.

These forms of energy can change from one form to the other.



- Energy is a scalar quantity
- **Dimensions :** $[M^1L^2T^{-2}]$
- **SI Unit :** joule
- **Other units :** 1 erg = 10^{-7} joules

  1kWh = 36×10^5 joules

  1 eV = 1.6×10^{-19} joules

  1 cal = 4.2 joules

In mechanics we are only concerned with the mechanical energy, which is of two types.

(a) Kinetic energy (b) Potential energy

8. Kinetic Energy

Kinetic energy is the internal capacity of doing work of an object by virtue of its motion.

OR

K.E. of a body can be calculated by the amount of work done in stopping the moving body or by the amount of work done in imparting the present velocity to the body from the state of rest.

If a particle of mass m is moving with velocity 'v' much less than the velocity of light, then the kinetic energy K.E. is given by

$$K.E. = \frac{1}{2}mv^2 = \frac{P^2}{2m} = \frac{1}{2}Pv$$

where P is momentum of the particle.

Example 15:

A body of mass 0.8 kg has initial velocity $(3\hat{i} - 4\hat{j})$ m/s and final velocity $(-6\hat{j} + 2\hat{k})$ m/s. Find the change in kinetic energy of the body?

Solution:

Change in kinetic energy

$$\Delta KE = \frac{1}{2}\ mv_f^2\ -\ \frac{1}{2}\ mv_i^2$$

where $v_f = \sqrt{6^2 + 2^2} = \sqrt{40}$ and $v_i = \sqrt{3^2 + 4^2} = \sqrt{25}$

$$= \frac{1}{2} \times 0.8 \left(\left(\sqrt{40}\right)^2 - \left(\sqrt{25}\right)^2\right) = 0.4\ [40 - 25] = 0.4\ (15) = 6 \text{ joules.}$$

Example 16:

In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with a speed of 200 m/s on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Solution:

Initial kinetic energy,

$$K_i = \frac{1}{2} \times \frac{50}{1000} \times 200 \times 200 = 1000 \text{ J}$$

Final kinetic energy,

$$K_f = \frac{10}{100} \times 1000 = 100 \text{ J}$$

If v_f is emergent speed of the bullet, then

$$\frac{1}{2} \times \frac{50}{1000} \times v_f^2 = 100 \Rightarrow v_f^2 = 4000 \Rightarrow v_f = 63.2 \text{ ms}^{-1}$$

Note that the speed is reduced by approximately 68% and not 90%.

Example 17:

Kinetic energy of a particle is increased by 300%. Find the percentage increase in its momentum.

Solution:

Kinetic energy $E = \frac{1}{2}\ mv^2$

momentum $p = mv$

When E is increased by 300%,

$$E' = E + 3E = 4E = 4\left(\frac{1}{2}mv^2\right) = 2mv^2$$

If v' is the new velocity of the body, then

$$\frac{1}{2}\ mv'^2 = 2mv^2 \ \Rightarrow\ v' = 2v$$

Hence, percentage change in momentum

$$= \frac{2mv - mv}{mv} \times 100 = 100\%.$$

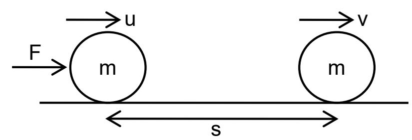
9. Work – Kinetic Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in its kinetic energy. So work done by all the forces = change in kinetic energy.

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Proof:

(i) For constant force :



$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad v^2 - u^2 = 2\left(\frac{F}{m}\right)s$$

$$\Rightarrow \quad s = \frac{(v^2 - u^2)m}{2F} = \frac{\Delta KE}{F}$$

$$\Rightarrow \quad Fs = \Delta KE$$

$$\Rightarrow \quad W = \Delta KE$$

(ii) For variable force:

$$W = \int F dx = \int madx = \int m\left(\frac{vdv}{dx}\right)dx$$

$$\int_u^v mvdv = m\left[\frac{v^2}{2}\right]_u^v = \frac{m}{2}[v^2 - u^2]$$

$$\Rightarrow W = \Delta KE$$

How to Apply Work-Kinetic Energy Theorem

The work-kinetic energy theorem is deduced here for a single body moving relative to an inertial frame, therefore it is recommended at present to use it for a single body in inertial frame. To use work-kinetic energy theorem the following steps should be followed.

- Identify the initial and final positions as position 1 and 2 and write expressions for kinetic energies, whether known or unknown.
- Draw the free body diagram of the body at any intermediate stage between positions 1 and 2. The forces shown will help in deciding their work. Calculate work by each force and add them to obtain the total work done $W_{1\to2}$ by all the forces.
- Use the work obtained in step 2 and kinetic energies obtained in step 1 in the equation $W_{1\to2} = K_2 - K_1$.

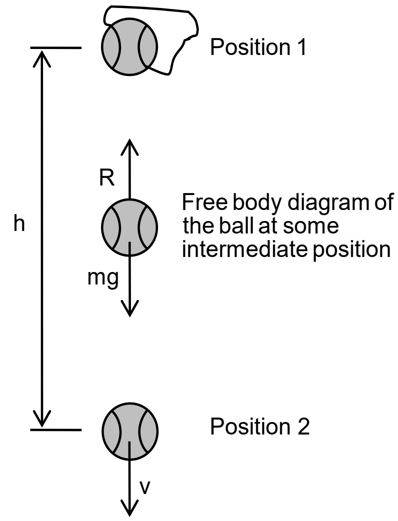
Example 18:

A 5 kg ball when falls through a height of 20 m acquires a speed of 10 m/s. Find the work done by air resistance.

Solution:

The ball starts falling from position 1, where its speed is zero; hence, kinetic energy is also zero.

$K_1 = 0$ J ...(i)



During the downward motion of the ball, constant gravitational force mg acts downwards and air resistance R of unknown magnitude acts upwards as shown in the free body diagram. The ball reaches position 20 m below the position-1 with a speed v = 10 m/s, so the kinetic energy of the ball at position 2 is

$K_2 = \frac{1}{2}mv^2 = 250$ J ...(ii)

Work done by gravity

$W_{g, 1 \to 2} = mgh = 1000$ J ...(iii)

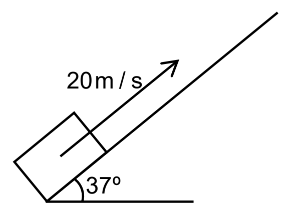
Denoting the work done by the air resistance as $W_{R,1\to2}$ and making use of eq. (i), (ii) and (iii) in work-kinetic energy theorem, we have

$W_{1\to2} = K_2 - K_1 \Rightarrow W_{g,\,1\to2} + W_{R,\,1\to2} = K_2 - K_1$

$\Rightarrow W_{R,\,1\to2} = -750$ J.

Example 19:

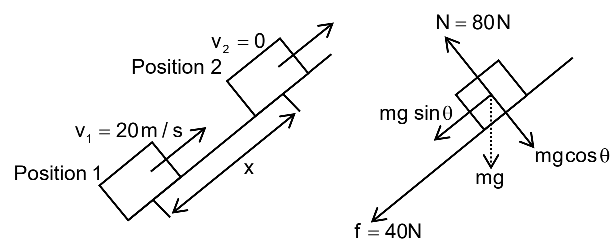
A box of mass m = 10 kg is projected up an inclined plane from its foot with a speed of 20 m/s as shown in the figure. The coefficient of friction μ between the box and the plane is 0.5. Find the distance travelled by the box on the plane before it stops for the first time.



Solution:

The box starts from position 1 with speed $v_1 = 20$ m/s and stops at position 2

Kinetic energy at position 1:



$K_1 = \frac{1}{2}mv_1^2 = 2000$ J

Kinetic energy at position 2 :

$K_2 = 0$

Work done by external forces as the box moves from position 1 to position 2 is,

$W_{1\to2} = W_{g,1\to2} + W_{f,1\to2} = -60x - 40x = -100\,x$ J

Applying work energy theorem for the motion of the box from position 1 to position 2, we have

$W_{1\to2} = K_2 - K_1 \Rightarrow -100x = 0 - 2000$

$\Rightarrow x = 20$ m

Example 20:

A particle of mass m moves with velocity v = $a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Solution:

Work done by all forces = W = ΔKE = $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Here $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$,

So $W = \frac{1}{2}ma^2 d - 0 = \frac{1}{2}ma^a d$.

Concept Builder-4

Q.1 The velocity of a ball (m = 2 kg) changes for to $3\hat{i} - 4\hat{j}$ to $4\hat{j} + 3\hat{i}$ after hitting a bat. What is the change in velocity & change in K.E. of the ball.

Q.2 By what percentage momentum of a ball will decrease if its kinetic energy decreases to 49% of initial.

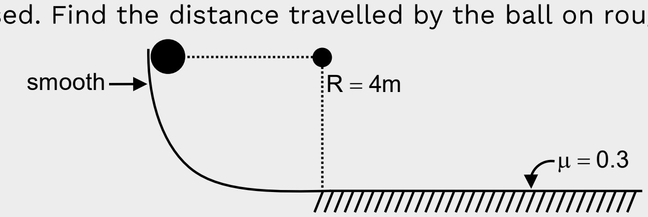
Q.3 If the linear momentum of a body is increased by 50%, then by what percentage will its kinetic energy increase?

Q.4 The kinetic energy of a body is numerically equal to thrice the momentum of the body. Find the velocity of the body.

Q.5 A bullet weighing 10 g is fired with a velocity 800 m/s. Its velocity decreases to 100 m/s after passing through a 1 m thick mud wall. Find the average resistance offered by the mud wall.

Q.6 A body of mass 10 kg is released from the top of a tower which acquires a velocity of 10 ms^{-1} after falling through the distance of 20 m. Calculate the work done by the drag force of the air on the body? (take g = 10 m/s^2)

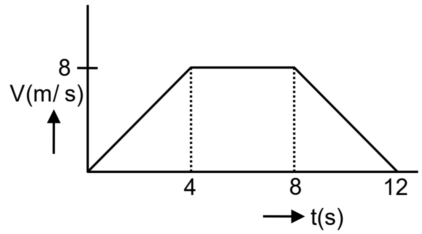
Q.7 If the ball is released. Find the distance travelled by the ball on rough track.



Q.8 A body of mass 'm' is dropped from a height H, it reaches the ground with a speed of $1.2\sqrt{gH}$ Calculate the work done by air-friction.

Example 21:

Velocity-time graph of a particle of mass 0.5 kg is shown in figure.



(i) Calculate work on the particle from t = 0 to t = 2s.

(ii) Calculate work on the particle from t = 4 to t = 8 sec.

(iii) Calculate work on the particle from t = 8 sec to t = 12 sec.

(iv) Work done on particle from t = 0 to t = 12 sec

(v) Calculate work on the particle from t = 2 sec to t = 11 sec.

Solution:

(i) At $t = 0$ s, $v_1 = 0$

at $t = 2$ s, $v_2 = 4$

Now work done = change in kinetic energy

$$W = \frac{1}{2} \times 0.5 \times (4^2 - 0^2) = 4 \text{ J}$$

(ii) Since change in kinetic energy is zero

So, work done zero.

(iii) At $t = 8$, $v_1 = 8$ m/sec

at $t = 12$, $v_2 = 0$

$$\Delta \text{ K.E.} = \frac{1}{2} m(v_2^2 - v_1^2) = -16 \text{ joule}$$

(iv) at $t = 0$, $v_1 = 0$

at $t = 12$, $v_2 = 0$

Δ K.E. = 0, work done = 0

(v) at $t = 2$, $v_1 = 4$ m/s

at $t = 11$, $v_2 = 2$

$$\Delta \text{ K.E.} = \frac{1}{2} m(v_2^2 - v_1^2) = -3 \text{ joule}$$

10. Conservative and Non–Conservative Forces

Conservative Force

- A force is said to be conservative if the work done by the force is independent of the path and depends only on initial and final positions.
- Work done by these forces in a closed path is always zero.

Examples of Conservative Force

All central forces are conservative like Gravitational, Electrostatic, Elastic force, Restoring force due to spring. Intermolecular force etc.

Central Force

A force whose line of action always passes through a fixed point (which is known as centre of force) and whose magnitude depends only on the distance from this point is known as central force.

$$\vec{F} = \frac{k}{r^2}\hat{r}$$

All forces following inverse square law are central forces.

is a central force such as Gravitational force and Coulomb force.

- All central forces are conservative forces.
- Central forces are functions of position.

Non Conservative Force

A force is said to be non-conservative if work done by the force in a moving body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non-conservative force field.

Frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which the body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Examples of Non-Conservative Force

All velocity-dependent forces such as air resistance, viscous force are non-conservative forces.

Difference Between Conservative Forces and Non-Conservative Forces

Conservative Forces

- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc, are conservative forces
- When a conservative force acts within a system the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system does not change.
- Work done is completely recoverable.

Non-Conservative Forces

- Work done depends upon path.
- Work done in a round trip is not zero.
- Forces which are velocity-dependent in nature e.g. friction, viscous force, etc.
- Work done against a non-conservative force may be dissipated as heat energy.
- Work done is not completely recoverable

11. Potential Energy and Change in Potential Energy

The energy stored in a body or system by virtue of its configuration or its position in a field is called potential energy.

In case of conservative force :

$$F = -\left(\frac{dU}{dr}\right)$$

i.e., $dU = -F\,dr$

So, $\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r}$

i.e., $U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$ .....(1)

Whenever and wherever possible, we take the reference point at ∞ and assume potential energy to be zero there, i.e., if we take $r_1 = \infty$ and $U_1 = 0$ then dropping suffix 2.

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W \quad .....(2)$$

In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from some reference position to given position. This is why in shifting a particle in a conservative field (say gravitational or electric), if the particle **moves opposite to the field**, work done by the field will be negative and so change in potential energy will be positive. i.e., potential energy will **increase** and when the particle **moves in the direction** of field, work will be positive and so change in potential energy will be negative, i.e., potential energy will **decrease**.

Regarding potential energy U it is worth noting that :

(1) Potential energy can be defined only for conservative forces. It does not exist for non-conservative forces.

(2) Potential energy can be positive or negative

(3) Potential energy depends on frame of reference.

(4) A moving body may or may not have potential energy.

(5) Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.

- Relationship between conservative force field and potential energy :

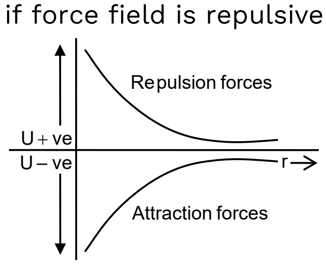
$$\vec{F} = -\nabla U = -\text{grad}(U) = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

- If force varies with only one dimension (say along x-axis) then $F = -\dfrac{dU}{dx} \Rightarrow dU = -F dx$

$$\Rightarrow \int_{U_1}^{U_2} dU = -\int_{x_1}^{x_2} F dx \Rightarrow \boxed{\Delta U = -W_C}$$

- Potential energy may be positive or negative or even zero.

(i) Potential energy is positive, if force field is repulsive in nature



(ii) Potential energy is negative, if force field is attractive in nature

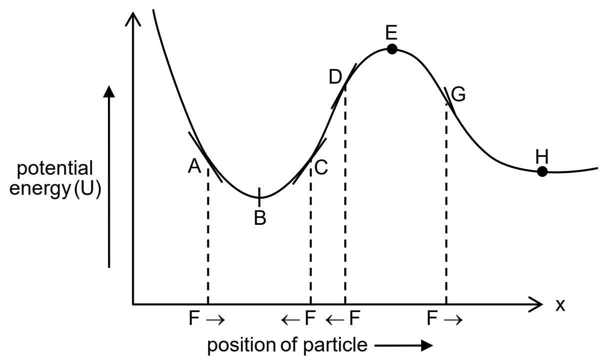
- If r ↑ (separation between body and force centre), U ↑, Force field is attractive or vice-versa.
- If r ↑, U ↓, force field is repulsive in nature.

Key Points

- The energy possessed by a body by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the external work done against the action of conservative force in order to shift it from a certain reference point (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the conservative force in moving the body from its present position to reference position.
- At a certain reference position, the potential energy of the body is assumed to be zero or the body is assumed to have lost the capacity of doing work.

12. Potential Energy Curve and Equilibrium

It is a curve which shows the change in potential energy with the position of a particle.



- **Stable Equilibrium**

After a particle is slightly displaced from its equilibrium position if it tends to come back towards equilibrium then it is said to be in stable equilibrium.

At point **A** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **C** : slope $\frac{dU}{dx}$ is positive so F is negative

$$\boxed{\text{At equilibrium } F = -\frac{dU}{dx} = 0}$$

At point **B** : It is the point of stable equilibrium.

At point **B** : $U = U_{min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = positive

- **Unstable Equilibrium**

After a particle is slightly displaced from its equilibrium position, if it tends to move away from equilibrium position then it is said to be in unstable equilibrium.

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative; At point **G** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **E** : it is the point of unstable equilibrium; At point **E** : $U = U_{max}$, $\frac{dU}{dx} = 0$ and $= \frac{d^2U}{dx^2}$ negative

- **Neutral Equilibrium**

  After a particle is slightly displaced from its equilibrium position if no force acts on it then the equilibrium is said to be neutral equilibrium.

  Point **H** corresponds to neutral equilibrium

  $\Rightarrow$ U = constant ; $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$.

Example 22:

The potential energy for a conservative force system is given by $U = ax^2 - bx$, where a and b are constants. Find out the (a) expression for force, (b) equilibrium position and (c) potential energy at equilibrium.

Solution:

(a) For conservative force

$$F = -\frac{dU}{dx} = -(2x - b) = -2ax + b$$

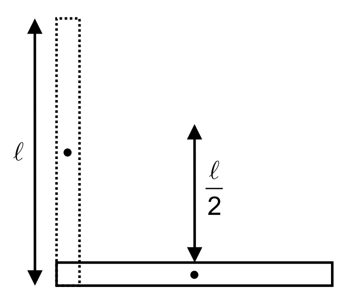
(b) At equilibrium

$$F = 0 \quad -2ax + b = 0 \qquad \Rightarrow x = \frac{b}{2a}$$

(c) $U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) \Rightarrow \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$.

Example 23:

A rod of mass 'm' and length 'ℓ' is placed in a horizontal position. Find the work done by the external force against gravity to alter its configuration from horizontal position to vertical position.



Solution:

Displacement between the positions of centre of mass of rod is $\frac{\ell}{2}$ (in a direction parallel to the force of gravity)

W = –change in PE of the rod = $mg\frac{\ell}{2}$.

13. Conservation of Mechanical Energy

We define the change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f \vec{F}\cdot d\vec{r}$$

where W is the work done by the internal force on the system as the system passes from the initial configuration i to the final configuration f.

We don't (or can't) define potential energy corresponding to nonconservative internal force.

Suppose only conservative internal forces operate between the parts of the system and the potential energy U is defined corresponding to these forces. There are either no external forces or the work done by them is zero. We have

$$U_f - U_i = -W = -(K_f - K_i)$$

or $$U_f + K_f = U_i + K_i.$$

The sum of the kinetic energy and the potential energy is called the total mechanical energy. We see from equation that the total mechanical energy of a system remains constant if the internal forces are conservative and the external forces do not work. This is called the principle of conservation of energy.

The total mechanical energy K + U is not constant if nonconservative forces, such as friction, act between the parts of the system, We can't apply the principle of conservation of energy in presence of nonconservative forces. The work-energy theorem is still valid even in the presence of nonconservative forces.

Work–Mechanical Energy Relation

If nonconservative internal forces operate within the system, or external forces do work on the system, the mechanical energy changes as the configuration changes. According to the work-energy theorem, the work done by all the forces equals the change in the kinetic energy. Thus,

$$W_c + W_{nc} + W_{ext} = K_f - K_i$$

where the three terms on the left denote the work done by the conservative internal forces, nonconservative internal forces and the external forces.

As $$W_c = -(U_f - U_i),$$

we get $$W_{nc} + W_{ext} = (K_f + U_f) - (K_i + U_i)$$

$$= E_f - E_i$$

where E = K + U is the total mechanical energy.

If the internal forces are conservative but external forces also act on the system and they do work,

$W_{nc} = 0$ and form,

$W_{ext} = E_f - E_i$.

The work done by the external forces equals to the change in the mechanical energy of the system.

Important Points about Work – Energy Relation

Let us summaries the concepts developed so far in this chapter.

(1) Work done on particle is equal to the change in its kinetic energy.

(2) Work done on a system by all the (external and internal) forces is equal to the change in its kinetic energy.

(3) A force is called conservative if the work done by it during a round trip of a system is always zero. The force of gravitation, Coulomb force, force by a spring etc. are conservative. If the work done by it during a round trip is not zero, the force is nonconservative. Friction is an example of nonconservative force.

(4) The change in the potential energy of a system corresponding to conservative internal forces is equal to negative of the work done by these forces.

(5) If no external forces act (or the work done by them is zero) and the internal forces are conservative, the mechanical energy of the system remains constant. This is known as the principle of conservation of mechanical energy.

(6) If some of the internal forces are nonconservative, the mechanical energy of the system is not constant.

(7) If the internal forces are conservative, the work done by the external forces is equal to the change in mechanical energy.

Mechanical Energy Conservation of a Freely Falling Body

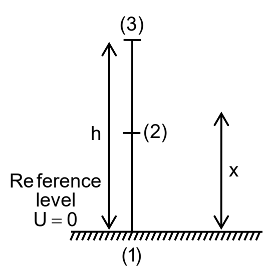
Let a ball of mass m be dropped from position (3) (as shown in figure)

At point 1

PE = 0; KE = $\frac{1}{2}$ mv^2

$\because$ v = $\sqrt{2gh}$ so KE = mgh

At point 2



PE = mgx; KE = $\frac{1}{2}$ mv^2

$\because$ v = $\sqrt{2g(h-x)}$ so KE = mg(h – x)

At point 3

PE = mgh; KE = 0

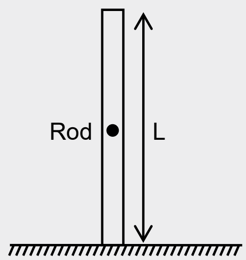
so during the motion of the ball at any position.

$ME_{(1)} = ME_{(2)} = ME_{(3)}$ and PE = mgx

Key Points

- Potential energy depends on the frame of reference but change in potential energy is independent of reference frame.
- Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
- It is a function of position and does not depend on the path.
- Whenever work is done by the conservative forces the potential energy decreases and whenever work is done against the conservative forces, potential energy increases.
- For regularly shaped uniform bodies, the potential energy change can be calculated by considering their mass to be centered at the geometrical center point.

For Example :



For a uniform vertical rod of length L

$PE = mg\ \frac{L}{2}$

Example 24:

A body is dropped from height 8 m. After striking the surface it rises to 6 m, what is the fractional loss in kinetic energy during impact? Assuming the frictional resistance to be negligible.

(1) 1/2 (2) 1/4 (3) 1/6 (4) 1/8

Solution:

Fractional loss in kinetic energy

$$= \frac{\text{loss kinetic energy}}{\text{initial kinetic energy}} \Rightarrow \frac{\text{loss in potential energy}}{\text{initial potential energy}} = \frac{mg(8-6)}{mg(8)} = \frac{2}{8} = \frac{1}{4}$$

Example 25:

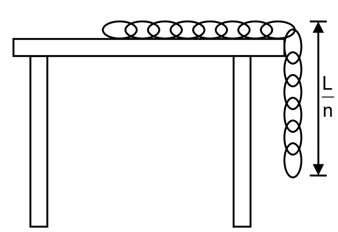
A chain of mass m and length L is held on a frictionless table in such a way that $\frac{1}{n}$th part is hanging below the edge of table. Calculate the work done to pull the hanging part of the chain.

Solution:

Required work done = change in potential energy of chain

Now, let Potential energy (U) = 0 at table level so potential energies of chain initially and finally are respectively

$$U_i = -mg\left(\frac{L}{2n}\right) = -\left(\frac{M}{L}\right)\frac{L}{n}\ g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2},$$



$U_f = 0$

Required work done = $U_f - U_i = \frac{MgL}{2n^2}$

Example 26:

The potential energy of a particle of mass 1 kg free to move along the x-axis is given by $U(x) = \left(\frac{x^2}{2} - x\right)$ joules. If total mechanical energy of the particle is 2 J, then find its maximum speed.

Solution:

Potential energy $U = \left(\frac{x^2}{2} - x\right) = \frac{x^2}{2}$

For minimum U, $\frac{d^2U}{dx^2} = 1$ = positive

and $\frac{d^2U}{dx^2} = 1$ = positive

so at x = 1, U is minimum.

Hence $U_{min} = -\frac{1}{2}$ J.

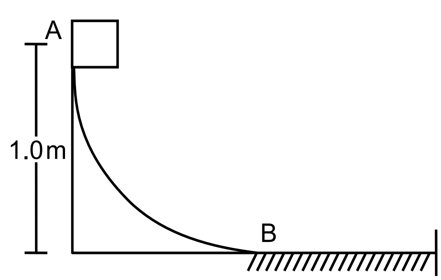
Total mechanical energy = Max KE + Min PE

Max KE = $\frac{1}{2} mv_{max}^2 = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$

$\Rightarrow \quad v_{max} = \sqrt{\frac{2}{1} \times \frac{5}{2}} = \sqrt{5}\ ms^{-1}$.

Example 27:

A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface. The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, then it would move a distance S on the rough surface. Calculate the value of S [g = 10 ms^{-2}]



Solution: $W_C + W_{nc} + W_{ext} = \Delta K$

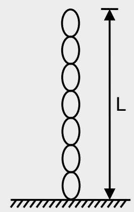
mgh – f.s + 0 = 0 ⇒ mgh – μ mg.s = 0

$\Rightarrow s = \frac{h}{\mu} = \frac{1}{0.2} = 5$ m

Concept Builder-5

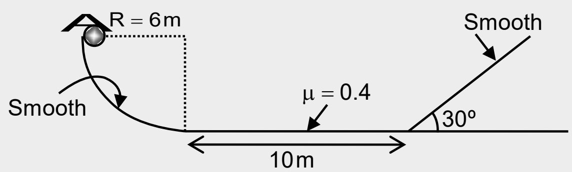
Q.1 A ball looses 20% of its kinetic energy while colliding with ground, If it is dropped from a height of 10 m, To what height it will rise ?

Q.2 What will be the loss in potential energy of chain (mass m) when half of its length is grounded.



Q.3 For a particle $U = x^3 - 3x^2$. If mass of particle is $\frac{1}{2}$ kg & its mechanical energy is 12 J then find the maximum speed of particle.

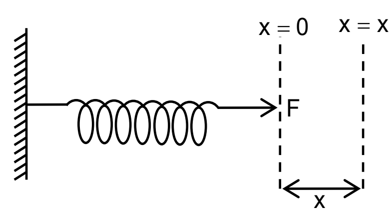
Q.4 If the ball is released from point A on the track find out the distance it will travel on inclined plane before it stops momentarily.



14. Spring Potential Energy and Spring – Block System

Work Done by Spring Force

- Work required to stretch a spring by a length x



$x_1 = 0$ $\quad x_2 = x$ and $\quad \theta = 0°$

$W = \int_0^x F dx$ $\quad \because F = kx$ $\quad W = \int_0^x kx\, dx$

$$\boxed{W = \frac{1}{2}kx^2}$$

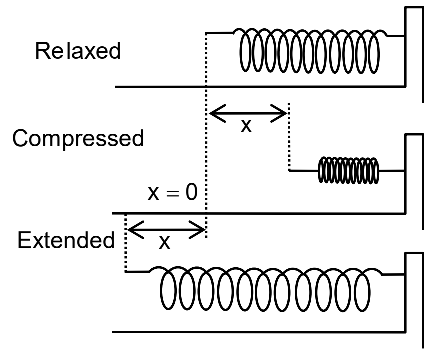
- Work required to stretch a spring from a length x_1 to x_2 :

Then $x_1 = x_1$ $\quad x_2 = x_2$ and $\quad \theta = 0°$

$W = \int_{x_1}^{x_2} F dx$ $\quad \because F = kx$ $\quad W = \int_{x_1}^{x_2} kx\, dx$

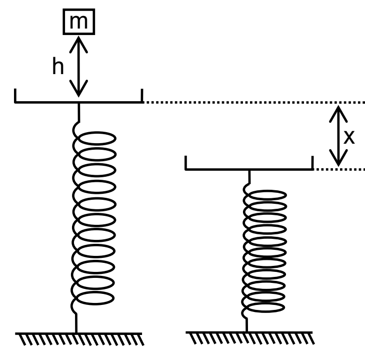
$$\boxed{W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)}$$

Potential Energy Associated with Spring Force



The potential energy of an ideal spring associated with a spring when compressed or elongated by a distance x from its natural length is given by the following expression $U = \frac{1}{2}kx^2$

Important Applications of Spring – Block System :

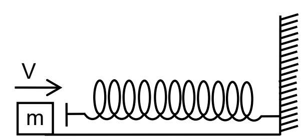


- A particle of mass m is freely released from a height h on a spring;

  if the spring gets compressed by x then:

  $\boxed{mg(h+x) = \frac{1}{2}kx^2}$ $mgh + 0 + 0_S = -mgx + \frac{1}{2}kx^2$

  $\boxed{mg(h+x) = \frac{1}{2}kx^2}$

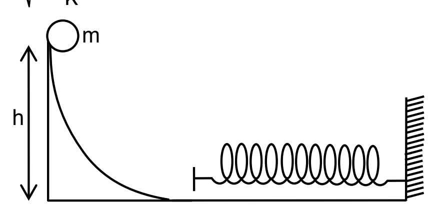


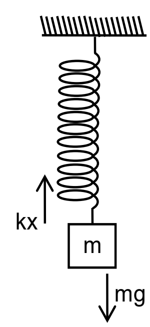
- If a body of mass m moving with speed v collides with a spring. The spring gets compressed by x_{max}, the

  $\frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2 \Rightarrow \boxed{\sqrt{\frac{m}{k}}v = x_{max}}$

- If a body is released from a height h and collides with a springs.

  $mgh = \frac{1}{2}kx_{max}^2 \Rightarrow \sqrt{\frac{2mgh}{k}} = x_{max}$





- If a block is attached to the lower end of a spring hanging from a fixed support

  Equilibrium condition :

  $F_{net} = 0$

  $mg - kx = 0$

  $\Rightarrow \boxed{\frac{mg}{k} = x}$

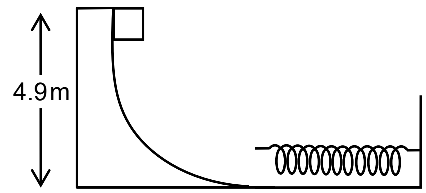
- **Maximum Extension**

  Decrease in G.P.E. of the block = increase in P.E. of the spring

  $$mgx = \frac{1}{2}kx_{max}^2 \qquad \Rightarrow \qquad \boxed{\frac{2mg}{k} = x_{max}}$$

Example 28:

Figure shows a smooth curved track terminating in a smooth horizontal part. A spring of spring constant 400 N/m is attached at one end to a wedge fixed rigidly with the horizontal part. A 40 g mass is released from rest at a height of 4.9 m on the curved track. Find the maximum compression of the spring.



Solution:

At the instant of maximum compression the speed of the 40 g mass reduces to zero. Taking the gravitational potential energy to be zero at the horizontal part, the conservation of energy shows,

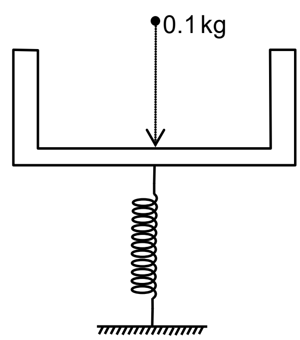
$$mgh = \frac{1}{2}kx^2$$

where m = 0.04 kg, h = 4.9 m, k = 400 N/m and x is the maximum compression.

Thus, $$x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2\times(0.04\,kg)\times(9.8\,m/s^2)(4.9\,m)}{(400\,N/m)}} = 9.8\text{ cm}$$

Example 29:

A massless platform is kept on a light elastic spring as shown in Fig. When a sand particle of 0.1 kg mass is dropped on the pan from a height of 0.24 m, the particle strikes the pan (in a perfectly inelastic manner) and the spring compresses by 0.01 m. From what height should the particle be dropped so as to generate a compression of 0.04 m ?



Solution:

$$mg(h + x) = \frac{1}{2}kx^2 \qquad ...(1)$$

& $$mg(h' + x') = \frac{1}{2}kx^2 \qquad ...(2)$$

Divide (1) by (2)

$$\frac{h+x}{h'+x'} = \frac{x^2}{x'^2} \Rightarrow \frac{0.24+0.04}{h'+0.04} = \left(\frac{0.01}{0.04}\right)^2$$

$$\Rightarrow \frac{0.25}{h'+0.04} = \frac{1}{16} \Rightarrow h' + 0.04 = 4$$

$\Rightarrow$ h' = 4 – 0.04 = 3.96 m.

Example 30:

A block strikes the free end of a horizontal spring with the other end fixed, placed on a smooth surface with a speed v. After compressing the spring by x, the speed of the block reduces to half. Calculate the maximum compression of the spring.

Solution:

Let maximum compression be x_{max} then by COME

$$= \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}kx_{max}^2}$$

$$\Rightarrow \frac{3}{4} = \frac{x^2}{x_{max}^2} \Rightarrow x_{max} = \frac{2}{\sqrt{3}}x$$

Example 31:

If in the above question KE of the block reduces to half, then find out the maximum compression of the spring.

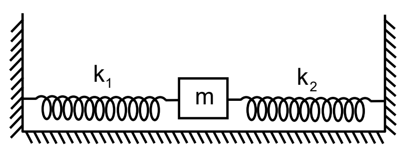
Solution:

$$KE - \frac{KE}{2} = \frac{1}{2}\ kx^2 \Rightarrow \frac{KE}{2} = \frac{1}{2}\ kx^2$$

$$KE = \frac{1}{2}\ kx_{max}^2 \Rightarrow kx^2 = \frac{1}{2}\ kx_{max}^2 \Rightarrow x_{max} = \sqrt{2}x$$

Example 32:

A block of mass m is attached to two springs of spring constants k_1 and k_2 as shown in figure. The block is displaced by x towards right and released. The velocity of the block when it is at x/2 will be:



(1) $\sqrt{\frac{(k_1+k_2)x^2}{2m}}$ (2) $\sqrt{\frac{3}{4}\frac{(k_1+k_2)x^2}{m}}$ (3) $\sqrt{\frac{(k_1+k_2)x^2}{m}}$ (4) $\sqrt{\frac{(k_1+k_2)x^2}{4m}}$

Solution: By COME $\Rightarrow K_1 + U_1 = K_2 + U_2$

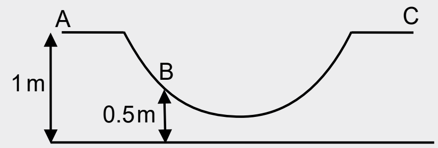
$$\Rightarrow 0 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 = \frac{1}{2}mv^2 + \frac{1}{2}k_1\left(\frac{x}{2}\right)^2 + \frac{1}{2}k_2\left(\frac{x}{2}\right)^2$$

$$\Rightarrow \frac{1}{2}\ (k_1 + k_2)x^2 = \frac{1}{2}mv^2 + \frac{1}{8}\ (k_1 + k_2)x^2$$

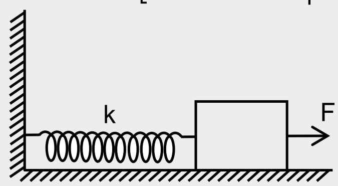
$$\Rightarrow v = \sqrt{\frac{3}{4}\frac{(k_1+k_2)x^2}{m}}$$

Concept Builder-6

Q.1 A particle is placed at the point A of a frictionless track ABC as shown in figure. It is pushed slightly towards right, Find its speed when it reaches the point B. Take $g = 10\ m/s^2$.



Q.2 A block attached to a spring kept on a smooth surface is pulled by a constant horizontal force, as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is : [Given that spring does not break]

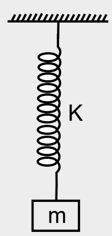


(1) $\frac{F^2}{k}$ (2) $\frac{2F^2}{k}$ (3) ∞ (4) $\frac{F^2}{2k}$

Q.3 A block of mass m is lowered slowly from the natural length position of a massless spring by an external agent to equilibrium position. The extension produced in the spring was asked to two students.

Student A : $\frac{1}{2}kx^2 = mgx \quad \therefore x = \frac{2mg}{k}$

Student B : $mg = kx \quad \therefore x = \frac{mg}{k}$



(1) Student A is incorrect, Student B is correct
(2) Student A is correct, Student B is incorrect
(3) Both are correct
(4) Both are incorrect

Q.4 A 20 newton stone falls accidentally from a height of 4 m on to a spring of stiffness constant 1960 Nm^{-1}. Write down the equation to find out the maximum compression (x_m) of the spring.

Q.5 A spring is initially compressed by x and then, it is further compressed by y. Find out the work done during the latter compression. (spring constant is k.)

Q.6 A spring of force constant 100 N/m is stretched by 5 cm. Find the work done by applied force.

Q.7 Two springs have their respective force constants K_1 and K_2. Both are stretched till their elastic potential energies are equal. If the stretching forces are F_1 and F_2 find $\frac{F_1}{F_2}$

Q.8 A long spring is stretched by 1 cm. If work done in this process is W, then find out the work done in further stretching it by 1 cm.

Q.9 A block of mass m moving with speed v compresses a spring through distance x before its speed is halved. What is the value of spring constant?

(1) $\frac{3mv^2}{4x^2}$ (2) $\frac{mv^2}{4x^2}$ (3) $\frac{mv^3}{2x^2}$ (4) $\frac{2mv^2}{x^2}$

15. Power

Average Power and Instantaneous Power

- When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile.

  In many cases, it is useful to know not just the total amount of work being done, but also how fast the work is being done. We define power as the rate at which work is being done.

  $$\text{Average Power} = \frac{\text{Work done}}{\text{Time taken to do work}}$$

  $$= \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

  If ΔW is the amount of work done in a time interval Δt, then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

- Instantaneous power is the scalar product of force and velocity at any instant.

  When work is measured in joules and t is in seconds, the unit for power is joules per second, which is called watt. For motors and engines, power is usually measured in horsepower, where 1 hp = 746 W. The definition of power is applicable to all types of energies also like mechanical, electrical, thermal.

  Instantaneous power $P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$

  Where *v* is the instantaneous velocity of the particle. Here dot product is used because only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

- For a system of varying mass

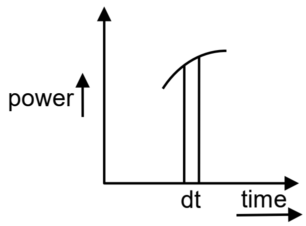
  $F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} + v\frac{dm}{dt}$

  If v = constant then

  $F = v\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$

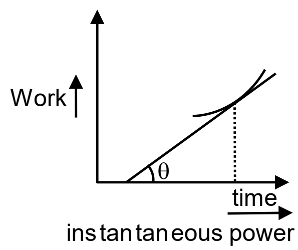
- Power is a scalar quantity with dimensions $[M^1L^2T^{-3}]$
- SI unit of power is J/s or watt.
- 1 horsepower = 746 watts = 550 ft- ℓ b/s.

Graphical Analysis



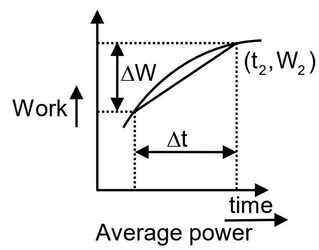
$W = \int Pdt$

Fig.(a)



$P = \frac{dW}{dt} = \tan\theta$

Fig.(b)



$\vec{P} = P_{av} = \frac{W_2 - W_1}{t_2 - t_1} = \frac{\Delta W}{\Delta t}$

Fig.(c)

- Area under power-time graph gives the work done. $W = \int Pdt$ (See Fig. a)
- The slope of tangent at a point on work-time graph, gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work-time graph gives average power between two points (See Fig. c)

Efficiency

Machines are designed to convert energy into useful work, however because of frictional effects and other dissipative forces work performed by the machine is always less than the energy supplied to the machine.

The efficiency of a machine is given by.

$$\eta = \frac{\text{work done}}{\text{energy input}}$$

Example 33:

A body of mass m starting from rest from the origin moves along the x-axis with a constant power (P). Calculate the :

(i) relation between velocity and time

(ii) relation between distance and time

(iii) relation between velocity and distance

Solution:

(i) $P = Fv = mav = m\frac{dv}{dt}v$

$$\Rightarrow \int_0^t \frac{P}{m}dt = \int_0^v v\,dv \Rightarrow \frac{P}{m}t = \frac{v^2}{2}$$

$$\Rightarrow v = \sqrt{\frac{2P}{m}t} \Rightarrow \boxed{v \propto t^{\frac{1}{2}}} \qquad ...(1)$$

(ii) $\frac{dx}{dt} = \sqrt{\frac{2P}{m} t^{\frac{1}{2}}} \Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}} \int_0^t t^{\frac{1}{2}} dt$

$\Rightarrow x = \sqrt{\frac{2P}{m}} \frac{2}{3} \Rightarrow t^{\frac{2}{3}}$ $\boxed{x \propto t^{\frac{2}{3}}}$ ...(2)

(iii) From (1) & (2),

$\Rightarrow$ $\boxed{x \propto v^3}$ ...(3)

Example 34:

A pump is used to deliver water at a certain rate from a given pipe. By what amount should velocity of water, force on the water and power of motor be increased to obtain n times water from the same pipe in the same time?

Solution:

Amount of water flowing per unit time $\frac{dm}{dt} = Av_{\rho}$

v = velocity of flow, A is area of cross-section,

ρ = density of liquid

To get n times water in the same time,

$\left(\frac{dm'}{dt}\right) = n \frac{dm}{dt} \Rightarrow Av' \rho = nAv \rho \Rightarrow v' = nv$

$F = \frac{vdm}{dt} \Rightarrow F' = v' \frac{dm'}{dt} = n^2 v \frac{dm}{dt} = n^2 F$

To gets n times water, force must be increased n^2 times.

$\because \quad P = v^2 \frac{dm}{dt}$

So $\frac{P'}{P} = \frac{v'^2(dm'/dt)}{v^2(dm/dt)} = \frac{n^2v^2n\ dm/dt}{v^2 dm/dt} = n^3 \Rightarrow P' = n^3 P$

Thus to get n times water, the power must be increased n^3 times.

Example 35:

An engine pumps water of density, through a hose pipe. Water leaves the hose pipe with a velocity v. Find the

(i) rate at which kinetic energy is imparted to water.

(ii) power of the engine.

Solution:

(i) Rate of change of kinetic energy

$= \frac{dE_k}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2 \frac{dm}{dt} = \frac{1}{2} v^2 \frac{d}{dt}(\rho Ax)$

$= \frac{1}{2}\rho Av^2 \frac{dx}{dt} = \frac{1}{2} \rho Av^3$

(ii) Power

$= Fv = \left(v\frac{dm}{dt}\right)v = v^2 \frac{dm}{dt} = v^2 (\rho Av) = \rho Av^3$

Example 36:

A pump can take out 7200 kg of water per hour from a 100 m deep well. Calculate the power of the pump assuming that its efficiency is 50%. ($g = 10\ m/s^2$)

Solution:

Output power = $\frac{mgh}{t} = \frac{7200 \times 10 \times 100}{3600}$ = 2000 W

Efficiency $\eta = \frac{\text{output power}}{\text{input power}}$

Input power = $\frac{\text{output power}}{\eta} = \frac{2000 \times 100}{50}$ = 4kW

Example 37:

The force required to tow a boat at constant velocity is proportional to the speed. If a speed of 4.0 km/h requires 7.5 kW, how much power does a speed of 12 km/h require?

Solution:

Let the force be $F = \alpha v$, where v is speed and α is a constant of proportionality.

The power required is

$P = Fv = \alpha v^2$

Let P_1 be the power required for speed v_1 and P_2 be the power required for speed v_2.

P_1 = 7.5 kW and $v_2 = 3v_1$,

$P_2 = \left(\frac{v_2}{v_1}\right)^2 P_1 \Rightarrow P_2 = (3)^2 \times 7.5$ kW = 67.5 kW.

Example 38:

A truck of mass 10,000 kg moves up an inclined plane rising 1 in 50 with a speed of 36 km/h. Find the power of the engine ($g = 10\ m/s^2$).

Solution:

Force against which work is done

$F = mg \sin\theta = 10{,}000 \times 10 \times \frac{1}{50}$ = 2000 N

Speed $v = \frac{36 \times 5}{18}$ = 10 m/s

so P = 2000 × 10 = 20 kW.

Example 39:

An engine can pull 4 coaches at a maximum speed of 20 m/s. Mass of the engine is twice the mass of every coach. Assuming resistive forces to be proportional to the weight, approximate maximum speeds of the engine when it pulls 12 and 6 coaches are :

(1) 8.5 m/s and 15 m/s respectively (2) 6.5 m/s and 8 m/s respectively

(3) 8.5 m/s and 13 m/s respectively (4) 10.5 m/s and 15 m/s respectively

Solution:

Since power of Engine will be Treated as same

So, $F_1v_1 = F_2v_2$

Let weight of coaches be w so weight of engine is 2w

For 12 coaches

Now, $[4(w) + 2w] (20) = [12(w) + 2w]v$

$\Rightarrow v = 8.5$ m/s

For 6 coaches

$[4(w) + 2w] (20) = [6(w) + 2(w)] v$

$\Rightarrow v = 15$ m/s

Concept Builder-7

Q.1 A body of mass 2 kg starts from rest with a constant power of 25 watt, if velocity of the body is 3 m/s then find the displacement.

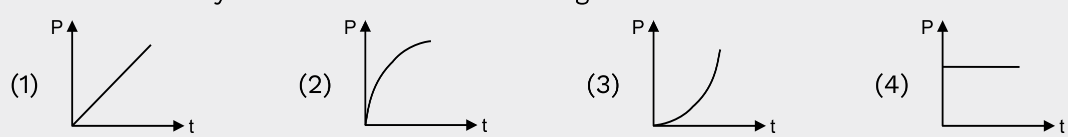
Q.2 A car of mass 2 kg has an engine which can deliver constant power 5 watt. Find the maximum speed that the car can attain in 5 seconds.

Q.3 To double the flow rate inside a pipe, by what factor, power of motor & force on water must be increased.

Q.4 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Q.5 An engine pumps up 100 kg of water through a height of 10 m in 5 s. If the efficiency of the engine is 60%, find out the power of the engine.
($g = 10$ m/s^2).

Q.6 A motor drives a body along a straight line with a constant force. The power P developed by the motor must vary with time t as shown in figure.



ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. 5 J **2.** –15 J

3. $\theta = 60°$ **4.** 1 J

5. (3) **6.** mgh

7. (1)

CONCEPT BUILDER-2

1. $W = x_2^2 - x_1^2$ **2.** 288 J

3. 21 J **4.** 10 Unit

5. 3 J **6.** 40 J

CONCEPT BUILDER-3

1. (2)

2. (2) Work done = Area under the F-S curve

3. (a) 75 J (b) –25 J (c) 50 J

CONCEPT BUILDER-4

1. $\Delta\vec{v} = 8\hat{j}$, ΔK = Zero

2. 30% **3.** 125%

4. 6 units **5.** 3150 N

6. –1500 J **7.** 13.3 m

8. –0.28 mgH

CONCEPT BUILDER-5

1. h = 8 m **2.** $\frac{3}{8}$ mgL

3. 8 m/s **4.** 4 m

CONCEPT BUILDER-6

1. $v_B = \sqrt{10}$ m/s. **2.** (2)

3. (1) **4.** $49x_m^2 - x_m - 4 = 0$

5. $\frac{1}{2}ky(y + 2x)$ **6.** 0.125 J

7. $\sqrt{\frac{K_1}{K_2}}$ **8.** 3 W

9. (1)

CONCEPT BUILDER-7

1. $\frac{18}{25}$ m **2.** 5 m/s

3. 8P & 4F **4.** 43.6 KW

5. 3.33 KW **6.** (1)

Exercise - I

Work Done by a Constant Force

1. A force $\vec{F} = \left(2\hat{i} - \hat{j} + 4\hat{k}\right)$ N displaces a particle upto $\vec{d}$ = $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$m, calculate work done.

(1) Zero (2) 8 J
(3) 4 J (4) 12 J

2. A body of mass m is displaced from point A(3, 1, 2) to point B(4, 3, 3) under the effect of a force $\vec{F} = \left(3\hat{i} + 2\hat{j} - \hat{k}\right)$N, calculate work done by the force.

(1) 57 J (2) 6 J
(3) 0 (4) 22 J

3. A person of mass m is standing on one end of a plank of mass M and length L and floating in water. The person moves from one end to another and stops. Work done by normal force is :

(1) MgL (2) mgL
(3) $\frac{mMgL}{M+m}$ (4) 0

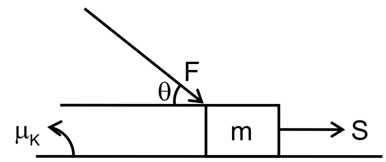
4. A 2 kg mass lying on a table is displaced in the horizontal direction through 50 cm. The work done by the normal reaction will be–

(1) 0 (2) 100 joule
(3) 100 erg (4) 10 joule

5. A body of mass M tied to a string is lowered at a constant acceleration of (g/4) through a vertical distance h. The work done by the string will be...

(1) $\frac{3}{4}$ Mgh (2) $\frac{1}{4}$Mgh
(3) $\frac{-3}{4}$Mgh (4) $\frac{-1}{4}$ Mgh

6. Find work done by friction for displacement 'S'?



(1) μ_K(mg + Fsinθ).S (2) $-\mu_K$(mg + Fsinθ).S
(3) μ_K(mg – Fsinθ).S (4) $-\mu_K$(mg – Fsinθ).S

7. The work done against gravity in taking 10 kg mass at 1 m height in 1 s will be –

(1) 49 J (2) 98 J
(3) 196 J (4) None of these

8. A force of 10N displaces an object by 10m. If work done is 50J then direction of force make an angle with direction of displacement will be :-

(1) 120° (2) 90°
(3) 60° (4) None of these

9. A body of mass 5 kg under a force which causes displacement in it given S = $\frac{t^2}{4}$ metres where 't' is time. The work done by the force in 4 seconds is :-

(1) 12J (2) 20J
(3) 6J (4) 10J

10. A force acts $\vec{F} = (5\hat{i} + 4\hat{j})$N on a body and produced a displacement 2m along x-axis. The work done will be

(1) 30 J (2) 40 J
(3) 10 J (4) 20 J

11. A stone of mass m is tied to a string of length l at one end and by holding second end it is whirled into a horizontal circle, then work done will be–

(1) 0 (2) $\left(\frac{mv^2}{\ell}\right)2\pi\ell$
(3) (mg) · $2\pi\ell$ (4) $\left(\frac{mv^2}{\ell}\right)\ell$

12. A particle is moved from a position $\vec{r}_1 = (3\hat{i} + 2\hat{j} - 6\hat{k})$ metre to a position $\vec{r}_2 = (14\hat{i} + 13\hat{j} - 9\hat{k})$ metre under the action of a force $\vec{F} = 4\hat{i} + \hat{j} + 3\hat{k}$. The work done is equal to :

(1) 32 J (2) 64 J
(3) 96 J (4) 46 J

13. A body of mass 10 kg is displaced from point A(2, 1, 3) to point B(3, 3, 4) under the effect of a force of magnitude 20 N in the direction of $\left(6\hat{i} + 8\hat{j}\right)$. Calculate W.D. by the force :-

(1) 22 J (2) $20\sqrt{6}$ J
(3) 44 J (4) Zero

14. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is :

(1) Mgx (2) $\frac{1}{2}Mgx^2$
(3) $-\frac{1}{2}Mgx$ (4) Mgx^2

Work Done by Variable Force and Graphical Method of Work Calculation

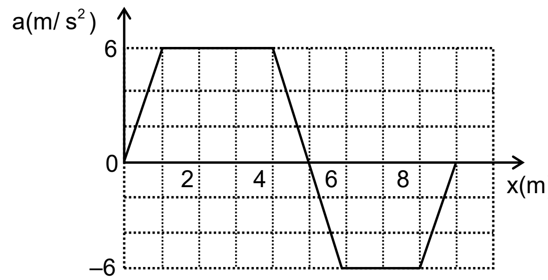
15. A force $\vec{F} = (3x^2 + 2x - 7)$N acts on a 2kg body as a result of which the body gets displaced from x = 0 to x = 5m. The work done by the force will be :

(1) 35 J (2) 70 J
(3) 115 J (4) 270 J

16. A force $F = Kx^2$ acts on a particle at an angle of 60° with the x-axis. The work done in displacing the particle from x_1 to x_2 will be :

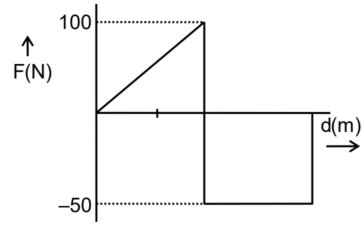
(1) $\frac{kx^2}{2}$ (2) $\frac{k}{2}\left(x_2^2 - x_1^2\right)$
(3) $\frac{k}{6}\left(x_2^3 - x_1^3\right)$ (4) $\frac{k}{3}\left(x_2^3 - x_1^3\right)$

17. Figure gives the acceleration of a 2.0 kg body as it moves from rest along x axis while a variable force acts on it from x = 0m to x = 9m. The work done by the force on the body when it reaches
(i) x = 4m and (ii) x = 7m shall be as given below

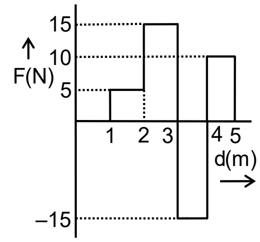


(1) 21 J and 33 J respectively
(2) 21 J and 15 J respectively
(3) 42 J and 60 J respectively
(4) 42 J and 30 J respectively

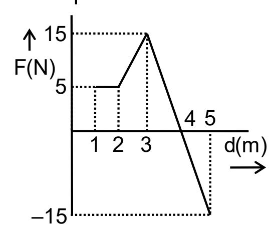
18. Calculate the work done for following F-d curves

(A)


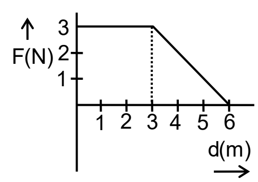
(P) 100 J

(B)


(Q) 13.5 J

(C)


(R) 15 J

(D)


(1) (A – P); (B – Q); (C – Q); (D – R)
(2) (A – P); (B – R); (C – R); (D – Q)
(3) (A – P); (B – P); (C – Q); (D – R)
(4) (A – P); (B – P); (C – R); (D – Q)

19. A force acts on a 30 gm particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 second is:

(1) 5.28 J (2) 450 mJ

(3) 490 mJ (4) 530 mJ

Kinetic Energy, Relation Between Kinetic Energy and Power

20. If the momentum of a body is increased n times, its kinetic energy increases.

(1) n times (2) 2n times

(3) $\sqrt{n}$ time (4) n^2 times

21. If K.E. increases by 3%. Then momentum will increase by–

(1) 1.5% (2) 9%

(3) 3% (4) 2%

22. If K.E. body is increased by 100%. Then percentage change in 'P'.

(1) 50% (2) 41.4%

(3) 10% (4) 20%

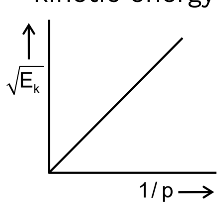
23. Two particles of mass 1 kg and 5 kg have same momentum, calculate ratio of their K.E. will be :

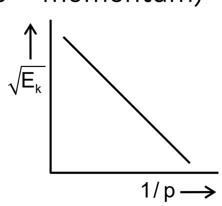
(1) 5 : 1 (2) 25 : 1

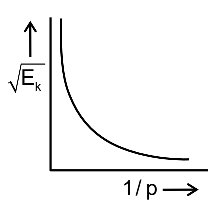
(3) 1 : 1 (4) 10 : 1

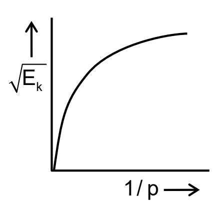
24. The graph between $\sqrt{E_k}$ and $\frac{1}{p}$ is

(E_k = kinetic energy and p = momentum) –

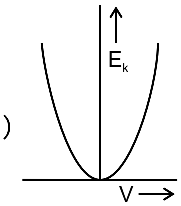
(1)


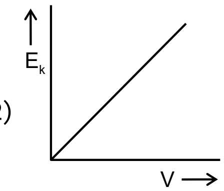
(2)


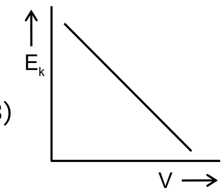
(3)


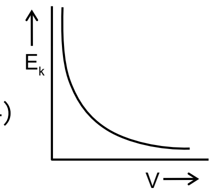
(4)


25. The graph between kinetic energy E_k and velocity V is -

(1)


(2)


(3)


(4)


26. If the kinetic energy of a body is double of its initial kinetic energy, then the momentum of the body will be :-

(1) $2\sqrt{2}$ times

(2) $\sqrt{2}$ times

(3) $\frac{1}{2}$ times

(4) none of these

27. A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each towards earth, their respective kinetic energies will be in the ratio of–

(1) 1 : 4 (2) 1 : 2

(3) 1 : $\sqrt{2}$ (4) $\sqrt{2}$: 1

28. If the kinetic energy is increased by 300%, the momentum will increase by :-

(1) 100% (2) 200%

(3) 150% (4) 300%

29. If the momentum of a certain body is increased by 50%, its kinetic energy will be increased by:-

(1) 25% (2) 50%

(3) 100% (4) 125%

Work Kinetic Energy Theorem

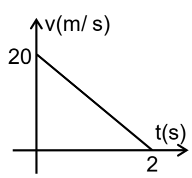
30. A force acts on a 100 g particle in such a way that the position of the particle as a function of time is given by $x = 6t - 2t^2 + 2$, where x is in metres and t is in seconds. The work done during the first 2 second is:

(1) –1.6 J (2) –1600 J
(3) 1.6 J (4) 1600 J

31. A constant force F is applied to a body of mass m moving with initial velocity u. If after the body undergoes a displacement S, its velocity becomes v, then the total work done is :-

(1) $m[v^2+u^2]$ (2) $\frac{m}{2}[u^2+v^2]$
(3) $\frac{m}{2}[v^2-u^2]$ (4) $m[v^2 - u^2]$

32. Velocity-time graph of a particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all the forces on the particle is:



(1) 400 J (2) – 400 J
(3) – 200 J (4) 200 J

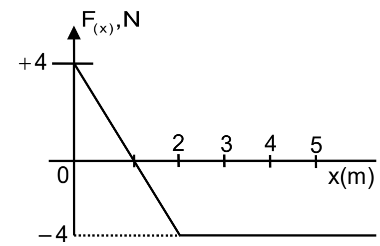
33. At time t = 0 s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :-

(1) $\sqrt{t}$
(2) constant
(3) t
(4) $\frac{1}{\sqrt{t}}$

34. A 0.5 kg ball is thrown up with an initial speed 14 m/s and reaches a maximum height of 8.0 m. How much energy is dissipated by air drag acting on the ball during the ascent?

(1) 19.6 joules (2) 4.9 joules
(3) 10 joules (4) 9.8 joules

35. The only force F_x acting on a 2.0 kg body as it moves along the x-axis varies as shown in the figure. The velocity of the body along positive x-axis at x = 0 is 4 m/s. The kinetic energy of the body at x = 3.0 m is:-



(1) 4 J (2) 8 J
(3) 12 J (4) 16 J

Conservative and Nonconservative Forces

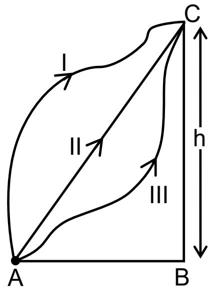
36. Which of the following statements is true for work done by conservative forces :-

(1) It does not depend on path
(2) It is equal to the difference of initial and final potential energy function
(3) It can be recovered completely
(4) All of the above

37. Which of the following statement is incorrect for a conservative field ?

(1) Work done is going from initial to final position is equal to change in kinetic energy of the particle.
(2) Work done depends on path but not on initial and final positions.
(3) Work done does not depend on path but depends only on initial and final positions
(4) Work done on a particle in the field for a round trip is zero.

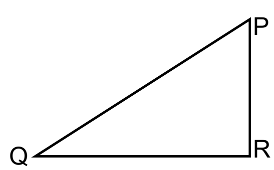
38. As shown in the diagram a particle is to be carried from point A to C via paths (I), (II) and (III) in gravitational field, then which of the following statements is correct:-



(1) Work done is same for all the paths
(2) Work done is minimum for path (II)
(3) Work done is maximum for path (I)
(4) None of the above

39. Which of the following is a non-conservative force:-
(1) Electric force
(2) Gravitational force
(3) Spring force
(4) Viscous force

40. For the path PQR in a conservative force field the amounts work done in carrying a body from P to Q and from Q to R are 5 Joule and 2 Joule respectively. The work done in carrying the body from P to R will be –

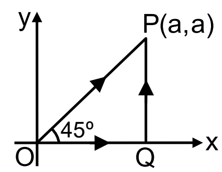


(1) 7 J (2) 3 J
(3) $\sqrt{21}$ J (4) Zero

41. A 10 kg satellite completes one revolution around the earth at a height of 100 km in 108 minutes. The work done by the gravitational force of earth will be–

(1) 108 × 100 × 10 J (2) $\frac{108 \times 10}{100}$ J
(3) 0 J (4) $\frac{100 \times 10}{108}$ J

42. A particle is moved from (0, 0) to (a, a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :



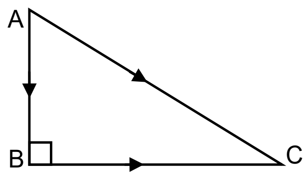
(1) $W_1 = W_2$ (2) $W_1 = 2W_2$
(3) $W_2 = 2W_1$ (4) $W_2 = 4W_1$

43. Which of the following statements is incorrect for work done by conservative forces :–
(1) Work done is path independent
(2) The work done to bring a body from infinity to a point is the potential energy at that point
(3) Work done by such forces is non zero if body returns to its initial position
(4) All of the above

44. Which of the following statement is correct for a non conservative field ?
(1) Work done on a particle in the field for a round trip is zero.
(2) Work done depends on path but not on initial and final positions.
(3) Work done does not depend on path but depends only on initial and final positions
(4) None of the above

45. The work done by the frictional force on a pencil in drawing a complete circle of radius r = 1/π metre on the surface by pencil of negligible mass with a normal pressing force N = 5 N (μ = 0.5) is :
(1) + 4J (2) –3 J
(3) – 2 J (4) – 5J

46. The work done in moving a particle under the effect of a conservative force, from position A to B is 3 joule and from B to C is 4 joule. The work done in moving the particle from A to C is :



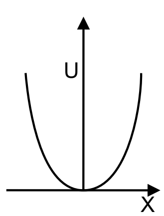
(1) 5 joule (2) 7 joule
(3) 1 joule (4) –1 joule

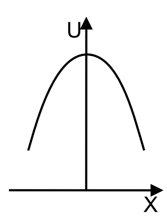
Potential Energy, Change in potential Energy, Potential Energy curve and Equilibrium

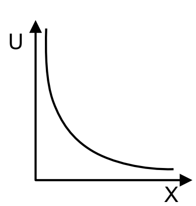
47. The mass of a bucket full of water is 15 kg. It is being pulled up from a 15m deep well. Due to a hole in the bucket 6 kg water flows out of the bucket. The work done in drawing the bucket out of the well will be ($g = 10 m/s^2$) –

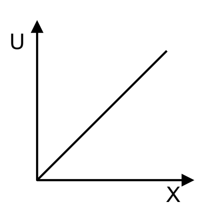
(1) 900 Joule
(2) 1500 Joule
(3) 1800 Joule
(4) 2100 Joule

48. The graph between potential energy U and displacement x in the state of stable equilibrium will be :-

(1) 

(2) 

(3) 

(4) 

49. A particle moves in a potential region given by $U = 8x^2 - 4x + 400$ J. Its state of equilibrium will be

(1) x = 25 m (2) x = 0.25 m
(3) x = 0.025 m (4) x = 2.5 m

50. If the potential energy of two molecules is given by, $U = \frac{A}{r^{12}} - \frac{B}{r^6}$ then at equilibrium position, its potential energy is equal to:

(1) $\frac{A^2}{4B}$ (2) $-\frac{B^2}{4A}$
(3) $\frac{2B}{A}$ (4) 3A

Conservation of Mechanical Energy

51. A body of mass 2 kg falls from a height of 20m. What is the loss in potential energy ?

(1) 400 J (2) 300 J
(3) 200 J (4) 100 J

52. 4 J of work is required to stretch a spring through 10 cm beyond its unstretched length. The extra work required to stretch it through additional 10 cm shall be

(1) 4 J (2) 8 J
(3) 12 J (4) 16 J

53. A body is dropped from a height h. When loss in its potential energy is U then its velocity is v. The mass of the body is –

(1) $\frac{U^2}{2v}$ (2) $\frac{2v}{U}$
(3) $\frac{2v}{U^2}$ (4) $\frac{2U}{v^2}$

54. A stone is projected vertically up to reach maximum height 'h'. The ratio of its kinetic energy to potential energy, at a height $\frac{4h}{5}$ will be :-

(1) 5 : 4 (2) 4 : 5
(3) 1 : 4 (4) 4 : 1

55. A uniform chain of length L and mass M is lying on a smooth table and $\frac{2}{3}$ of its length is hanging down over the edge of the table. If g is the acceleration due to gravity, the work done to pull the hanging part on the table is:-

(1) MgL (2) $\frac{MgL}{3}$

(3) $\frac{MgL}{9}$ (4) $\frac{2MgL}{9}$

56. A projectile is fired at 30° with momentum p, neglecting friction, the change in kinetic energy, when it returns back to the ground, will be :-

(1) zero (2) 30%

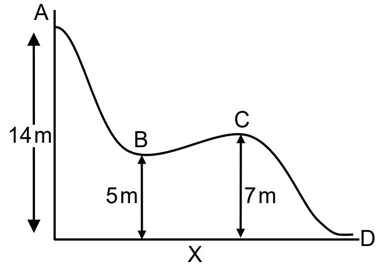
(3) 60% (4) 100%

57. A ball is dropped from a height of 10m. If 40% of its energy is lost on collision with the earth then after collision the ball will rebound to a height of–

(1) 10 m (2) 8 m

(3) 4 m (4) 6 m

58. Figure shows the vertical section of frictionless surface. A block of mass 2 kg is released from the position A; its KE as it reaches the position C is :-



(1) 180J (2) 140J

(3) 40J (4) 280J

59. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is

(1) – 0.5 J (2) –1.25 J

(3) 1.25 J (4) 0.5 J

Spring Potential Energy and Spring Block System

60. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is:-

(1) 16 J (2) 8 J

(3) 32 J (4) 24 J

61. If a spring extends by x on loading then energy stored by the spring is :

(T is tension in the spring, K = spring constant)

(1) $\frac{T^2}{2x}$ (2) $\frac{T^2}{2k}$

(3) $\frac{2k}{T^2}$ (4) $\frac{2T^2}{k}$

62. In stretching a spring by 2cm energy stored is given by U, then stretching by 10 cm energy stored will be :-

(1) U (2) 5U

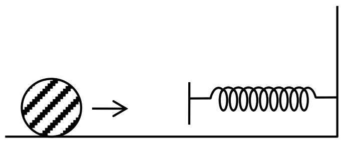
(3) $\frac{U}{25}$ (4) 25U

63. A block of mass 16 kg is moving on a frictionless horizontal surface with velocity 4m/s and comes to rest after pressing a spring. If the force constant of the spring is 100 N/m then the compression in the spring will be :-

(1) 3.2 m (2) 1.6 m

(3) 0.6 m (4) 6.1 m

64. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k=50N/m. The maximum compression of the spring would be :-



(1) 0.12 m (2) 1.5 m
(3) 0.5 m (4) 0.15 m

65. An ideal spring with spring–constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is :

(1) $\frac{4Mg}{k}$
(2) $\frac{2Mg}{k}$
(3) $\frac{Mg}{k}$
(4) $\frac{Mg}{2k}$

66. A spring of force constant 100 N/m has an extension of 10 cm. The work done in extending it from 10 cm to 20 cm and then bringing it back to 15 cm is :

(1) 6250 J (2) 0.625 J
(3) −0.625 J (4) −6250 J

67. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is-

(1) 12.50 N-m (2) 18.75 N-m
(3) 25.00 N-m (4) 6.25 N-m

Power

68. An electric motor produces a tension of 4500N in a load lifting cable and rolls it at the rate of 2 m/s. The power of the motor is:-

(1) 9 kW (2) 15 kW
(3) 225 kW (4) 9×10^3 hp

69. A motor vehicle of 100 hp is moving a car with a constant velocity of 72 km/hour. The forward force exerted by the engine on the car is :

(1) 3.73×10^3 N
(2) 3.73×10^2 N
(3) 3.73×10^1 N
(4) None of the above

70. A crane lifts 300 kg weight from earth's surface upto a height of 2m in 3 seconds. The average power generated by it will be:

(1) 1960 W (2) 2205 W
(3) 4410 W (4) 0 W

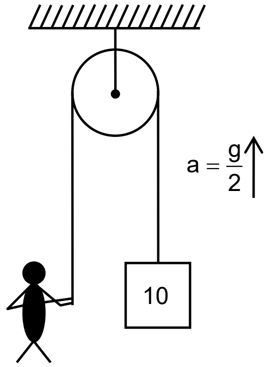
71. Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is :-

(1) $\frac{15}{11}$ (2) $\frac{11}{15}$
(3) $\frac{11}{9}$ (4) $\frac{9}{11}$

72. A car is moving with a speed of 40 km/hr. If the car engine generates 7 kilowatt power, then the resistance in the path of motion of the car will be

(1) 360 newton
(2) 630 newton
(3) Zero
(4) 280 newton

73. Calculate power generated by tension in the string in first 2 seconds of motion :-



(1) 250 W (2) 500 W
(3) 750 W (4) 1000 W

74. A body of mass m starting from rest from origin moves along x-axis with constant power (P). Calculate relation between velocity–distance.

(1) $x \propto v^{1/2}$ (2) $x \propto v^2$
(3) $x \propto v$ (4) $x \propto v^3$

75. A pump is used to deliver water at a certain rate from a given pipe. To obtain 3 times water from the same pipe in the same time, by what factor, the force of the motor should be increased?

(1) 3 times
(2) 9 times
(3) $\sqrt{3}$ times
(4) $\frac{1}{3}$ times

76. A body of mass 4 kg is moving up an inclined plane rising 1 in 40 with velocity 40 m/sec if efficiency is 50% the calculate power required.

(1) 38.4 W
(2) 55 W
(3) 78.4 W
(4) 108 W

77. A 1.0 hp motor pumps out water from a well of depth 20 m and fills a water tank of volume 2238 liters at a height of 10 m from the ground. The running time of the motor to fill the empty water tank is ($g = 10\ ms^{-2}$)

(1) 5 minutes (2) 10 minutes
(3) 15 minutes (4) 20 minutes

78. Water is falling on the blades of a turbine at a rate of 100 kg/s from a certain spring. If the height of the spring be 100 metres, the power transferred to the turbine will be :-

(1) 100 kW (2) 10 kW
(3) 1 kW (4) 1000 kW

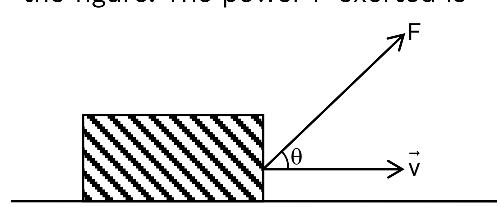
79. If the force applied is F and the velocity gained is v, then the average power developed is:-

(1) $\frac{F}{v}$ (2) $\frac{v}{F}$
(3) $\frac{Fv}{2}$ (4) Fv^2

80. What average horsepower is developed by an 80kg man while climbing in 10 s flight of stairs that rises 6 m vertically?

(1) 0.63 hp (2) 1.26 hp
(3) 1.8 hp (4) 2.1 hp

81. A constant force is acting on a body of mass m with constant velocity as shown in the figure. The power P exerted is



(1) $Fv \cos \theta$ (2) $\frac{F \cos \theta}{mg}$
(3) $\frac{Fmg \cos \theta}{v}$ (4) $\frac{mg \sin \theta}{F}$

82. A force $\vec{F} = (3\hat{i} + 4\hat{j})$ N acts on a 2 kg movable object that moves from an initial position $\vec{d}_i = (-3\hat{i} - 2\hat{j})$ m to final position $\vec{d}_f = (5\hat{i} + 4\hat{j})$ in 6s. The average power delivered by the force during the interval is equal to :

(1) 8 watt
(2) $\frac{50}{6}$ watt
(3) 15 watt
(4) $\frac{50}{3}$ watt

83. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to-

(1) $t^{3/4}$
(2) $t^{3/2}$
(3) $t^{1/4}$
(4) $t^{1/2}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	1	2	4	1	3	2	2	3	4	3	1	4	3	3	3	3	4	2	1	4	1	2	1	3	1
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	2	2	1	4	1	3	2	4	4	3	4	2	1	4	1	3	1	3	2	4	2	2	1	2	2
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	1	3	4	3	4	1	4	2	2	2	2	4	2	4	2	2	2	1	1	1	1	2	3	4	2
Que.	76	77	78	79	80	81	82	83																	
Ans.	3	3	1	3	1	1	1	2																	

Exercise - II

1. A variable force, given by the 2-dimensional vector $\vec{F} = (3x^2\hat{i} + 4\hat{j})$, acts on a particle. The force is in newton and x is in metre. What is the change in the kinetic energy of the particle as it moves from the point with coordinates (2, 3) to (3, 0)?
(The coordinates are in meters)

(1) –7 J
(2) Zero
(3) +7 J
(4) +19 J

2. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is:-

(1) $\frac{mv_1t}{t_1}$
(2) $\frac{mv_1^2t}{t_1^2}$
(3) $\frac{mv_1t^2}{t_1}$
(4) $\frac{mv_1^2t}{t_1}$

3. Work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by :-

(1) $\frac{1}{2}m\frac{v}{t_1}t^2$
(2) $m\frac{v}{t_1}t^2$
(3) $\frac{1}{2}\left(\frac{mv}{t_1}t\right)^2t^2$
(4) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$

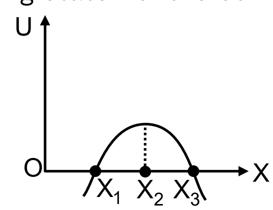
4. A particle in a certain conservative force field has a potential energy given by $U = \frac{20xy}{z}$ The force exerted on it is

(1) $\left(\frac{20y}{z}\right)\hat{i} + \left(\frac{20x}{z}\right)\hat{j} + \left(\frac{20xy}{z^2}\right)\hat{k}$
(2) $-\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} + \left(\frac{20xy}{z^2}\right)\hat{k}$
(3) $-\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} - \left(\frac{20xy}{z^2}\right)\hat{k}$
(4) $\left(\frac{20y}{z}\right)\hat{i} + \left(\frac{20x}{z}\right)\hat{j} - \left(\frac{20xy}{z^2}\right)\hat{k}$

5. A particle moves on a rough horizontal ground with some initial velocity say v_0. If $\frac{3}{4}$ of its kinetic energy is lost due to friction in time t_0 then coefficient of friction between the particle and the ground is :-

(1) $\frac{v_0}{2gt_0}$
(2) $\frac{v_0}{4gt_0}$
(3) $\frac{3v_0}{4gt_0}$
(4) $\frac{v_0}{gt_0}$

6. In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct :



(1) x_1 is in stable equilibrium
(2) x_2 is in stable equilibrium
(3) x_3 is in stable equilibrium
(4) None of these

7. A machine which is 75% efficient, uses 12 J of energy in lifting 1 kg mass through a certain distance. The mass is then allowed to fall through the same distance, the velocity at the end of its fall is:-

(1) $\sqrt{12}$ m / s
(2) $\sqrt{18}$ m / s
(3) $\sqrt{24}$ m / s
(4) $\sqrt{32}$ m / s

8. The potential energy of a 1 kg particle free to move along the x-axis is given by $U(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$J. The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is-

(1) $3/\sqrt{2}$
(2) $\sqrt{2}$
(3) $1/\sqrt{2}$
(4) 2

9. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10m. The work done against friction is :

(take g = 10 m/s^2)

(1) zero (2) 100 J

(3) 200 J (4) 300 J

10. A body of mass 3 kg is under a constant force which causes a displacement s in metres in it, given by the relation s = $\frac{1}{3}t^2$, where t is in seconds. Work done by the force in 2 seconds is:-

(1) $\frac{5}{19}$ J (2) $\frac{3}{8}$ J

(3) $\frac{8}{3}$ J (4) $\frac{19}{5}$ J

11. A block of mass M is attached to the lower end of a vertical spring. the spring is hung from a ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched, the maximum extension produced in the length of the spring will be:-

(1) Mg/2k (2) Mg/k

(3) 2 Mg/k (4) 4 Mg/k

12. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water :-

(1) $\frac{1}{2}m^2 v^2$

(2) $\frac{1}{2}$ mv^3

(3) mv^3

(4) $\frac{1}{2}$ mv^2

13. A body of mass 1 kg is thrown upwards with a velocity 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? (g = 10 m/s^2)

(1) 10 J

(2) 20 J

(3) 30 J

(4) 40 J

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	3	2	4	2	1	4	2	1	2	3	3	2	2

Exercise – III (Previous Year Question)

1. An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?

[AIPMT-2010]

(1) 800 W

(2) 400 W

(3) 200 W

(4)100 W

2. The potential energy of a system increases if work is done : **[AIPMT-2011]**

(1) Upon the system by a nonconservative force

(2) By the system against a conservative force

(3) By the system against a nonconservative force

(4) Upon the system by a conservative force

3. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest : **[AIPMT-2011]**

(1) At the highest position of the body

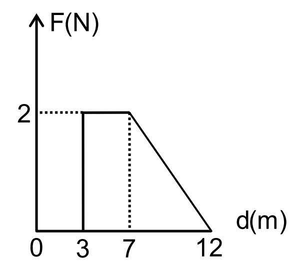
(2) At the instant just before the body hits the earth

(3) It remains constant all through

(4) At the instant just after the body is projected

4. Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is:

[AIPMT-2011]



(1) 18 J (2) 21 J

(3) 26 J (4) 13 J

5. The potential energy of a particle in a force field is : $U = \frac{A}{r^2} - \frac{B}{r}$

where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is :

[AIPMT (Pre) 2012]

(1) A/B (2) B/A

(3) B/2A (4) 2A/B

6. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to:

[AIPMT (Mains) 2012]

(1) $t^{-1/2}$ (2) $t/\sqrt{m}$

(3) t^2P_0 (4) $t^{1/2}$

7. A uniform force of $(3\hat{i}+\hat{j})$ newton acts on a particle of mass 2kg. Hence the particle is displaced from position $(2\hat{i}+\hat{k})$ meter to position $(4\hat{i}+3\hat{j}-\hat{k})$ meter. The work done by the force on the particle is:

[NEET (UG) 2013]

(1) 15 J (2) 9 J

(3) 6 J (4) 13 J

8. A block of mass 10 kg, moving in x direction with a constant speed of 10 ms^{-1}, is subjected to a retarding force F = 0.1x J/m during its travel from x = 20 m to 30 m. Its final KE will be: **[AIPMT-2015]**

(1) 450 J (2) 275 J

(3) 250 J (4) 475 J

9. A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest the force on the particle at time t is :- **[AIPMT-2015]**

(1) $\sqrt{mk}\,t^{-1/2}$ (2) $\sqrt{2mk}\,t^{-1/2}$

(3) $\frac{1}{2}\sqrt{mk}\,t^{-1/2}$ (4) $\sqrt{\frac{mk}{2}}\,t^{-1/2}$

10. A boy of mass 50 kg jumps to a height of 0.8 m from the ground then momentum transferred by the ground to boy is: **[AIIMS-2015]**

(1) 400 kg m/s (2) 200 kg m/s

(3) 800 kg m/s (4) 500 kg m/s

11. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis. What power will be developed by the force at the time t ? **[NEET- I - 2016]**

(1) $(2t^2 + 3t^3)$ W (2) $(2t^2 + 4t^4)$ W

(3) $(2t^3 + 3t^4)$ W (4) $(2t^3 + 3t^5)$ W

12. A particle moves from a point $(-2\hat{i} + 5\hat{j})$ to $(4\hat{j} + 3\hat{k})$ when a force of $(4\hat{i} + 3\hat{j})$ N is applied. How much work has been done by the force ? **[NEET- II - 2016]**

(1) 5 J (2) 2 J

(3) 8 J (4) 11 J

13. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take 'g' constant with a value 10 m/s^2. The work done by the (i) gravitational force and the (ii) resistive force of air is : **[NEET - 2017]**

(1) (i) – 10 J (ii) – 8.25 J

(2) (i) 1.25 J (ii) – 8.25 J

(3) (i) 100 J (ii) 8.75 J

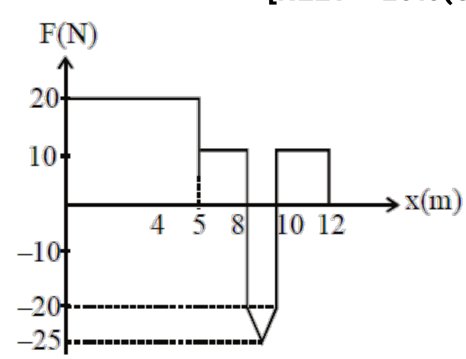
(4) (i) 10 J (ii) – 8.75 J

14. A force F = 20 + 10y acts on a particle in y-direction where F is in newton and y in meter. Work done by this force to move the particle from y = 0 to y = 1 m is : **[NEET - 2019]**

(1) 30 J (2) 5 J

(3) 25 J (4) 20 J

15. An object of mass 500g, initially at rest is acted upon by a variable force, whose X component varies with x in the manner shown. The velocities of the object at point X = 8 m and X =12 m, would be the respective values of (nearly) **[NEET – 2019(Odisha)]**



(1) 18 m/s and 24.4 m/s

(2) 23 m/s and 24.4 m/s

(3) 23 m/s and 20.6 m/s

(4) 18 m/s and 20.6 m/s

16. A particle is released from height S from the surface of the Earth. At a certain height its kinetic energy is three times its potential energy. The height from the surface of earth and the speed of the particle at that instant are respectively :

[NEET - 2021]

(1) $\frac{S}{4}, \frac{3gS}{2}$

(2) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$

(3) $\frac{S}{2}, \frac{\sqrt{3gS}}{2}$

(4) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$

17. Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine? ($g = 10\ m/s^2$)

[NEET - 2021]

(1) 10.2 kW (2) 8.1 kW

(3) 12.3 kW (4) 7.0 kW

18. An electric lift with a maximum load of 2000 kg (lift + passengers) is moving up with a constant speed of 1.5 ms^{-1}. The frictional force opposing the motion is 3000N. The minimum power delivered by the motor to the lift in watts is : ($g = 10\ ms^{-2}$) **[NEET - 2022]**

(1) 23000 (2) 20000

(3) 34500 (4) 23500

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	1	2	2	4	4	4	2	4	4	2	4	1	4	3	3	4	2	3