CBSE 12th Maths

Chapter - 2 Inverse Trigonometric Functions

Competency-Based Questions 2024-25

Multiple Choice Questions :

Q1. What is the domain of the function $y = \sec^{-1} x + \sin^{-1} x$?

1. -1 and 1

2. [-1, 1]

3. (-∞, -1] U [1, ∞)

4. Φ

Ans. 1. -1 and 1

Q2.

Given: $\tan^{-1}\left(\sqrt{1+\tan^2\theta} \times \sin(\pi - \theta)\right) = \dfrac{1-a^2}{(1+a)^2}$

where $a \in R$ and $a \neq -1$

Which of the following gives the value of θ in terms of a?

$\tan\left(\dfrac{a-1}{a+1}\right)$	$\left(\dfrac{1+a}{1-a}\right)$	$\dfrac{1-a^2}{(1+a)^2}$	$\left(\dfrac{a-1}{1+a}\right)$
Expression 1	Expression 2	Expression 3	Expression 4

1. Expression 1

2. Expression 2

3. Expression 3

4. Expression 4

Ans. 3. Expression 3

Q3. Which of the following is the domain of the function given below?

$y = \cos^{-1}(1/x-3)$

1. [-1, 1]

2. [2, 4]

3. (-∞, 2] U [4, ∞)

4. (-∞, -1] U [1, ∞)

Ans. 3. (-∞, 2] U [4, ∞)

Q4. The domain of f (x) = $\sin^1$ x - $\cos^1$ x is [-1, 1] while its range is [-3π/2, π/2].

If g (x) is the inverse of f (x), which of the following is true about the domain of the function g (x)?

1. It is [-1, 1].

2. It is [-3π/2, π/2].

3. It is independent of the domain and range of f (x).

4. (cannot be said without knowing g(x))

Ans. 2. It is [-3π/2, π/2].

Q5. If $\sec^{-1}$ (- x) = π/8, which of the following could be the value of sec-1 (x)?

1. (-π/8)

2. 7π/8

3. 9π/8

4. (cannot be determined without knowing the value of x)

Ans. 2. 7π/8

Q6. Which of the following is equal to -$\tan^{-1}$(2/3π)?

1. $\cot^{-1}$ (3π/2)

2. -$\cot^{-1}$ (3π/2)

3. π/2 - $\cot^{-1}$ (3π/2)

4. π/2 + $\cot^{-1}$ (3π/2)

Ans. 2. -$\cot^{-1}$ (3π/2)

Free Response Questions :

Q7. Akash says that, since the domain of the sine function is (-∞, ∞), $\sin^{-1}$ x is well defined in the domain (-∞, ∞).

Is Akash right or wrong? Justify your answer.

Ans. Writes that Akash is wrong.

Writes that $\sin^{-1}$ x cannot be defined in the domain (-∞, ∞) as sin x is not one-one in that domain.

(Award full marks for any other valid reason.)

Q8. Simplify:

cos(π/2 + $\sin^{-1}$ 1/√3)

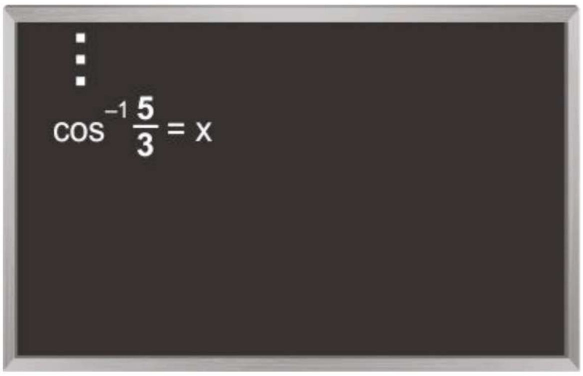
Show your work.

Ans. Simplifies the above expression as:

cos(π/2 + $\sin^{-1}$ 1/√3) = -sin($\sin^{-1}$ 1/√3)

Simplifies the above expression as (- 1/√3).

Q9. While solving an inverse trigonometry problem on the blackboard, Satish wrote the following as part of his solution:



His teacher stopped him and said that he must have made a mistake in the solution.

How did the teacher recognise that Satish had made a mistake? Justify your answer.

Ans. Writes that the $\cos^{-1}$ function is not defined for 5/3 as it is outside the domain of the inverse cosine function. Therefore, the teacher stopped Satish here.

Q10. cosec π/6 = cosec 5π/6 = 2, but π/6 ≠ 5π/6·

Since the cosecant function is not one-one, how can it be made invertible? Give a reason for your answer.

Ans. Writes that the cosecant function can be made invertible when its domain is restricted to [n π- π/2, n π + π/2] - {0}, where n is an integer.

Q11. Prove that:

$$\sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right] = 2\tan^{-1}(2^{x}),\ \text{where } x \le 0$$

Ans. Rewrites the LHS of the given equation as:

$$\sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right] = \sin^{-1}\left[\frac{2.\,2^{x}}{1+(2^{x})^{2}}\right]$$

Uses the property of inverse trigonometric functions and writes:

$$\sin^{-1}\left[\frac{2.\,2^{x}}{1+(2^{x})^{2}}\right] = 2\tan^{-1}(2^{x})$$

Q12. Chirag asked his students to find the value of:

$\tan^{-1}(-x) - \tan^{-1}(1/x)$

Rahul said that the value of the above expression can be found ONLY if x is known.

Is Rahul correct? Justify your answer.

Ans. Writes that Rahul is incorrect.

Writes that $\tan^{-1}(-x) - \tan^{-1} 1/x$ can be written as $-(\tan^{-1} x + \cot^{-1} x)$.

Finds the value of the given expression as $(-\pi/2)$ as $\tan^{-1} x + \cot^{-1} x = \pi/2$.

Q13. Considering the principal value branch, prove that the property below is true ONLY for xy > (-1).

$\tan^{-1} x - \tan^{-1} y = \tan^{-1} x-y/1+xy$

Ans. Assumes $xy < (-1)$, $x = \tan\theta$ and $y = \tan\Theta$.

Rewrites the above inequality as $\tan\theta < \tan(\Theta - \pi/2)$.

Writes that, since tangent is an increasing function in the principal value branch, $0 < (\Theta - \pi/2)$.

Uses steps 1 and 3 to write $\tan^{-1} x - \tan^{-1} y < -\pi/2$.

Uses the above step to conclude that:

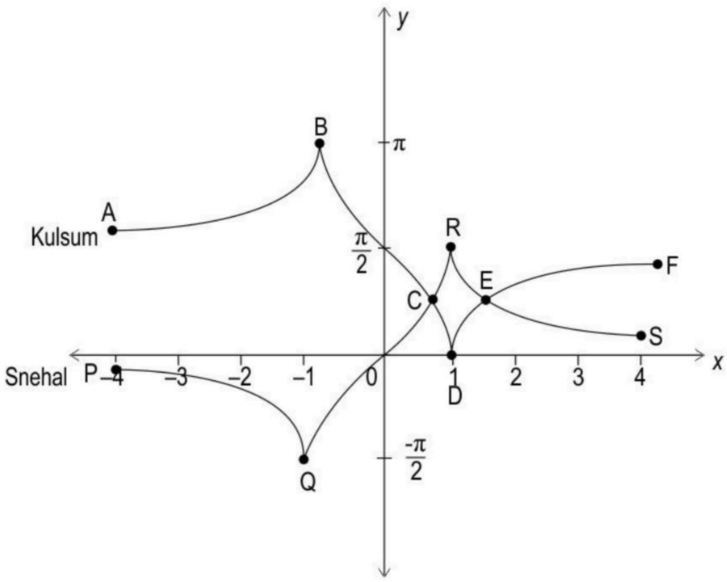
If $xy < (-1)$, the value of $\tan^{-1} x - \tan^{-1} y \notin (-\pi/2, \pi/2)$.

Hence, proves that the given property is true only for $xy > (-1)$.

Case-Based Type Questions :

Q14. Answer the questions based on the given information.

Madhu created a design on his floor using a combination of graphs of inverse trigonometric functions in the domain [-4, 4]. He also represented the coordinate axes for reference. He asked his friends, Kulsum and Snehal, to choose a path to walk on. Kulsum chose path ABCDEF, while Snehal chose path PQCRES. They both started walking at the same time and with the same speed.



1. Write the range of each of the two functions that Kulsum chose as her path to walk on.

Ans. Identifies two functions that Kulsum chose to walk on as $\cos^{-1}(x)$ and $\sec^{-1}(x)$.

Writes that the range of $\cos^{-1}(x)$ is $[0, \pi]$.

Writes that the range of $\sec^{-1}(x)$ is $[0, \pi] - \pi/2$.

2. The graphs of which of the two functions are combined to form the path that Snehal chose?

Ans. Identifies the trigonometric functions that Snehal chose to walk on as $\sin^{-1}(x)$ and $\operatorname{cosec}^{-1}(x)$.

3. What is the x -coordinate of Kulsum's and Snehal's first meeting point? Show your steps.

Ans. Mentions that both friends met at point C which is the point of intersection of two 0.5 functions, $\sin^{-1}(x)$ and $\cos^{-1}(x)$ and writes:

$\cos^{-1}(x) = \sin^{-1}(x)$

Simplifies the above equation as:

$\cos(\cos^{-1}(x)) = \cos(\sin^{-1}(x))$

$\Rightarrow x = \cos(\sin^{-1}(x))$

Substitutes $\sin^{-1}(x)$ as u and simplifies the above equation as:

$\sin u = \cos u$

$\Rightarrow \sin u = \sin(\pi/2 - u)$

$\Rightarrow u = \pi/4$

(Award full marks if $\cos^{-1}(x)$ and $\sin^{-1}(x)$ are shown to be equal at $y = \pi/4$ directly.)

Finds the x -coordinate of Kulsum's and Snehal's first meeting point as:

$x = \sin u = \sin(\pi/4) = 1/\sqrt{2}$