Chapter 9

PAIR OF STRAIGHT LINES

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. The equation $(x+y-6)(xy-3x-y+3)=0$ represents the sides of a triangle then the equation of the circumcircle of the triangle is

(a) $x^2+y^2-5x-9y+20=0$
(b) $x^2+y^2-4x-8y+18=0$
(c) $x^2+y^2-3x-5y+8=0$
(d) $x^2+y^2+2x-3y-1=0$

2. If θ_1 and θ_2 be the angles which the lines $(x^2+y^2)(\cos^2\theta\sin^2\alpha+\sin^2\theta)=(x\tan\alpha-y\sin\theta)^2$ make with the axis of x, and $\theta=\frac{\pi}{6}$, then $\tan\theta_1+\tan\theta_2$ is equal to

(a) $-\frac{8}{3}\sin\alpha$
(b) $-\frac{8}{3}\operatorname{cosec}2\alpha$
(c) $-8\sqrt{3}\operatorname{cosec}2\alpha$
(d) $-4\operatorname{cosec}2\alpha$

3. If a and b are positive numbers ($a<b$), then the range of values of K for which a real λ can be found such that the equation $ax^2+2\lambda xy+by^2+2K(x+y+1)=0$ represents a pair of straight lines is :

(a) $a<K^2<b$
(b) $a\le K^2\le b$
(c) $K^2\le a$ or $K^2\ge b$
(d) $K\le 2a$ or $K\ge 2b$

4. The line $y=3x$ bisects the angle between the lines $a^2x^2+2axy+y^2=0$ if a is equal to

(a) -3 or 3
(b) 3 or $\frac{1}{3}$
(c) $-\frac{1}{3}$ or $\frac{1}{3}$
(d) $\frac{1}{3}$ or -3

5. The area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is

(a) $\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$
(b) $\frac{n^2\sqrt{h^2-ab}}{al^2-2hlm+bm^2}$
(c) $\frac{n^2\sqrt{h^2-ab}}{al^2+2hlm+bm^2}$
(d) None of these

6. If the equation $2hxy+2gx+2fy+c=0$ represents two straight lines, then the area of a rectangle it forms with the coordinates axes is

(a) $\left|\frac{f}{g}\right|$
(b) $\frac{|fg|}{h^2}$
(c) $\frac{f^2+g^2}{h^2}$
(d) $\left|\frac{f+g}{h}\right|$

7. The image in the x-axis of the pair of lines represented by $ax^2+2hxy+by^2=0$ is given by the equation

(a) $ax^2-2hxy+by^2=0$
(b) $ax^2+2hxy-by^2=0$
(c) $ax^2-2hxy-by^2=0$
(d) $ax^2+2hxy+by^2=1$

8. The angle between the lines $(x^2+y^2)\sin^2\alpha=(x\cos\beta-y\sin\beta)^2$ is

(a) α
(b) 2α
(c) $\alpha+\beta$
(d) $2(\alpha-\beta)$

9. The lines $y=mx$ bisects the angle between the lines $ax^2+2hxy+by^2=0$ if

(a) $h(1+m^2)=m(a+b)$
(b) $h(1-m^2)=m(a-b)$
(c) $h(1+m^2)=m(a-b)$
(d) None of these

10. Two of the straight lines given by $3x^3+3x^2y-3xy^2+dy^3=0$ are at right angles if

(a) $d=-\frac{1}{3}$
(b) $d=\frac{1}{3}$
(c) $d=-3$
(d) $d=4$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ

11. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines,then the product of the perpendicular drawn from the origin to the lines is

(a) $\dfrac{|c|}{\sqrt{(a+b)^2+4h^2}}$ (b) $\dfrac{|c|}{\sqrt{(a-b)^2+4h^2}}$

(c) $\dfrac{4|c|}{\sqrt{(a-b)^2+4h^2}}$ (d) $\dfrac{|c|}{\sqrt{(a-b)^2+h^2}}$

12. If $u=ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, then equation of the third pair of straight lines through the points where these meet the axes is

(a) $au+4(fg-ah)xy=0$

(b) $bu+4(fg-bh)xy=0$

(c) $cu+4(fg-ch)xy=0$

(d) $(a+b)u+4(fg-ch)xy=0$

13. The line $lx+my=1$ intersects the circle $x^2+y^2=a^2$ at points A, B, If AB subtends 45° at the origin then $a^2(l^2+m^2)=$

(a) $4\pm2\sqrt{2}$ (b) $4\pm2\sqrt{6}$

(c) $2\sqrt{6}$ (d) $4-\sqrt{6}$

14. The equation $x^3-6x^2y+11xy^2-6y^3=0$ represents three straight lines passing through the origin, the slopes of which form *a*/*an*

(a) A.P. (b) G.P.

(c) H. P. (d) None of these

15. The pair of lines joining origin to the intersection of the curve $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ by the line $lx+my+n=0$ are coincident if

(a) $a^2l^2+b^2m^2=n^2$ (b) $\dfrac{a^2}{l^2}+\dfrac{b^2}{m^2}=\dfrac{1}{n^2}$

(c) $\dfrac{l^2}{a^2}+\dfrac{m^2}{b^2}=n^2$ (d) None of these

16. A point moves so that the distance between the feet of perpendiculars drawn from it to the lines $ax^2+2hxy+by^2=0$ is a constant $2k$. The equaiton of its locus is

(a) $(x^2+y^2)\{(a-b)^2+4h^2\}=\mathrm{k}^2(\mathrm{h}^2-\mathrm{ab})$

(b) $k^2(x^2+y^2)=\dfrac{h^2-ab}{(a-b)^2+h^2}$

(c) $x^2+y^2=\dfrac{k^2\{(a-b)^2+4h^2\}}{h^2-ab}$

(d) $(a+b)^2(x^2+y^2)=h^2-ab$

17. If two of the lines $ax^3+bx^2y+cxy^2+dy^3=0$ $(a\neq0)$ make complementary angles with x-axis in anticlockwise sense then

(a) $a(a-c)+d(b-d)=0$ (b) $a^2+ac+bd+d^2=0$

(c) $\dfrac{a^2}{c}+\dfrac{d^2}{d}=1$ (d) $(a+b+c+d)\,a\,b\,c\,d=1$

18. The equation $a(x^4+y^4)-4bxy(x^2-y^2)+6c\,x^2y^2=0$ represents two pairs of lines at right angles. The two pairs will concide if

(a) $b^2=a+3c$ (b) $a^2=b^2-3ac$

(c) $a^2+b^2=3ac$ (d) $2b^2=a^2+3ac$

19. The value of λ for which the lines joining the points of intersection of curves C_1 and C_2 to the origin are equally inclined to the axis of x, where $C_2:\lambda x^2+3y^2-2\lambda xy+9x=0$ and $C_1: 3x^2-4y^2+8xy-3x=0$, is

(a) $\lambda=\dfrac{4}{3}$ (b) $\lambda=12$

(c) $\lambda=1$ (d) $\lambda=\dfrac{5}{6}$

20. The pair of straight lines joining the origin to the points of inersection of the circles $x^2+y^2=a^2$ and $x^2+y^2+2(gx+fy)=0$ is

(a) $a^2(x^2+y^2)-2(gx+fy)^2=0$

(b) $a^2(x^2+y^2)-4(gx+fy)^2=0$

(c) $a^2(x^2+y^2)+4(gx+fy)^2=4$

(d) $a^2(x^2+y^2)-(gx+fy)^2=a^2$

21. If the slope of one of the lines represented by $ax^2+2hxy+by^2=0$ be the square of the other, then $\dfrac{a+b}{h}+\dfrac{8h^2}{ab}=$

(a) 4 (b) 6

(c) 8 (d) None of these

22. If a pair of variable straight lines $x^2+4y^2+\alpha xy=0$ (where α is a real parameter) cut the ellipse $x^2+4y^2=4$ at two points A and B, then the locus of the point of intersection of tangents at A and B is

(a) $x^2-4y^2+8xy=0$ (b) $(2x-y)(2x+y)=0$

(c) $x^2-4y^2+4xy=0$ (d) $(x-2y)(x+2y)=0$

MARK YOUR RESPONSE					
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ			

23. The pair of straight lines represented by the equation $2x^2+2y^2+8xy+2\sqrt{3}x+2\sqrt{3}y+1=0$ intersect at the point

(a) $\left(\frac{1}{2\sqrt{3}},\frac{1}{2\sqrt{3}}\right)$ (b) $\left(\frac{1}{3\sqrt{2}},\frac{1}{3\sqrt{2}}\right)$

(c) $\left(\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{4},\frac{1}{3}\right)$

24. If the pair of lines $6x^2-\alpha xy-3y^2-24x+3y+\beta=0$ intersect on x-axis, then α is equal to

(a) 3/2 (b) –5/2

(c) – 18 (d) – 7

MARK YOUR RESPONSE	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ			

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE

Suppose two equations representing pair of straight lines have identical portion of quadratic terms, then the pair of lines represented by one equation will be parallel to the pair of lines represented by the second equation. So, if each of the following equations

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \ldots(1)$$

and

$$ax^2+2hxy+by^2-2gx-2fy+c=0 \quad \ldots(2)$$

represents a pair of straight lines then these four enclose a parallelogram.

Now answer the following questions :

1. If θ be the angle between two non-parallel sides, then $\sin\theta=$

(a) $\frac{2\sqrt{h^2-ab}}{a+b}$ (b) $\frac{2\sqrt{h^2-ab}}{\sqrt{(a-b)^2+4h^2}}$

(c) $\frac{2\sqrt{h^2-ab}}{\sqrt{g^2-ac}}$ (d) $\frac{2\sqrt{h^2-ab}}{\sqrt{f^2-bc}}$

2. The area enclosed by the parallelogram is

(a) $\frac{2c}{\sqrt{f^2-bc}}$ (b) $\frac{2c}{\sqrt{h^2-ab}}$

(c) $\frac{4c}{\sqrt{(a-b)^2+4h^2}}$ (d) $\frac{(a-b)^2+4h^2}{\sqrt{h^2-ab}}$

3. The equation (1) and (2) represent coincident pair of real lines if $(a\neq 0)$

(a) $ab-h^2=0$ (b) $ab-h^2=0, c>0$

(c) $ab-h^2\le 0$ (d) $ab-h^2=0$, $ac\le 0$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. **Statement-1** : The combined equation of ℓ_1,ℓ_2 is $2x^2+6xy+y^2=0$ and that of m_1,m_2 is $4x^2+18xy+y^2=0$. If the angle between ℓ_1,m_2 is α then angle between the lines ℓ_2,m_1 is α.

Statement-2 : If the pairs of lines $\ell_1\ell_2=0$, and $m_1m_2=0$ are equally inclined then angle between ℓ_1 and m_2 = angle between ℓ_2 and m_1.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ				

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. The line $x + y = 2$ intersects the circle $x^2 + y^2 = 8$ at two points. The equation(s) of the straight line(s) joining the origin and the points of intersection is (are)

(a) $x+\left(2+\sqrt{3}\right)y=0$ (b) $x+\left(2-\sqrt{3}\right)y=0$

(c) $\left(2-\sqrt{3}\right)x+y=0$ (d) $\left(2+\sqrt{3}\right)x+y=0$

2. If one of the lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between the coordinate axes, then which of the following can be true

(a) $a+b=2h$ (b) $a+b=-2h$

(c) $(a-b)^2=4h^2$ (d) $(a+b)^2=4h^2$

3. If $4x^2 + ay^2 + 2by = c^2$ represents a pair of perpendicular straight lines then

(a) $a=-4, b=c$ (b) $a=-4, b=2c$

(c) $a=-4, b=-2c$ (d) $a=-4, b=-c$

4. The diagonals of a square are along the pair of lines whose equation is $2x^2 - 3xy - 2y^2 = 0$. If (2, 1) is a vertex of the square, then the other vertices of the square are

(a) $(1,2)$ (b) $(-1,2)$

(c) $(1,-2)$ (d) $(-2,-1)$

5. Let $0 < p < q$ and $a \neq 0$ such that the equation

$$px^2+4\lambda xy+qy^2+4a(x+y+1)=0$$

represents a pair of straight lines, then a can lie in the interval

(a) $(-\infty,\infty)$ (b) $(-\infty,p]$

(c) $[p,q]$ (d) $[q,\infty)$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Match the following column :

Column-I		**Column-II**
(A)	The pair of lines $3x^2-5xy+2y^2=0$, $9x^2-25xy+4y^2=0$ are	p. parallel
(B)	The pair of lines $x^2-y^2+8xy=0$, $2x^2-2y^2-xy=0$	q. perpendicular
(C)	The pair of lines $2x^2-3xy+y^2=0$, $x^2+3y+2y^2=0$	r. equally inclined
(D)	The pair of lines $x^2-7xy+12y^2=0$, $x^2-7xy+12y^2+y-9x+20=0$	s. one pair bisects the angles between theother

MARK YOUR RESPONSE

1.

	p	q	r	s
A	○	○	○	○
B	○	○	○	○
C	○	○	○	○
D	○	○	○	○

2. Let $ax + by = 1$ be a chord of the curve $3x^2 - y^2 - 2x + 4y = 0$ intersecting the curve at the points A and B such that AB subtends a right angle at the origin O. Match the entries from the following two columns :

	Column-I	Column-II
(A)	$a - 2b + 1$ is equal to	p. 2
(B)	the distance from the origin of the farthest chord cannot exceed	q. $\sqrt{5}$
(C)	if the triangle OAB is isoceless then the area of the triangle cannot exceed	r. 3
(D)	The number of chords such that triangle OAB is isoceless cannot exceed	s. 5

3. **Match the following column :**

	Column-I	Column-II
(A)	If the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + c = 0$ represents a pair of straight lines and θ be the angle between them, then $7\lvert\tan\theta\rvert$ is equal to	p. –2
(B)	If the straight line $\frac{x}{a}+\frac{y}{b}=1$ intersects the curve $\frac{x^2}{b^2}+\frac{y^2}{a^2}$ at the points P and Q and the chord PQ subtends angle θ at the origin then $\tan\frac{\theta}{2}$ is equal to	q. 2
(C)	If the lines $x^2 + 4xy - 2y^2 + 4x + 2fy + c^2 = 0$ intersect on the x-axis then f is equal to	r. 4
(D)	In the equation given in (C) the value of c is equal to	s. 1

MARK YOUR RESPONSE

2.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

3.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

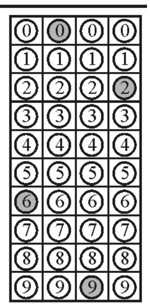
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. If the centroid (x', y') of the triangle whose sides are represented by the equations $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$, is given by

$$\frac{x'}{bl - hm} = \frac{y'}{am - hl} = \frac{p}{q(am^2 - 2hlm + bl^2)},$$ then $p + q$ is equal to

2. One of the bisectors of the angle between the lines $a(x - 1)^2 + 2h(x - 1)(y - 2) + b(y - 2)^2 = 0$ is $x + 2y - 5 = 0$. If the other bisector passes through the point $(\alpha, \alpha - 4)$ then α is equal to

3. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + 10 = 0$ represents a pair of straight lines, equidistant from the origin, then $\dfrac{f^4 - g^4}{bf^2 - ag^2}$ is equal to

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MARK YOUR RESPONSE

1.				2.				3.			
0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(b)	**5**	(a)	**9**	(b)	**13**	(a)	**17**	(a)	**21**	(b)
2	(b)	**6**	(b)	**10**	(c)	**14**	(c)	**18**	(d)	**22**	(d)
3	(d)	**7**	(a)	**11**	(b)	**15**	(a)	**19**	(b)	**23**	(a)
4	(d)	**8**	(b)	**12**	(c)	**16**	(c)	**20**	(b)	**24**	(a)

B COMPREHENSION TYPE

1	(b)	**2**	(b)	**3**	(d)

C REASONING TYPE

1 (a)

D MULTIPLE CORRECT CHOICE TYPE

1	(a,b,c,d)	**2**	(a,b,d)	**3**	(b,c)	**4**	(b,c,d)	**5**	(b,d)

E MATRIX-MATCH TYPE

1. A - r; B - s; C - q; D - p

2. A - p; B - q,r,s; C - s; D - r, s

3. A - s, B - s, C - r, D - p,q

F NUMERIC/INTEGER ANSWER TYPE

1	5	**2**	4	**3**	10

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(b)** The separate equation of lines are $x + y = 6$

$xy - y - 3x + 3 = 0 \Rightarrow y(x-1) - 3(x-1) = 0$

$\Rightarrow (x-1)(y-3) = 0$

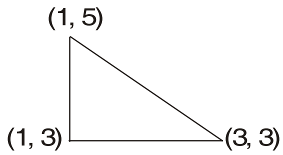
Equation of the sides of the triangle are

$x + y = 6$ ...(1)

$y = 3$ ...(2)

$x = 1$ ...(3)

Triangle is right angled at $(1, 3)$.



$\therefore$ The cricumcircle is the circle on $(3, 3)$ and $(1, 5)$ as ends of a diameter and its equation is

$(x-3)(x-1) + (y-3)(y-5) = 0$

i.e., $x^2 + y^2 - 4x - 8y + 18 = 0$

2. **(b)** The given equation can be written as

$(x^2 + y^2)(\cos^2\theta \sin^2\alpha + \sin^2\theta) = x^2\tan^2\alpha - 2xy\tan\alpha\sin\theta + y^2\sin^2\theta$

or $(\cos^2\theta\sin^2\alpha + \sin^2\theta - \tan^2\alpha)x^2 + 2(\tan\alpha\sin\theta)xy + \cos^2\theta\sin^2\alpha\, y^2 = 0$

Since the slope of these lines are given as $\tan\theta_1$ and $\tan\theta_2$

Sum of the slopes $= \dfrac{-2\tan\alpha\sin\theta}{\cos^2\theta\sin^2\alpha}$ $\quad \left(\because\ \theta = \dfrac{\pi}{6}\right)$

$$\Rightarrow \tan\theta_1 + \tan\theta_2 = \frac{-2\tan\theta \times \frac{1}{2}}{\frac{3}{4}\times\sin^2\alpha} = -\frac{8}{3}\operatorname{cosec}2\alpha$$

3. **(d)** We have $ax^2 + 2\lambda xy + by^2 + 2Kx + 2Ky + 2K = 0$

$h = \lambda, g = K, c = 2K, f = K$

$= abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$ab.(2K) + 2\lambda K^2 + aK^2 - bK^2 - 2\lambda^2 K = 0$

$2K\lambda^2 - 2K^2\lambda + (a+b)K^2 - 2abK = 0$

For real λ, $B^2 - 4AC \geq 0$

$4K^4 - 4\,2K[(a+b)K^2 - 2aK] \geq 0$

$K^2 - 2(a+b)K + 4ab \geq 0, (K-2a)(K-2b) \geq 0$

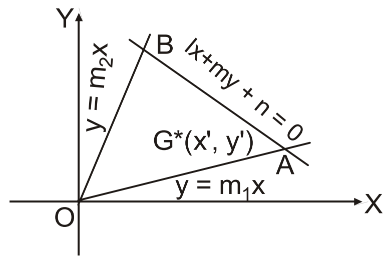
$K \leq 2a$ or $K \geq 2b$.

4. **(d)** Equation of the bisectors ofthe angles between the lines $a^2x^2 + 2axy + y^2 = 0$ is $\dfrac{x^2 - y^2}{a^2 - 1} = \dfrac{xy}{a}$ which is satisfied by $y = 3x$ if $\dfrac{1-9}{a^2-1} = \dfrac{3}{a}$

i.e., if $3a^2 + 8a - 3 = 0$ or if $a = -3, \dfrac{1}{3}$.

5. **(a)** Let the equation of lines represented by $ax^2 + 2hxy + by^2 = 0$ be $y = m_1x$ and $y = m_2x$, then,

$m_1 + m_2 = -\dfrac{2h}{b}$ amd $m_1m_2 = \dfrac{a}{b}$.



Co-ordinates of A and B are $\left(\dfrac{-n}{l+mm_1}, \dfrac{-nm_1}{l+mm_1}\right)$

and $\left(\dfrac{-n}{l+mm_2}, \dfrac{-nm_2}{l+mm_2}\right)$ respectively.

Then required area

$$= \frac{1}{2}\left|\left(\frac{-n}{l+mm_1}\right)\left(\frac{-nm_2}{l+mm_2}\right) - \left(\frac{-n}{l+mm_2}\right)\left(\frac{-nm_1}{l+mm_1}\right)\right|$$

($\because$ If co-ordinates $(0, 0)$, (x_1, y_1) and (x_2, y_2) then area $\frac{1}{2}|x_1y_2 - x_2y_1|$)

$$= \frac{1}{2}\left|\frac{n^2(m_2 - m_1)}{l^2 + lm(m_1+m_2) + m^2m_2m_2}\right|$$

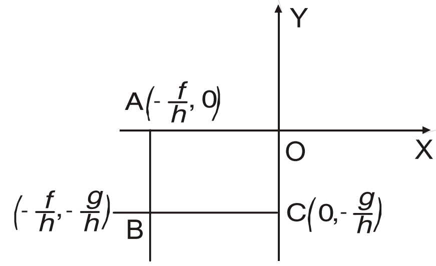
$$= \frac{1}{2}\left|\frac{n^2\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{l^2 + lm(m_1+m_2) + m^2m_1m_2}\right|$$

$$= \frac{1}{2}\left|\frac{n^2\sqrt{\left(\frac{4h^2}{b^2} - \frac{4a}{b}\right)}}{l^2 - \frac{2hlm}{b} + \frac{m^2a}{b}}\right| = \frac{n^2\sqrt{(h^2 - ab)}}{(am^2 - 2hlm + bl^2)}$$

6. **(b)** The equation $2hxy + 2gx + 2fy + c = 0$ represents a pair of straight line if

$$0 + 2fgh - 0 - 0ch^2 = 0 \Rightarrow c = \frac{2fg}{h}$$

$$\therefore\ 2hxy + 2gx + 2fy + \frac{2fg}{h} = 0$$

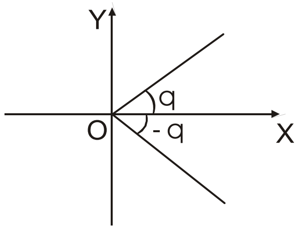


$$\Rightarrow (hx + f)(hy + g) = 0 \Rightarrow x = -\frac{f}{h}, y = -\frac{g}{h}$$

$$\therefore \text{ Required area of rectangle} = \left|-\frac{f}{h}\right|\left|-\frac{g}{h}\right| = \frac{|fg|}{h^2}$$

7. **(a)** Let the slopes of the lines be m_1 and m_2 Then $m_1 + m_2 = -\frac{2h}{b}$ and $m_1 m_2 = \frac{a}{b}$. Clearly the image of the line $y - mx = 0$ in the x-axis is $y + mx = 0$

$\therefore$ The required pair of image lines is $(y + m_1 x)(y + m_2 x) = 0$



$$\Rightarrow y^2 + (m_1 + m_2)\,xy + m_1 m_2 x^2 = 0$$

$$\Rightarrow y^2 - \frac{2h}{b}xy + \frac{a}{b}x^2 = 0 \Rightarrow ax^2 - 2hxy + by^2 = 0$$

8. **(b)** Given equation is $(x^2 + y^2)\sin^2\alpha = (x\cos\beta - y\sin\beta)^2$

$\Rightarrow x^2(\sin^2\alpha - \cos^2\beta) + 2xy\sin\beta\cos\beta$
$+ y^2(\sin^2\alpha - \sin^2\beta) = 0 \quad \ldots(1)$

Let the angle between the lines representing by (1) is θ

$$\therefore\ \tan\theta = 2\left|\frac{\sqrt{h^2 - ab}}{a+b}\right|$$

$$= 2\frac{\sqrt{\sin^2\beta\cos^2\beta - (\sin^2\alpha - \cos^2\beta)(\sin^2\alpha - \sin^2\beta)}}{|\sin^2\alpha - \cos^2\beta + \sin^2\alpha - \sin^2\beta|}$$

$$= 2\frac{\sqrt{\begin{array}{r}\{\sin^2\beta\cos^2\beta - \sin^4\alpha + \sin^2\alpha\sin^2\beta \\ + \sin^2\alpha\cos^2\beta - \sin^2\beta\cos^2\beta\}\end{array}}}{|(2\sin^2\alpha - 1)|}$$

$$= 2\frac{\sqrt{\sin\alpha(1 - \sin^2\alpha)}}{|-\cos 2\alpha|} = \frac{2\sin\alpha\cos\alpha}{|-\cos 2\alpha|} = \tan 2\alpha$$

$\Rightarrow \theta = 2\alpha$

9. **(b)** Equation of pair of bisectors of angles between lines $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{xy} = \frac{a-b}{h} \Rightarrow h(x^2 - y^2) = (a-b)xy \cdots (1)$$

But $y = mx$ is one of these lines, then it will satisfy it. Substituting $y = mx$ in (1)

$$h(x^2 - m^2x^2) = (a-b)x.mx$$

Dividing by x^2, $h(1 - m^2) = m(a-b)$

10. **(c)** Let the pair of $\perp$ lines be represented by $x^2 + 2pxy - y^2 = 0$ and the third line is $y = mx$, then

$3x^3 + 3x^2y - 3xy^2 + dy^3 = \lambda\,(mx - y)\,(x^2 + 2pxy - y^2)$

Comparing the coefficients of various terms from both the sides, we get $3 = \lambda m$, $3 = (2pm\lambda - \lambda)$;

$-3 = (-\lambda m - 2p\lambda)$ and $d = \lambda$ taking $\lambda = d$, everywhere we get

$md = 3 \Rightarrow 2pmd - d = 3 \Rightarrow 6p - d = 3$; Also $md + 2pd = 3 \Rightarrow 2pd = 0 \Rightarrow p = 0$ or $d = 0$

$\because d \neq 0$ (otherwise md = 3 becomes invalid)

$\therefore p = 0 \Rightarrow d = -3$

11. **(b)** Let the given equation represents the straight lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$

Then $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$
$= (l_1x + m_1y + n_1)\,(l_2x + m_2y + n_2)$

Comparing the coefficient, we get

$l_1l_2 = a,\ m_1m_2 = b,\ n_1n_2 = c$

$l_1m_2 + l_2m_1 = 2h,\ l_1n_2 + l_2n_1 = 2g$ and $m_1n_2 + m_2n_1 = 2f$

The product of perpendiculars drawn from the origin to the lines is

$$= \frac{|n_1|}{\sqrt{l_1^2 + m_1^2}} \times \frac{|n_2|}{\sqrt{l_2^2 + m_2^2}}$$

$$= \frac{|n_1n_2|}{\sqrt{l_1^2l_1^2 + l_1^2m_2^2 + l_2^2m_1^2 + m_1^2m_1^2}}$$

$$= \frac{|n_1n_2|}{\sqrt{l_1^2l_1^2 + (l_1m_2 + l_2m_1)^2 - 2l_1l_2m_1m_2 + m_1^2m_2^2}}$$

$$\frac{|c|}{\sqrt{a^2 + 4h^2 - 2ab + b^2}} = \frac{|c|}{\sqrt{(a-b)^2 + 4h^2}}$$

12. **(c)** $u = 0$ represents a pair of straight lines if

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ...(1)

combined equation of coordinate axes is $xy = 0$

$\therefore$ The curve through the intersection of $u = 0$ and $xy = 0$ is $u + \lambda xy = 0$ ...(2)

$\Rightarrow ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \lambda xy = 0$

$\Rightarrow ax^2 + (2h + \lambda) xy + by^2 + 2gx + 2fy + c = 0$

It is represents a pair of straight lines, then

$$abc + 2fg\left(h + \frac{\lambda}{2}\right) - af^2 - bg^2 - c\left(h + \frac{\lambda}{2}\right)^2 = 0$$

$$\Rightarrow (abc + 2fgh - af^2 - bg^2 - ch^2) + \lambda(fg - ch) - \frac{c\lambda^2}{4} = 0$$

$$\Rightarrow \lambda = \frac{4(fg - ch)}{c} \text{ [from (1)]}$$

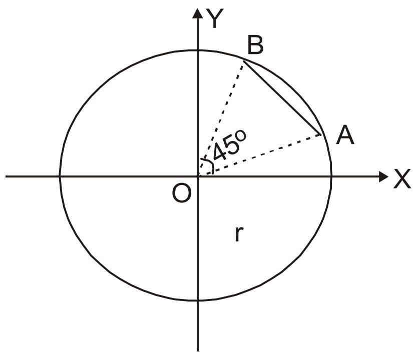
Hence, from (2) the equation of the third pair is $cu + 4(fg - ch)xy = 0$

13. **(a)** Homogenise the equation of the circle with the help of the equation of the line

$x^2 + y^2 = a^2 (1)^2 = x^2 + y^2 = a^2 (lx + my)^2$

$\Rightarrow (a^2l^2 - 1) x^2 + 2lma^2xy + (a^2m^2 - 1) y^2 = 0$

$$\therefore \tan 45° = \frac{2\sqrt{l^2m^2a^4 - (a^2l^2 - 1)(a^2m^2 - 1)}}{a^2l^2 - 1 + a^2m^2 - 1}$$



squaring, we get $\quad 1 = \dfrac{4(a^2l^2 + a^2m^2 - 1)}{(a^2l^2 + a^2m^2 - 2)^2}$

$\Rightarrow a^4 (l^2 + m^2) - 8a^2(l^2 + m^2) + 8 = 0$

$$\therefore l^2 + m^2 = \frac{8a^2 \pm \sqrt{64a^4 - 32a^4}}{2a^4}$$

$$a^2(l^2 + m^2) = 4 \pm 2\sqrt{2}$$

14. **(c)** The given equation is equivalent to

$(x - y)(x - 2y)(x - 3y) = 0$

which represent three straight lines

$y = x,\ y = \dfrac{1}{2}x,\ y = \dfrac{1}{3}x$ whose slopes are 1, 1/2, 1/3

which from an H.P.

15. **(a)** $lx + my + n = 0 \Rightarrow \dfrac{lx + my}{-n} = 1$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 = \left(\frac{lx + my}{-n}\right)^2$$

$$\Rightarrow \left(\frac{n^2}{a^2} - l^2\right)x^2 + \left(\frac{n^2}{b^2} - m^2\right)y^2 - 2lmxy = 0$$

This represent a pair of coincident line if

$$l^2m^2 - \left(\frac{n^2}{a^2} - l^2\right)\left(\frac{n^2}{b^2} - m^2\right) = 0 \Rightarrow$$

$$\frac{n^4}{a^2b^2} = \frac{n^2m^2}{a^2} + \frac{n^2l^2}{b^2} \Rightarrow a^2l^2 + b^2m^2 = n^2$$

Alternatively, the given line must be a tangent to the given curve

16. **(c)** $\angle ONP = \dfrac{\pi}{2}$ and $\angle OMP = \dfrac{\pi}{2}$. So O, M, P, N are concyclic, OP being diameter.

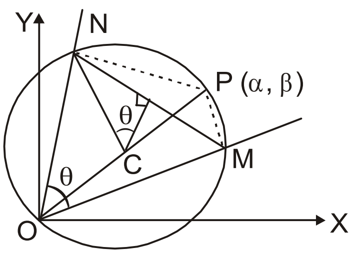
Let P be (α, β). Then equation of the circle is $(x - 0)(x - \alpha) + (y - \beta) = 0$

$\Rightarrow x^2 + y^2 - \alpha x - \beta y = 0$. If θ be the angle between the lines then $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a + b}$

$$\therefore \sin\theta = \frac{2\sqrt{h^2 - ab}}{\sqrt{(a - b)^2 + 4h^2}} = \frac{k}{\sqrt{\dfrac{\alpha^2}{4} + \dfrac{\beta^2}{4}}}$$

[$\because MN = 2k$]

$$\therefore \sqrt{\alpha^2 + \beta^2}\ \sqrt{h^2 - ab} = k\sqrt{(a - b)^2 + 4h^2}$$



$$\Rightarrow (\alpha^2+\beta^2)(h^2-ab)=k^2\{(a-b)^2+4h^2\}$$

Locus of $P(\alpha,\beta)$ is $x^2+y^2=\dfrac{k^2\{(a-b)^2+4h^2\}}{h^2-ab}$

17. **(a)** Dividing by x^3 and put $\dfrac{y}{x}=m$, we get

$dm^3+cm^2+bm+a=0$. Let its roots be m_1, m_2, m_3

then $m_1\, m_2\, m_3=-\dfrac{a}{b}$.

If $m_1=\tan\alpha$, then $m_2=\tan(90-\alpha)=\cot\alpha$

$\therefore m_1 m_2=1 \Rightarrow m_3=-\dfrac{a}{b}$.

Since it is the root of the equation

$$\therefore d\left(-\frac{a}{d}\right)^3+c\left(-\frac{a}{d}\right)^2+b\left(-\frac{a}{d}\right)+a=0$$

$$\Rightarrow -a^3+ca^2-abd+ad^2=0$$

$$\Rightarrow a(a-c)+d(b-d)=0$$

18. **(d)** The equation is homogeneous equation of fourth degree it must represent four straight lines passing through origin. The lines are perpendicular in pair. So $a(x^4+y^4)-4xy(x^2-y^2)+6cx^2y^2=(ax^2+pxy-ay^2)(x^2+qxy-y^2)$, p and q being constants.
On comparing similar powers, we get
$p+aq=-4b$ ...(1)
and $-2a+pq=6c$ ...(2)

Again, if two pairs coincide $\dfrac{p}{a}=q \Rightarrow p=aq$ ...(3)

From (1) and (3) $q=-\dfrac{2b}{a}$ and $p=-2b \Rightarrow$ from (2)

$$-2a+\frac{4b^2}{a}=6c \Rightarrow 2b^2=a^2+3ac$$

19. **(b)** To find the equation of pair of lines through origin we eliminate the terms containing x from the equations of C_1 and C_2. Thus,
$C_1+3C_2=0 \Rightarrow (\lambda+9)x^2+(24-2\lambda)xy-9y^2=0$
These lines are equally inclined to axes if $24-2\lambda=0 \Rightarrow \lambda=12$

20. **(b)** The equation of lines joining the origin to the points of intersection will be obtained by making one equation homogeneous with the help of the other. we have

$$x^2+y^2=a^2 \quad \text{and} \quad x^2+y^2+2(gx+fy)=0$$

From (1) and (2), $2(gx+fy)=-a^2$ squaring both the sides, we get

$4(gx+fy)^2=a^4=a^2(x^2+y^2)$
$\Rightarrow a^2(x^2+y^2)-4(gx+fy)^2=0$, which is the required equation.

21. **(b)** Let m and m^2 be the slopes of the lines represented by $ax^2+2hxy+by=0$

Then, $m+m^2=-\dfrac{2h}{b}$ ...(1)

$m.m^2=\dfrac{a}{b}$ or $m^3=\dfrac{a}{b}$ ...(2)

from (1) $(m+m^2)^3=\left(-\dfrac{2h}{b}\right)^3$

$$\Rightarrow m^3+m^6+3.\,m.m^2(m+m^2)=-\frac{8h^3}{b^3}$$

$$\Rightarrow \frac{a}{h}+\frac{a^2}{b^2}+\frac{3a}{b}\left(-\frac{2h}{b}\right)=-\frac{8h^3}{b^3} \quad \{\text{from (1) and (2)}\}$$

$$\Rightarrow \frac{a}{b^2}(a+b)\frac{8h^3}{b^3}=\frac{6ah}{b^2} \text{ or } \frac{(a+b)}{h}+\frac{8h^2}{ab}=6$$

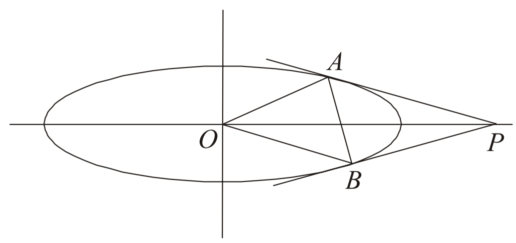
22. **(d)** Let the point of intersection of tangents A and B be $P(h,k)$, then equation of AB is

$\dfrac{xh}{4}+\dfrac{yk}{1}=1$ ...(1)

Homogenizing the ellipse using (1)

$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$

$$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2(k^2-1)+\frac{2kh}{4}xy=0 \quad \text{...(2)}$$



Given equation of OA and OB is

$x^2+4y^2+\alpha xy=0$ ...(3)

$\because$ (2) and (3) represent same line

Hence $\dfrac{h^2-4}{16}=\dfrac{k^2-1}{4}=\dfrac{hk}{2\alpha}$

$\Rightarrow h^2 - 4 = 4(k^2-1) \Rightarrow \ h^2 - 4k^2 = 0$

$(h-2k)(h+2k) = 0$

$\therefore$ locus $(x-2y)(x+2y) = 0.$

23. **(a)** The pair of lines given by

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ intersect at the point

$$\left(\sqrt{\frac{f^2-bc}{h^2-ab}}, \sqrt{\frac{g^2-ac}{h^2-ab}}\right)$$

$\therefore$ Required point of intersection $= \left(\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}\right).$

24. **(a)** Put $y = 0$, then equation should be a perfect square.

$\Rightarrow 6x^2 - 24x + \beta = 0$

$f(x)$ should be a perfect square.

$\therefore\ \Delta = (24)^2 - 4.6\beta = 0 \Rightarrow \beta = 24$

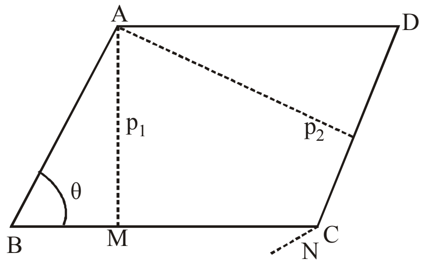
Now the given equation represents a pair of lines, so

$$6\times -3\times\beta + 2\times\frac{3}{2}\times -12\times -\frac{\alpha}{2} - 6\times\left(\frac{3}{2}\right)^2 + 3\times(-12)^2$$

$$-\beta\left(-\frac{\alpha}{2}\right)^2 = 0 \Rightarrow \left(\alpha - \frac{3}{2}\right)^2 \Rightarrow \alpha = \frac{3}{2}$$

B COMPREHENSION TYPE

1. **(b)** Let the first equation represents lines $(lx + my + n)$ $(l_1 x + m_1 y + n_1) = 0$
then the second equation represents lines $(lx + my - n)\,(l_1 x + m_1 y - n_1) = 0$
comparing the coefficents, we get
$l\,l_1 = a, m\,m_1 = b, n\,n_1 = c, m\,n_1 + m_1\,n = 2f, n\,l_1 + n_1\,l = 2g, l\,m_1 + l_1\,m = 2h$



Let the angle between two non paralled sides be θ then

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b} \ \Rightarrow \sin\theta = \frac{2\sqrt{h^2-ab}}{\sqrt{(a-b)^2+4h^2}}$$

2. **(b)** $\therefore\ p_1 = \frac{n-(-n)}{\sqrt{l^2+m^2}}$ and $p_2 = \frac{n_1-(-n_1)}{\sqrt{l_1^2+m_1^2}}$

$$\Rightarrow\ p_1p_2 = \frac{4n\,n_1}{\sqrt{l^2l_1^2 + l^2m_1^2 + m^2m_1^2}}$$

$$= \frac{4c}{\sqrt{a^2+b^2+4h^2-2ab}} = \frac{4c}{\sqrt{(a-b)^2+4h^2}}$$

Now, the area of parallelogram $= \frac{P_1P_2}{\sin\theta}$,

where, p_1 and p_2 are distance between two parallel lines

$\therefore$ Required area $= \frac{p_1p_2}{\sin\theta} = \frac{2c}{\sqrt{h^2-ab}}$

3. **(d)** If the equation represent coincident pair, then $h^2 - ab = 0$. Clearly both equations represent the same line. So, $g = f = 0$.

The equations become $(\sqrt{a}x + \sqrt{b}y)^2 + c = 0$. For real solution $ac \le 0$.

C REASONING TYPE

1. **(a)** The pair of bisectors of $2x^2 + 6xy + y^2 = 0$

and $4x^2 + 18xy + y^2 = 0$ coincides

$\Rightarrow$ angle between ℓ_1, m_2 is same as angle between ℓ_2, m_1.

Both are true and it is correct reason.
Hence (a) is correct choice.

D MULTIPLE CORRECT CHOICE TYPE

1. **(a,b,c,d)** Homogenising the equation of the circle we get

$$x^2+y^2-8\left(\frac{x+y}{2}\right)^2=0$$

$$\Rightarrow x^2+y^2+4xy=0$$

$$\Rightarrow y=\frac{-4x\pm\sqrt{16x^2-4x^2}}{2}$$

$$\therefore\ y=\left(-2\pm\sqrt{3}\right)x\ \Rightarrow y+\left(2\pm\sqrt{3}\right)x=0,$$

which can also be written as $\left(2\mp\sqrt{3}\right)y+x=0$

2. **(a,b,d)** One of the lines must be $y=x$ or $y=-x$. So, $a+b=2h$ or $a+b=-2h \Rightarrow (a+b)^2=4h^2$

3. **(b,c)** We have, $4+a=0$ and $4\times a\times -c^2+0-0-4b^2+0=0$
$\Rightarrow a=-4$ and $16c^2-4b^2=0 \Rightarrow a=-4$
and $b=\pm 2c$

4. **(b,c,d)** Separate equations of the diagonals are $x-2y=0$ and $2x+y=0$
Since, origin is the centre, so one of the vertex is $(-2,-1)$. Diagonal length of the square is $2\sqrt{5}$. So the other vertices are given by

$$\frac{x}{-\frac{1}{\sqrt{5}}}=\frac{y}{\frac{2}{\sqrt{5}}}=\pm\sqrt{5}\ \Rightarrow (-1,2)\text{ and }(1,-2).$$

5. **(b,d)** $p\,.\,q\,.\,4a+2\,.\,2a\,.\,2a\,.\,2\lambda-p\,.\,4a^2-q\,.\,4a^2-4a\,.\,4\lambda^2=0$

$$\Rightarrow\ 4\lambda^2-4a\lambda+\{(p+q)\,a-pq\}=0\qquad (\because a\neq 0)$$

$$\because\ \lambda\in R,\ 16a^2-4.4\,\{(p+q)a-pq\}\geq 0$$

$$or\ (a-p(a-q)\geq 0$$

$$\therefore\ a\leq p\ \text{ or }\ a\geq q$$

E MATRIX-MATCH TYPE

1. **A - r; B - s; C - q; D - p**

(A) Pair of bisectors of $3x^2-5xy+2y^2=0$ is $5x^2-5y^2+2xy=0$
Pair of bisectors of $9x^2-25xy+4y^2=0$ is $5x^2-5y^2+2xy=0$
Therefore given pair of lines are equally inclined.

(B) Pair of bisectors of $x^2-y^2+8xy=0$ is $2x^2-2y^2-xy=0$
Pair of bisectors of $2x^2-2y^2-xy=0$ is $x^2-y^2+8xy=0$
Therefore one pair bisects the angles between other.

(C) $2x^2-3xy+y^2=0,\ x^2+3xy+2y^2=0$

$$\Rightarrow\ (y-x)(y-2x)=0,\ (y+x)(2y+x)=0$$

$\Rightarrow$ The pairs are perpendicular

(D) $x^2-7xy+12y^2=0,\ x^2-7xy+12y^2+y-9x+20=0$ are parallel as their quadratic terms are same

2. **A - p; B - q,r,s; C - s; D - r, s**

Homogenising we get $3x^2-y^2+2(2y-x)(ax+by)=0$

$$\Rightarrow (3-2a)x^2+(4a-2b)xy+(4b-1)y^2=0\ ......(1)$$

(A) These are perpendicular lines if $3-2a+4b-1=0 \Rightarrow a-2b=1$.

(B) $a-2b=1 \Rightarrow ax+by=1$ passes through the point $(1,-2)$ so it represents a family of concurrent lines. Clearly the distance of the farthest chord cannot exceed the distance of the point $(1,-2)$ from the origin.

(C) Length of perpendicular from origin to the line $=$

$$\frac{1}{\sqrt{a^2+b^2}}$$

$\therefore$ area of isosceles triangle $=\dfrac{1}{a^2+b^2}$

Now, $a^2+b^2=(2b+1)^2+b^2=5b^2+4b+1\geq\dfrac{1}{5}$

$$\Rightarrow\frac{1}{a^2+b^2}\leq 5$$

(D) Equation of perpendicular from the origin to the chord is $bx-ay=0$ ....(2)
Equation of bisectors of equation (1) is

$$\frac{x^2-y^2}{xy}=\frac{3-2a-4b+1}{2a-b}=\frac{3-2(1+2b)-4b+1}{2(1+2b)-b}$$

Also, (2) must satisfy above, thus

$$\frac{1-\left(\frac{b}{1+2b}\right)^2}{\frac{b}{1+2b}}=\frac{2-8b}{2+3b}\Rightarrow\frac{3b^2+4b+1}{b(1+2b)}=\frac{2-8b}{2+3b}$$

which is cubic in b, so at most three values of b are possible.

3. **A - s, B - s, C - r, D - p,q**

(A) $|\tan\theta|=\dfrac{2\sqrt{25-24}}{12+2}=\dfrac{1}{7}$

(B) Homogenising we get

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=\left(\frac{x}{a}+\frac{y}{b}\right)^2 \Rightarrow \left(x^2-y^2\right)\left(\frac{1}{b^2}-\frac{1}{a^2}\right)-\frac{2xy}{ab}=0$$

$\Rightarrow$ The lines are perpendicular, so $\theta=\frac{\pi}{2}$

(C) Put $y = 0$, we get $x^2 + 4x + c^2 = 0$, which gives equal roots if $c^2 = 4$, then equation becomes

$x^2+4xy-2y^2+4x+2fy+4=0$ which pepresents *a* pair of st. lines if $f = 4$

(D) As obtained in (C) $c^2 = 4$

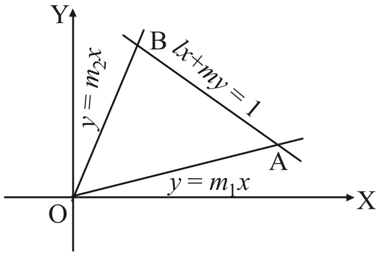
F NUMERIC/INTEGER ANSWER TYPE

1. **Ans : 5**

Let the lines represented by

$ax^2 + 2hxy + by^2 = 0$ be $y = m_1x$ and $y = m_2x$.

Therefore, $m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$



Coordinates of A and B are $\left(\frac{1}{l+mm_1},\frac{m_1}{l+mm_1}\right)$ and $\left(\frac{1}{l+mm_2},\frac{m_2}{l+mm_2}\right)$.

The centroid (x', y') of $\Delta\, OAB$ is given by

$$x'=\left(\frac{\frac{1}{l+mm_1}+\frac{1}{l+mm_2}+0}{3}\right)$$

$$=\left(\frac{2l+m(m_1+m_2)}{3\{l^2+ml(m_1+m_2)+m^2m_1m_2\}}\right)$$

$$=\left(\frac{2l-\frac{2hm}{b}}{3\left(l^2-\frac{2hml}{b}+\frac{m^2a}{b}\right)}\right)=\frac{2}{3}\frac{(bl-hm)}{(am^2-2hlm+bl^2)} \quad \ldots(1)$$

$$y'=\left(\frac{\frac{m_1}{l+mm_1}+\frac{m_2}{l+mm_2}+0}{3}\right)$$

$$=\left(\frac{l(m_1+m_2)+2mm_1m_2}{3(l^2+lm(m_1+m_2)+m^2m_1m_2}\right)$$

$$=\left(\frac{-\frac{2hl}{b}+\frac{2ma}{b}}{3\left(l^2-\frac{2hlm}{b}+\frac{m^2a}{b}\right)}\right)=\frac{2}{3}\cdot\frac{(am-hl)}{(am^2-2hlm+bl^2)} \quad \ldots(2)$$

From (1) and (2) ,we have

$$\frac{x'}{bl-hm}=\frac{y'}{am-hl}=\frac{2}{3(am^2-2hlm+bl^2)}$$

2. **Ans : 4**

Pair of bisectors of

$a(x-1)^2 + 2h(x-1)(y-2) + b(y-2)^2 = 0$ is

$$\frac{(x-1)^2-(y-2)^2}{a-b}=\frac{(x-1)(y-2)}{h}$$

$\Rightarrow h\{x^2 - y^2 - 2x + 4y - 3\} = (a - b)(xy - 2x - y + 2)$

$\Rightarrow hx^2 - hy^2 - (a - b)xy + 2x(a - b - h) + y(a - b + 4h) - 2(a - b) - 3h = 0$

Given one bisector is $x + 2y - 5 = 0$

then let other bisector is $2x - y + \lambda = 0$

$\therefore$ pair of bisector is $(x + 2y - 5)(2x - y + \lambda) = 0 \Rightarrow 2x^2 - 2y^2 + 3xy - 10x + \lambda x + 2\lambda y + 5y - 5\lambda = 0$

Comparing (1) and (2) we get

$h = 2$, $a - b = -3$, $2a - 2b - 2h = -10 + \lambda$

$\therefore 2(-3) - 4 = -10 + \lambda \Rightarrow \lambda = 0$

$\therefore$ The other bisector is $2x - y = 0$

So, $2\alpha - (\alpha - 4) = 0 \Rightarrow \alpha = 4$

3. **Ans : 10**

We have $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, where $c = 10$

Let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c \equiv (l_1x + m_1y + n_1)(l_2x + m_2y + n_2)$

Comparing the coefficient of similar terms, we get

$l_1l_2 = a$, $m_1m_2 = b$, $n_1n_2 = c$

$l_1m_2 + l_2m_1 = 2h$, $l_1n_2 + l_2n_1 = 2g$, $m_1n_2 + m_2n_1 = 2f$

Now, the two lines are equidistant from origin

$$\therefore \frac{0.l_1+0m_1+n_1}{\sqrt{l_1^2+m_1^2}}=\frac{0.l_2+0.m_2+n_2}{\sqrt{l_1^2+m_2^2}}$$

$$\Rightarrow n_1^2(l_2^2-m_2^2)=n_2^2(l_1^2-m_1^2)$$

$\Rightarrow n_1^2l_2^2-n_2^2l_1^2=n_2^2m_1^2-n_1^2m_2^2$. On squaring, we get

$$(n_1l_2+n_2l_1)^2[(n_1l_2+n_2l_1)^2-4n_1n_2l_1l_2]$$

$$=(m_1n_2+m_2n_1)^2.[(m_1n_2+m_2n_1)^2-4m_1m_2n_1n_2]$$

$\therefore 4g^2[4g^2 - 4ac] = 4f^2[4f^2 - 4bc]$

$$\Rightarrow f^4-g^4=c(bf^2-ag^2) \Rightarrow \frac{f^4-g^4}{bf^2-ag^2}=c=10$$